Virial Theorem Simplest

- $2T + U = 0$
- By re-arranging the above equation and making some simple assumptions about T~$(Mv^2/2)$) and U~(GM^2/R) for galaxies one gets
- $M \sim v^2 R/G$
  - M is the total **mass** of the **galaxy**, v is the mean **velocity** of **object** in the galaxy/cluster, G is Newton's **gravitational constant** and R is the **effective radius** (size) of the object.
- This equation is extremely important, as it relates two observable properties of galaxies (velocity dispersion and effective **half-light radius**) to a fundamental, but unobservable, property – the mass of the galaxy. Consequently, the virial theorem forms the root of many galaxy **scaling relations**.
- Therefore, we can estimate the **Virial Mass** of a system if we can observe:
  - The true overall extent of the system R_{tot}
  - The mean square of the velocities of the individual objects in the system

Virial Theorem

- Another derivation following Bothun http://ned.ipac.caltech.edu/level5/Bothun2/Bothun4_1_1.html- please read this
- Moment of inertia, I, of a system of N particles

  $I=\Sigma m_i r_i^2$ sum over i=1,N (express r_i^2 as $(x_i^2+y_i^2+z_i^2)$)
- take the first and second time derivatives ; let d^2x/dt^2 be symbolized by x,y,z
- $\frac{1}{2} d^2I/dt^2 = \Sigma m_i (dx_i/dt)^2+(dy_i/dt)^2+(dz_i/dt)^2+\Sigma m_i(x_i x+y_i y+z_i z)$

mv^2 (2 KE)+Potential energy (W) ; $W \sim 1/2GN^2m^2/R=1/2GM^2_{tot}/R_{tot}$

after a few dynamical times, if unperturbed a system will come into Virial equilibrium–time averaged inertia will not change so $2<T>+W=0$

For self gravitating systems $W=-GM^2/2R_H$; R_H is the harmonic radius- the sum of the distribution of particles appropriately weighted [$1/R_H = 1/N\ \Sigma_i\ 1/r_i$]

The virial mass estimator is $M=2\sigma^2R_H/G$; for many mass distributions $R_H \sim 1.25\ R_{eff}$ where R_{eff} is the half light radius, σ is the 3-d velocity dispersion

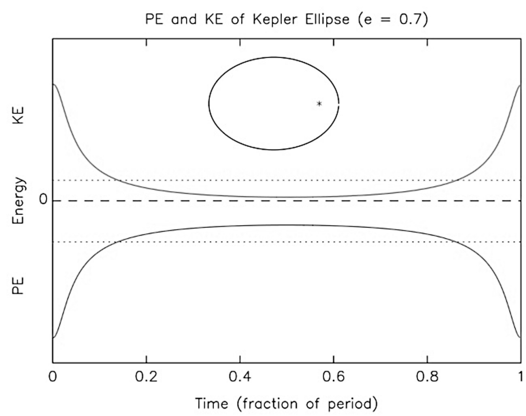
Virial Theorem - Simple Cases

- Circular orbit:

$mV^2/r=GmM/r^2$

Multiply both sides by r, $mV^2=GmM/r$

$mV^2=2KE$; $GmM/r=-W$ so $2KE+W=0$



- Time averaged Keplerian orbit

define U=KE/|W|; as shown in figure i clearly changes over the orbit; but take averages:

$-W=\langle GMm/r\rangle=GMm\langle 1/r\rangle$

$=GMm(1/a)$

$KE=\langle 1/2mV^2\rangle=GMm\langle 1/r-1/2a\rangle$

$=1/2GMm(1/a)$

So again $2KE+W=0$

Red: kinetic energy (positive) starting at perigee

Blue: potential energy (negative)

Virial Thm

- If I is the moment of inertia
- $\frac{1}{2}d^2I/dt^2 =2KE+W+\Sigma$
  - where Σ is the work done by external pressure
  - KE is the kinetic energy of the system
  - W is the potential energy (only if the mass outside some surface S can be ignored)
- For a static system ($d^2I/dt^2 =0$) $2KE+W+\Sigma =0$- almost always $\Sigma=0$
- Using the virial theorem, masses can be derived by measuring characteristic velocities over some characteristic scale size. In general, the virial theorem can be applied to any gravitating system after one dynamical timescale has elapsed.

