Lecture 12: Black Holes

- Old ideas for black holes
- Theory of black holes
- Real-life black holes
  - Stellar mass
  - Supermassive
- Speculative stuff

"The last I heard, Medwick was working on a model black hole in his lab."

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I : OLD IDEAS FOR BLACK HOLES

- "What goes up must come down"… or must it?
- **Escape velocity**, V_{esc}
  - Critical velocity object must have to just escape the gravitational field of the Earth
    - $V<V_{esc}$: object falls back to Earth
    - $V>V_{esc}$: object never falls back to Earth
- In fact, escape velocity given in general by

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

when the mass of an object is M and the distance from the center is R- G is the gravitational constant

- Starting from Earth's surface, V_{esc}=11 km/s
- Starting from Sun's surface, V_{esc}= 616 km/s

18^{th} Century ideas

- By making M larger and R smaller, V_{esc} increases
- Idea of an object with gravity so strong that light cannot escape first suggested by Rev. John Mitchell in 1783

- Laplace (1798) – "*A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible.*"

II : MODERN IDEAS

- Karl Schwarzschild (1916)
  - First solution of Einstein's equations of GR
  - Describes gravitational field in (empty) space around a non-rotating mass
- Space-time interval in Schwarzschild's solution (radial displacements only) is rather complex- see text book

- Features of Schwarzschild's solution:
  - Yields Newton's law of gravity, with flat space, at large R
  - Space-time curvature becomes infinite at center (R=0; this is called a **space-time singularity**)
  - Gravitational time-dilation effect becomes infinite on a spherical surface known as the **event horizon**, where coefficient of Δt is zero
  - Radius of the sphere representing the event horizon is called the **Schwarzschild radius,** $R_s = 2GM/c^2$

Event horizon

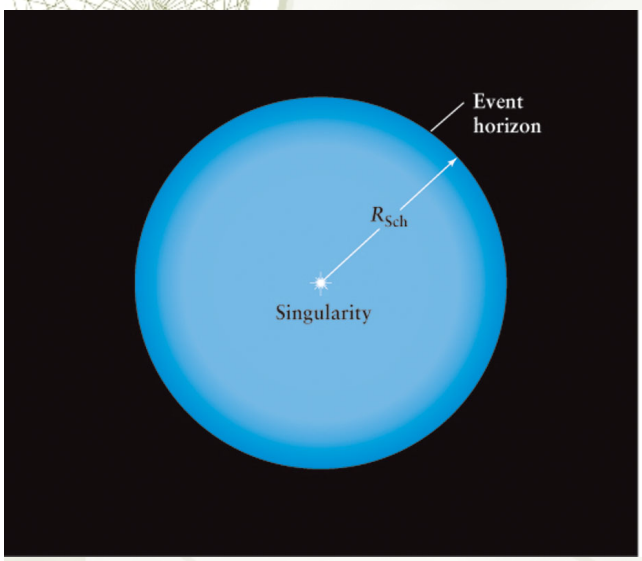
- **Point of no-return**… the location where the escape velocity equals the speed of light
- The **gravitational redshift becomes infinite** here (as seen by an outside static observer)
- Nothing occurring inside can be seen from outside (or have any causal effect on the external Universe!)
- So… as a practical matter, astrophysics never need concern themselves with the Universe interior to the event horizon
- Radius corresponding to event horizon for a non-spinning black hole is known as the **Schwarzschild radius**

$$R_{Sch} = \frac{2GM}{c^2} \approx 3\left(\frac{M}{M_\odot}\right) \text{ km}$$

⊙ symbol for sun, $M_\odot$ =solar mass

Derived from GR treatment of problem… but same formula results from using Newtonian treatment of escape velocity

Modern (GR) ideas of black holes



Gravitational redshift outside of a spherical object with mass M is

$$\nu_{obs} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \nu_{emit}$$

Gravitational length contraction

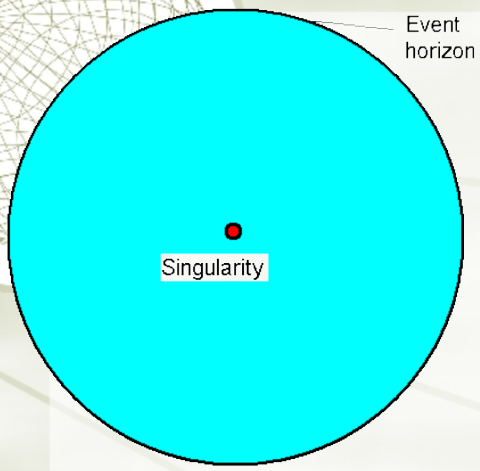
$L' = L\,(1-R_s/R)^{1/2}$

Gravitational time dilation

$t' = t/(1-R_s/R_s)^{1/2}$

$R_s = 2GM/c^2$

As $R \rightarrow R_s$ time goes to ∞ length goes to zero, wavelength of emitted radiation goes to zero



Schwarschild radius is NOT the singularity

At the Schwarzschild radius the gravitational time dilation goes to infinity and lengths are contracted to zero

•For a body of the Sun's mass, **Schwarzschild radius**

$$R_S = \frac{2GM}{c^2} \rightarrow 3\text{km}$$

• **Singularity** – spacetime curvature is infinite. Everything destroyed. Laws of GR break down.

• **Event horizon** – gravitational time-dilation is infinite as observed from large distance.

• Any light emitted at R_s would be infinitely redshifted - hence could not be observed from outside

More features of Schwarzschild black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)
- Observer who enters event horizon would only "feel" "strange" gravitational effects if the black hole mass is small, so that R_s is comparable to observer's size
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
- Thus black hole would produce "shadow" on sky