

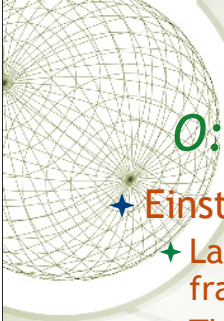


Lecture 10: General Relativity I

- ★ Einstein Tower Experiment
- ★ Gravitational redshifting
- ★ Strong Equivalence Principal

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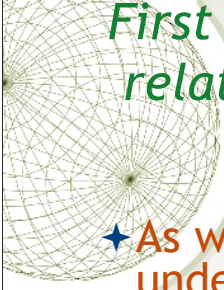


O: RECAP OF SPECIAL RELATIVITY

- ★ **Einstein's postulates**
 - ★ Laws of physics look the same in any inertial frame of reference.
 - ★ The speed of light is the same in any inertial frame of reference
- ★ **Strange consequences**
 - ★ Time dilation and length contraction
 - ★ Relativity of simultaneity and ordering of events
 - ★ Equivalence and conversion of mass and energy
- ★ **Why have we been so carefully avoiding gravity until now?**



★ Discussion: Suppose we simply supplement the Special Theory of Relativity with Newton's law of gravity... what things might go wrong?

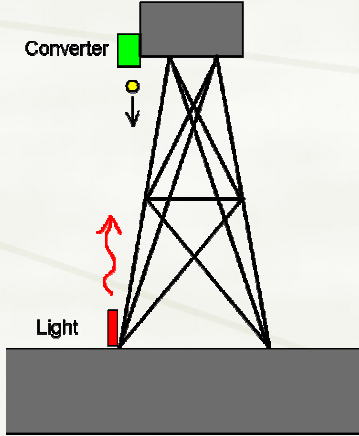


First Newtonian mechanics (special relativity), now his law of gravity (general relativity)

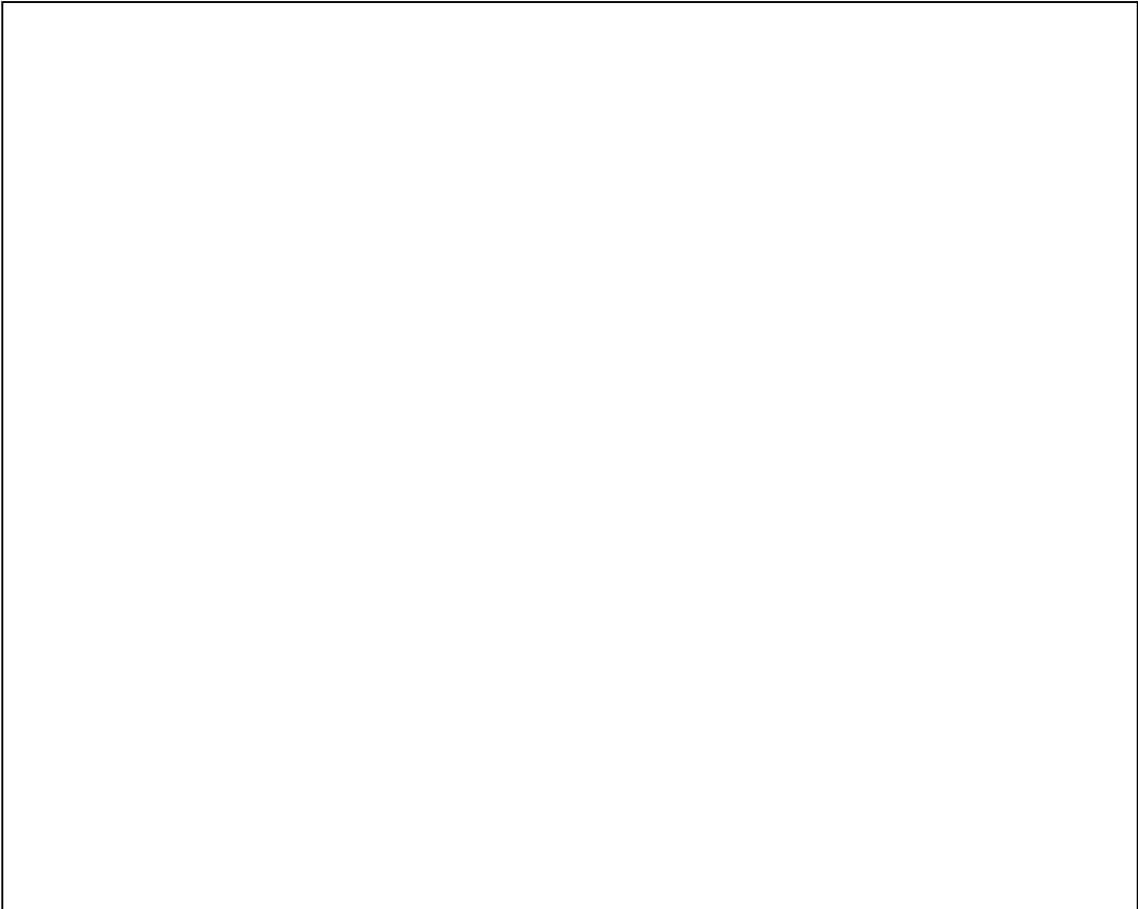
- ★ As we have just learned we have to understand
 - ★ In whose frame do we measure r ?
 - ★ Does the force depend based on your reference frame?
 - ★ Can gravity information travel (communicate) faster than c ? If not, shouldn't there be some reference to c in the equations for gravitational force?

