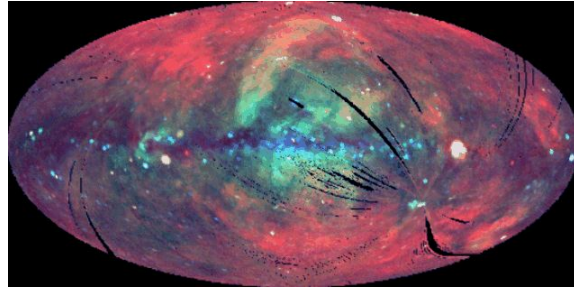
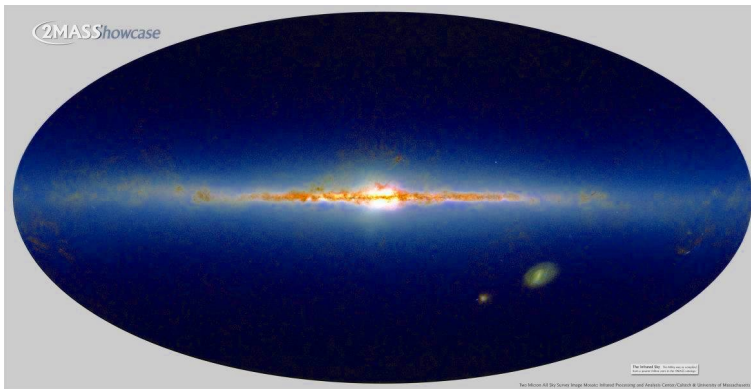


- Why study the MW?
 - its big, bright, close
 - Allows detailed studies of stellar kinematics, stellar evolution, star formation, direct detection of dark matter??
- Problems
 - We are in it
 - Distances are hard to determine
 - Dust is a serious issue

Milkyway



Milky Way in X-rays- Image of the Hot ISM



Milky Way in near IR
www.milkywayproject.org

Recent reviews

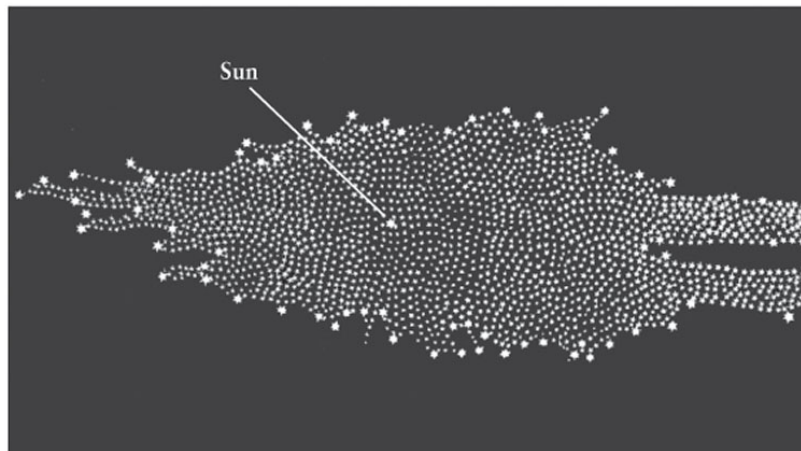
The Milky Way's stellar disk-Mapping and modeling the Galactic disk Hans-Walter Rix · Jo Bovy *Astron Astrophys Rev* (2013) 21:61

Detailed study of the MW is a very active field

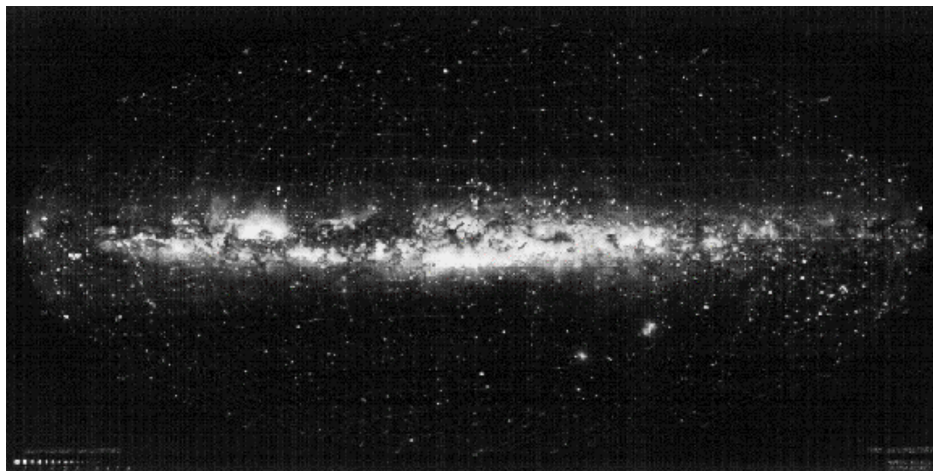
- The stellar halo of the Galaxy Amina Helmi *The Astronomy and Astrophysics Review* 2008, Volume 15, Issue 3, pp 145-188,
- Freeman KC, Bland-Hawthorn J (2002) *ARA&A* 40:487 The New Galaxy: Signatures of Its Formation

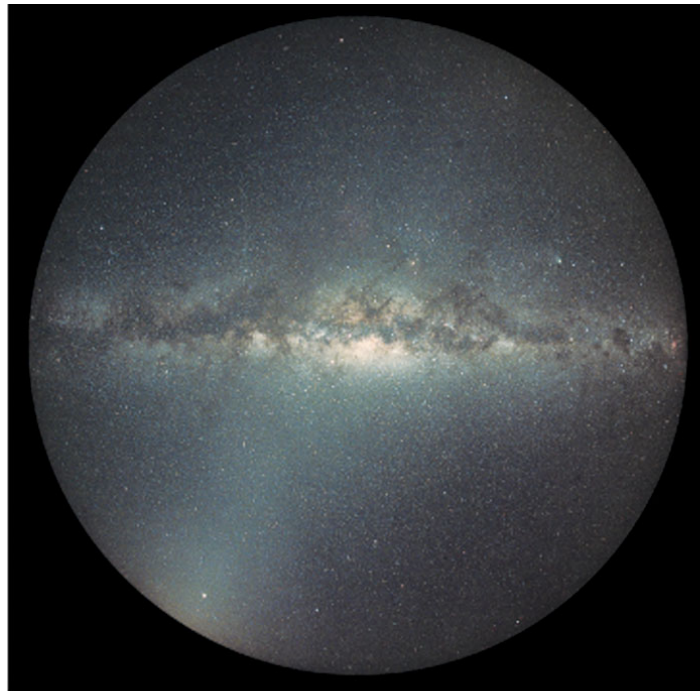
Our place in the Galaxy

- We live in a large disk galaxy of average mass
 - The sun is in the disk, towards the edge (8kp from center)
 - Projected onto the sky, this disk of stars looks like a band of light that rings the sky... the Milky Way
- This realization came somewhat slowly...
 - Disk-like nature of galaxy realized by Thomas Wright (1780); refined by Kant
 - First attempt to map out galaxy made by William Herschel (1785); refined by Kapteyn in 1920
 - Herschel came to the conclusion that we sit at the center of the Galactic disk. In fact, **he was wrong**... had not accounted absorption by dust! (*something that he did not know about*)



Herschel's map of the Galaxy



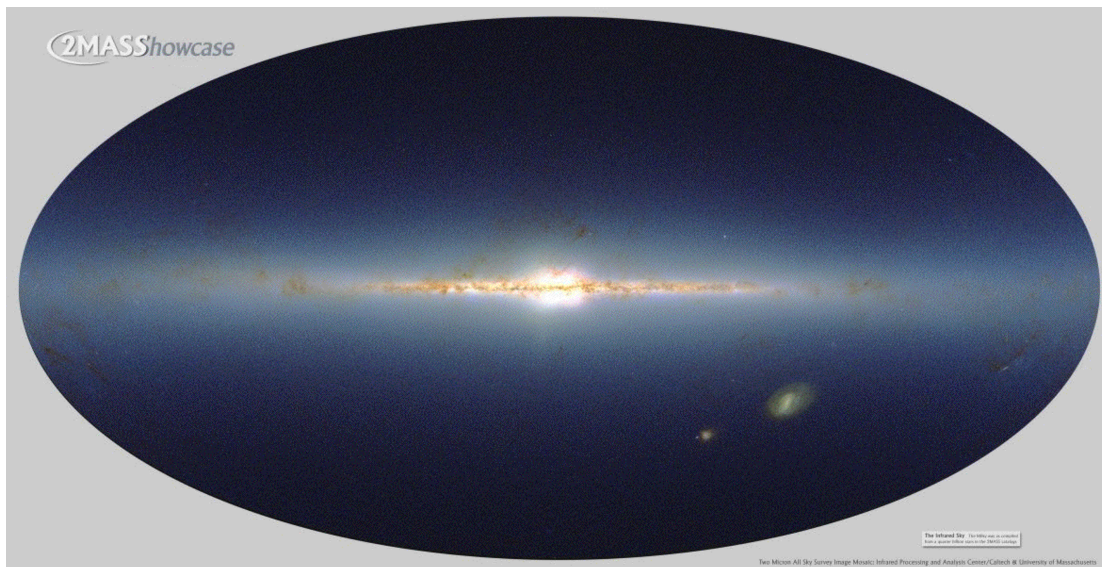


← View out of the plane of our Galaxy

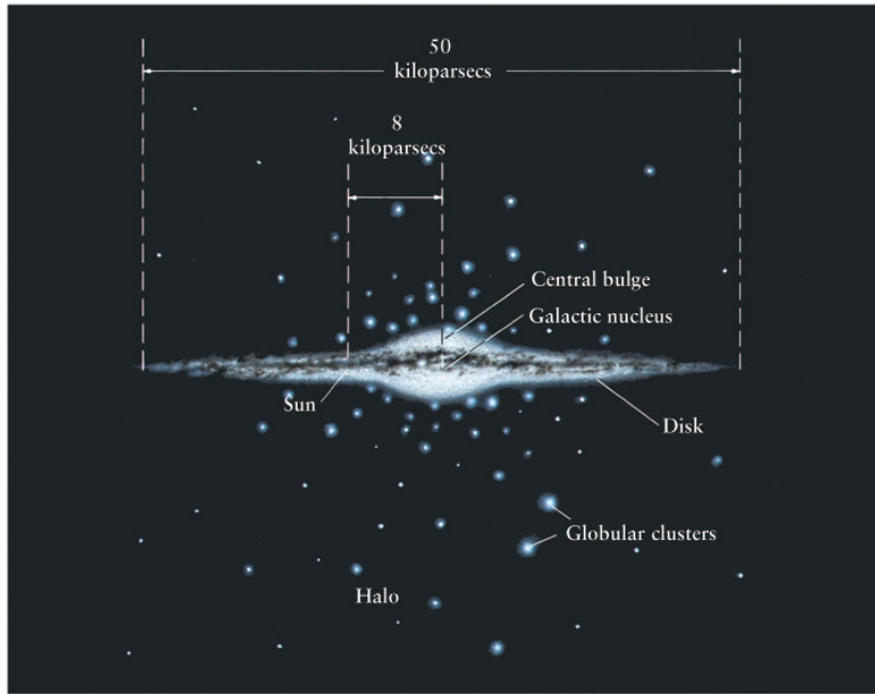
← View within the plane of our Galaxy

← View out of the plane of our Galaxy

(b)

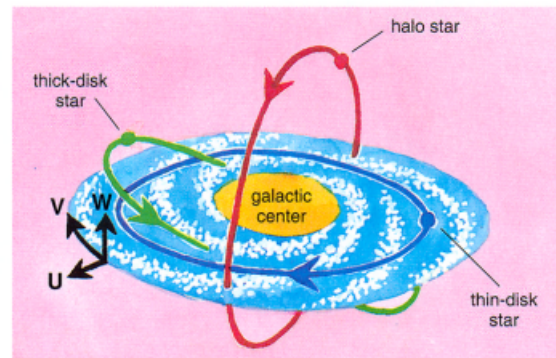
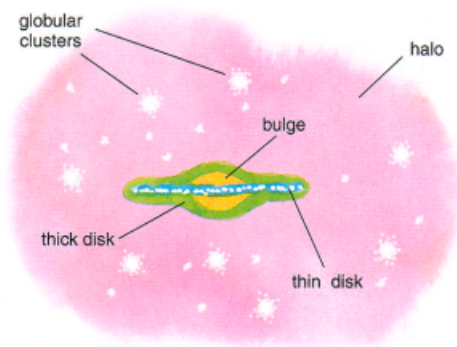


The MW galaxy as seen by an infrared telescope- IR light is much less sensitive to 'extinction' by dust than optical light



1 kiloparsec = 3.26×10^3 lightyears = 3.08×10^{19} m

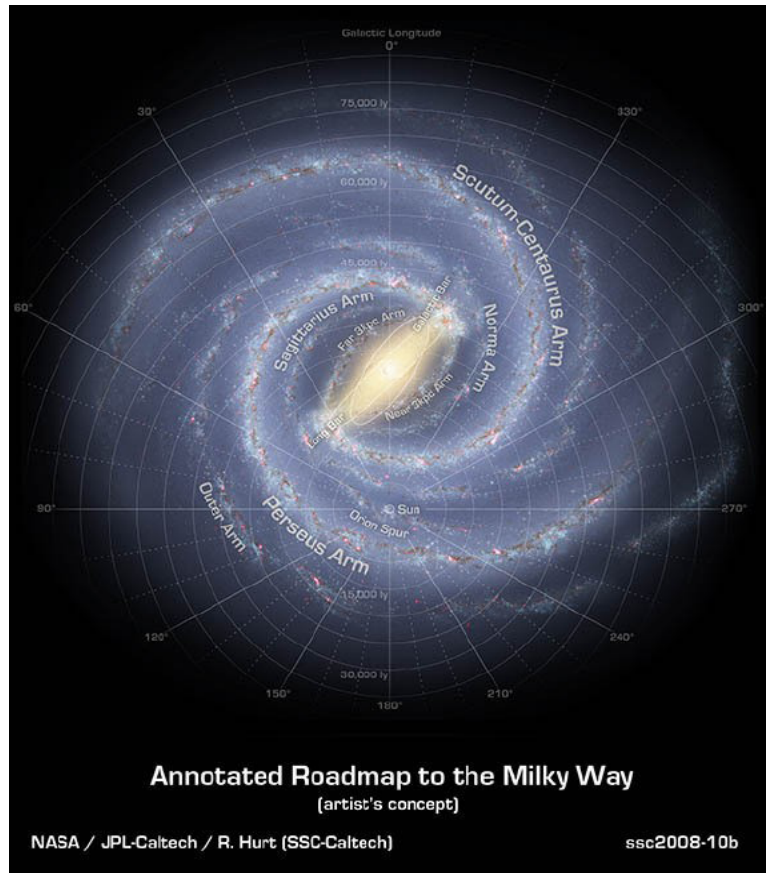
Schematic Image and Dynamics of MW



Cristina Chiappini

- Its only in the MW and a few other nearby galaxies that fossil signatures of galaxy formation + evolution (ages dynamics and abundances for individual stars) is possible.

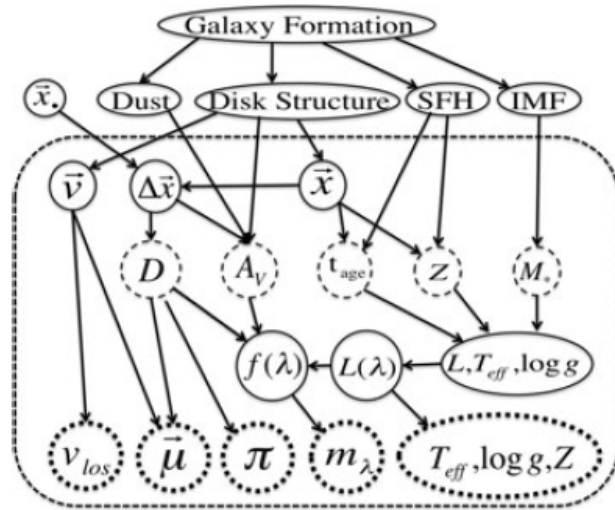
These signatures allow a probe back to early epochs and constraints on theories of galaxy formation



- The positions, velocities, chemical abundances, and ages of MW stars are very strongly and systematically correlated
- In the disk:
 - younger and/or more metal-rich stars tend to be on more nearly circular orbits with lower velocity dispersions.
- Subcomponents of the Disk can be defined on the basis of the spatial distribution, kinematics, or chemical abundances.
- Most common has been to describe the Disk in terms of a dominant thin disk and a thick disk, with thin-thick disk samples of stars defined spatially, kinematically, or chemically

Observables and What we Want to Learn

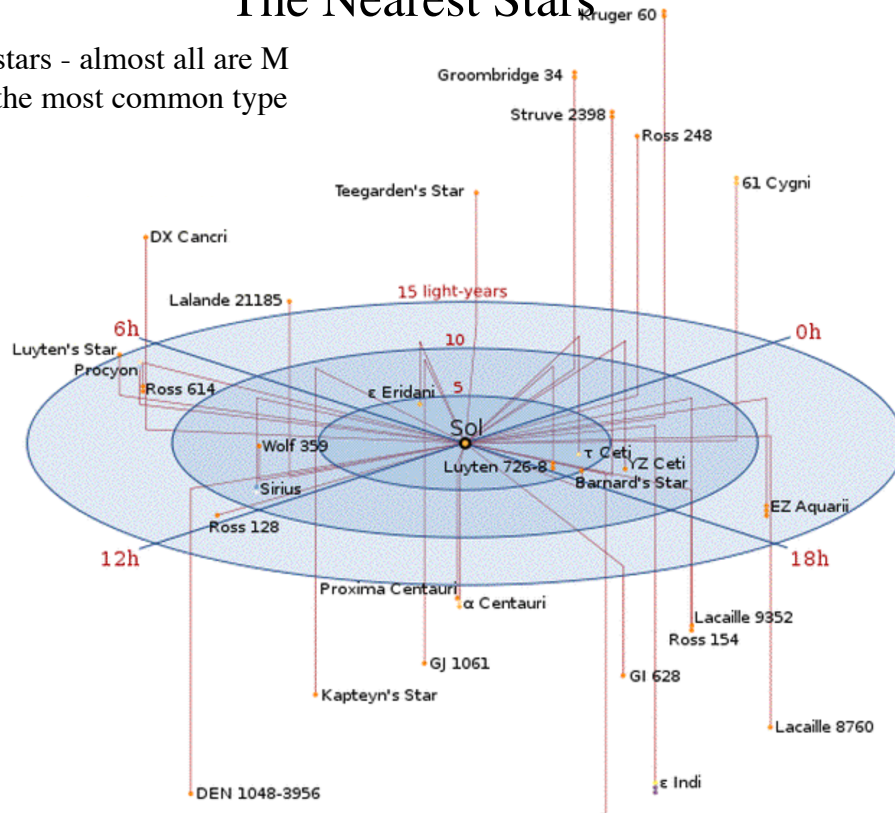
- Observables are in dotted and desired information in solid ellipses
- Observables
line-of-sight-velocity, v_{los} , proper motions, μ , parallax π , multi-band photometry m_λ , and stellar parameters derived from spectra (T_{eff} , $\log g$, abundances, Z); most of them depend on the Sun's position x , Δx .
- Desired information is stellar masses M , age t_{age} and abundances Z , distance D from the Sun and the (dust) extinction along the line of sight, A_V .



