Chapter 15

Theories of Spiral Structure

That rotating disk galaxies should exhibit spiral structure is not surprising, but the nature of the spiral patterns is not completely understood – probably because there is no *unique* cause of spiral structure.

15.1 Material Arms & Density Waves

Because disk galaxies rotate differentially, the orbital period is an increasing function of radius R. Thus *if* spiral arms were material features then differential rotation would would soon wind them up into very tightly-coiled spirals. The expected pitch angle of material arms in a spiral galaxy like the Milky Way is only about 0.25 degrees (BT87, Ch. 6.1.2). In fact, pitch angles measured from photographs range from about 5 degrees for Sa galaxies to 20 degrees for Sc galaxies (Kennicutt 1981). The most likely implication is that *spiral arms are not material features*.

The other possibility is that spiral arms are *density waves*; in this case the stars which make up a given spiral arm are constantly changing. Observational and numerical evidence lends strong support to the idea of spiral density waves.

15.2 Epicyclic Theory

Just as water molecules in the ocean do not move very far in response to a passing wave, the stars in a disk galaxy need not move far from their unperturbed orbits to create a spiral density wave. To describe the *local* motions of stars in a disk we study the equations of motion for small perturbations from a circular orbit. The result is a description of stellar motion in terms of *epicycles*.

Let x and y be coordinates for a 'not-quite-Cartesian' frame of reference which revolves about the center of the galaxy with the angular velocity $\Omega_0 = \Omega(R_0)$ of a circular orbit at radius R_0. In terms of R and θ,

$$x \equiv R - R_0\,, \qquad y \equiv R_0(\theta - \Omega_0 t)\,; \tag{15.1}$$

thus x increases outward from the center, and y increases in the direction of rotation.

In this coordinate system, the linearized equations of motion for a star near the guiding center are

$$\frac{d^2x}{dt^2} - 2\Omega_0 \frac{dy}{dt} = 4\Omega_0 A_0 x\,, \tag{15.2}$$

$$\frac{d^2y}{dt^2} + 2\Omega_0 \frac{dx}{dt} = 0\,, \tag{15.3}$$

where A_0 is Oort's 'constant' evaluated at R_0. These linearized equations have a solution of the form

$$x(t) = \alpha \cos(\kappa t)\,, \qquad y(t) = -\sin(\kappa t)\,, \tag{15.4}$$

which describe an ellipse about the guiding center. The sign of $y(t)$ is such that the motion about the ellipse is retrograde with respect to the galactic rotation. This follows from conservation of angular momentum: when the star is at radii $R > R_0$ it must drift backward with respect to the guiding center since both have the same specific angular momentum.

Substituting (15.4) into (15.3), we obtain

$$\alpha = \frac{\kappa}{2\Omega_0} \tag{15.5}$$

for the axial ratio of the ellipse. Substituting (15.4) and (15.5) into (15.2), we obtain

$$\kappa^2 = 4(\Omega_0^2 - A_0\Omega_0)\,, \tag{15.6}$$

which is equivalent to the formula given in the previous chapter.

The Sun and nearby disk stars make about 1.3 radial oscillations per orbit about the Galactic Center. In the solar neighborhood, $\alpha \simeq 0.7$; thus the Sun and nearby disk stars are moving on epicycles which are squashed by about 30% in the radial direction (BT87, Ch. 3.2.3).

15.3 Kinematic Spiral Waves

One application of epicycles is the construction of *kinematic spiral waves*. For example, consider a ring of test particles on similar epicyclic orbits with their guiding centers at the same radius R_0. Let the initial phases of the epicycles be such that at $t = 0$ the particles define an oval. As time moves forward the guiding centers travel around the galaxy with angular velocity Ω_0, but the stars at the ends of the oval are being carried backward with respect to their guiding centers, so the form of the oval advances more slowly. The precession rate or 'pattern speed' of the oval is

$$\Omega_{\rm p} = \Omega - \kappa/2\,. \tag{15.7}$$

This point is illustrated by Fig. 2 of Toomre (1977; hereafter T77).