Using the Virial Theorm- (from J. Huchra)

- It is hard to use for distant galaxies because individual test particles (stars) are too faint
- **However it is commonly used for clusters of galaxies**

Assume the system is spherical. The observables are (1) the l.o.s. time average velocity:

$$< v^2_{R,i} >_{\Omega} \; = \; 1/3 \; v_i^2$$

projected radial v   averaged over solid angle

i.e. we only see the radial component of motion &

$$v_i \sim \sqrt{3}\, v_r$$

Ditto for position, we see projected radii R,

$R = \theta\, d$,   d = distance, θ = angular separation

Thus after taking into account all the projection effects, and if we assume masses are the same so that $M_{sys} = \Sigma\, m_i = N\, m_i$ we have

$$M_{VT} = \frac{3\pi}{2G} N \frac{\Sigma\, v_i^2}{\sum_{i<j} (1/R_{ij})}$$

this is the Virial Theorem Mass Estimator

$\Sigma\, v_i^2$ = Velocity dispersion

$[\sum_{i<j} (1/R_{ij})]^{-1}$ = Harmonic Radius

Time Scales for Collisions (S&G 3.2)

- N particles of radius r_p; Cross section for a direction collision $\sigma_d=\pi r^2_p$
- **Definition of mean free path: $\lambda=1/n\sigma_d$**

where n is the number density of particles (particles per unit volume), $n=N/(4\pi\ell^3/3)$

The characteristic time between collisions (Dim analysis) is $t_{collision}=\lambda/v\sim[(\ell/r_p)^2 t_{cross}/N]$ where v is the velocity of the particle. for a body of size ℓ, $t_{cross}=\ell/v$= crossing time

Time Scales for Collisions

So lets consider a galaxy with $\ell\sim10$kpc, $N=10^{10}$ stars and v~200km/sec

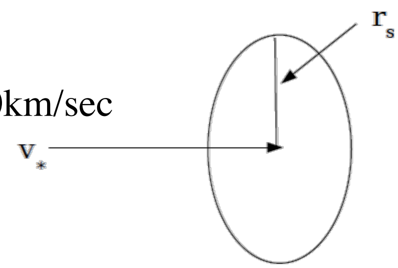
- if $r_p = R_{sun}$, $t_{collision}\sim10^{21}$ yrs Therefore direct collisions among stars are completely negligible in galaxies.

- For indirect collisions the argument is more complex (see S+G sec 3.2.2, MWB pg 231-its a long derivation-see next few pages) but the answer is the same - **it takes a very long time for star interactions to exchange energy (relaxation).**

- $t_{relax}\sim Nt_{cross}/10\ln N$
- It's only in the centers of the densest globular clusters and galactic nuclei that this is important

How Often Do Stars Encounter Each Other (S&G 3.2.1)

Definition of a 'strong' encounter, $GmM/r > 1/2mv^2$
potential energy exceeds KE of incoming particle
<u>So a critical radius is $r<r_s=2GM/v^2$</u> eq 3.48

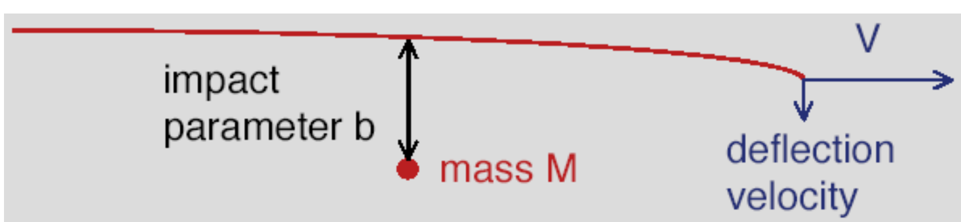


Putting in some typical numbers $m\sim 1/2M_\odot$ $v=30$km/sec
$r_s=1$AU
So how often do stars get that close?