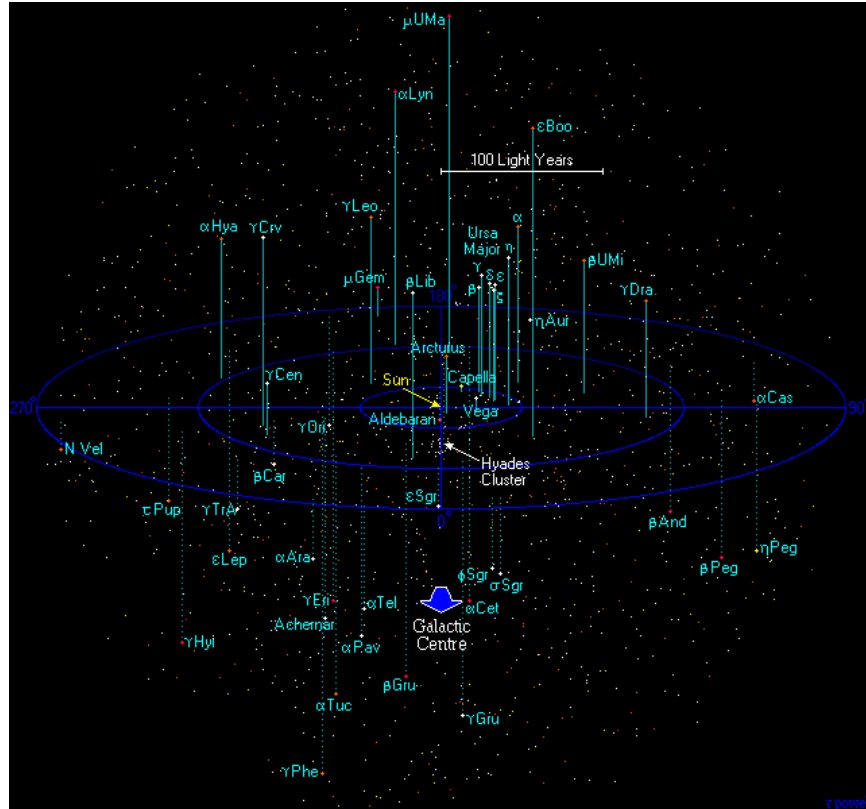
Rix and Bovy 2013

The Nearest Stars

- Nearest stars - almost all are M dwarfs- the most common type of star



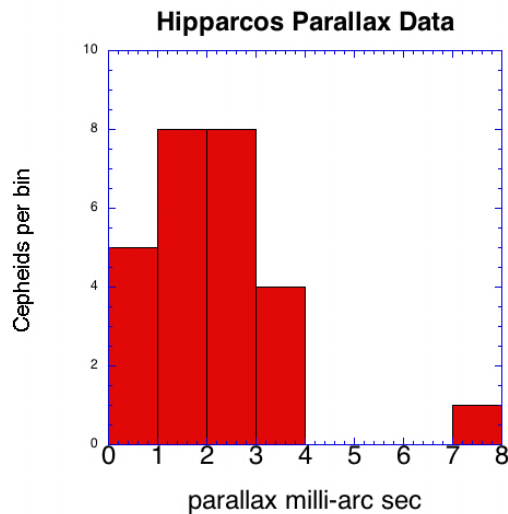
Stars Within 250pc



Relative vs Absolute Distances

- H. Levitt determined that Cepheids were very good relative distance determiners- how to connect with stellar parallax absolute distance measurements??
- This was done in 1913 by Hertzsprung using statistical parallax-Traditional annual parallax techniques are not capable of determining distances to even the closest Cepheids because the 2AU base line is not long enough.

It wasn't until 1997 that the parallax was directly to the nearest Cepheids using the Hipparcos satellite- the reason is that the closest Cepheids have parallaxes of **milliarc** secs- now HST has 13 - see A 3% Solution: Determination of the Hubble Constant with the Hubble Space Telescope and Wide Field Camera 3 Reiss et al 2012



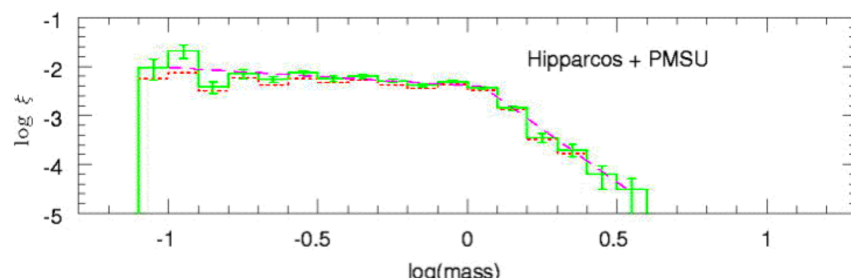
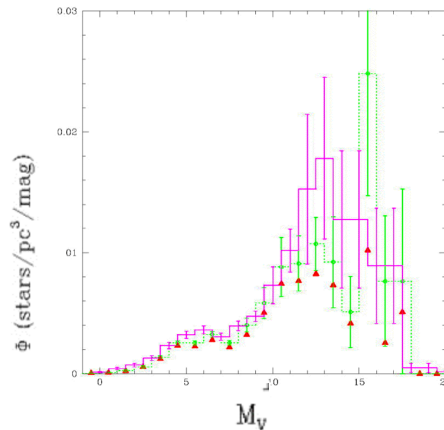
- Can measure 3D structure from star counts
- Can measure dynamics from individual stars
- Can only do in the MW: (a lot more later)
- Need to find a good tracer of potential

that probes the disk ($z \sim 1 \text{ kpc}$)

- numerous sufficiently old, well-mixed well-calibrated distances good radial velocity measurements
- Lower main sequence stars (G - K dwarfs)
- Parameterize possible potentials
- Known star populations + gas + dark disk + halo
- Bottom line No convincing evidence for ‘cold’ DM component in the disk .

Luminosity and Mass Function

- A fundamental property of stars is how they are distributed in mass and luminosity- the mass and luminosity functions
- One has to transform the observables (flux, color etc) into physical units (luminosity in some band, temperature) using theoretical stellar models and distances determined via a variety of means
- The best set of distances are from parallax and the largest data set is for the solar neighborhood ($R \sim 25 \text{ pc}$) from the Hipparchos satellite set by its ability to measure small parallaxes



Star Counts

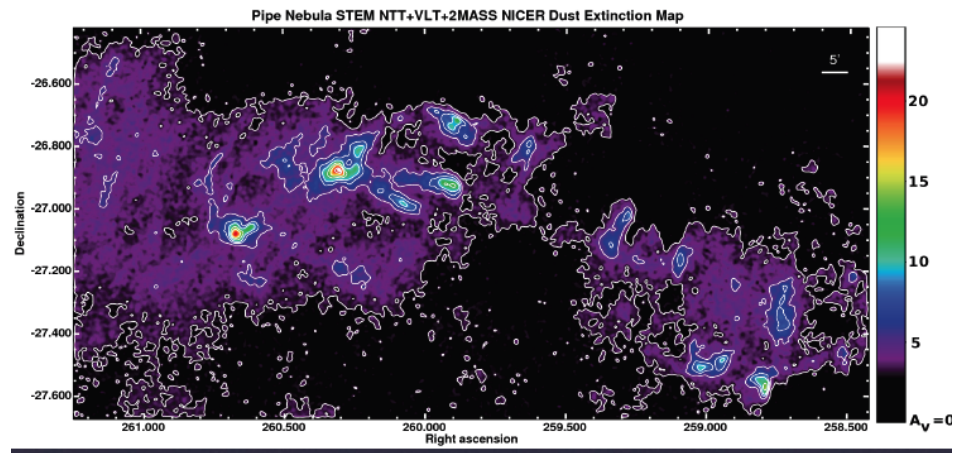
- We wish to determine the structure of the MW
 - Define 2 functions
 - $A(m,l,b)$: # of stars at an **apparent mag m** , at galactic coordinates l,b per sq degree per unit mag.
 - $N(m,l,b)$: cumulative # of stars with mag $< m$, at galactic coordinates l,b per sq degree per unit mag.
 - Then clearly $dN(m,l,b)/dm=A(m,l,b)$
 - or $N(m,l,b)=\int A(m',l,b) dm$
 - Simplest galaxy model : uniform and infinite
- if ρ_* = density of stars and Ω = solid angle of the field, the volume of a shell at distance r is $\Omega r^2 dr$ and the number of stars is $N(r)=\int_0^R \Omega r^2 \rho_* dr= 1/3 \Omega R^3 \rho_*$
- Now if all the stars have the same luminosity (e.g. absolute magnitude) M and utilize (from the definition of absolute mag $m-M=5 \log r - 5$)
 - (e.g. $r=10^{(0.2(m-M)+1)}$ pc) then $dr=(0.2)(\ln 10)10^{(0.2(m-M)+1)} dm$
 - and thus $N(m)=\int_{-\infty}^m \Omega \rho_*=(0.2)(\ln 10)(10^{(0.2(m-M)+1)})^3 dm$; oh the pain of magnitudes

Star Counts

- $N(m)=\int_{-\infty}^m \Omega \rho_*=(0.2 \times 10^3)(\ln 10) \int_{-\infty}^m (10^{(0.6(m-M))}) dm'$
- or (finally)
- $N(m)= 333 \Omega \rho_* 10^{(0.6(m-M))}$
- This is not what is observed
 - finite size of **disk** (not sphere)
 - effects of dust
- Olbers paradox: if galaxy (universe) was infinite the total light would diverge
- Goal is to find the true space density of stars as a function of distance, galactic coordinates, luminosity, spectral type, age, metallicity etc
- Luminosity function of stars $f(m,etc)$

Need to Measure Extinction Accurately

Galactic extinction maps



Luminosity Function

- Simplest form $f(m)=\#$ of stars per unit volume with luminosity (absolute mag) between M and $M+dm$
- Observationally it is a time dependent quantity (since stars evolve and are born and die and since stellar ages are function of mass)
 - thus the luminosity function, while an observable, has to be carefully defined.
- Observational issues
 - incompleteness due to flux limited samples in a given bandpass,
 - uncertainty in distances (need to transform from observed flux to true luminosity)
 - effects of dust
 - need a large volume (stars are very rare at high luminosities)
- Many of these problems were overcome by Hipparchos (large number of parallax distances) and near IR surveys (relatively free from effects of dust);
 - major advance expected with launch of GAIA in Oct 2013

IMF of MW Stars

- Observing the IMF is tricky, 3 approaches
- Observe a young cluster and count the stars in it as a function of mass. (e.g. the Pleiades) straightforward, but limited by the number of young clusters where we can directly measure individual stars down to low masses. get a clean measurement, but the statistics are poor.
- field stars in the solar neighborhood whose distances are known. statistics are much better, but only use this technique for low mass stars, few massive stars in local volume and numbers controlled by star formation history
- get limits on the IMF from the integrated light and colors of stellar populations
- despite these problems most results show that the IMF is very similar from place to place

Luminosity and Mass Function

- There are several 'nasty' problems
 - since stars evolve the 'initial' mass function can only be observed in very young systems
 - none of these are close enough for parallax measurements

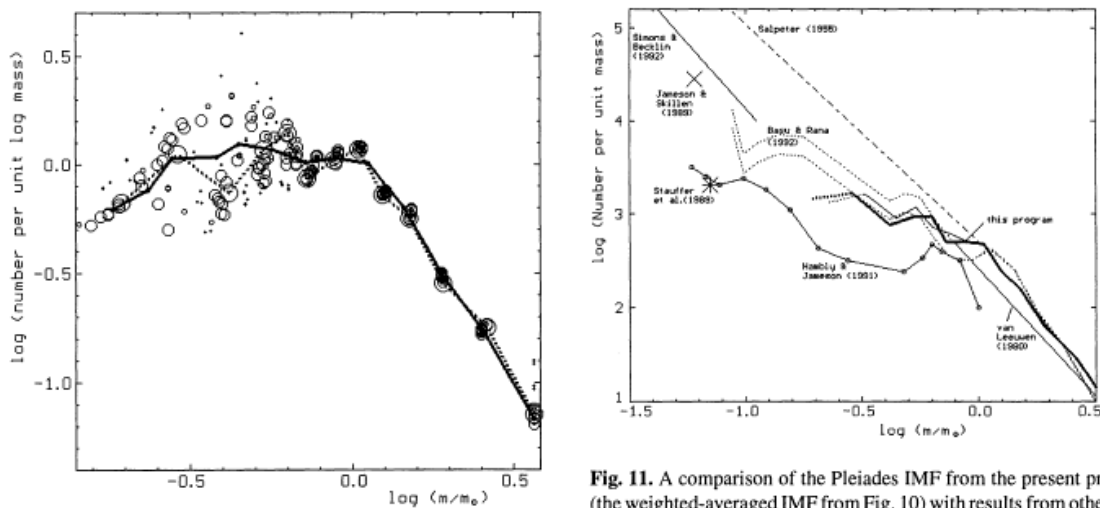
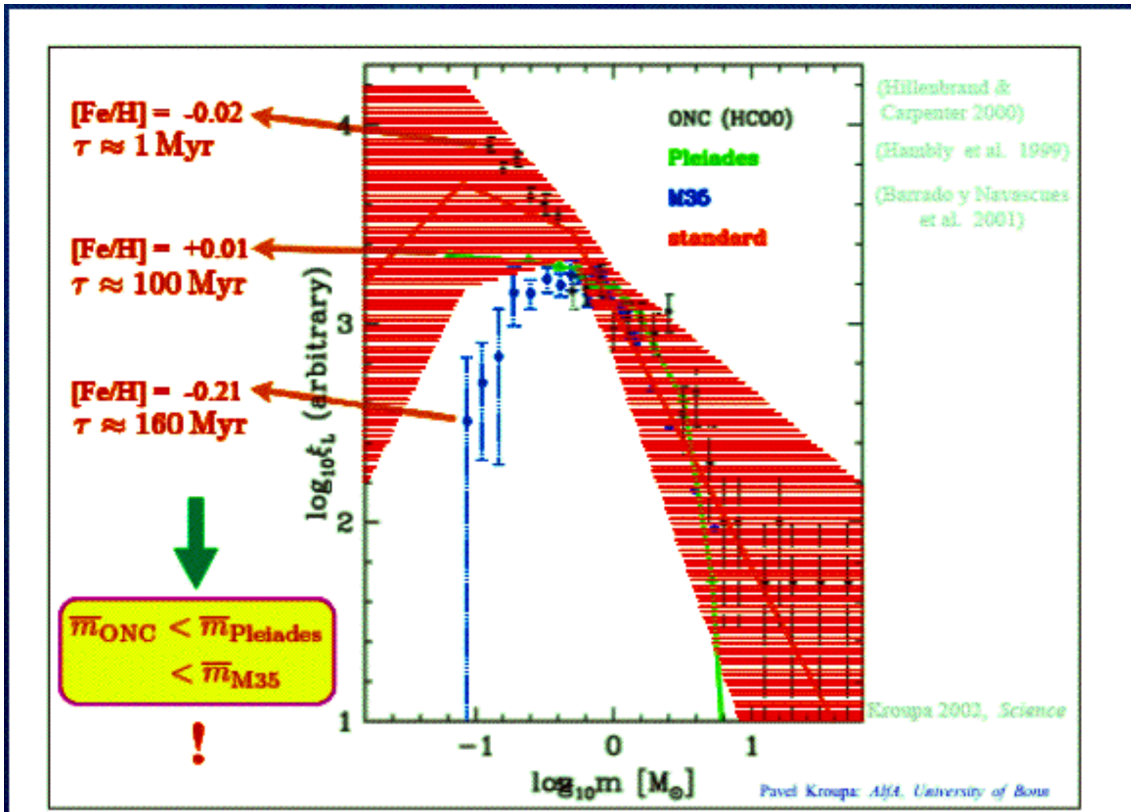


Fig. 11. A comparison of the Pleiades IMF from the present program (the weighted-averaged IMF from Fig. 10) with results from other stud-

IMF



Open Star Clusters- A SSP

- the individual stars of the Galactic plane differ not only in the masses and angular momenta, but also in their ages and in their chemical compositions at birth.
- This multiplicity of free-parameters complicates the study of stars. For instance, the initial mass, the initial chemical composition, and the age of a star determining the star's color and luminosity.
- Open star clusters are sets of stars that differ only in their masses at birth and in their angular momenta. They formed at the same time from the same molecular cloud with \sim the same chemical composition at birth and the same age.
- The stars of a single open cluster show how initial mass alone affects color and luminosity, and the comparison of stars from two different clusters shows how initial chemical composition affects color and luminosity and how stars evolve over time.
- The extent to which the massive stars deviate from the main sequence defines an age for the cluster. The Hyades cluster is estimated to be 625 ± 50 million years old
- Over 1 billion years, encounters with molecular clouds cause an open cluster to totally dissipate.

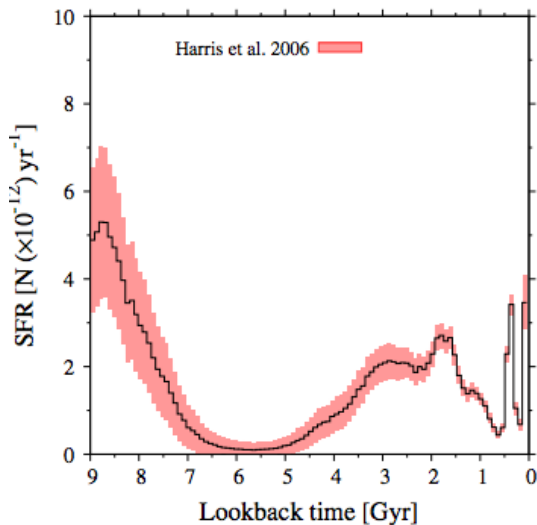
Nearby Stars

- Historically one dealt with flux (magnitude) limited samples of stars
- the Hipparchos satellite measured the absolute distances to many stars via parallax - now have a proper census of the stars at <100pc (at this close distance effects of dust are small)- Major change coming up SOON with the launch of GAIA- in the mean time
 - Local Group and Star Cluster Dynamics from HSTPROMO (The Hubble Space Telescope Proper Motion Collaboration) R. P. van der Marel arxiv 1309.2014
 - Goal to determine fully three-dimensional velocities, need to determine Proper Motions. If get to $DPM \approx 50 \text{ mas/yr}$, corresponds to a velocity accuracy $dv \approx (D/4) \text{ km/s}$ at distance D kpc.
 - RAVE and SEGUE velocity surveys: SEGUE will observe $\sim 240,000$ stars in the range $15 < V < 21$, while RAVE aims at 10^6 stars with $9 < I < 12$. The average velocity errors that these surveys can achieve are of the order of 10 and 1 km/s, respectively.