By superimposing ovals of different sizes, one can produce a wide variety of spiral patterns. If $\Omega - \kappa/2$ were independent of R, such patterns would persist indefinitely because all the superimposed ovals would precess at the same rate. In fact, plausible disk galaxy models have circular velocity profiles which yield $\Omega - \kappa/2$ fairly constant over a range of radii (*e.g.*, Fig. 6-10 of BT87). Compared to material arms, density waves in the Milky Way should wind up about six times less rapidly, yielding predicted pitch angles of about 1.4 degrees. This is an improvement, but still inconsistent with most observed pitch angles. Moreover, this kinematic model has neglected the self-gravity of spiral structures, so it can't be telling the whole story.

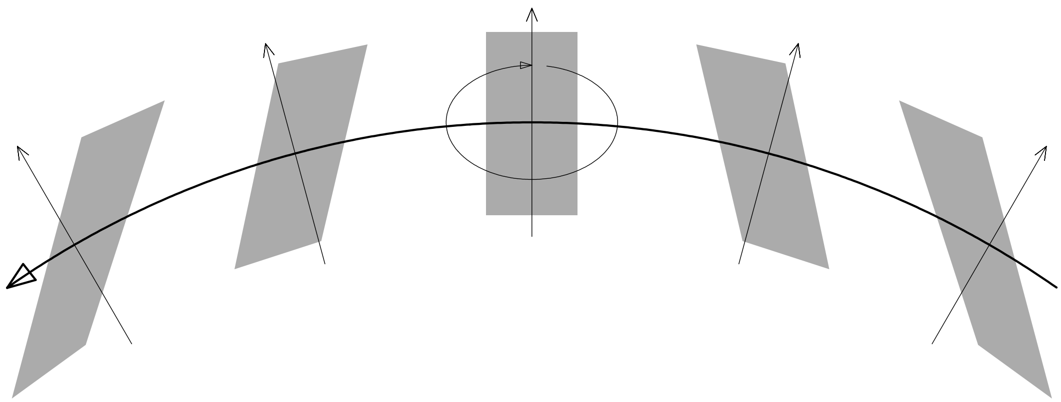

Figure 15.1: Evolution of an overdense perturbation in a shearing disk. The disk rotates counter-clockwise, as indicated by the heavy arc; a typical star moves around an elliptical epicycle in a clockwise direction. The perturbation (grey patch) initially has the form of a *leading* spiral (right), but is sheared into a *trailing* spiral (left) by the differential rotation of the disk. The epicycle and the perturbation rotate in the same direction, so stars stay in the perturbation longer than they would under other conditions.

15.4 Swing Amplification

As we saw in Chapter 14, disks are stablized on small scales by random motion, and on large scales by rotation. Under some circumstances, the stabilizing effects of random motion can be temporarily suppressed. If this happens, small perturbations the the disk can be *swing amplified* (eg., Toomre 1981).

Fig. 15.1 illustrates the mechanism of swing amplification. In this diagram, the grey region represents a density wave created by the coherent excitation of epicyclic motions in a uniform disk. The wave initially has the form of a leading spiral, but it's sheared into a trailing spiral by the disk's differential rotation. In a stable disk, random motions normally disrupt an overdense region before it has time to collapse. Here, however, the specific form of the density wave insures that individual stars remain within the overdense region for a substantial fraction of the epicyclic period κ; as a consequence, the wave is amplified by a modified form of the Jeans instability.

The gain of the swing amplifier depends on the value of Q and on the vratio $X \equiv \lambda/\lambda_{\rm crit}$, where $\lambda_{\rm crit} = 4\pi G\Sigma\kappa^{-2}$ is the shortest wavelength nominally stabilized by rotation. Toomre (1981) compares amplification factors estimated using different techniques, and finds fairly good agreement between a local WKB analysis and a global numerical calculation. The amplification factor peaks at $X \sim 1.5$, reaching values in excess of 10^2 for relatively cold disks. Swing amplification is much less effective for $X > 2$; perturbations with such long wavelengths are stabilized by rotation, which remains effective even when random motions fail to do their part.