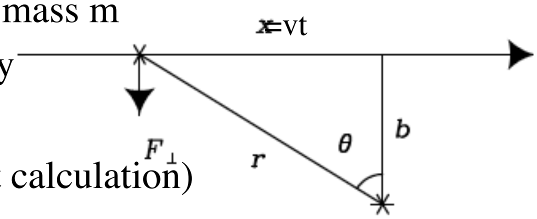
consider a cylinder Vol$=\pi r^2_s vt$; if have n stars per unit volume than on average the encounter occurs when
$n\pi r^2_s vt=1$, $t_s=v^3/ 4\pi nG^2M^2$
Putting in typical numbers $=\mathbf{4x10^{12}}(v/10\text{km/sec})^3(M/M_\odot)^{-2}(n/pc^3)^{-1}$ **yr**- a very long time (universe is only 10^{10}yrs old) eq 3.55
- galaxies are essentially collisionless

What About Collective Effects ? sec 3.2.2

For a weak encounter $b >> r_s$
Need to sum over individual interactions- effects are also small



Relaxation Times

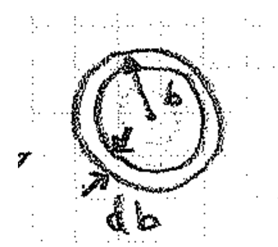


- Star passes by a system of N stars of mass m
- assume that the perturber is stationary during the encounter and that $\delta v/v<<1$
(B&T pg 33-sec 1.2.1. sec 3.1 for exact calculation)
- So δv is perpendicular to v
  - assume star passes on a straight line trajectory
- The force perpendicular to the motion is

$F_p=Gm^2\cos\theta/(b^2+x^2)=Gbm^2/(b^2+x^2)^{3/2}=(Gm^2/b^2)(1+(vt/b)^2)^{-3/2}=m(dvdt)$

so $\delta v=1/m \int F_p dt = (Gm^2/b^2)\int^{\infty}_{-\infty} dt(1+(vt/b)^2)^{-3/2}= 2GM/bv$

Relaxation Times



- In words, δv is roughly equal to the acceleration at closest approach,
Gm/b^2, times the duration of this acceleration $2b/v$.

The surface density of stars is $\sim N/\pi R^2$
N is the number of stars and R is the galaxy radius

let δn be the number of interactions a star encounters with impact parameter
between b and δb crossing the galaxy once
$\delta n\sim(N/\pi r^2)2\pi b\delta b=\sim(2N/r^2)b\delta b$

each encounter produces a dv but are randomly oriented to the stars intial velocity v and thus their mean is zero (vector) HOWEVER
the mean square is NOT ZERO and is
$\Sigma\delta v^2\sim\delta v^2\delta n= (2Gm/bv)^2(2N/R^2)\ bdb$

Relaxation...continued

- now integrating this over all impact parameters from b_{min} to b_{max}
- one gets $\delta v^2 \sim 8\pi n(Gm)^2/v \ln \Lambda$; where r is the galaxy radius eq (3.54)

  ln Λ is the Coulomb logarithm = $\ln(b_{max}/b_{min})$ (S&G 3.55)

- For gravitationally bound systems the typical speed of a star is roughly $v^2 \sim GNm/r$

  (from KE=PE) and thus $\delta v^2/v^2 \sim 8 \ln \Lambda/N$

- For each 'crossing' of a galaxy one gets the same δv so the number of crossing for a star to change its velocity by order of its own velocity is $n_{relax} \sim N/8 \ln \Lambda$

Relaxation...continued

So how long is this??

- Using eq 3.55

$t_{relax} = V^3/[8\pi n(Gm)^2 \ln \Lambda] \sim [2\times10^9 \text{ yr}/\ln\Lambda](V/10\text{km/sec})^3(m/M_\odot)^{-2}(n/10^3\text{pc}^{-3})^{-1}$

Notice that this has the same form and value as eq 3.49 (the strong interaction case) with the exception of the 2lnΛ term

- Λ**~N/2** **(S&G 3.56)**
- $\mathbf{t_{relax} \sim (0.1N/\ln N) t_{cross}}$; if we use $N \sim 10^{11}$; t_{relax} is much much longer than t_{cross}
- Over much of a typical galaxy the dynamics over timescales $t < t_{relax}$ is that of a **collisionless** system in which the constituent particles move under the influence of the gravitational field generated by a smooth mass distribution, rather than a collection of mass points
- However there are *parts* of the galaxy which 'relax' much faster