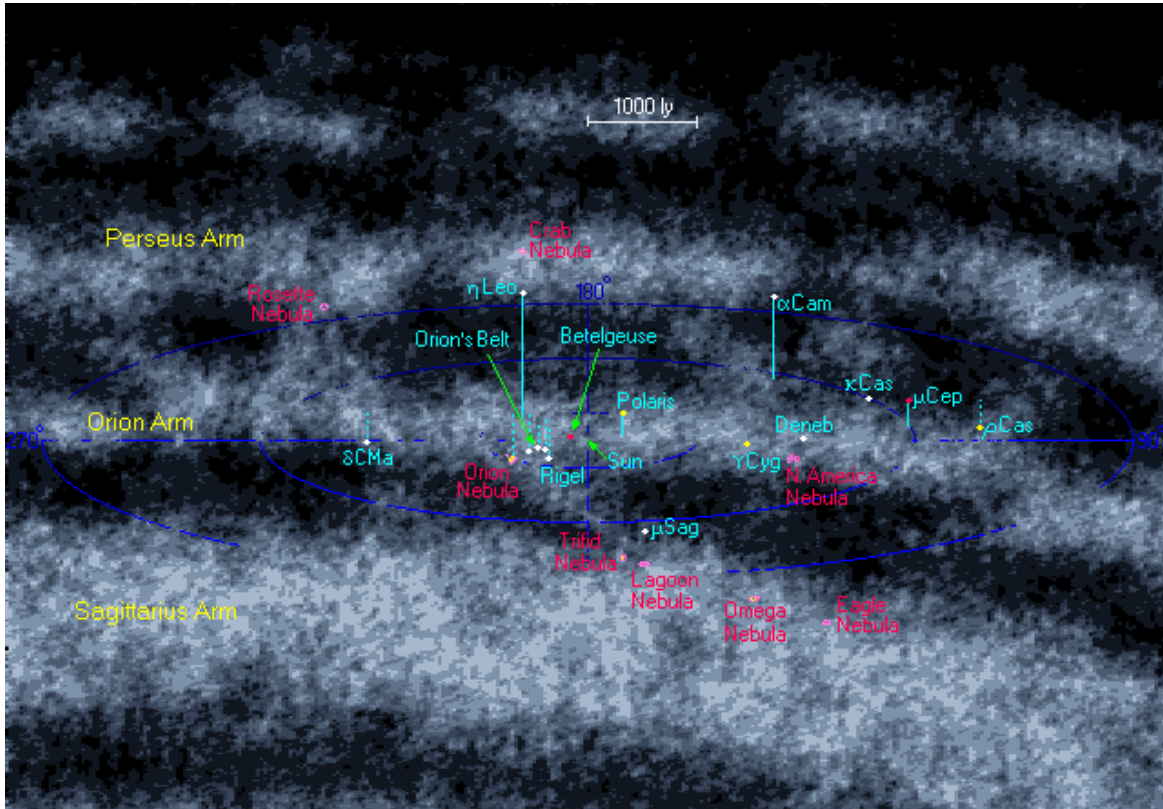


SFR In Solar Neighborhood

- By modeling the white dwarf age/density distribution one can estimate the SFR rate 'nearby' (Rowell 2012)
- We will later compare this to the overall rate of SF of the universe and find significant differences
 - is it because the local neighborhood is not representative of the whole MW?
 - or because the MW is not representative of the average of the universe??

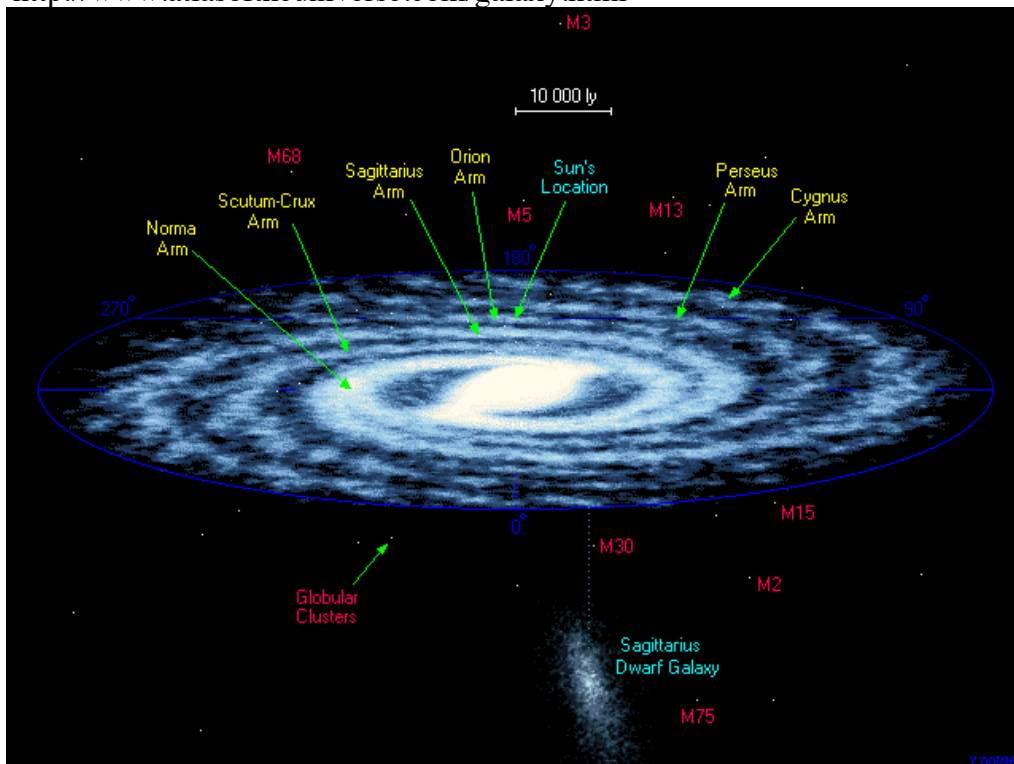


5kpc- Orion Arm



The MW

- <http://www.atlasoftheuniverse.com/galaxy.html>

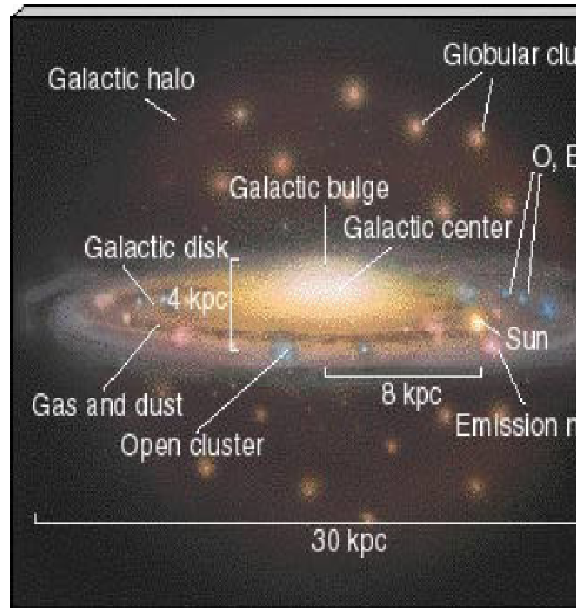


Basic Structure of Milky Way

Bulge is quite spherical and is dominated by old stars

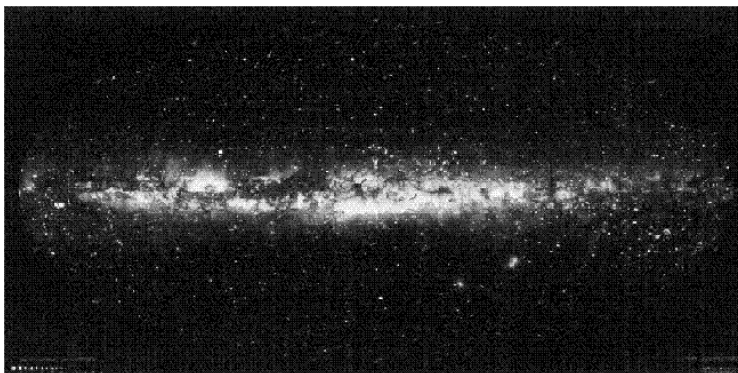
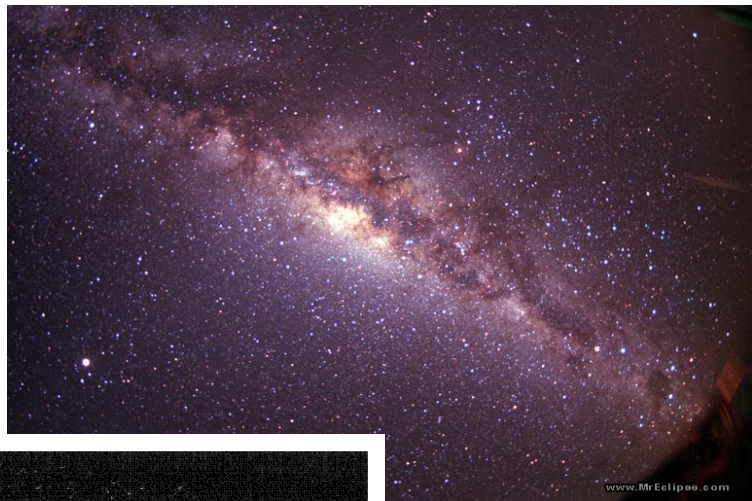
Disk- location of almost all the cold gas and most of the HI- site of star formation and thus young stars- wide range in metallicity

Halo- globular clusters, most of MW dark matter, only 1% of stars



MW in Optical

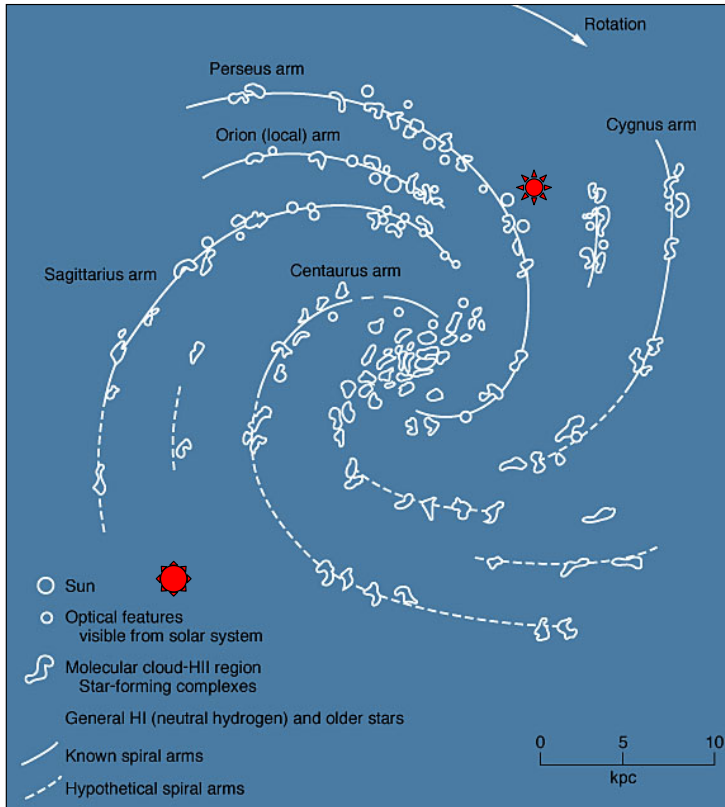
- Notice the strong effect of dust.



this is a drawing of the MW all sky- state of the art 1950's

Milky Way, Sbc-galaxy (all-sky projection in optical)

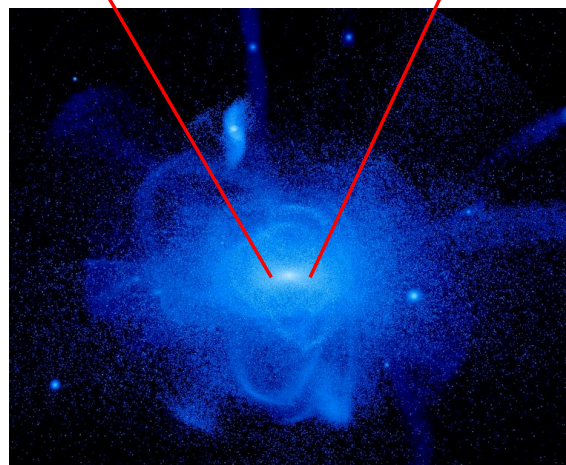
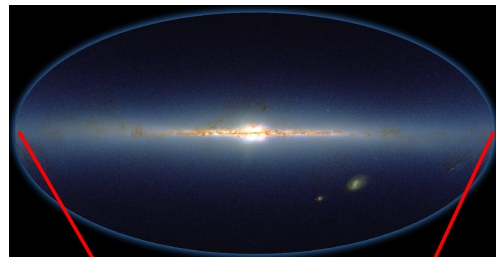
Map of the Milky Way Galaxy



The map has been using HI velocity data sec 2.3.1 in S+G

Theorists View of Dynamics of Stars in MW

- In cold dark matter theories of structure formation many mergers have occurred - it takes a VERY long time for the orbits to 'relax' and thus there should be dynamical signatures of the mergers
- Only in MW and LMC/SMC is there any chance to determine the 3-D distribution of velocities and positions to constrain such models in DETAIL.
- Look for signs of assembly of MW galaxy in our stellar halo (and thin/thick disk)
 - Stellar halo is conceivably all accreted material
 - Stellar streams in the solar neighborhood



H. Rix

Simplified View of Streams

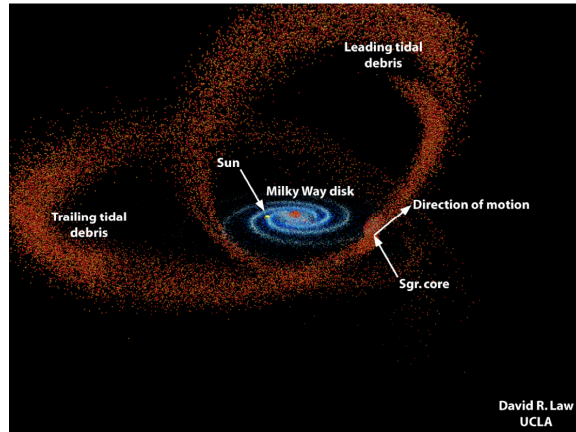
- galactic haloes are threaded with the phase-mixed remains of dwarf satellites and globular clusters that have been destroyed by the tides of their host's gravitational potential (Law and Majewski 2009)

These tidally disrupted stars may make a significant fraction of the halo

- these dynamical tracers can provide constraints on the mass distribution of the baryonic and dark matter components of the Milky Way

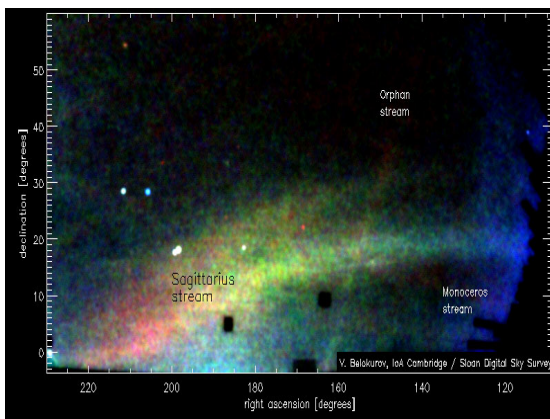
- Tidal disruption radius

dwarf has mass m and radius r , MW mass M and separation between the 2 is R - consider the dwarfs gravitational binding force Gm^2/r^2



Disrupting force due to MW is $(GMm/2)[(1/(R-r)^2 - 1/(R+r)^2)] \sim$ when $r \ll R$ this is $\sim GMmR/r^3$

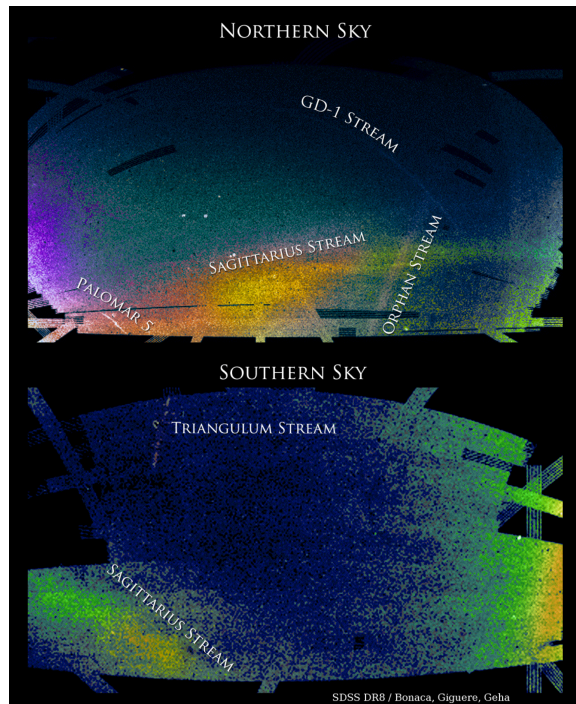
2 are equal when $r \sim R(m/kM)^{1/3}$
k depends on structure of object
 See B&T sec 8.3 or Roche limit



- map of stars in the outer regions of the Milky Way (1/4 of sky). The trails and streams that cross the image are stars torn from disrupted Milky Way satellites. The color corresponds to distance, with **red being the most distant** and **blue being the closest**. The large, forked feature is the Sagittarius stream, further away from us (lower left) and closer to us (middle right). Other features marked are the Monoceros ring

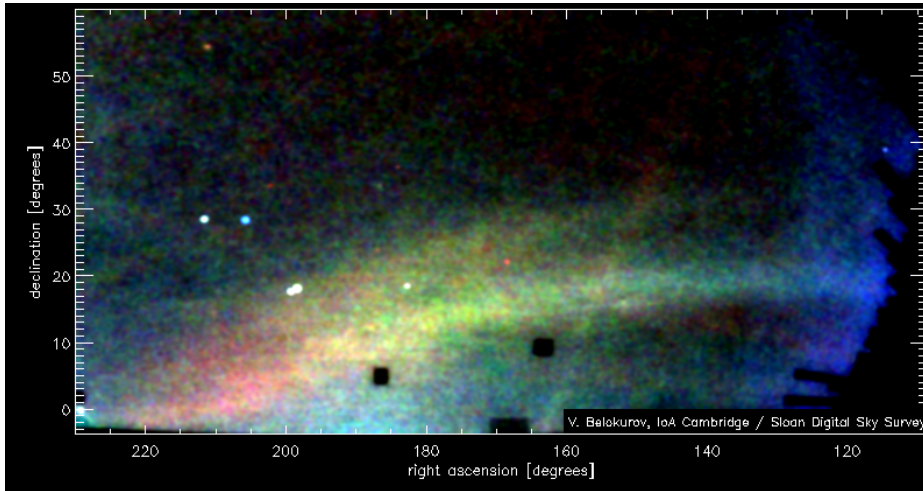
V Belokurov, SDSS-II Collaboration)

Streams in the MW

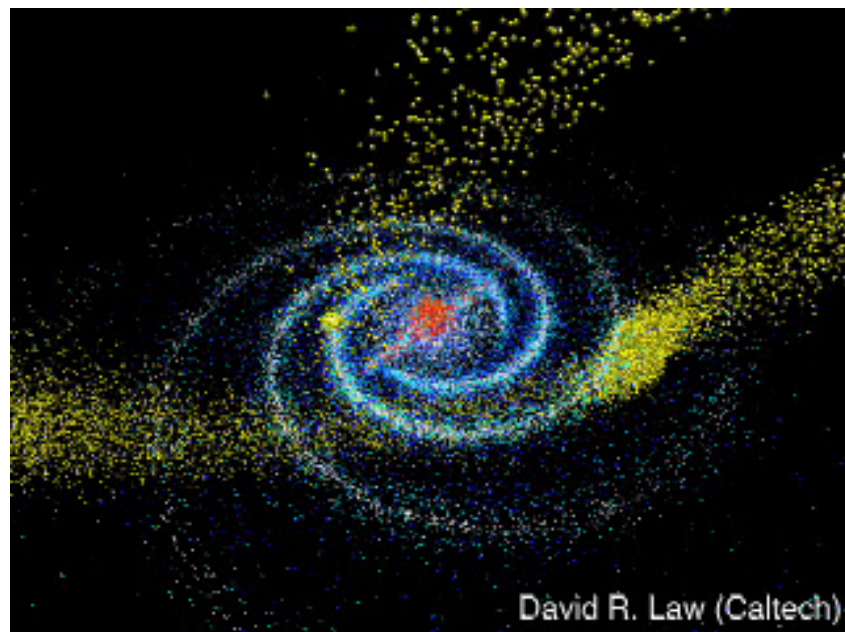


Stellar halo : fossil record of assembly?