Motivation for General Relativity: Einstein's tower

- ✦ So far, we have ignored the effects of gravity. Is this really okay??
- ✦ Consider another thought experiment to test whether **light** can be **affected by gravity**.
- ✦ Consider a tower on Earth
 - ✦ Shine a light ray from bottom to top
 - ✦ When light gets to top, turn its energy into mass.
 - ✦ then drop this mass to bottom of tower in Earth's gravity field
 - ✦ then turn it back into energy- BUT now have more mass (the sum of the original mass+ the energy gained by 'falling' from the height of the tower)



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Perpetual motion?

- ★ If we could do this, then we could get energy from nothing!
 - ★ Original energy in light beam= E_{start}
 - ★ Turn photon into mass at top created mass is $m=E_{\text{start}}/c^2$
 - ★ then drop mass...at bottom of tower it has picked up speed (and energy) due to the effects of gravitational field *.
 - ★ When we turn it back into energy, we have $E_{\text{end}}=E_{\text{start}}+E_{\text{grav}}$
 - ★ But, we started off with only E_{start} -we have made energy! We're rich!
- * a little physics.. the energy due to falling in a gravitational field is $E_{\text{fall}}=mgh$ (h=height you fall, m is the mass falling, and g is the local acceleration due to gravity)

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★ details

- ★ Suppose original photon energy E
- ★ *By assumption*, it is not affected by gravity so it has energy E once it reaches top
- ★ Thus, mass created at top is $m=E/c^2$
- ★ Then drop mass... at bottom of tower, it has picked up energy due to the conversion of gravitational potential energy ($E_{\text{grav}}=mgh$)
- ★ When we convert it back into energy, we made energy!
We're rich!!!!

$$E_{\text{new}} = E + mgh = E \left(1 + \frac{gh}{c^2} \right)$$

★ **Clearly something is wrong with our assumptions...**

- ★ Only way we can conserve energy is to suppose that light is affected by gravity...
- ★ We need the photon to lose energy as it climbs upwards... at top of tower, to balance things

$$E_{top} = E \left(1 + \frac{gh}{c^2} \right)^{-1}$$

- ★ This is known as **gravitational redshift**-
for light the energy of a photon is related to its
frequency, ν , (alternatively its wavelength, λ)

$$E = h \cdot \nu = hc / \lambda \rightarrow \text{lower energy, longer wavelength - redder}$$

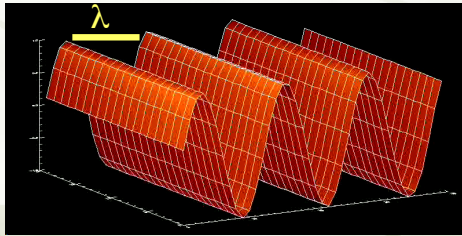
★ The profound nature of gravitational redshift...

- ★ Imagine a clock based on the frequency of light
- ★ Place the clock at the base of the tower... observe it from the top.
- ★ Photons lose energy... so they decrease frequency
- ★ Thus, we see the clock running slowly!
- ★ Time passes at a slower rate in a gravitational field! (units of ν is cycles per sec)

*h= Planck's constant- notice 2 uses of the variable h in this page!