Relaxation

- Values for some representative systems

	<m>	N	r(pc)	t_{relax}(yr)	age(yrs)
Pleiades	1	120	4	1.7×10^7	$<10^7$
Hyades	1	100	5	2.2×10^7	4×10^8
Glob cluster	0.6	10^6	5	2.9×10^9	10^9-10^{10}
E galaxy	0.6	10^{11}	3×10^4	4×10^{17}	10^{10}
Cluster of gals	10^{11}	10^3	10^7	10^9	10^9-10^{10}

Scaling laws $t_{relax} \sim t_{cross} \sim R/v \sim R^{3/2} / (Nm)^{1/2} \sim \varrho^{-1/2}$

- Numerical experiments (Michele Trenti and Roeland van der Marel 2013 astro-ph 1302.2152) show that even globular clusters never reach energy **equipartition** (!) to quote 'Gravitational encounters within stellar systems in virial equilibrium, such as globular clusters, drive evolution over the two-body relaxation timescale. The evolution is toward a thermal velocity distribution, in which stars of different mass have the same energy). This thermalization also induces mass segregation. As the system evolves toward energy equipartition, high mass stars lose energy, decrease their velocity dispersion and tend to sink toward the central regions. The opposite happens for low mass stars, which gain kinetic energy, tend to migrate toward the outer parts of the system, and preferentially escape the system in the presence of a tidal field"

So Why Are Stars in Rough Equilibrium?

- Another process, 'violent relaxation' (MBW sec 5.5), is crucial.
- This is due to rapid change in the gravitational potential (e.g., collapsing protogalaxy)
- Stellar dynamics describes in a statistical way the collective motions of stars subject to their mutual gravity-The essential difference from celestial mechanics is that each star contributes more or less equally to the total gravitational field, whereas in celestial mechanics the pull of a massive body dominates any satellite orbits
- The long range of gravity and the slow "relaxation" of stellar systems prevents the use of the methods of statistical physics as stellar dynamical orbits tend to be much more irregular and chaotic than celestial mechanical orbits-....woops.
- to quote from MBW pg 248
- Triaxial systems with realistic density distributions are therefore difficult to treat analytically, and one in general relies on numerical techniques to study their dynamical structure

Collisionless Boltzmann Eq (= Vlasov eq)
S+G sec 3.4

- When considering the structure of galaxies, one cannot follow each individual star (10^{11} of them!),
- Consider instead stellar density and velocity distributions. However, a fluid model is not really appropriate since a fluid element has a single velocity, which is maintained by particle-particle collisions on a scale much smaller than the element.
- For stars in the galaxy, this is **not true** - stellar collisions are very rare, and even encounters where the gravitational field of an individual star is important in determining the motion of another are very infrequent
- So taking this to its limit, treat each particle as being collisionless, moving under the influence of the mean potential generated by all the other particles in the system $\phi(\mathbf{x},t)$

Collisionless Boltzmann Eq s S&G 3.4

- The distribution function is defined such that $f(\mathbf{r},\mathbf{v},t)d^3\mathbf{x}d^3\mathbf{v}$ specifies the number of stars inside the volume of phase space $d^3\mathbf{x}d^3\mathbf{v}$ centered on $(\mathbf{x},\mathbf{v})$ at time t-
- We can describe a many-particle system by its distribution function f(x,v,t) = density of stars (particles) within a phase space element

At time t, a full description of the state of this system is given by specifying the number of stars

$$f(x, v, t)d^3xd^3v$$

Then $f(\mathbf{x}, \mathbf{v}, t)$ is called the "distribution function" (or "phase space number density") in 6 dimensions (**x** and **v**) of the system.

$f \geq 0$ since no negative star densities

Since the potential is smooth, nearby particles in phase space move together-- fluid approx.

See S&G sec 3.4

- The collisionless Boltzmann equation (CBE) is like the equation of continuity,

$$dn/dt = \partial n/\partial t + \partial(nv)/\partial x = 0$$

but it allows for changes in velocity and relates the changes in $f(x, v, t)$ to the forces acting on individual stars

- In one dimension, the CBE is

$$df/dt = \partial f/\partial t + v\partial f/\partial x - [\partial\phi(x, t)/\partial x]\, \partial f/\partial v = 0$$

Collisionless Boltzman cont

- Starting point: Boltzmann Equation (= phase space continuity equation)
- It says: if I follow a particle on its gravitational path (=Lagrangian derivative) through phase space, it will always be there.

$$\boxed{\frac{Df(\vec{x}, \vec{v}, t)}{Dt} = \frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \vec{\nabla}\Phi_{grav}\frac{\partial f}{\partial \vec{v}} = 0}$$

- A rather ugly partial differential equation!