- Dwarf galaxies are disrupting and contributing to the stellar halo
 - 1% of stellar mass of our galaxy
 - takes ~5Gyr for MW to 'digest' a merging dwarf
 - See such effects in nearby galaxies (see later lecture on mergers)



- Milky Way Galaxy (blue/white points and orange bulge) with the Sun (yellow sphere), inner and outer Sgr stream models (yellow/red points respectively), Monoceros tidal stream model (violet points), and observed Triangulum-Andromeda structure (green points).



<http://www.astro.caltech.edu/~drlaw/MWstreams.html>

Streams Originating from Globular Clusters

- Our colloquium this week by N. Kallivayalil is on 'streams'
- Useful in reconstructing the accretion history of the Galaxy,
- and as sensitive probes of the Galactic potential
- The streams tend to be 'cold' since they originate from infalling structures that are much less massive than the MW
- A large sample of such streams will ultimately allow a map of the distribution of Galactic dark matter with higher spatial resolution than presently possible and allow a search for dark matter 'sub-halos' (see Kupper, A, Lane, R., & Heggie, D. C. 2012, MNRAS, 420, 2700)

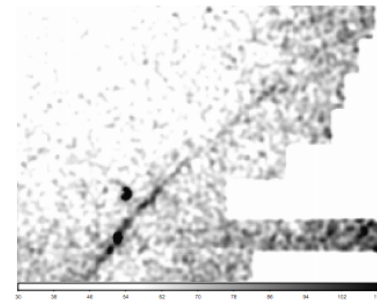
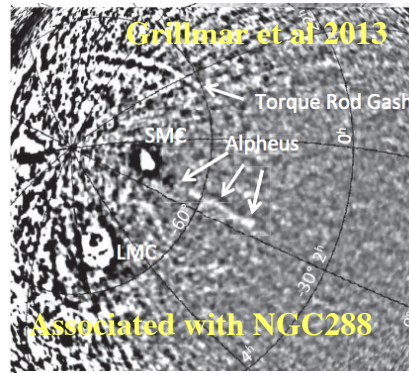


FIG. 1.— The match filtered star densities in the region of the Pal 5 stream in the SDSS λ and η co-ordinate system. The raw image has been smoothed with a 3 pixel Gaussian. The object above the stream is the foreground cluster M5.

stream associated with
Pal 5
Grillmar et al 2012

Future Problems for Analytic Methods for Detailed Mass Measurements

- Tidal stripping of dark matter from subhalos falling into the Milky Way produces narrow, "cold" tidal streams as well as more spatially extended “debris flows” in the form of shells, sheets, and plumes.
- The matter in the solar neighborhood is commonly assumed to be smoothly distributed in space and to have a Maxwellian velocity distribution- but....
- Tidal effects tend to make the density distribution smooth, but these tidal disruption processes are sources of **velocity substructure**.
- the speed distributions measured in high resolution numerical simulations exhibit deviations from the standard Maxwellian assumption, especially at large speeds.

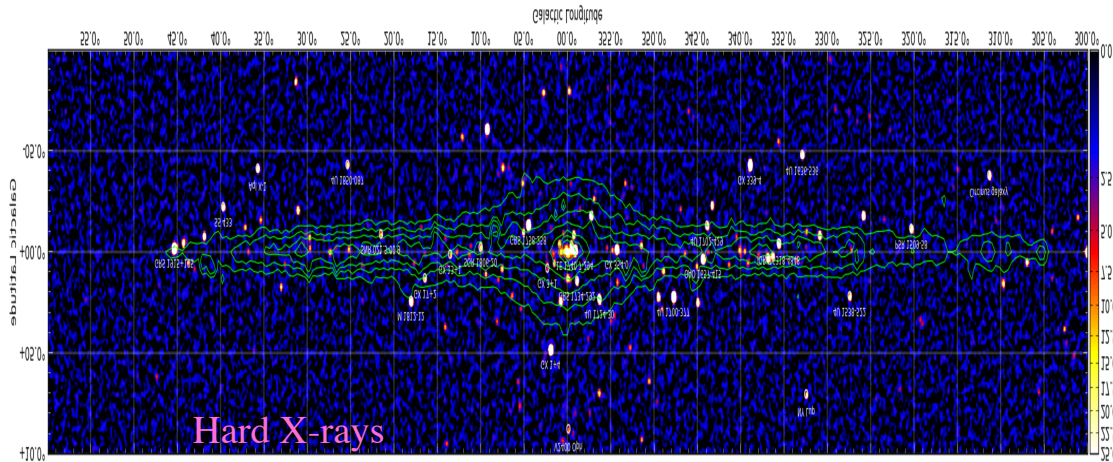
Kuhlen et al 2012

Other Wavelengths

In 'hard' (2-10 keV) x-rays one sees
accreting x-ray binaries Neutron stars and black holes

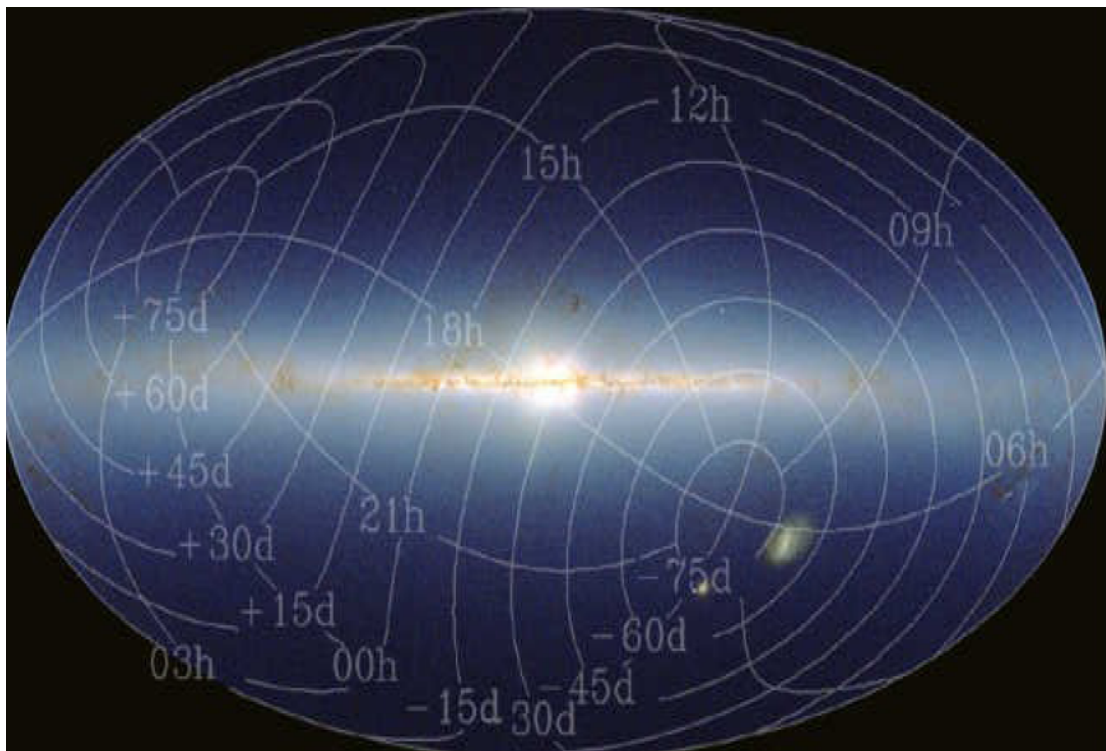
- 2 Populations companions

- 1) are massive and young (high mass x-ray binaries) POP I
- 2) old (Low mass x-ray binaries) POP II



Coordinate Systems

- Galactic (l,b) and celestial (Ra and Dec) see S+G pg 34-37 for a quick refresher



Coordinate Systems

The stellar velocity vectors are
z: velocity component perpendicular
to plane

θ : motion tangential to GC with
positive velocity in the direction
of rotation

π : radial velocity wrt to GC

With respect to galactic coordinates GC

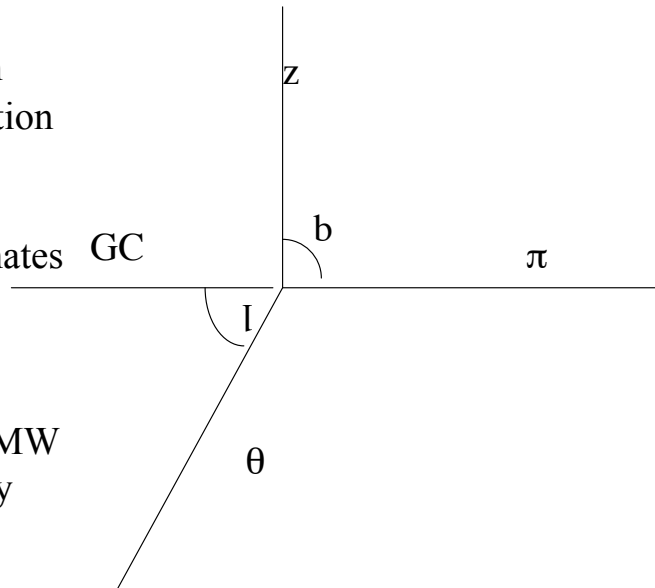
+ π = ($l=180, b=0$)

+ θ = ($l=30, b=0$)

+z= ($b=90$)

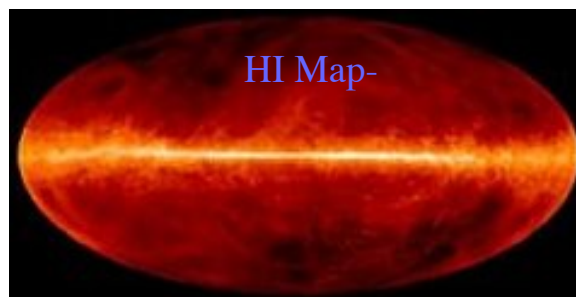
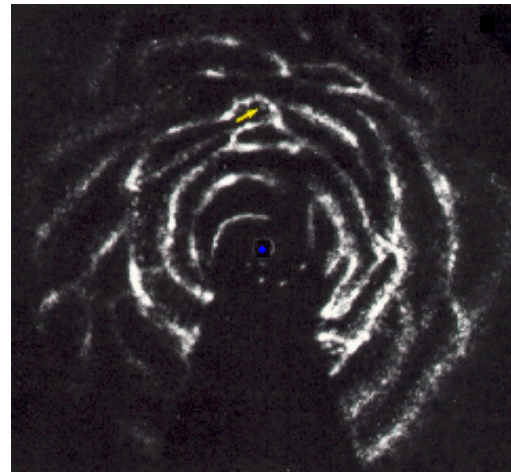
Local standard of rest: assume MW
is axisymmetric and in steady
state

$(\pi, \theta, z)_{\text{LSR}} = (0, \theta_0, 0)$;



HI Maps

- HI lies primarily in the plane- maps have velocity data associated with them allows dynamics to be determined
 - deproject HI velocity and intensity map to show total structure of the galaxy
- Not affected by dust- shows detailed structures.
- see review article by Kalbela and J. Kerp on the web page
- Neutral atomic hydrogen (HI) traces the interstellar medium (ISM) over a broad range of physical conditions.
- 21-cm emission line is a key probe of the structure and dynamics of the Milky Way Galaxy.



HI Observables

- Observed intensity $T_B(l, b, \nu)$ observed in Galactic coordinates longitude l and latitude b need to be converted into volume densities $n(R, z)$ (Burton & de Lintell Hekkert 1986, Diplas & Savage 1991).
- Assuming that most of the gas follows an axisymmetric circular rotation yields a relation for the differential rotation velocity (e.g., Burton 1988)

$$v(R, z) = \left[\frac{R_\odot}{R} \Theta(R, z) - \Theta_\odot \right] \sin l - \cos b$$
 where v is the radial velocity along a line of sight (directly measurable); and Θ is the tangential velocity
- for $R < R_\odot$, distances are ambiguous,
- for $R > R_\odot$, one needs to know the Galactic constants R_\odot and Θ_\odot and the form of $\Theta(R, z)$ e.g. the rotation curve shape.

R_\odot is the distance of the sun from the galactic center and Θ_\odot is the velocity of rotation at the sun . (a lot more later)

Plane is not totally flat

- The HI distribution is slightly warped- see in edge on galaxies also
- HI distribution is roughly exponential with $N(R, z) \sim n_0 \exp(-R/R_0)$; $R_0 \sim 3.5$ kpc, $n_0 \sim 1$
- Quick quiz: integrate the density distribution to get the total surface mass distribution of gas

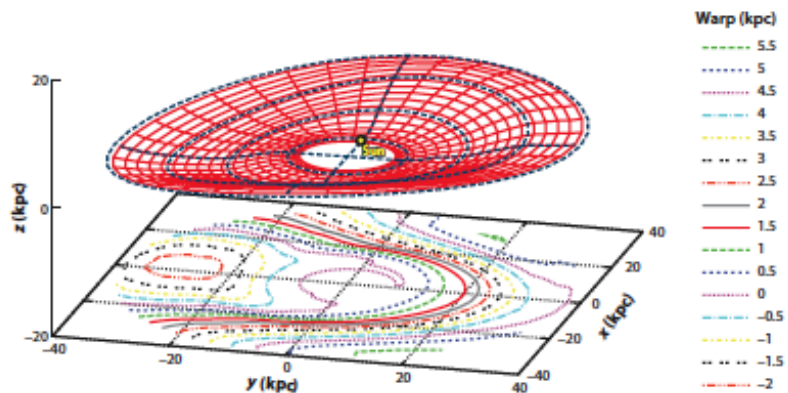
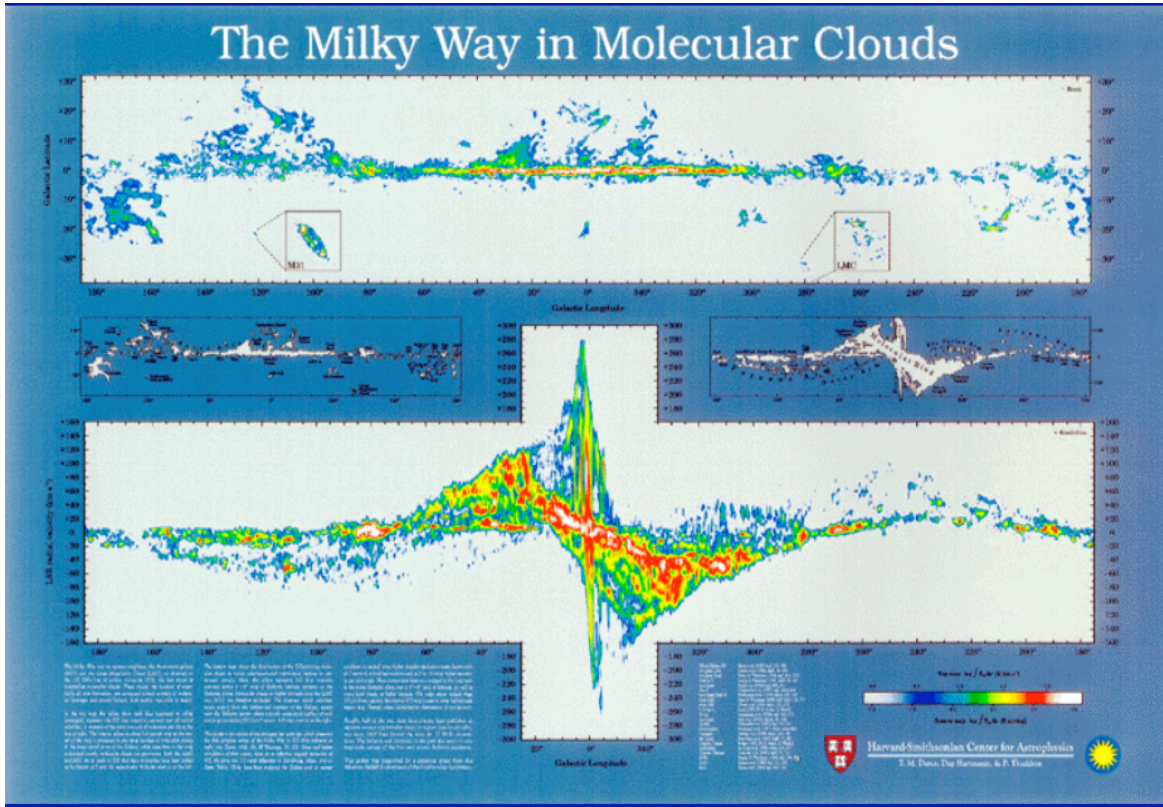
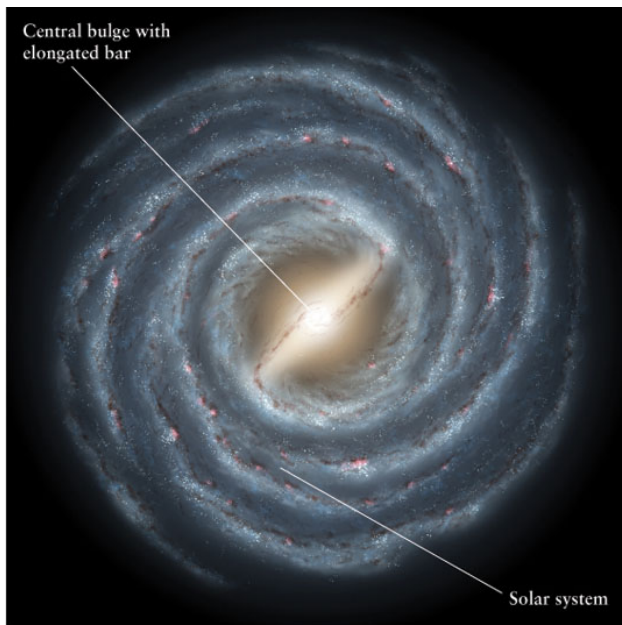


Figure 3

CO Maps



MW is a Barred Galaxy



(a) The structure of the Milky Way's disk



(b) Closeup of the Sun's galactic neighborhood

The **MW bar**, consists of relatively old red stars, roughly 9 kpc in length oriented at about a 45-degree angle relative to a line joining the sun and the center of the galaxy.