In N-body experiments, particle noise creates a spectrum of perturbations of all sizes and shapes, including both leading and trailing spirals. As the differential rotation of the disk shears leading spirals into trailing configurations, swing amplification boosts the amplitudes of those which have wavelengths of order $\sim 1.5\lambda_{\rm crit}$, thereby creating a multi-armed pattern of trailing spirals with a characteristic spacing. Shown in Fig. 15.2 is an N-body simulation of a disk galaxy. Apart from

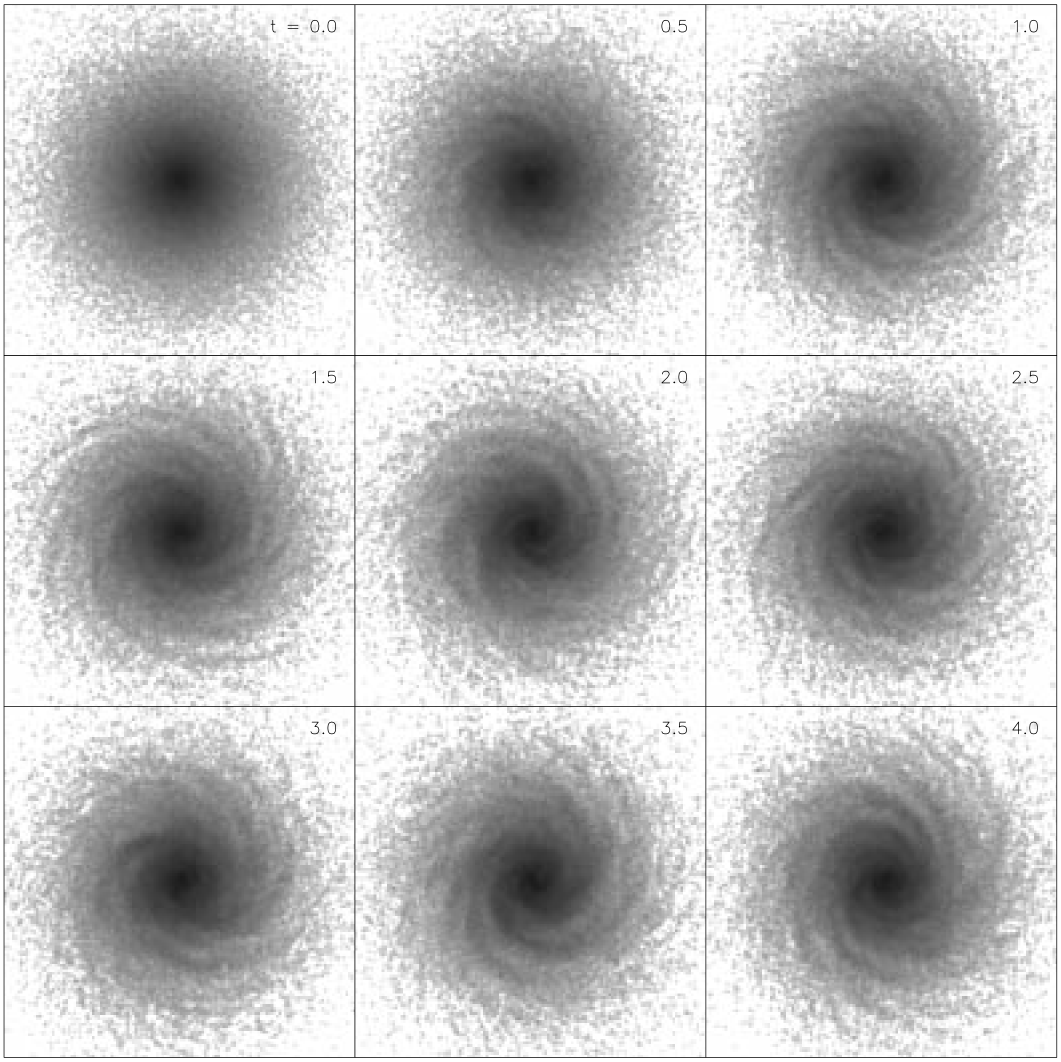

Figure 15.2: Stable disk.simulation. Initially, this disk has $Q \simeq 1.26$, suffcient to curb local instabilities. The galaxy model includes a bulge and a halo (not shown); the disk is 15% of the total mass. Each frame is 15 disk scale lengths on a side; times are given in units of the rotation period at ~ 3 disk scale lengths.

Poissonian fluctuations, this disk was initially featureless; the spiral pattern which develops is due to swing-amplified particle noise.

With time, however, the spiral patterns in numerical simulations tend to fade away as perturbations due to spiral features boost the random velocities of disk stars (e.g. Sellwood & Carlberg 1984). Once the disks become too 'hot', random stellar velocities reduce the gain of the swing-amplifier and prevent the amplification of small fluctuations. In this respect, N-body experiments fall short of explaining the spiral patterns of real galaxies, which persist for many tens of rotations.

15.5 Alternatives

15.5.1 Quasi-Stationary Spiral Structure

The key assumption of the QSSS hypothesis is that spiral structures simply rotate at constant pattern speed Ω_p without significant evolution (Lin & Shu 1964, 1966). To arrange such a spiral, we require the effective precession speed

$$\Omega_\mathrm{eff} = \Omega - |\nu|\frac{\kappa}{2}, \tag{15.8}$$

to be independent of R, where $|\nu| = \omega/\kappa$ is the dimensionless frequency given by the WKB dispersion relation for nearly-axisymmetric density waves (T77, Fig. 4). This is possible, in principle, because $|\nu|$ depends on the local radial wavelength λ.