Collisionless Boltzmann Eq

- This results in (S+G pg 143)

$$\boxed{\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0,}$$

- the flow of stellar phase points through phase space is incompressible – the phase-space density of points around a given star is always the same

- The distribution function f is a function of seven variables (t, **x**, **v**), so solving the collisionless Boltzmann equation in general is hard. So need either simplifying assumptions (usually symmetry), or try to get insights by taking moments of the equation.
- Take moments of an eq-- multipying f by powers of v

- For a collisionless stellar system in dynamic equilibrium, the gravitational potential,ϕ , relates to the phase-space distribution of stellar tracers $f(\mathbf{x}, \mathbf{v}, t)$, via the collisionless Boltzmann Equation

- number density of particles: $n(\mathbf{x},t)=\int f(x, v, t)d^3\mathbf{v}$

average velocity: $\langle v(\mathbf{x},t)\rangle=\int f(x, v, t)\, \mathbf{v} d^3\mathbf{v}/\int f(x, v, t)d^3\mathbf{v}=(1/n(\mathbf{x},t))\int f(x, v, t)\, \mathbf{v}d^3\mathbf{v}$

- **bold variables are vectors**

Analogy with Gas- continuity eq see MBW sec 4.1.4

- $\partial\rho/\partial t + \nabla\bullet(\rho v) = 0$ which is equiv to
- $\partial\rho/\partial t + v\bullet\nabla\rho = 0$
- In the absence of encounters f satisfies the continuity eq, flow is smooth, stars do no jump discontinuously in phase space

- Continuity equation :

define w=(x,v) pair (generalize to 3-D)

$dw/dt = (v, -\nabla\phi)$ – 6-dimensional space

- $df/dt = 0$
- $\partial f/\partial t + \nabla_6(f\ \ dw/dt) = 0$

Jeans Equations MBW sec 5.4.3

- Since f is a function of 7 variables, obtaining a solution is challenging
- Take moments (e.g. integrate over all **v**)
  - let n be the space density of 'stars'

$\partial n/\partial t + \partial(n\langle v_i\rangle)/\partial x_i = 0$; continuity eq. zeroth moment

first moment (multiply by v and integrate over all velocities)

$\partial(n\langle v_j\rangle/\partial t) + \partial(n\langle v_i v_j\rangle)/\partial x_i + n\partial\phi/\partial x_j = 0$

equivalently

$n\partial(\langle v_j\rangle/\partial t) + n\langle v_i\rangle\, \partial\langle v_j\rangle/\partial x_i = -n\partial\phi/\partial x_j - \partial(n\sigma^2_{ij})/\partial x_i$

where

n is the integral over velocity of f ; $n = \int f\, d^3v$

$\langle v_i\rangle$ is the mean velocity in the i^{th} direction $= (1/n)\int f\, v_i\, d^3v$

$\sigma^2_{ij} = \langle (v_i - \langle v_i\rangle)(v_j - \langle v_j\rangle)\rangle$ "stress tensor"

$= \langle v_i v_j\rangle - \langle v_i\rangle\langle v_j\rangle$

Collisionless Boltzmann Eq

astronomical structural and kinematic observations provide information only about the **projections of phase space distributions along lines of sight**,
limiting knowledge about f and hence also about ϕ.

Therefore all efforts to translate existing data sets into constraints on CBE involve simplifying assumptions.
- dynamic equilibrium,
- symmetry assumptions
- particular functional forms for the distribution function and/or the gravitational potential.

Collisionless Boltzmann Eq- Moments

define n(x,t) as the number density of stars at position x

then the zeroth moment is:
$\partial n/\partial t+\partial/\partial x(nv)=0$; the same eq as continuity equation of a fluid

first moment:
$n\partial v/dt+nv\partial v/dx=-n\partial\phi/\partial x-\partial/\partial x(n\sigma^2)$

σ is the velocity dispersion
But unlike fluids, we do not have thermodynamics to help out....
nice math but not clear how useful