Basic Properties of MW

Diameter $\sim 23\text{kpc}$ - at sun orbital period

$\sim 2.5 \times 10^8$ yrs

Mass $\sim 2 \times 10^{11} M_{\odot}$ (details later)

M/L ~ 10 (on average)

Official distance of sun from GC is 8.5kpc,

$v_{\text{circular}} \sim 220\text{km/sec}$

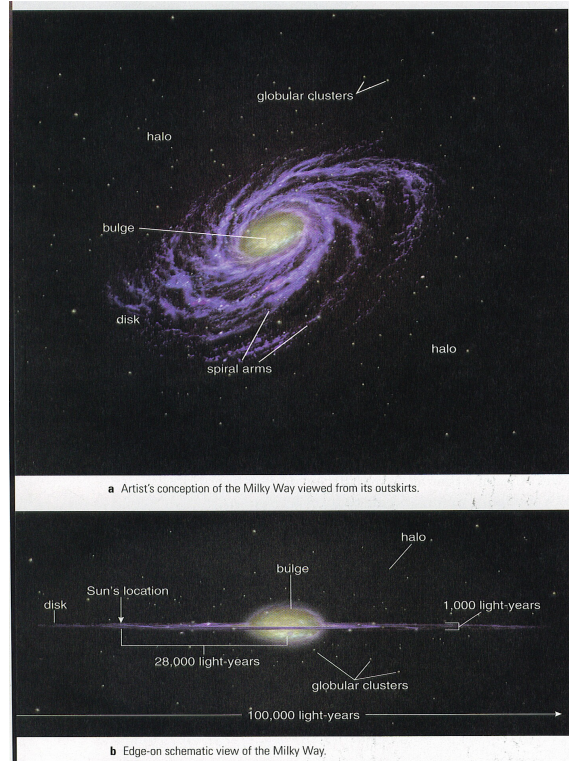
the Milky Way is a barred galaxy

Perpendicular to the disk the stellar distribution(s) can each be 'well' described as

$n(x) \sim \exp(-z/h)$; h =scale height

The disk is NOT simple and has at least 2 components

- 1) thin disk has the largest fraction of gas and dust in the Galaxy, and star formation is taking place ; $h \sim 100\text{pc}$, $\sigma_z \sim 20\text{km/sec}$
- 2) thick disk $h \sim 1.5\text{kpc}$ older, lower metallicity population, less gas- only makes up 2% of mass density at $z \sim 0$.



Components of MW

HII scale height: 1 kpc

CO scale height: 50-75 pc

HI scale height: 130-400 pc

Stellar scale height: 100 pc in spiral arm, 500 pc in disk

Stellar mass: $\sim 5 \times 10^{10} M_{\odot}$

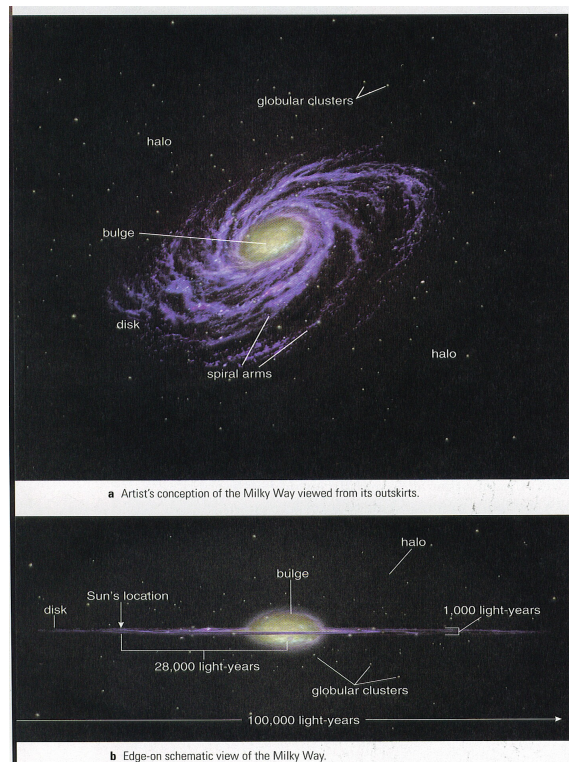
HI mass: $\sim 3 \times 10^9 M_{\odot}$

H_2 mass (inferred from CO mass): $\sim 0.8 \times 10^9 M_{\odot}$

Total MW mass within virial radius

dark-halo mass within the virial radius is $\sim 8 \times 10^{11} M_{\odot}$: About 50% dark matter

The mass values depend on the radius within which they are estimated



Mass Distribution near Sun

- The (surface) density distributions can be derived from dynamical studies (much more later in class)
- The total surface mass density of all gravitating matter within 1.1 kpc of the centerline of the disk at the position of the sun is $67 \pm 6 M_{\odot} \text{pc}^{-2}$ and that of all identified matter (stars and gas) is $42 \pm 5 M_{\odot} \text{pc}^{-2}$
- The local density of **dark matter** is $0.0075 \pm 0.0023 M_{\odot} \text{pc}^{-3}$ (Zhang et al 2012) (see next lecture for how this is done)
- This dark matter density is consistent with fits to the MW halo models
- However this is very technically challenging and the amount of dark matter is rather uncertain.
- This analysis is done using the vertical distribution of stars and their velocities (more later)

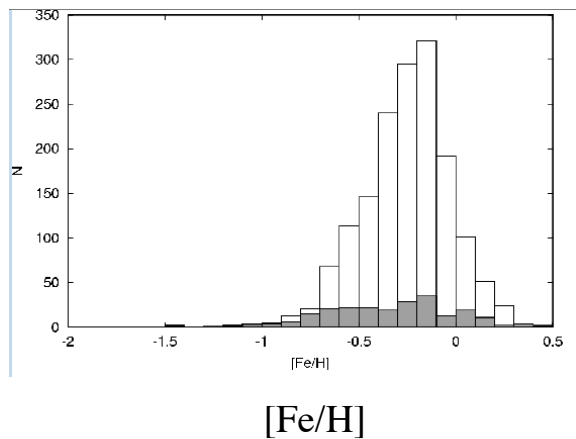
RESULTS FOR GALACTIC PARAMETERS A

Parameter	Flat rotation curve
$V_c(R_0)$ [km s ⁻¹]	218±6
A [km s ⁻¹ kpc ⁻¹]	13.5 ^{+0.2} _{-1.7}
B [km s ⁻¹ kpc ⁻¹]	-13.5 ^{+1.7} _{-0.2}
$(B^2 - A^2)/(2\pi G)$ [$M_{\odot} \text{pc}^{-3}$]	...
Ω_0 [km s ⁻¹ kpc ⁻¹]	27.0 ^{+0.3} _{-3.5}
R_0 [kpc]	8.1 ^{+1.2} _{-0.1}
$V_{R,\odot}$ [km s ⁻¹]	-10.5 ^{+0.5} _{-0.8}
$V_{\phi,\odot}$ [km s ⁻¹]	242 ⁺¹⁰
$V_{\phi,\odot} - V_c$ [km s ⁻¹]	23.9 ^{+3.1} ₋₃
$\mu_{\text{Sgr}} A^*$ [mas yr ⁻¹]	6.32 ^{+0.07} _{-0.70}
$\sigma_R(R_0)$ [km s ⁻¹]	31.4 ^{+0.1} _{-3.2}
$R_0/h\sigma$	0.03 ^{+0.01} _{-0.27}
$\chi^2 \equiv \sigma_{\phi}^2/\sigma_R^2$	0.70 ^{+0.30} _{-0.01}

Thin and Thick Disk -Details Composition

- Each of the 'components' of the MW has a 'different' (but overlapping) chemical composition (Metallicity)
- stars in the thin disk have a higher metallicity and M/L (~3). than those in the thick disk, high M/L~15 (age and metallicity effect)
- Thin disk $M_{\text{stars}} \sim 6 \times 10^{10} M_{\odot}$;
 $M_{\text{gas}} \sim 0.5 \times 10^{10} M_{\odot}$. Stellar luminosity $L_B \approx 1.8 \times 10^{10} L_{\odot}$
- Thick disk has low mass and luminosity $M \sim 3 \times 10^9 M_{\odot}$ and $L_B \approx 2 \times 10^8 L_{\odot}$
- the metallicity of stars in the Galactic halo and in the bulge is even lower. - in the older literature one has 'Pop I' and 'Pop II'
- Pop I is the component which dominates the disk O,B stars, open clusters, dust HII regions
- Pop II - bulge; old relatively metal poor

thin disk-open
thick disk shaded



"The Formation and Evolution of the Milky Way," by Cristina Chiappini;2001

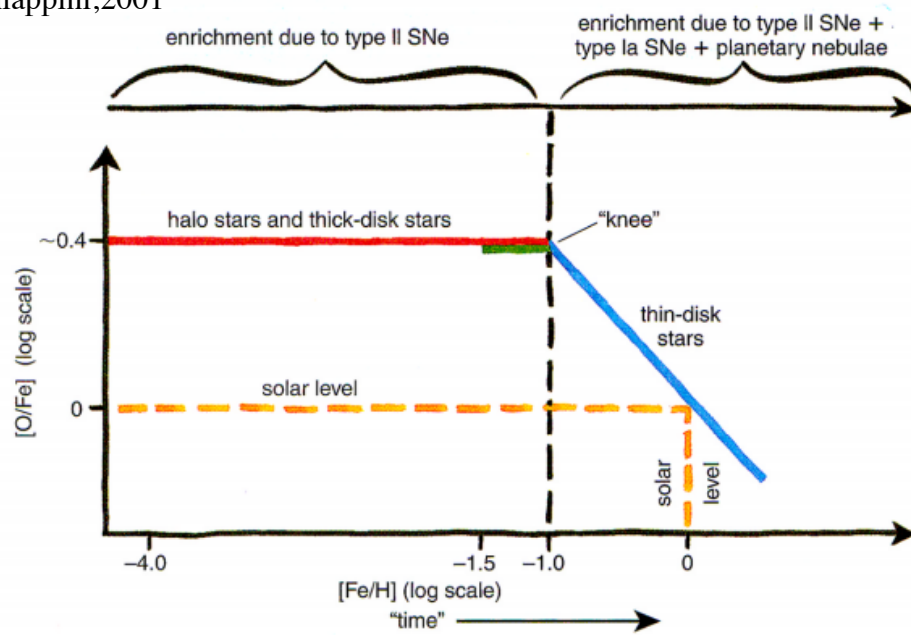
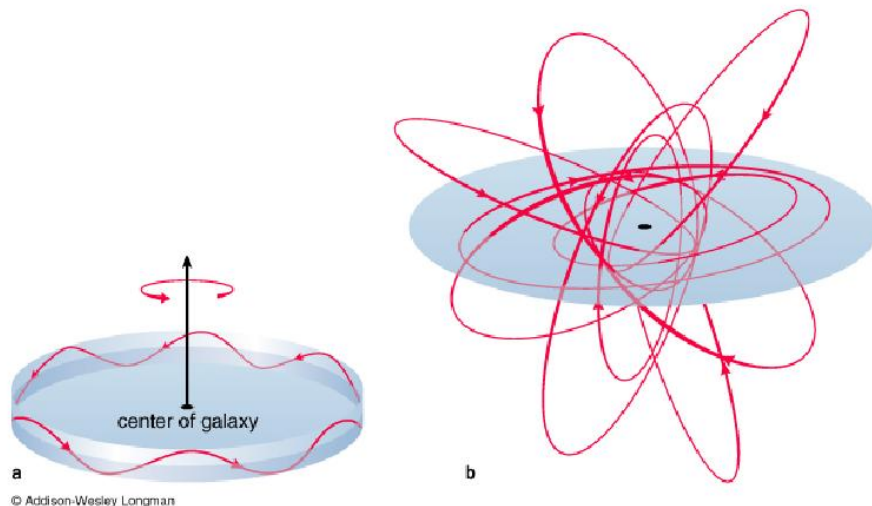


Figure 5. The general metallicity of the Galaxy—as measured by the abundance of iron (Fe), compared with hydrogen (H)—increases with time (*abscissa*) and so serves as a basis for comparing the relative abundances of two elements (such as oxygen (O) and iron; *ordinate*) that are created on different timescales. A plot of these quantities reveals a "plateau" of metal-poor stars (metallicity less than -1) that drops at a "knee" as the relative proportion of iron in the Galaxy increases. Since type Ia supernovae (SNe) are the primary source of iron, astronomers believe that the "knee" occurred about one billion years after the Galaxy began to form (see Figure 4). The halo stars (red line) and some of the thick-disk stars (green line) tend to occupy the "plateau," whereas thin-disk stars (blue line) occupy the descending slope. These observations suggest that the halo, and part of the thick disk were formed in the first billion years of the Galaxy's evolution, and the thin disk formed later.

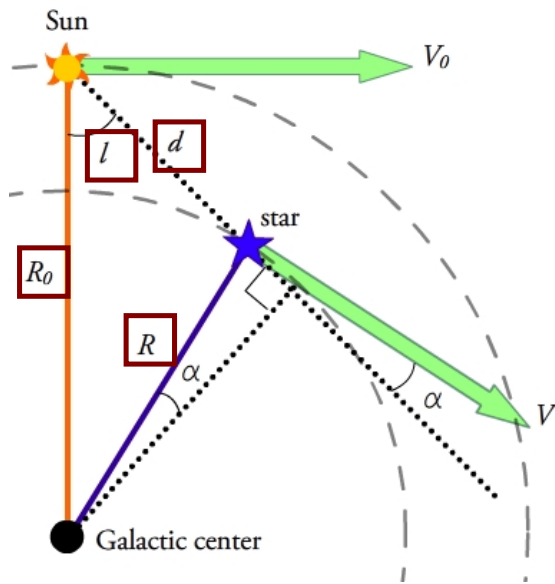
Zeroth Order Dynamics

- Stars in disk have mostly rotational velocity- very little random or r or z components
- Stars in bulge and halo mostly random orbits, but some rotation.
- Need to use different techniques to estimate the mass of these '2' components



Galactic Rotation- S+G sec 2.3, B&T sec 3.2

- Consider a star in the midplane of the Galactic disk with Galactic longitude, l , at a distance d , from the Sun. Assume circular orbits radii of R and R_0 from the galactic center and rotational velocities of V and V_0
- The 2 components of velocity- radial and transverse are then for circular motion
- $V_{\text{observed, radial}} = V(\cos \alpha) - V_0 \sin(l)$
- $V_{\text{observed, tang}} = V(\sin \alpha) - V_0 \cos(l)$
- using the law of sines
- $\sin l / R \sim \cos \alpha / R_0$



wikipedia

Nearly Circular Orbits in Axisymmetric Potentials- B&T sec 3.2.3- see

<http://www.astro.utu.fi/~cflynn/galdyn/>

- Coordinate System
- Galactic Standard of Rest:
 - Origin GC (center of mass)

Velocity of HI

- In the plane of the disk the velocity and intensity of HI gas (Sparke and Gallagher fig 2.20)

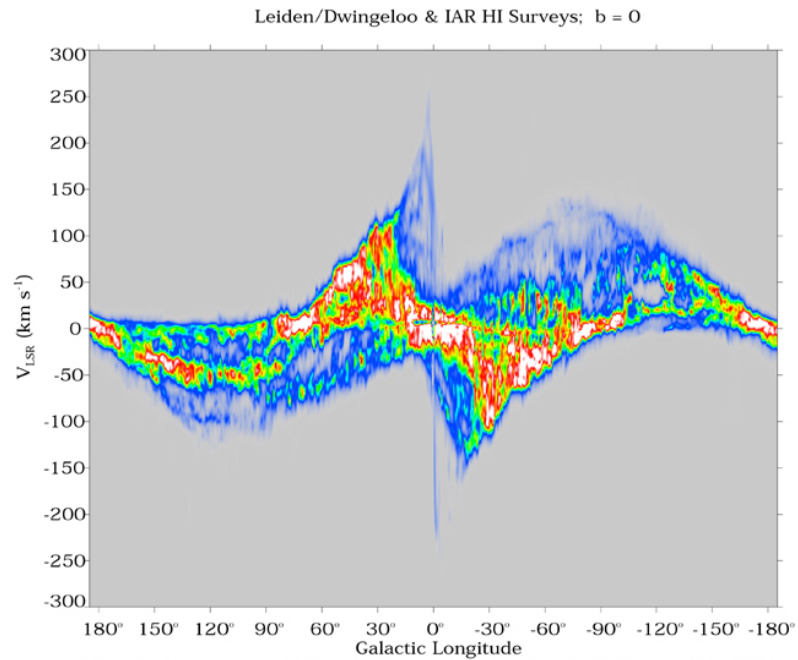


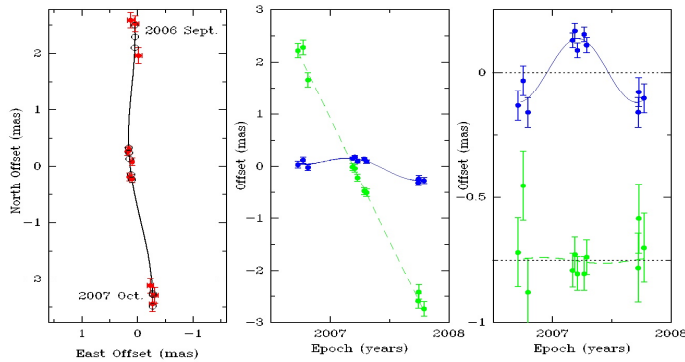
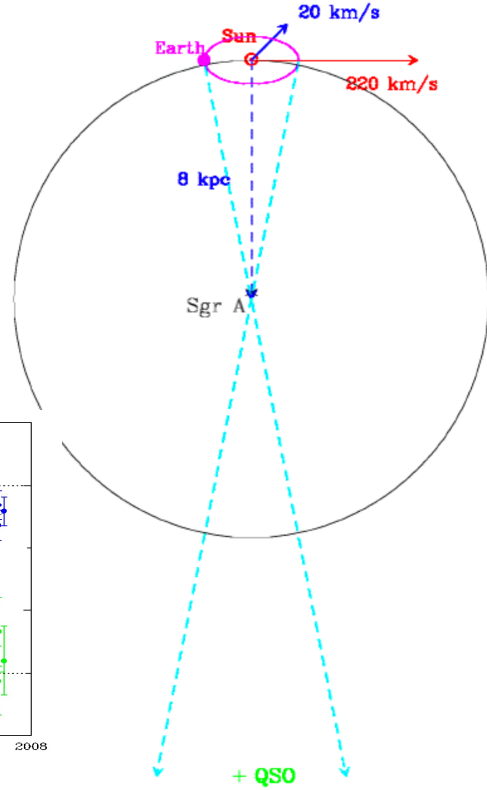
Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Timescales