The mathematical details are pretty tricky; suffice it to say that this is a self-consistent problem, and that where a solution can be found it is unique. Thus the real advantage of the QSSS is that it provides a definite set of predictions for a given spiral system.

The WKB analysis of QSSS gets into trouble at *resonances* where responses become very large and linear theory breaks down. The three most important resonances are the *Outer Lindblad Resonance* (OLR), where $\Omega_\mathrm{p} = \Omega + \kappa/2$, the *Corotation Resonance* (CR), where $\Omega_\mathrm{p} = \Omega$, and the *Inner Lindblad Resonance(s)* (ILR), where $\Omega_\mathrm{p} = \Omega - \kappa/2$. In particular, the ILR can *absorb* the inward-propagating density waves, much like ocean waves break and dissipate energy when they reach a beach (T77).

15.5.2 Chaotic Spirals

To maintain spiral structure in N-body experiments it's necessary to counteract the increasing random motions of disk stars. One way to do this is to mimic the effects of star formation by constantly adding new stars on circular orbits. Sellwood & Carlberg (1984) present simulations in which the disk is assumed to grow by ongoing gas accretion; the accreted mass is added to the model in the form of particles on initially circular orbits. If the mass accreted per rotation is about 1.5% of the disk's initial mass, the disk can maintain an open spiral pattern similar to the spiral patterns of typical Sc galaxies. Further implications of this accretion hypothesis are reviewed by Toomre (1990).

15.5.3 Tidal Spirals

Tides between galaxies provoke a two-sided response, much like the ocean's response to the tidal pull of the Moon. Since the classic two-armed 'grand-design' spiral galaxies M51 and M81 are clearly interacting with close companions, it's very likely that these galaxies owe their symmetric spirals to tidal interactions (Toomre & Toomre 1972). Tidal perturbations, swing-amplified in differentially-rotating disks, can indeed produce striking 'grand-design' spiral patterns. In the experiment shown in Fig. 15.3, a disk galaxy is perturbed by the parabolic passage of a smaller companion. In the aftermath of this passage, the disk develops a two-armed spiral pattern which persists for several rotation periods.