Jeans Eq

- $n\partial(\langle v_j\rangle/\partial t) + n\langle v_i\rangle \partial\langle v_j\rangle/\partial x_i = -n\partial\phi/\partial x_j - \partial(n\sigma^2_{ij})/\partial x_i$
- So what are these terms??
- Gas analogy: Euler's eq of motion

  $\rho\, \partial \mathbf{v}/\partial t + \rho\, (\mathbf{v} \cdot \nabla)v = -\nabla P - \rho\nabla\Phi$
- $n\partial\phi/\partial x_j$: gravitational pressure gradient
- $n\sigma^2_{ij}$ "stress tensor" is like a pressure, but may be anisotropic, allowing for different pressures in different directions - important in elliptical galaxies and bulges 'pressure supported' systems (with a bit of coordinate transform one can make this symmetric e.g. $\sigma^2_{ij}=\sigma^2_{ji}$)

Jeans Eq Cont

- $n\partial\mathbf{v}/dt+n\mathbf{v}\partial\mathbf{v}/d\mathbf{x}=-n\partial\phi/\partial\mathbf{x}-\partial/\partial\mathbf{x}(n\boldsymbol{\sigma}^2)$
- Simplifications: assume isotropy, steady state, non-rotating

  → terms on the left vanish
- Jean Eq becomes: $-n\,\nabla\phi = \nabla(n\sigma^2)$
- using Poisson eq: $\nabla^2\phi = 4\pi G\rho$
- Generally, solve for ρ (mass density)

Jeans Equations: Another Formulation

- Jeans equations follow from the collisionless Boltzmann equation; Binney & Tremaine (1987), MBW 5.4.2. S+G sec 3.4 .

cylindrical coordinates and assuming an axi-symmetric and steady-state system, the accelerations in the radial (R) and vertical (Z) directions can be expressed in terms of observable quantities:

the stellar number density distribution ν_*

And 5 velocity components

- a rotational velocity v_ϕ
- 4 components of random velocities (velocity dispersion components) $\sigma_{\phi\phi}, \sigma_{RR}, \sigma_{ZZ}, \sigma_{RZ}$

$$a_R = \sigma_{RR}^2 \times \frac{\partial(\ln \nu)}{\partial R} + \frac{\partial \sigma_{RR}^2}{\partial R} + \sigma_{RZ}^2 \times \frac{\partial(\ln \nu)}{\partial Z} + \frac{\partial \sigma_{RZ}^2}{\partial Z} + \frac{\sigma_{RR}^2}{R} - \frac{\sigma_{\phi\phi}^2}{R} - \frac{\overline{v_\phi}^2}{R},$$

$$a_Z = \sigma_{RZ}^2 \times \frac{\partial(\ln \nu)}{\partial R} + \frac{\partial \sigma_{RZ}^2}{\partial R} + \sigma_{ZZ}^2 \times \frac{\partial(\ln \nu)}{\partial Z} + \frac{\partial \sigma_{ZZ}^2}{\partial Z} + \frac{\sigma_{RZ}^2}{R}.$$

where a_Z, a_R are accelerations in the appropriate directions-

given these values (which are the gradient of the gravitational potential), the dark matter contribution can be estimated after accounting for the contribution from visible matter

Use of Jeans Eqs: Surface mass density near Sun

- Poissons eq $\nabla^2\phi = 4\pi\rho G = -\nabla\bullet\mathbf{F}$
- Use cylindrical coordinates, axisymmetry

  $(1/R)\partial/\partial R(RF_R) + \partial F_z/\partial z = -4\pi\rho G$

- $F_R = -v_c^2/R$ v_c = circular velocity (roughly constant near Sun) –

  F_R = force in R direction

So $\rho = (-1/4\pi G)\partial F_z/\partial z$; **only vertical gradients count**

since the surface mass density $\Sigma = 2\int \rho dz$ (integrate 0 to $+\infty$ thru plane)

$\Sigma = -F_z/2\pi G$

Now use Jeans eq: $nF_z = -\partial(n\sigma^2_z)/\partial z + (1/R)\partial/\partial R(Rn\sigma^2_{zR})$; if R+z are separable, e.g $\phi(R,z) = \phi(R) + \phi(z)$ then $\sigma^2_{zR} \sim 0$ and voila! (eq 3.94 in S+G)

$\Sigma = -(1/2\pi Gn)\ \partial(n\sigma^2_z)/\partial z$; need to observe the number density distribution of some tracer of the potential above the plane [goes as $\exp(-z/z_0)$] and its velocity dispersion distribution perpendicular to the plane

Spherical systems- Elliptical Galaxies and Globular Clusters

- while apparently simple we have 3 sets of unknowns $<v^2_r>$, $\beta(r)$, $n(r)$
- and 2 sets of observables $I(r)$- surface brightness of radiation (in some wavelength band) and the lines of sight projected velocity field (first moment is velocity dispersion)
- It turns out that one has to 'forward fit'- e.g. propose a particular form for the unknowns and fit for them.