- crossing time $t_c = 2R/\sigma$
- dynamical time $t_d = \sqrt{3\pi/16G\rho}$ - related to the orbital time; assumption homogenous sphere of density ρ
- relaxation time- the time for a system to 'forget' its initial conditions
 $t_r \sim N t_c / 48 f^2$: N objects carrying f of total mass :
 $S+G$ gives $t_r = V^3 / 8\pi G^2 m^2 n \ln \Lambda \sim 2 \times 10^9 \text{ yrs} / [(V/10)^3 (m_\odot)^{-2} (n/10^3 \text{ pc}^{-3})^{-1}]$
 major uncertain is in $\ln \Lambda$ – numerical simulations

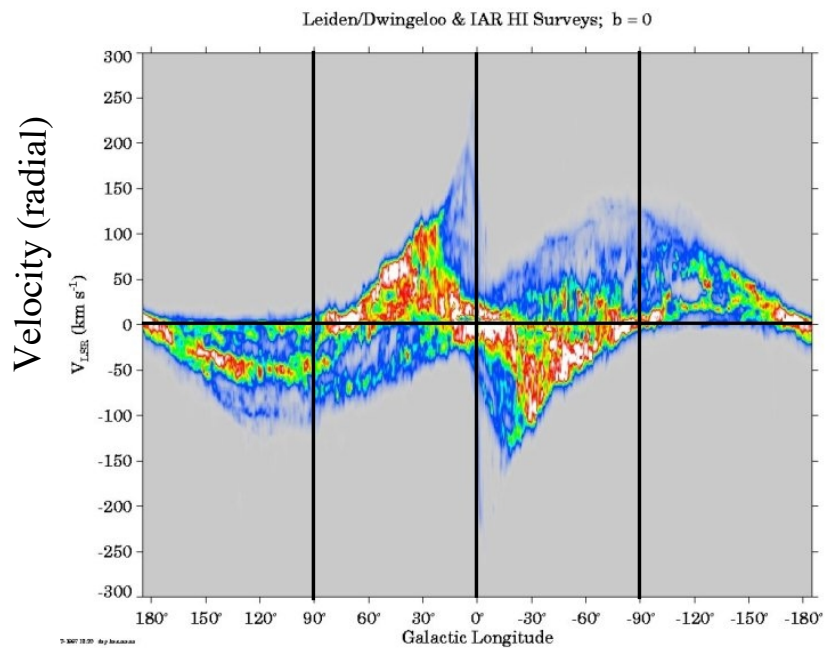
Distances From Motions

- Distance to the galactic center (R_0) is rather important; in problem 2.6 (S&G) discusses one way to use the observed positions and velocities of stars in orbit around the galactic center to get the distance
- Another way of doing this: measure the proper motion+parallax of SgrA* caused by the velocity of the sun
- East in blue, north in green -right panel has proper motion removed. left panel motion on sky



Galactic Rotation Curve HI data

- Velocity, longitude, intensity graph of HI in the MW fig 2.20 in S+G
- The HI probes very large scales and **so many of the approximations in the derivation of the Oort constants are not correct and one must use the full up equations.**



Galactic Longitude

Distribution of Light in Disk (S+G eq 2.8)

the thin disk and the thick disk has a similar form but different scale height and density of stars

Radial scale length of a spiral disk
 $\Sigma(r) = \Sigma_0 \exp(-R/R_d)$; integrate over r to get total mass $M_d = 2\pi \Sigma_0 R_d^2$

Vertical density distribution is also an exponential $\exp(-z/z_0)$ so total distribution is product of the two

$\rho(R,z) = \rho_0 \exp(-R/R_d) \exp(-z/z_0)$
 while we may know the scale length of the stars, that of the dark matter is not known.
 Also the nature of the dark matter halo is not known:- disk/halo degeneracy

Somewhat more precisely the luminosity distribution is
 $L(R,z) = L_0 \exp(-R/h) / \cosh^2(z/z_0)$
 with $L_0 = 0.05 L_\odot / \text{pc}^3$

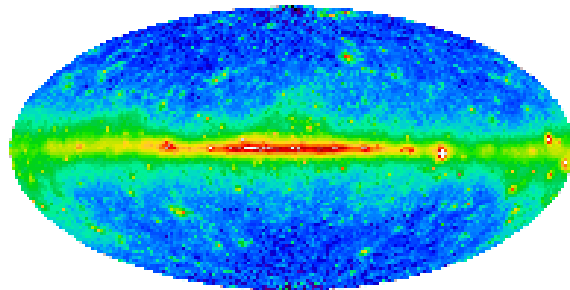
Even more detail

Each spectral type can be characterized by a scale height, a possible indicator of age. The older the star, the more dynamical interactions it has had (Spitzer and Schwarzschild 1951).
 The result is an increase in the spatial velocity of older stars (particularly along the vertical axis of the disk).
 M dwarfs have relatively large scale heights, ~ 300 pc, in contrast to the younger A-type stars with ~ 100 pc (see table 2.1 in S+G)

Cosmic Rays-100th Anniversary of their Discovery

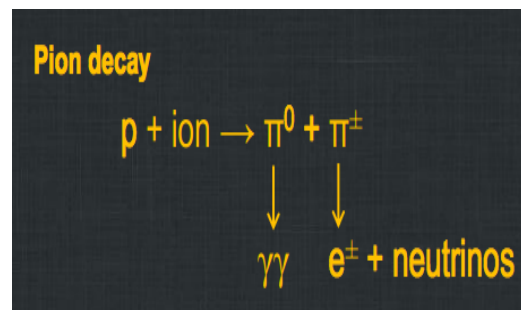
<http://www.aps.org/publications/apsnews/201004/physicshistory.cfm>

- These are very hard to study in other galaxies
 - they are visible by the synchrotron emission emitted by electrons spiraling in the magnetic field
 - γ -rays emitted by relativistic particles hitting gas
- MW
 - direct measures of CRs e.g. in situ
 - detailed γ -ray maps of MW
 - convolution of cosmic ray energy spectrum and intensity with target (gas) density
 - Very detailed radio maps



Fermi map of MW

Origin: acceleration of particles in supernova shocks via first order Fermi process - total power $\sim 10^{41}$ ergs/sec $\sim 10\%$ of SN shock energy

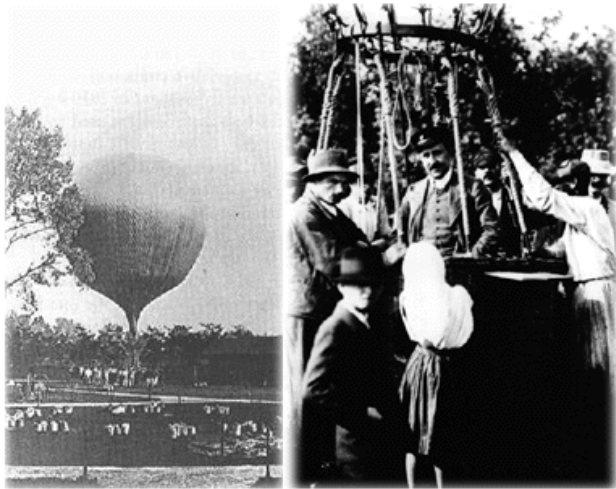


Cosmic Rays-100th Anniversary of their Discovery

Why Did He do This

- scientists had been puzzled by the levels of ionizing radiation measured on the earth and in the atmosphere.
- The assumption was that the radiation from the earth and would decrease as one went away from the surface.
- Hess greatly increasing the precision of the electroscopes*and then by personally taking the equipment aloft in a balloon. He measured the radiation at altitudes up to 5.3 km during 1911-12 without oxygen. The daring flights were made both at day and during the night, at significant risk to himself and showed that the level of radiation **increased** as one went higher-observed durir
- *He concluded*

*they spontaneous electroscopes is



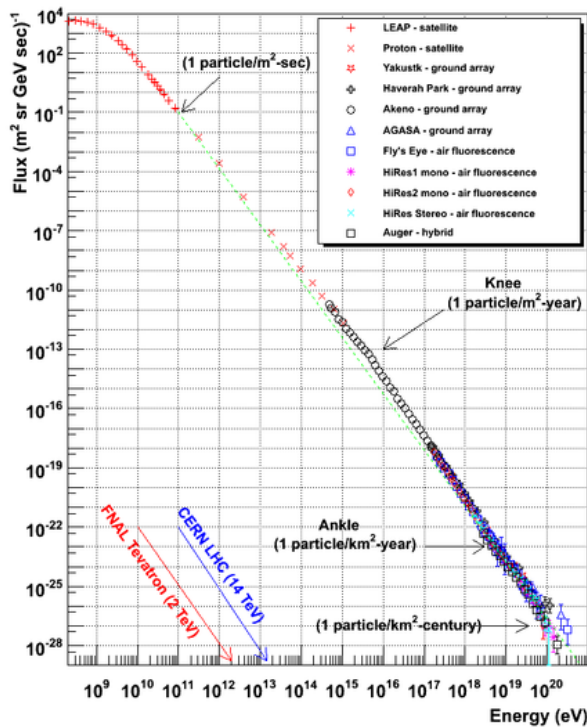
prize 1936)

of discharge of an

100 Years of Cosmic Rays

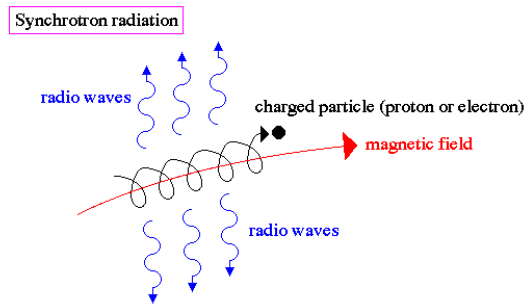
- In August 1912, the Austrian physicist Victor Hess flew in a balloon to altitudes of 5.3 km, measuring the flux of particles in the sky. The expectation was that the flux would decrease with altitude, precisely the opposite of what Hess found. **The shocking conclusion was that particles were raining down on Earth from space.**
- <http://www.npr.org/blogs/13.7/2012/07/25/157286520/cosmic-rays-100-years-of-mystery>

Cosmic Ray Spectra of Various Experiments

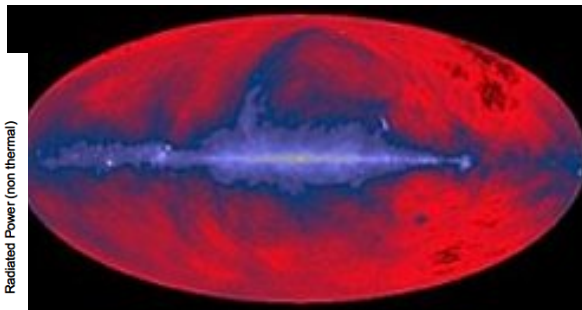
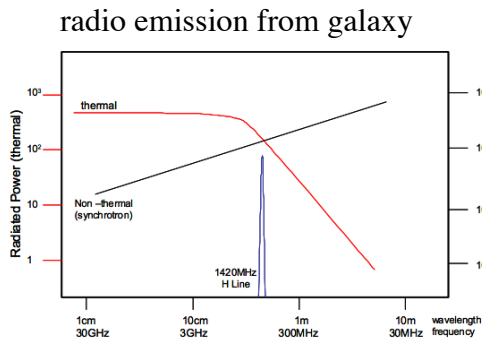


Cosmic Rays

- Have appreciable energy density $\sim 1 \text{ eV/cm}^3$
- Synchrotron emission is convolution of particle spectrum and magnetic field- also emission from 'non-thermal' bremsstrahlung
- Can ionize deeply into molecular clouds



http://abyss.uoregon.edu/~js/glossary/synchrotron_radiation.html



Cosmic Rays

- Accelerated particles propagate through the Galaxy where, due to the magnetic field, they move along complicated helical tracks.
- Therefore, the direction from which a particle arrives at Earth cannot be identified with the direction to its source of origin (Larmor radius, $m_e c(\sqrt{\gamma^2 - 1})/eB$; $3.3 \times 10^6 \text{ km}$ for $1 \mu\text{G}$, 100 MeV)
- The magnetic field is also the reason why particles do not leave the Milky Way along a straight path, but instead are stored for a long time ($\sim 10^7 \text{ yr}$) before they eventually diffuse out, an effect also called confinement



γ -ray Imaging of Star Forming Regions

- Fermi has imaged the γ -rays coming from star forming regions and γ -ray spectra show that this is due to cosmic rays interacting with dense gas (Lingenfelter 2012) in superbubbles (places of high massive star formation rate and thus high S/N rate).

γ -rays come from the interaction of CRs and dense gas- Fermi has imaged sites of CR creation !



Fig. 1 Typical ~1pc Star Forming Region Shown by Bright O & B Stars

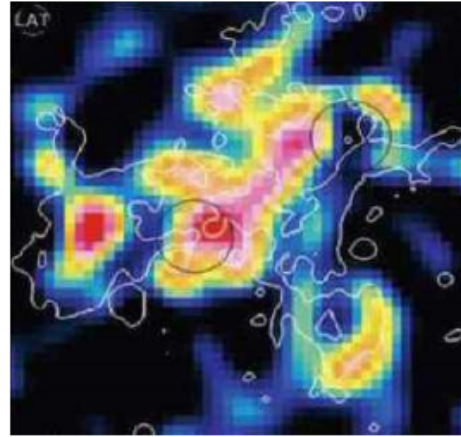


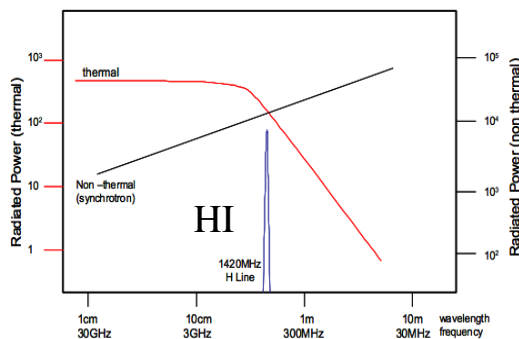
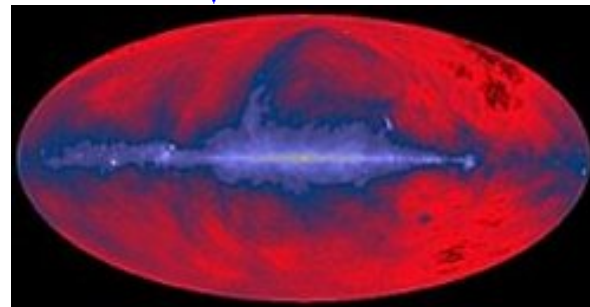
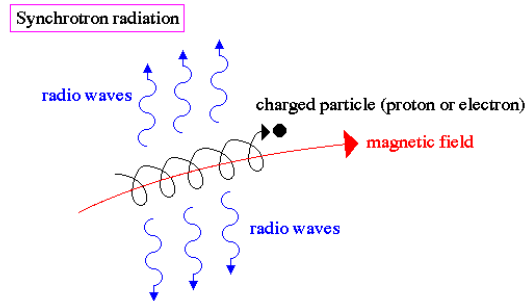
Fig. 2 ~100 pc Cygnus Superbubble in 10-100 GeV γ -Rays from Fermi [11]

Radio Continuum Emission

- Synchrotron emission: convolution of particle spectrum and magnetic field- power law spectrum- power law spectrum $F_\nu \sim A\nu^{-\alpha}$

slope, α depends on spectrum of CRs and intensity on magnetic field

- Thermal bremsstrahlung: fast, non-relativistic particles running by gas (breaking radiation)-exponential spectrum
- Relative intensity of the two components changes greatly with position.