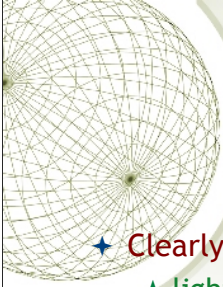
GRAVITATIONAL TIME DILATION

- ★ Recall properties of waves:
- ★ Waves characterized by (refer to Ch. 4 for review):
 - ★ Wavelength (λ) = distance between crests- (units length)
 - ★ Frequency (f or ν) = number of crests passing a given point per second (units cycles/time)
- ★ Speed of a crest; $s_c = \lambda\nu$ (this is generic to waves, for light $s_c=c$)
- ★ Energy of a wave is proportional to frequency ν , $E = h\nu$.



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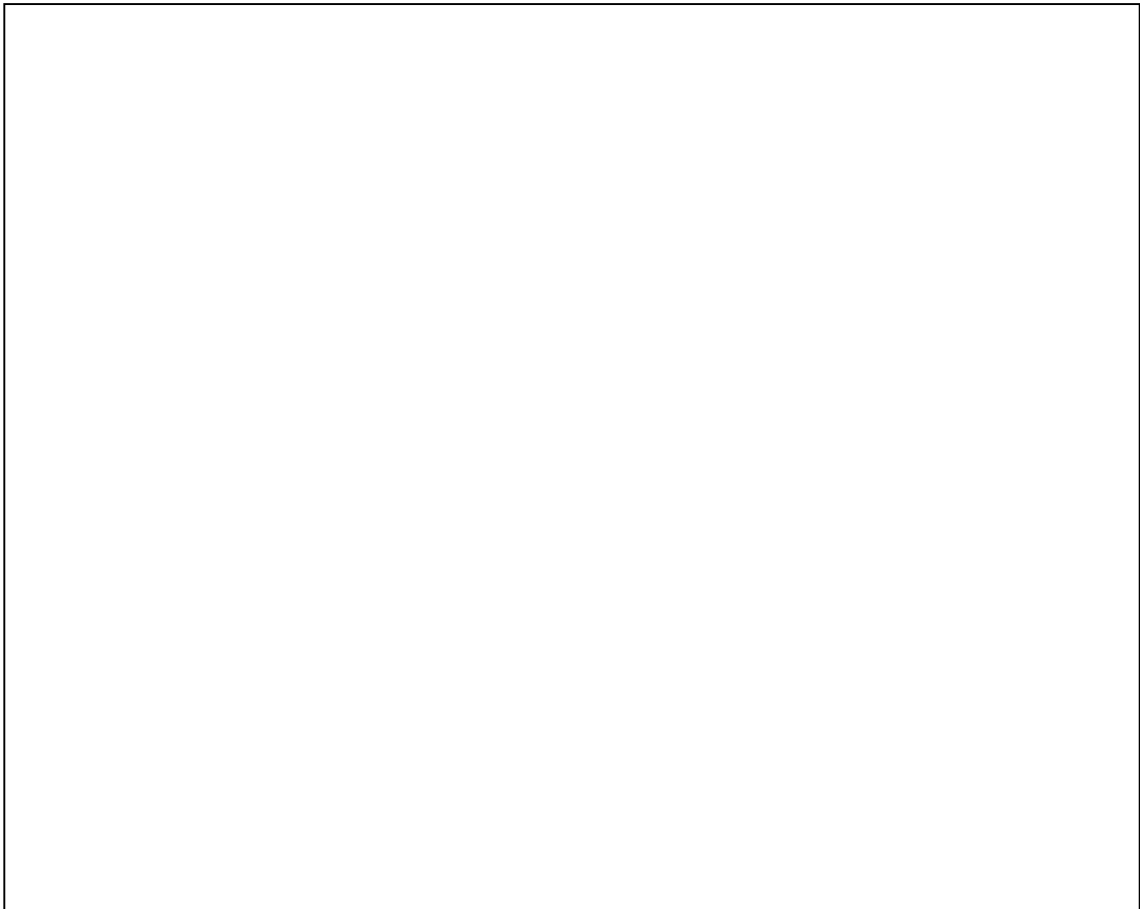
Maxwell and gravity

- ✦ Clearly, our assumption must be wrong...
 - ✦ light must be affected by gravity.
 - ✦ But gravity does not appear in Maxwell's equations, which govern light
 - ✦ Thus, Maxwell's equations are not complete* and are not exactly valid in the reference frame of Earth's surface, where there is gravity.
 - ✦ the Earth's surface must not be an inertial frame of reference!