Use of Jeans Eq For Galactic Dynamics

- Accelerations in the z direction from the Sloan digital sky survey for

1) all matter (top panel)

2) 'known' baryons only (middle panel)

3) ratio of the 2 (bottom panel)

Based on full-up numerical simulation from cosmological conditions of a MW like galaxy-this 'predicts' what a_Z should be near the Sun (Loebman et al 2012)

Compare with results from Jeans eq (ν is density of tracers, v_ϕ is the azimuthal velocity (rotation))

$$a_R = \sigma^2_{RR} \times \frac{\partial(\ln\nu)}{\partial R} + \frac{\partial\sigma^2_{RR}}{\partial R} + \sigma^2_{RZ} \times \frac{\partial(\ln\nu)}{\partial Z} + \frac{\partial\sigma^2_{RZ}}{\partial Z} + \frac{\sigma^2_{RR}}{R} - \frac{\sigma^2_{\phi\phi}}{R} - \frac{\overline{v_\phi}^2}{R}, \quad (1)$$

$$a_Z = \sigma^2_{RZ} \times \frac{\partial(\ln\nu)}{\partial R} + \frac{\partial\sigma^2_{RZ}}{\partial R} + \sigma^2_{ZZ} \times \frac{\partial(\ln\nu)}{\partial Z} + \frac{\partial\sigma^2_{ZZ}}{\partial Z} + \frac{\sigma^2_{RZ}}{R}. \quad (2)$$

Given accelerations $a_R(R,Z)$ and $a_Z(R,Z)$, *i.e.* the gradient of the gravitational potential, the dark matter contribution can be estimated after accounting for the contribution from visible matter.

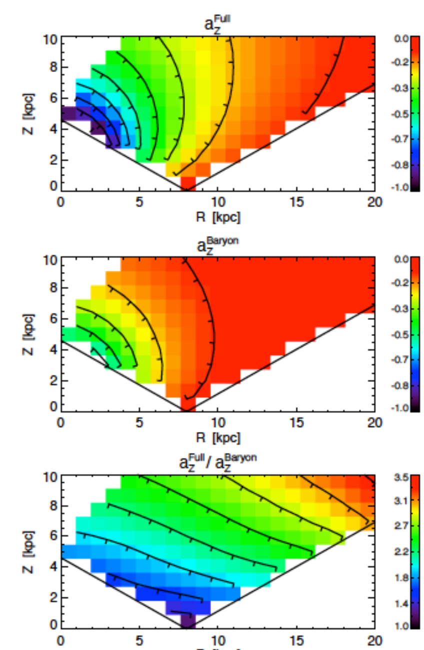
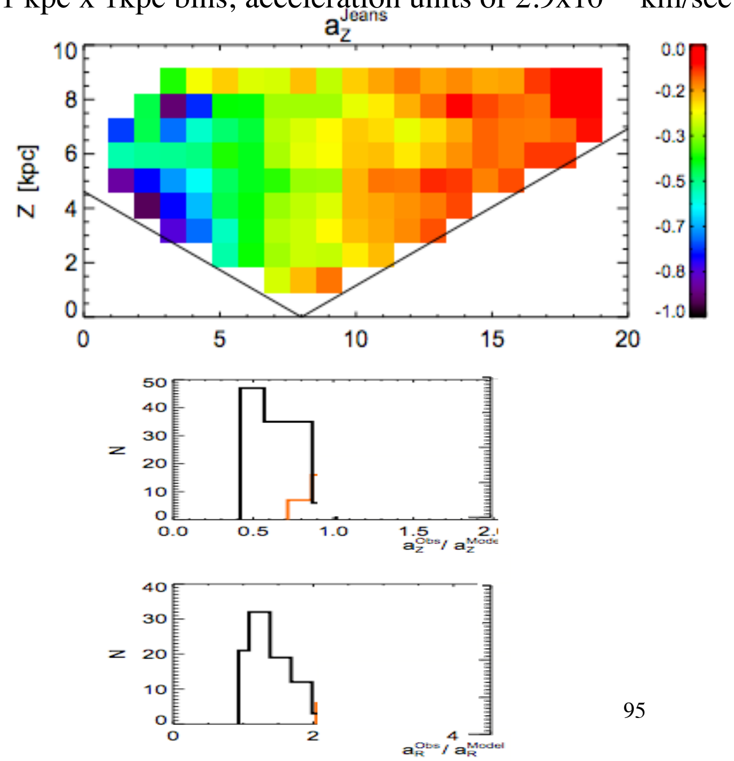