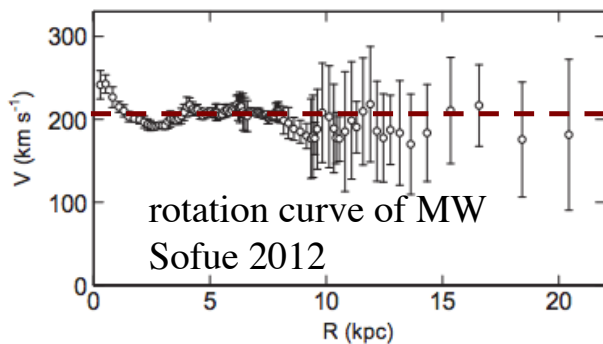


Simple Estimate of Mass of Milky Way

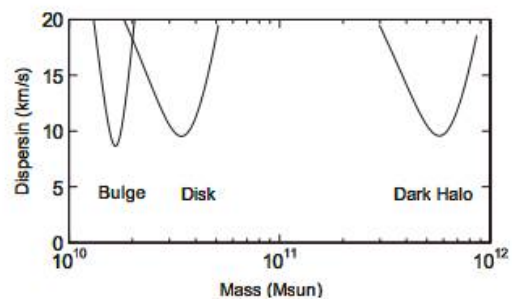
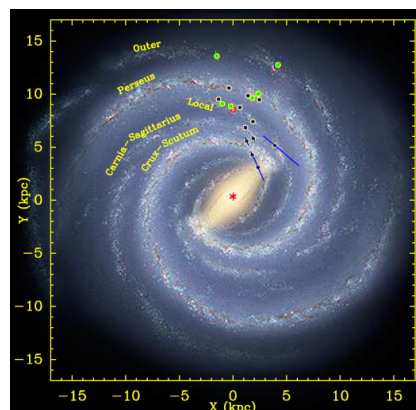
- If we follow problem S&G 2.18 and use $M \sim RV^2/G$ - of course this is for a sphere ... ignore the details (discuss later what is correct for a disk+sphere)
- suns distance from center $R_0 \sim 8\text{kpc}$ and rotational velocity $\sim 220\text{km/sec}$
 $M = 9 \times 10^{10} M_\odot$ - corresponds to a density of $\sim 4 \times 10^{-3} M_\odot/\text{pc}^3$ (uniform sphere) - mass within 8kpc; if extend to 350kpc (virial radius) get $4 \times 10^{12} M_\odot$; factor of 2-4 too high but right 'order'
- critical density of universe today $\rho_{\text{crit}} = 3H_0^2/8\pi G \sim 1.45 \times 10^{-7} M_\odot/\text{pc}^3$
- So the MW is 'overdense' by $\sim 2.7 \times 10^5$ at solar circle and 600 at virial radius (using above simple formula) and 150 using a more correct mass.
 - In CDM theories the size of a virialized system is when the overdensity is >200

Mass of Milky Way

- This turns out to be rather hard to determine- there is a degeneracy between velocity and distance- use rotation curve fitting and 'proper' potentials
- New data allows absolute distance to be determined for several star forming regions (Reid et al 2009)
- Stellar mass of MW is $\sim 6 \times 10^{10} M_\odot$
- DM mass is $1-2 \times 10^{12} M_\odot$; $M/L \sim 30$
- DM inside overdensity of 200 $1-2 \times 10^{12} M_\odot$



Locations of star-forming regions (dots) artist's Milky Way.

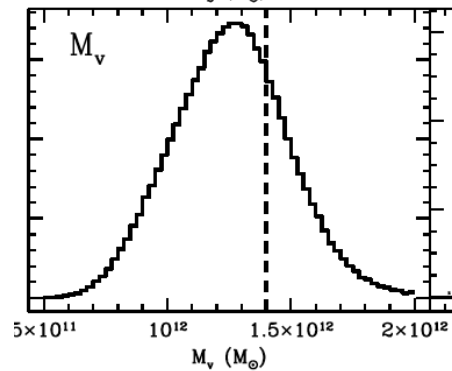
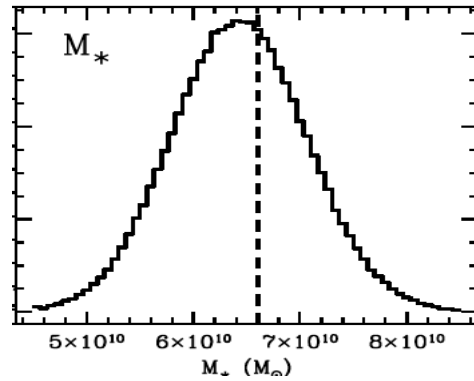


Mass of Milky Way

- The majority of the mass of the Galaxy is expected to lie in the CDM halo, which is only observable through its gravitational effect on luminous components of the Galaxy. Most recent estimates (McMillian 2012, van den Maerl 2012) differ

McMillian 2012 find

- disc scale lengths of 3.00 ± 0.22 kpc and 3.29 ± 0.56 kpc for the thin and thick discs respectively;
- at sun thin disk has 90% of the mass and thick disk 10%
- R_0 Solar radius of 8.29 ± 0.16 kpc
- a circular speed at the Sun of 239 ± 5 km/s
- total stellar mass of $6.43 \pm 0.63 \times 10^{10} M_{\odot}$
- bulge mass $M_b = 8.9 \times 10^9 M_{\odot}$
- virial mass of $1.26 \pm 0.24 \times 10^{12} M_{\odot}$
- a **local** dark matter density of $0.40 \pm 0.04 \text{ GeV cm}^{-3}$ (or in more normal units $0.01 M_{\odot}/\text{pc}^3$)



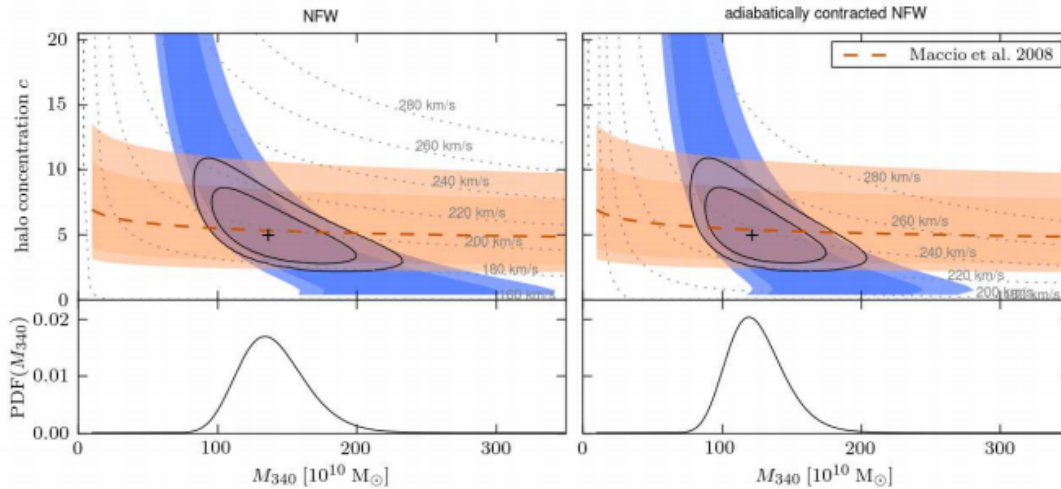
distribution functions of parameters
McMillian 2012

Mass of MW (Bovy and Tremaine 2012)

- The flatness of the Milky Way's circular-velocity curve at < 20 kpc (e.g., Xue et al. 2008) shows that the visible Galactic disk is embedded in a massive dark halo.
- The disk is composed of gas and stars (baryons), while the dark halo is believed to be dominated by dark matter.
- it remains unclear whether there is any need for a substantial amount of dark matter in the disk itself (Binney et al 2012)
- One way to determine the local density of dark matter is through a determination of the dependence of the gravitational potential on distance above the mid-plane of the disk ("height"), from measuring the kinematics of stars (e.g., Kapteyn 1922; Oort 1932; Bahcall 1984) - a lot more later.
- But, a major obstacle is that the uncertainty in the amount of baryonic matter in the disk makes it hard to determine the relative contributions from dark and baryonic matter to the density near the mid-plane.
- The contributions from baryonic and dark matter can be disentangled by measuring the gravitational potential out to larger heights. At heights of several times the disk thickness, the dark halo and the baryonic disk contributions to the potential have a different vertical dependence (e.g., Kuijken & Gilmore 1989; Garbari et al. 2011).

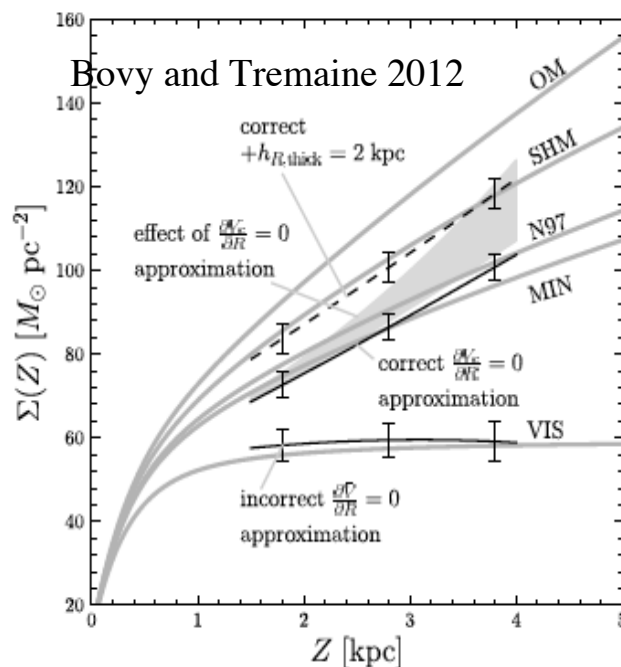
RAVE Sample

- In September 2013 another detailed analysis of the MW mass was determined (Binney et al 2013. Piffl 2013; <http://arxiv.org/pdf/1309.4293.pdf>)



Mass Density of MW Perpendicular to the Disk

- The breakdown of the assumptions made in this simple, “model-independent” Jeans analysis are such that the measurement has a systematic uncertainty reaching 10 to 20% at $|Z| = 4$ kpc.
- Therefore, a precise determination of the local dark matter density from observations at large Z using a Jeans analysis requires data that span a wide range in R such that the radial gradient of the velocity moments, can be determined.
- The Gaia mission (Perryman et al. 2001) will provide such measurements



- The line labeled VIS is the mass density of 'visible material'
- The grey lines are including the effects of different dark matter halo models

Thin Disk- Thick Disk

- There are a variety of stellar populations in the disk.
- There is a strong tendency for age, metallicity, velocity dispersion and scale height to be correlated.
- It used to be that this was parameterized as a 'thin' and 'thick' disk.
- Of course things are more complex (Bovy et al 2013) and there seems to be a more continuous distribution.

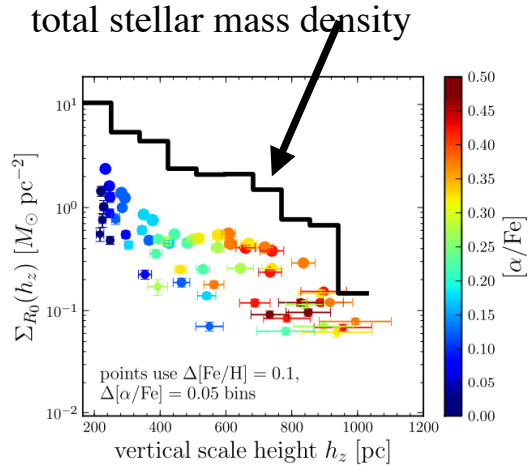
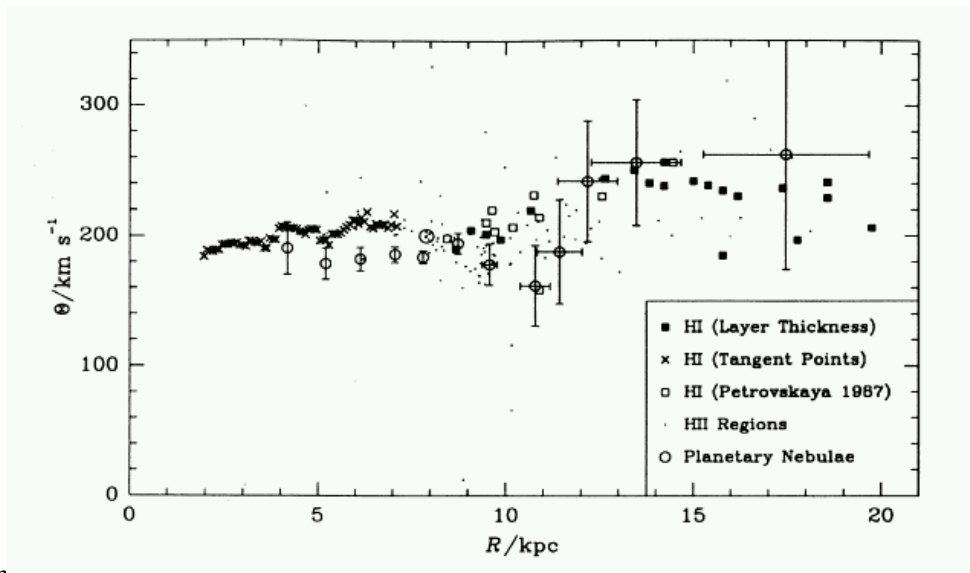


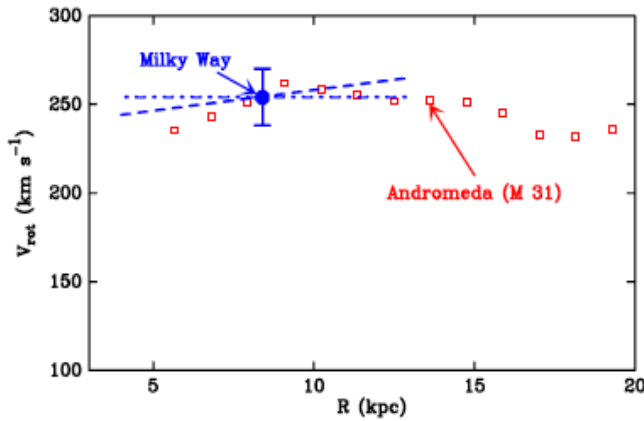
FIG. 2.— Distribution of stellar surface-mass density at the Solar radius $\Sigma_{R_0}(h_z)$ as a function of vertical scale height h_z . The thick black histogram shows the total stellar surface-mass density in bins in h_z , calculated by summing the total stellar masses of sub-populations in bins in $[\alpha/\text{Fe}]$ and $[\text{Fe}/\text{H}]$. The stellar surface-mass densities of the individual elemental-abundance bins in $[\text{Fe}/\text{H}]$ and $[\alpha/\text{Fe}]$ are shown as dots, with values for $\Sigma_{R_0}([\text{Fe}/\text{H}], [\alpha/\text{Fe}])$ on the y -axis. The points are color-coded by the value of $[\alpha/\text{Fe}]$ in each bin and the size of the points is proportional to the square root of the number of data points that the density fits are based on. Some of the errorbars are smaller than the points. Elemental abundance bins have a width of 0.1 in $[\text{Fe}/\text{H}]$ and 0.05 in $[\alpha/\text{Fe}]$.

MW Rotation Curve



- Flynn, Sommer-Larsen, Christensen-Dalsgaard

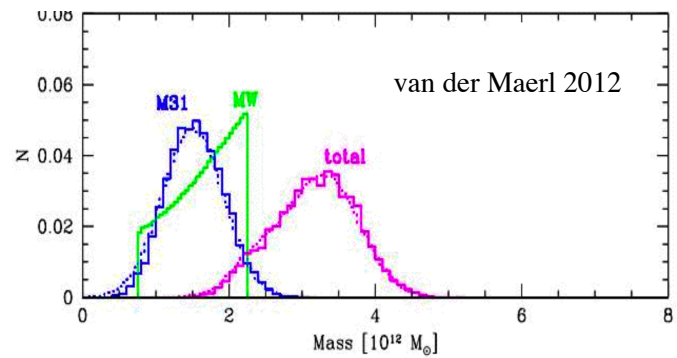
Comparison with M31



Blue line is from Reid 2009
notice it disagrees with
previous figure-
this is due to difficulties in
assigning accurate distances to
different tracers
and correcting for non-circular
motions

Probability that M31 and MW
have a given mass and for the sum

the Milky Way has a
significantly higher rotational
speed (or, equivalently, lower
baryonic mass) than the Tully-
Fisher relation predicts- more
later



- The light (yellow) arrows are for IAU standard values of $R_0 = 8.5$ kpc and $V_r = 220$ km/s and a flat rotation curve, black arrows for $V_r = 254$ km/s
- high mass star forming regions orbit the Galaxy slower than the Galaxy rotates!

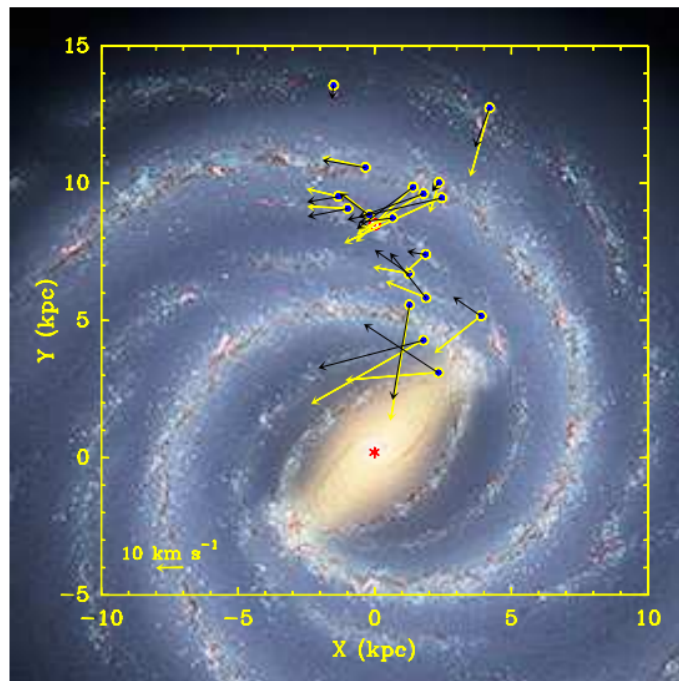
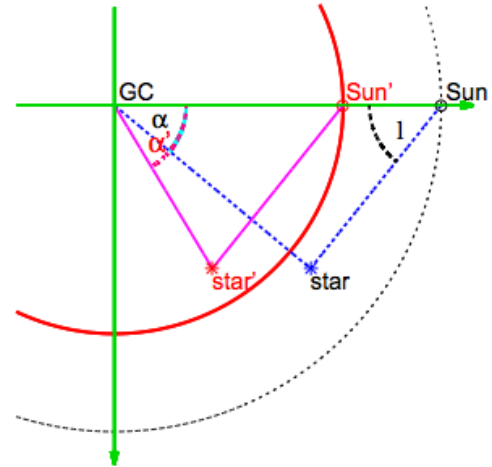


Fig. 3.— Peculiar motion vectors of high mass star forming regions (superposed on an artist conception) projected on the Galactic plane after transforming to a reference frame rotating with the Galaxy. A 10 km s^{-1} motion scale is in the lower left. The Galaxy is viewed from the north

Why Different Rotation Curves for MW

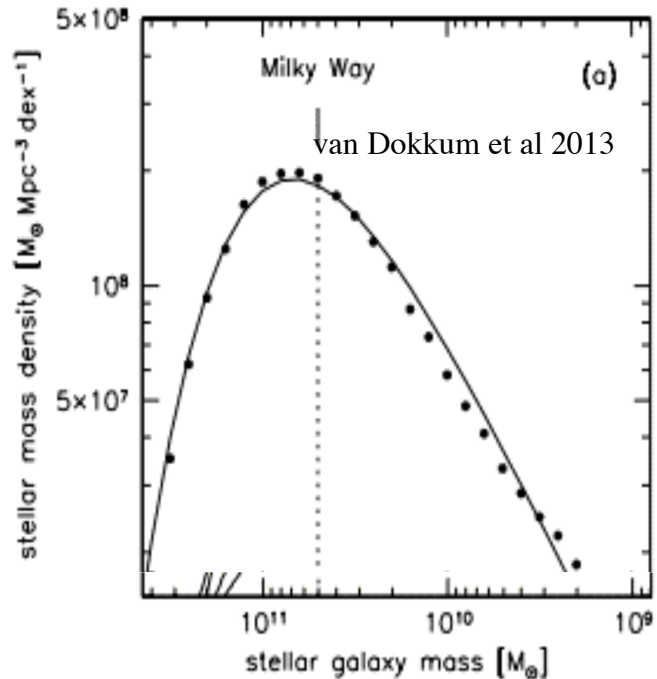
- Changing R_0 's effect on determination of the rotation curve
- Since the galactic longitude of the data source (star, gas) does not change the angle, α , must grow as R_0 lessens
- This reduces the rotation speed estimated from the sources radial velocity