* Einstein-Maxwell equations

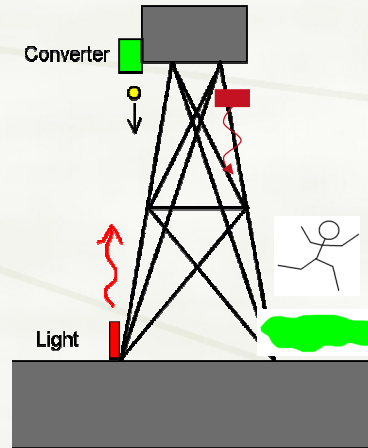
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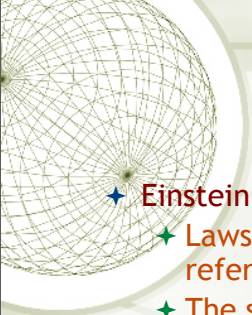
Resolving the tower problem

- ★ Now consider light ray aimed from top to bottom of tower
- ★ Free-falling (FF) observer sees light ray travel **unaffected** by gravity, since freefall is an inertial frame
- ★ From "Earth's" frame...
 - ★ Free-falling (FF) observer is traveling faster and faster
 - ★ Falling observer would see an increasing **redshift** of light source according to special relativity
 - ★ If FF observer is **supposed to see a constant frequency light beam**, then light must get relatively **blueshifted** as it falls in a gravitational field, to compensate
 - ★ Light beam aimed upward must conversely be increasingly **redshifted** with height
 - ★ **Gravitational redshifting removes just the right amount of energy to solve the tower paradox!**



(remember that the energy of a photon is $E=hc/\lambda$)

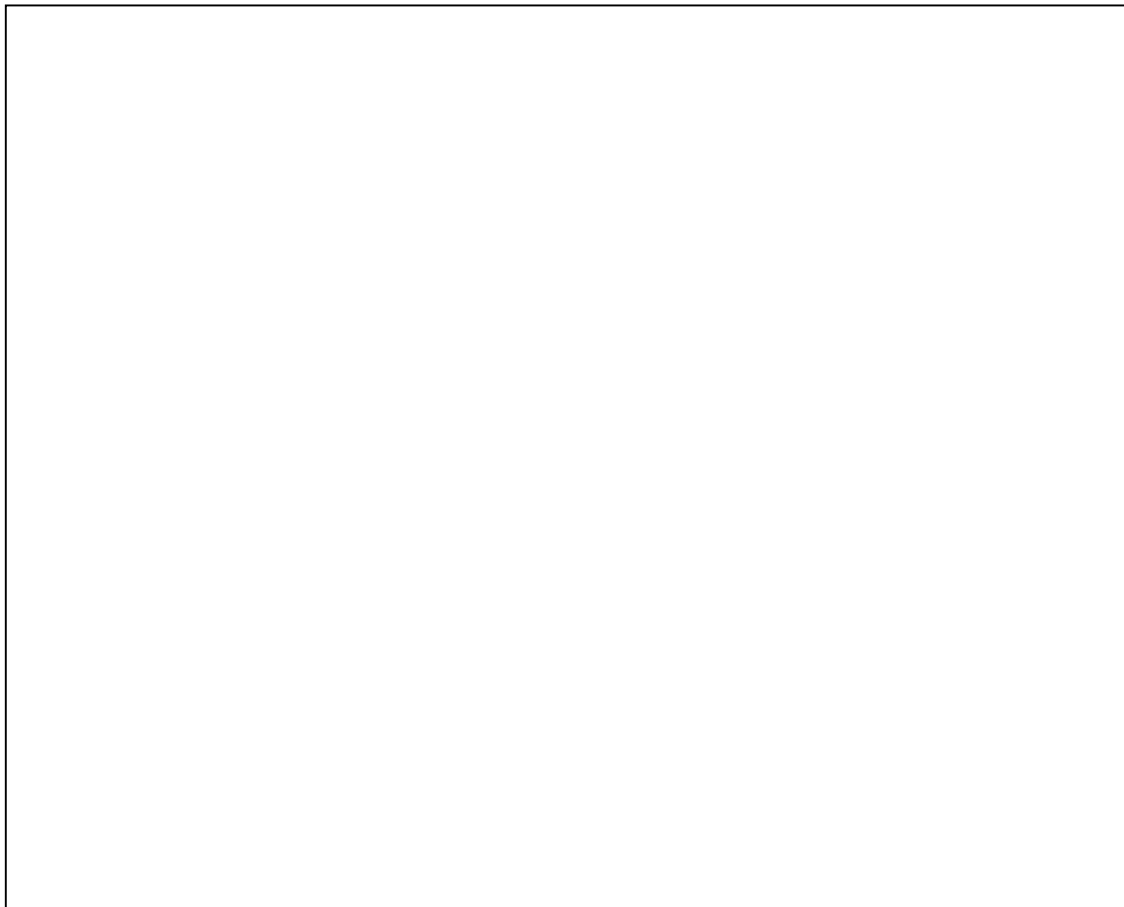
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RECAP