Figure 1. A comparison of the acceleration in the Z di[r...] when all contributions are included (star, gas, and dark [...] particles; top panel) to the result without dark matter ([...] panel). The acceleration is expressed in units of 2.9×10^{-13} [...] The ratio of the two maps is shown in the bottom panel. [...] importance of the dark matter increases with the distance fr[...] origin; at the edge of the volume probed by SDSS ($R \sim 2$[...]

What Does One Expect The Data To Look Like

- Now using Jeans eq
- **Notice that it is not smooth or monotonic**
  and that the simulation is neither perfectly rotationally symmetric nor steady state..
- errors are on the order of 20-30%- figure shows comparison of true radial and z accelerations compared to Jeans model fits

1 kpc x 1kpc bins; acceleration units of 2.9×10^{-13} km/sec^2



Jeans (Continued)

- Using dynamical data and velocity data, get estimate of surface mass density in MW

Σ_{total}~70 +/- 6$M_\odot/pc^2$

Σ_{disk}~48+/-9 $M_\odot/pc^2$

Σ_{star}~35$M_\odot/pc^2$

Σ_{gas}~13$M_\odot/pc^2$

we know that there is very little light in the halo so direct evidence for dark matter

Full Up Equations of Motion- Stars as an Ideal Fluid

(S+G pgs140-144, MBW pg 163)

Continuity equation (particles not created or destroyed)

$d\rho/dt+\rho\nabla\cdot v=0$; $d\rho/dt+d(\rho v)/dr=0$

Eq's of motion (Eulers eq)

$dv/dt = -\nabla P/\rho - \nabla\Phi$

Poissons eq

$\nabla^2\Phi(r) = -4\pi G\rho(r)$ (example potential)

Analogy of Stellar Systems to Gases

- Discussion due to Mark Whittle

- **Similarities :**

comprise many, interacting objects which act as points (separation >> size)

can be described by distributions in space and velocity eg Maxwellian velocity distributions; uniform density; spherically concentrated etc.

Stars or atoms are neither created nor destroyed -- they both obey continuity equations- not really true, galaxies are growing systems!

All interactions as well as the system as a whole obeys conservation laws (eg energy, momentum) **if isolated**

- **But** :
- The relative importance of short and long range forces is radically different :
  - atoms interact only with their neighbors
  - stars interact continuously with the entire ensemble via the long range attractive force of gravity
- eg uniform medium : $F \sim G (\rho\, dV)/r^2$; $dV \sim r^2 dr$; $F \sim \rho\, dr$

~ equal force from all distances

Analogy of Stellar Systems to Gases

- Discussion due to Mark Whittle

- The relative frequency of strong encounters is radically different :
  - -- for atoms, encounters are frequent and all are strong (ie $\delta V \sim V$)
  - -- for stars, pairwise encounters are very rare, and the stars move in the smooth global potential (e.g. S+G 3.2)

- Some parallels between gas (fluid) dynamics and stellar dynamics: many of the same equations can be used as well as :
  - ---> concepts such as Temperature and Pressure can be applied to stellar systems
  - ---> we use analogs to the equations of fluid dynamics and hydrostatics
- there are also some interesting differences
  - ---> pressures in stellar systems can be anisotropic
  - ---> self-gravitating stellar systems have negative specific heat

    $2K + U = 0 \rightarrow E = K + U = -K = -3NkT/2 \rightarrow C = dE/dT = -3Nk/2<0$

    and evolve away from uniform temperature.

Next Time

- The Local Group