R. Schonrich



Stellar Mass of MW compared to Local Galaxy Mass Function

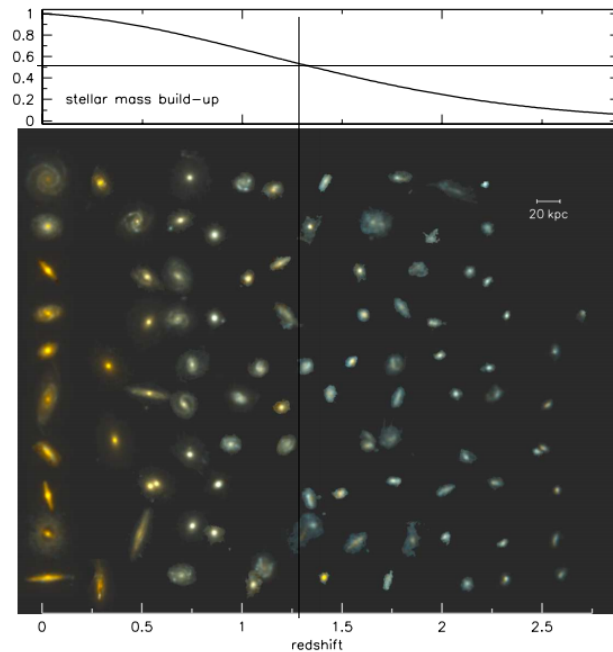
- The stellar mass of the MW is near the peak of the local galaxy mass function (not number density). (notice mass scale runs backwards....astronomers)



Progenitors of the MW

- What did the progenitors of the MW look like- van Dokkum et al 2013 present images of galaxies with the same mass density of the MW at a variety of redshifts using the average stellar mass build-up as a guide

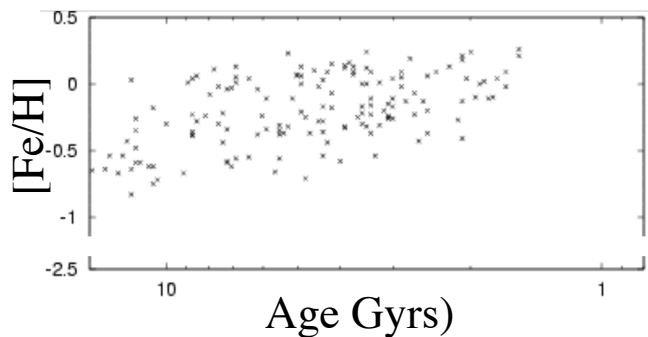
Notice that organized spirals appear only at $z < 1$ and that at higher redshift galaxies had a very different surface brightness profile. Galaxies also become redder with time (general drop of SF with redshift) and mergers are not required to explain the mass evolution of large spiral galaxies.



Age Metallicity

- Older stars **tend** to be metal poor: only in the MW and local group can this be studied with great detail (SG 4.3.2)
- However the metallicity history of the MW is very hard to unfold
- Older stars (in the MW) tend to be metal poor
 - logic is that metals are created in SN over cosmic time, next generation of stars if formed from this enriched gas, so more metal rich

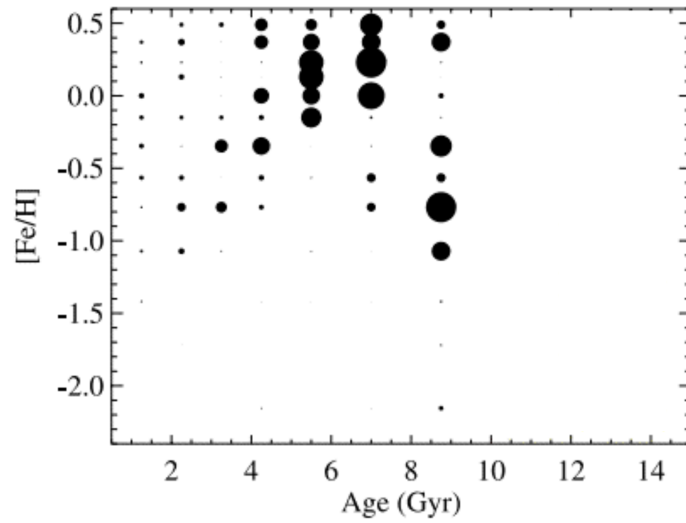
- Actually much more complex;
 - galaxy is not a closed box, gas flows in and out
 - galaxy mergers can mix things up
 - Two types of SN (type I produces mostly Fe, type II mostly O)
 - stars can move a long way from their regions of birth
 - star formation rate is not constant



Huge scatter- see <http://arxiv.org/pdf/1308.5744.pdf>
 8.2Gyr old sun like star with $\text{Fe}/\text{H} = -0.013 \pm 0.004$ and a solar abundance pattern

Age Metallicity

- Now can do this in M31 with HST data (!)
- Pattern seems to be more variance at younger ages rather than a trend.
- In M31 spheroid things are very different than in MW; 40% of the stars are metal-rich and younger than 10 Gyr ! (M31 has undergone a major merger MW has not)
- Lesson: MW may not be representative of spirals



Size of symbol is \sim # of stars in box; Brown et al 2006

MW as Model for Other Galaxies

- the Milky Way experienced very few minor mergers and no major merger during the last ~ 10 Gyrs- unexpected in a cosmological scenario
- The old stellar content of the thick disk indicates a possible a merger origin at an early epoch.
- The Milky Way is presently absorbing the Sagittarius dwarf though this is a very tiny event ($< 1\%$ of the Milky Way mass)
- Stars do not 'stay put' ; they can migrate long distances from their origin.
- Detailed analysis of stellar distributions in 7-D time, position and velocity) allow measure of mass and dynamical history-see next lecture

How Typical is the MW??

- the Milky Way is systematically offset by $\sim 1\sigma$ showing a significant deficiency in stellar mass, angular momentum, disk radius, and $[\text{Fe}/\text{H}]$ at a given V_{rot}
- The Milky Way had an exceptionally quiet formation history having escaped any major merger during the last 10 Gyr;
- Milky Way like galaxies correspond to only 7% of local spirals, - so onto the rest of the universe!
- But first, some detailed dynamics...

Galactic Rotation

- Then using a bit of trig

$$R(\cos \alpha) = R_0 \sin(l)$$

$$R(\sin \alpha) = R_0 \cos(l) - d$$

so

$$V_{\text{observed,radial}} = (\omega - \omega_0) R_0 \sin(l)$$

$$V_{\text{observed,tang}} = (\omega - \omega_0) R_0 \cos(l) - \omega d$$

then following the text expand $(\omega - \omega_0)$

around R_0 and using the fact that most of the velocities are local e.g. $R - R_0$ is small and d is smaller than R or R_0 (not TRUE for HI) and some more trig

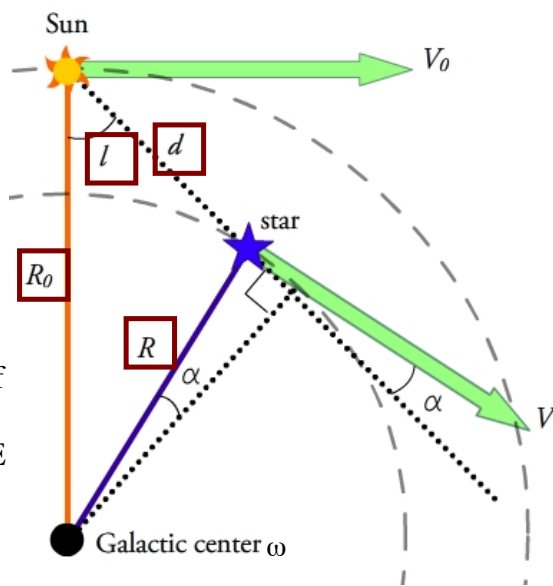
get

$$V_{\text{observed,radial}} = A \sin(2l); V_{\text{obs,tang}} = A \cos(2l) + B d$$

Where

$$A = -1/2 R_0 d \omega / dr \text{ at } R_0$$

$$B = -1/2 R_0 d \omega / dr - \omega$$



Galactic Rotation Curve- sec 2.3.1 S+G

Assume gas/star has a perfectly circular orbit

At a radius R_0 orbit with velocity V_0 ; another star/parcel of gas at radius R has a orbital speed $V(R)$

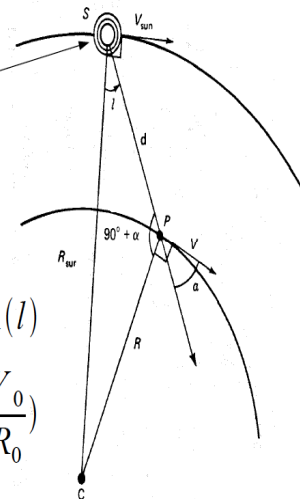
since the angular speed V/R drops with radius $V(R)$ is positive for nearby objects with galactic longitude $l < l < 90$ etc etc (pg 91 bottom)

- Galactic Rotation Curve

- At R_{sun} the lsr has a velocity of V_0
- A star at P has an apparent velocity of

$$1) V_r = V \cos(\alpha) - V_0 \sin(l)$$

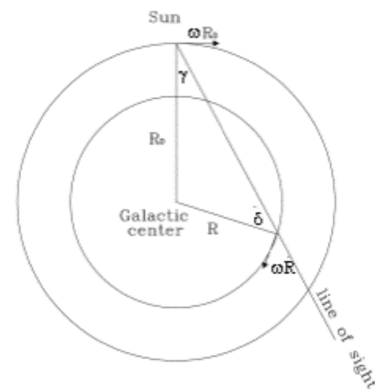
$$2) V_r = R_0 \sin(l) \left(\frac{V}{R} - \frac{V_0}{R_0} \right)$$



- Convert to angular velocity ω
- $V_{observed,radial} = \omega R (\cos \alpha) - \omega_0 R_0 \sin(l)$
- $V_{observed,tang} = \omega R (\sin \alpha) - \omega_0 R_0 \cos(l)$

In terms of Angular Velocity

- model Galactic motion as circular motion with monotonically decreasing angular rate with distance from center.
- Simplest physics: if the mass of the Galaxy is all at center angular velocity ω at R is $\omega = M^{-1/2} G^{1/2} R^{-3/2}$
- If looking through the Galaxy at an angle l from the center, velocity at radius R projected along the line of site minus the velocity of the sun projected on the same line is
- $V = \omega R \sin d - \omega_0 R_0 \sin l$
- $\omega =$ angular velocity at distance R
 $\omega_0 =$ angular velocity at a distance R_0
 $R_0 =$ distance to the Galactic center
 $l =$ Galactic longitude
- Using trigonometric identity $\sin d = R_0 \sin l / R$ and substituting into equation (1)



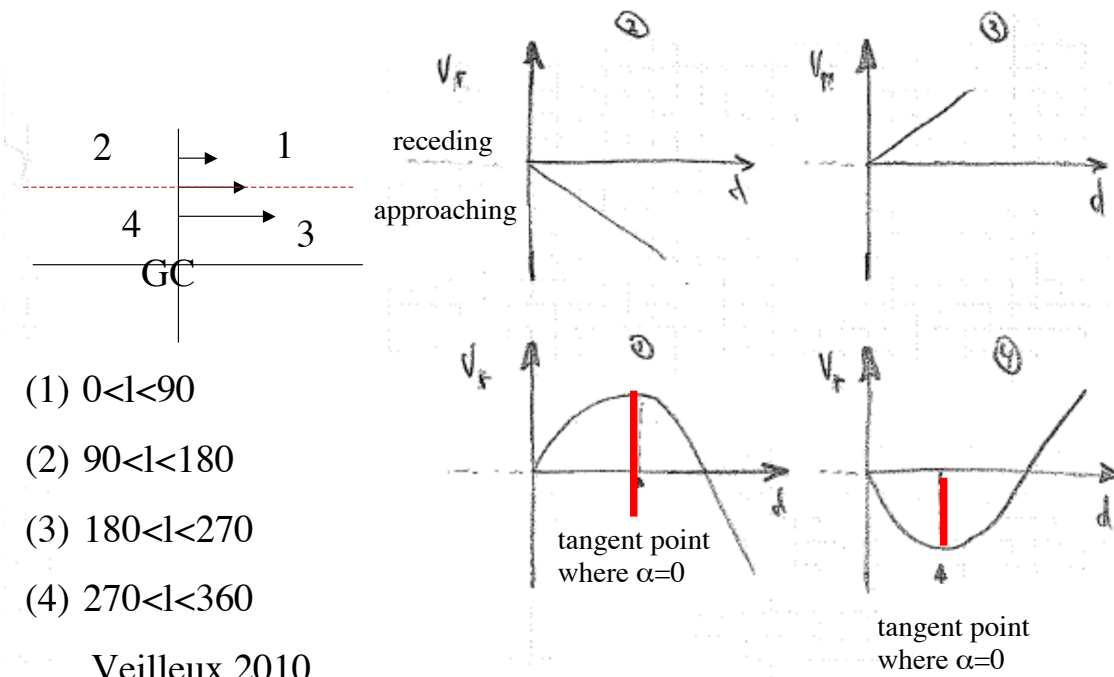
- $V = (\omega - \omega_0) R_0 \sin l$

<http://www.haystack.mit.edu/edu/undergrad/srt/SRTProjects/rotation.html>

Continued

- The tangential velocity $v_T = V_o \sin \alpha - V_o \cos l$
- and $R \sin \alpha = R_o \cos l - d$
- a little algebra then gives
- $V_T = V/R(R_o \cos l - d) - V_o \cos l$
- re-writing this in terms of angular velocity
- $V_T = (\omega - \omega_o)R_o \cos l - \omega d$

- For a reasonable galactic mass distribution we expect that the angular speed $\omega = V/R$ is monotonically decreasing at large R (most galaxies have flat rotation curves (const V) at large R) then get a set of radial velocities as a function of where you are in the galaxy
- V_T is positive for $0 < l < 90$ and nearby objects- if $R > R_o$ it is negative
- For $90 < l < 180$ V_T is always negative
- For $180 < l < 270$ V_T is always positive (S+G sec 2.3.1)



Oort Constants

- Derivation:
- for objects near to sun, use a Taylor series expansion of $\omega - \omega_0$

$$\omega - \omega_0 = d\omega/dR (R - R_0)$$

$$\omega = V/R; \quad d\omega/dR = d/dr(V/R) = (1/R)dV/dr - V/R^2$$

then to first order $V_r = (\omega - \omega_0)R_0 \sin l = [dV/dr - V/R](R - R_0) \sin l$; when $d \ll R_0$

$R - R_0 = d \cos l$ which gives

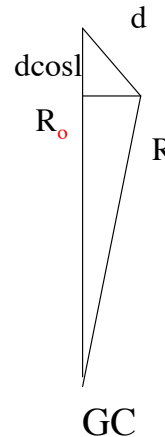
$$V_r = (V_0/R_0 - dV/dr) d \sin l \cos l$$

using trig identity $\sin l \cos l = 1/2 \sin 2l$

one gets the Oort formula

$V_r = A d \sin 2l$ where

$$A = \frac{1}{2} \left[\frac{V_0}{R_0} - \left(\frac{dV}{dR} \right)_{R_0} \right]$$



One can do the same sort of thing for V_T

Oort Constants

- For nearby objects ($d \ll R$) then (l is the galactic longitude)
 - $V(R) \sim R_0 \sin l (d(V/R)/dr)(R - R_0)$
 - $\sim d \sin(2l) [-R/2(d(V/R)/dr)] \sim d A \sin(2l)$
- A is one of 'Oorts constants'
- The other (pg 93 S+G) is related to the tangential velocity of a object near the sun $V_T = d[A \cos(2l) + B]$
- So, stars at the same distance r will show a systematic pattern in the magnitude of their radial velocities across the sky with Galactic longitude.
- A is the Oort constant describing the shearing motion and B describes the rotation of the Galaxy

$$A = \frac{1}{2} \left[\frac{V_0}{R_0} - \left(\frac{dV}{dR} \right)_{R_0} \right]$$

$$B = -\frac{1}{2} \left[\frac{V_0}{R_0} + \left(\frac{dV}{dR} \right)_{R_0} \right]$$

$$A + B = - \left(\frac{dV}{dR} \right)_{R_0} ; \quad A - B = \frac{V_0}{R_0}$$

$$A = -1/2 [R d\omega/dr]$$

Useful since if know A get kinematic estimate of d

Radial velocity $v_r \sim 2AR_0(1 - \sin l)$
 only valid near $l \sim 90$ measure
 $AR_0 \sim 115 \text{ km/s}$

Oort 'B'

- B measures 'vorticity' $B = -(\omega + \frac{1}{2} [R d\omega/dr]) = -\frac{1}{2} (V/R + dV/dR)$
 $\omega = A - B = V/R$; angular speed of Local standard of rest (sun's motion)

Oort constants are local description of differential rotation

Vaues

$A = 14.8 \text{ km/s/kpc}$

$B = -12.4$

Velocity of sun $V_0 = R_0(A - B)$

I will not cover epicycles: stars not on perfect circular orbits: see sec 3.2.3 in B&T

important point $\sigma_y^2 / \sigma_x^2 = -B/A - B$

Bovy and Tremaine continued- modern example of using Jeans and Poisson eq

- The radial Jeans equation for the disk is
- $$FR(R, Z) = \partial\Phi(R, Z)/\partial R = (1/v) (\partial (v\sigma_u^2)/\partial R) + (1/v) (\partial (v\sigma_{uW}^2)/\partial Z) + (\sigma_u^2 - \sigma_v^2 - V^2)/R$$
- where Φ is the gravitational potential, v is the tracer-density profile, σ_u^2 and σ_v^2 are the radial and azimuthal velocity dispersions squared, σ_{uW}^2 is the off-diagonal radial-vertical entry of the dispersion-squared matrix, and V is the mean azimuthal velocity;
- all of these quantities are functions of R and Z .
- The mean azimuthal velocity of a population of stars differs from the circular velocity due to the asymmetric drift. This offset arises because both the density of stars and the velocity dispersion typically decline with radius. This means that more stars with guiding centers at $R < R_0$ are passing through the solar neighborhood than stars with guiding centers $R > R_0$; the former are on the outer parts of their orbits, where their azimuthal velocity is less than the circular velocity