- ★ Einstein's postulates for Special Relativity
 - ★ Laws of physics look the same in any inertial frame of reference.
 - ★ The speed of light is the same in any inertial frame of reference
- ★ Strange consequences of Special Relativity
 - ★ Time dilation and length contraction
 - ★ Relativity of simultaneity and ordering of events
 - ★ Equivalence and conversion of mass and energy
- ★ Behavior of light in gravity field ("tower" experiment)
 - ★ Energy of light must vary in a gravitational field to ensure that free-fall is an inertial frame

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*What makes free-fall an inertial frame?
Think back to the astronauts...*



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Apollo 10: Lots of gravity, but everything is falling together ¹³



EQUIVALENCE PRINCIPLES

- ◆ Recall the “weak” equivalence principle:
 - ◆ All objects are observed to accelerate at the same rate in a given gravitational field (Galileo tower of Pisa experiment).
 - ◆ Therefore, the inertial and gravitational masses must be the same for any object.
 - ◆ This has been verified experimentally, with fractional difference in masses $<10^{-11}$
- ◆ As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot, locally, be distinguished

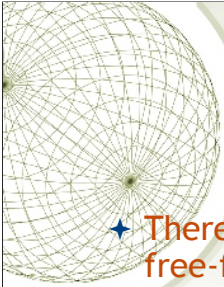
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II: **STRONG EQUIVALENCE PRINCIPLE**

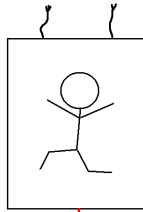
- ★ **Recap of the weak equivalence principle**
 - ★ All objects accelerate at the same rate in a given gravitational field.
 - ★ In other words, inertial and gravitational masses are the same for any object.
- ★ Einstein introduced the **strong equivalence principle** - when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- ★ What does this mean???



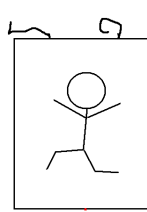
Free Fall vs 'No' Gravity

- ★ There is no way of telling the difference between a free-falling frame in a gravitational field and an inertial frame in no gravitational field... the two are equivalent.

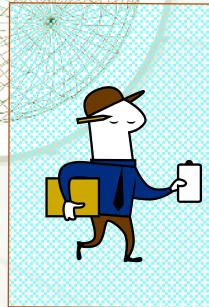
Free-falling elevator



Elevator in deep space



Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space

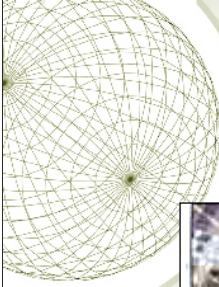


$$a = 9.8 \text{ m/s}^2$$

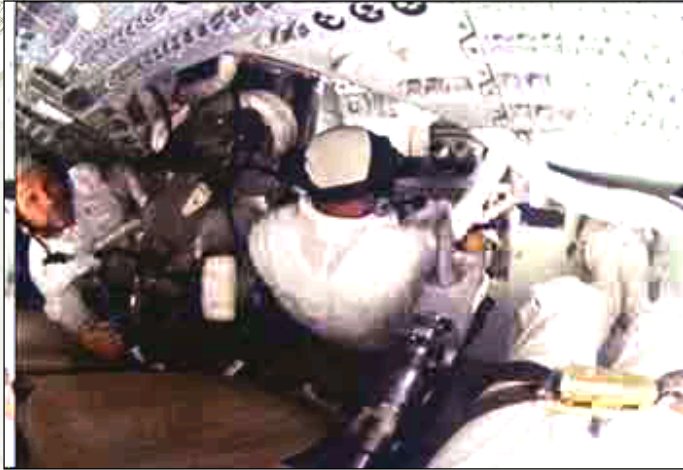


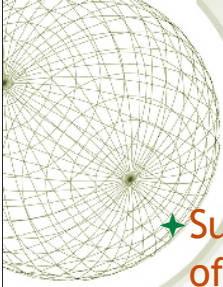
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Back to the Astronauts...