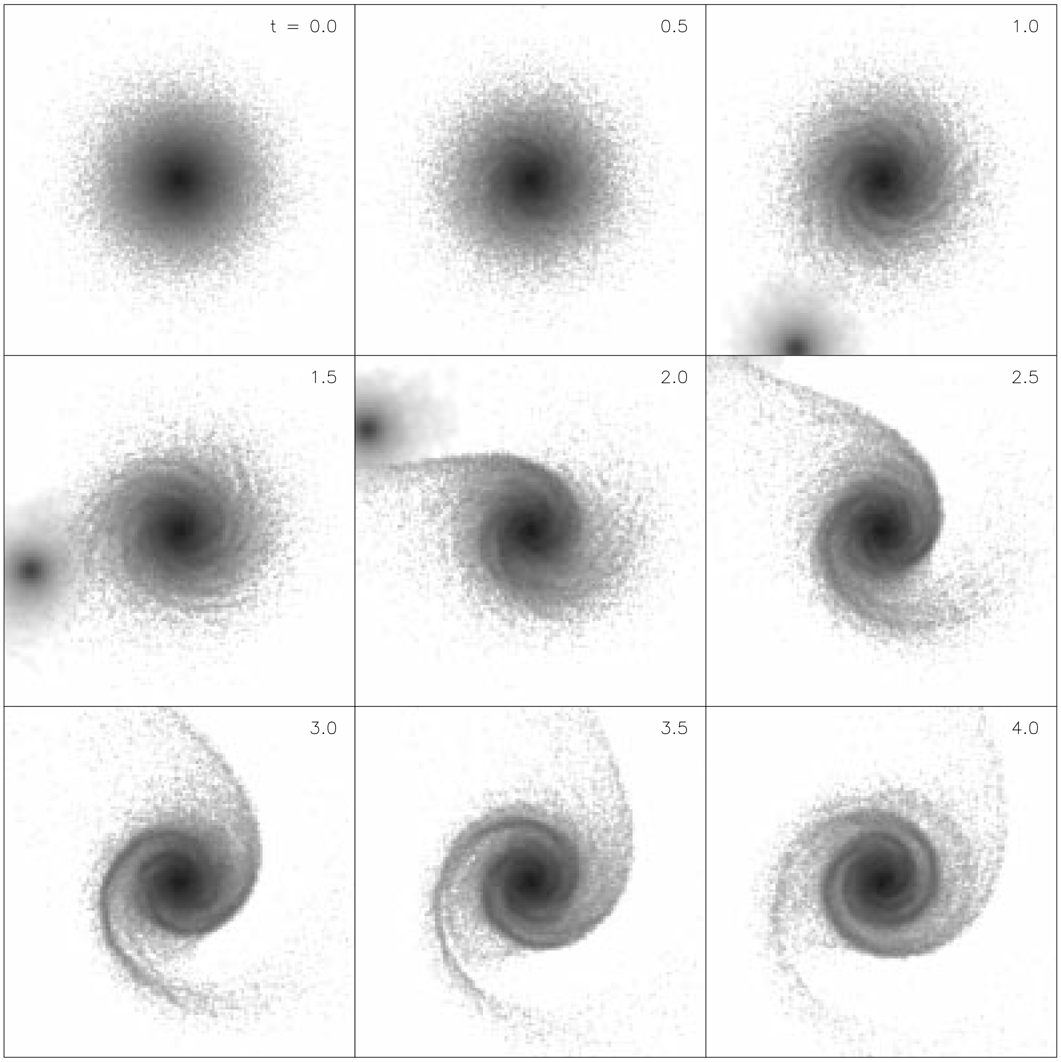

Figure 15.3: Tidal encounter between the disk in Fig. 15.2 and a companion of 10 the total mass. This compaion approached on a parabolic initial orbit and reached an apocenter of ~ 9 disk scale lengths at $t = 1.5$. Each frame is 24 scale lengths on a side; times are given in units of the rotation period at ~ 3 scale lengths.