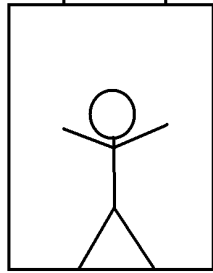


What about gravity?

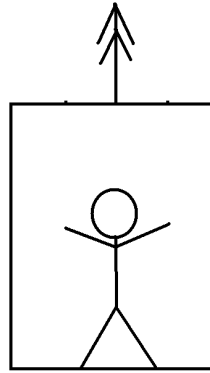
- ✦ Suppose that you decide that your frame of reference is not inertial...
 - ✦ Freely moving bodies change velocity
 - ✦ Is it because of gravity or is the frame accelerating?
 - ✦ Einstein says: *you cannot tell the difference!*
 - ✦ Is Gravity a “fictitious force”?? - i.e., a force which appears to exist because we are living in a non-inertial frame of reference.

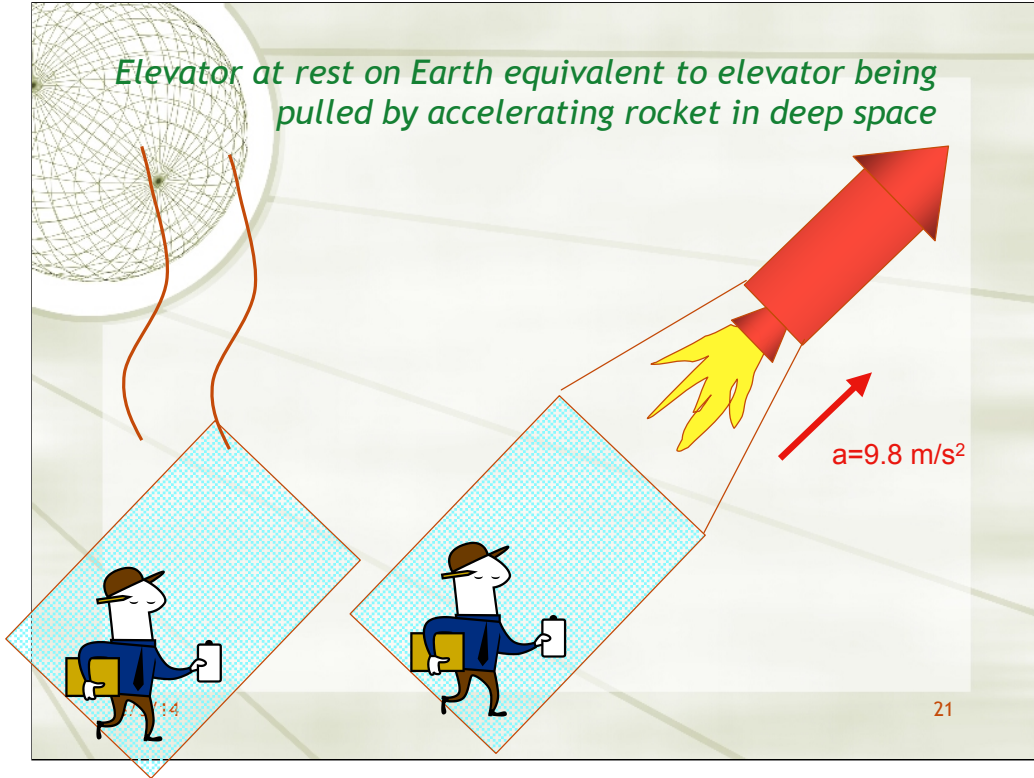
Acceleration vs Gravity

Elevator in gravitational field



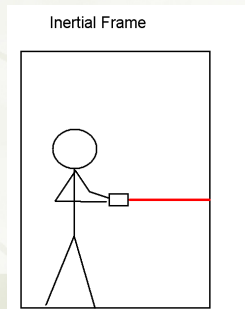
Elevator accelerating upwards



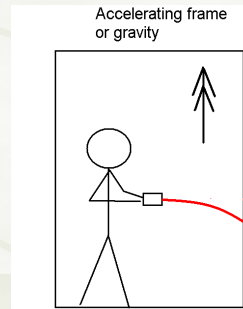


What about light? It “falls”, too!

- ★ Astronaut in inertial frame with flashlight
 - Inertial frame, so light goes in straight lines
 - It doesn't matter whether this is free fall or far from masses
- ★ What if we now put flashlight in a gravitational field (accelerated frame)?
 - Light beam will bend: it must accelerate at the same rate and direction as the elevator
 - Strong equivalence principle \Rightarrow frame with gravity acts the same
 - Important conclusion - light “falls” due to gravity!- **how can we test this idea?**



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Can Gravity Bend Light?

- ★ If light, does not have mass, how could it possibly be affected by gravity?
- ★ After all, the force of gravity - - is directly dependent upon the mass of two objects.
- ★ Thus, light could not be bent by gravity.
- ★ <http://isaacmmcphee.suite101.com/albert-einstein-and-bending-light-a43865#ixzz1nboyQEaH>

The Eddington Test

✦ 1919 - the first “accessible” total Solar eclipse since Einstein postulated the Strong Equivalence Principle

✦ Arthur Eddington

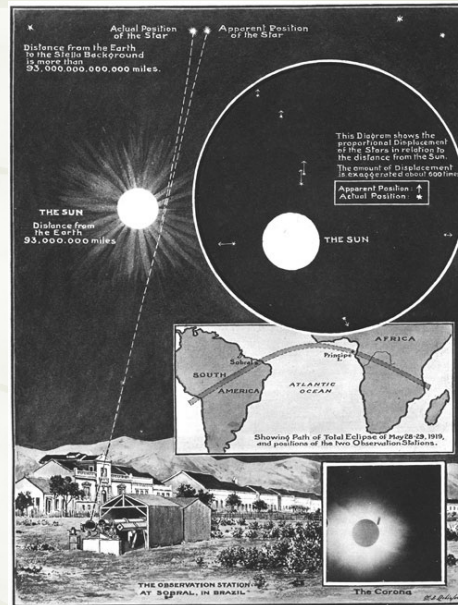
- ✦ Famous British Astronomer
- ✦ Led expedition to South Africa to observe eclipse
- ✦ Was looking for effects of gravitational light bending by searching for shifts in positions of stars just next to the Sun*.
- ✦ The shifts were exactly as Einstein calculated! (1.75 arc seconds)
Extremely important



*Einstein predicted that the magnitude of the shift depended on how large the angle was between the sun and the background star and the mass of the sun

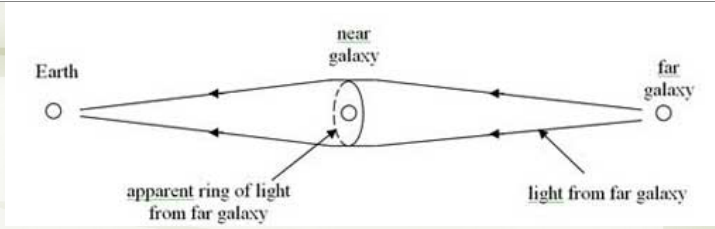
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- The total eclipse of 29th May 1919 gave scientists the chance to test the theory for the first time. Eddington travelled to Príncipe to observe the eclipse and measure the apparent locations of stars near the Sun. Heavy clouds parted minutes before the eclipse and, with the Sun almost directly in front of them, the stars appeared to be shifted from their positions - direct evidence that our nearest star shapes the space around it.
- 'Without Eddington's clever experiment and the fortuitous timing of this total eclipse it might have taken ages before Einstein's theory of gravity, first proposed in 1915, was proven to be correct,' (Pedro Ferreira Oxford University)

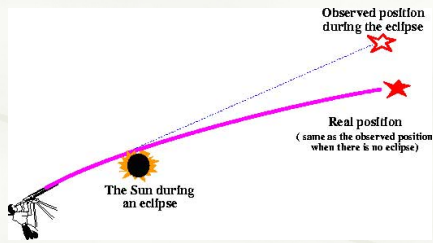


- ★ Prior to this, the GR was untested - just an interesting, intelligent, idea.
- ★ After Eddington, Einstein (and his science) was finally be taken seriously, and even the non-scientists of the world somehow found themselves standing in awe of the genius of this Swiss-German physicist.





★ Extraordinary claims require extraordinary evidence, and it's hard to think of something more extraordinary than the idea that massive objects warp space and time.



Angle is very small

<http://sunearthday.nasa.gov/2006/events/einstein.php>