Expansion of the Universe

- Homogeneity + Isotropy
  - Relative position and orientation of galaxies cannot be changed by the expansion of the Universe

This means that if the present separation of two galaxies is d_0, then at time t we can write the separation as,

$$d(t) = d_0\, a(t)$$

where a(t) is the scale factor and is independent of position and only depends upon time.

The relative velocity of the two galaxies is

$$v = \dot{d} = d_0\, \dot{a}(t) = \frac{\dot{a}}{a} d$$

The definition of the Hubble constant is
v = Hd so,

$$H = \frac{\dot{a}}{a}$$

The present day value of the Hubble constant is H_0.

Sometimes you'll hear the term comoving coordinates. This is just the distances divided by a(t). So in these coordinates the position of a given galaxy's coordinates do not change with The expansion of the universe.

We can derive the evolution of the scale factor a using familiar Newtonian dynamics with two modifications from GR.

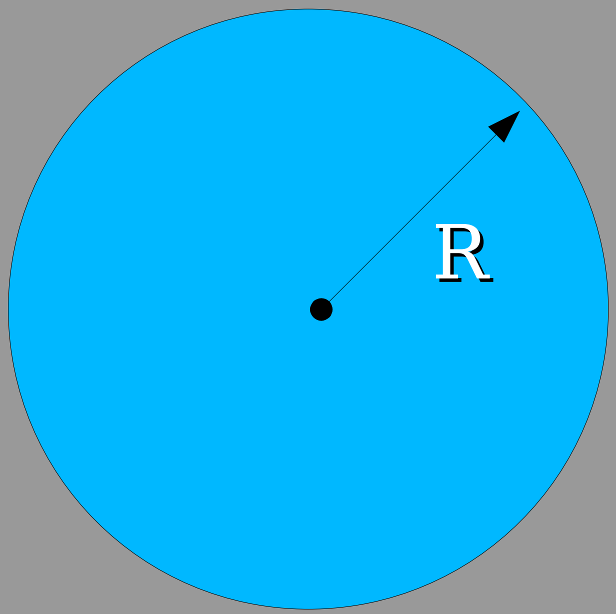
1) Birkoff's Theorem: The force due to gravity at a radius r is only determined by the mass interior to r.

2) Energy also contributes to the gravitational field.

$$\rho_m + \frac{u}{c^2}$$

Matter density   Energy density

Lets look at they evolution of a sphere in the universe with radius R,



Since the sphere will expand with the Universe R is a function of time
$R(t) = R_0 a(t)$

Since the time dependence is contained entirely in the scale factor a(t) the sphere can have any radius so we can make the radius small enough that space is approximately Euclidian.

The expansion of the sphere will slow due to the matter and energy enclosed,

$$\frac{d^2 R}{d^2 t} = -\frac{GM}{R^2}$$

Recall that the mass M must contain the contribution from the energy density. Radiation with energy density u has a pressure

$$P = \frac{1}{3} u$$

So we can write the total gravitating mass density as as

$$\rho = \rho_m + \frac{3P}{c^2}$$

The mass within our sphere is

$$M = \rho V = \frac{4}{3} \pi R^3 \rho$$

We can now substitute this into the equation for acceleration

$$\frac{d^2 R}{d^2 t} = -\frac{G}{R^2} \left(\frac{4}{3} \pi R^2 \rho\right)$$

Since R = R_0(t) we can write this as an equation describing the evolution of the scale factor

$$\ddot{a} = -\frac{4\pi G}{3} \left(\rho_m + \frac{3P}{c^2}\right) a$$

Notice that ρ_m is always > 0

Radiation pressure is also > 0

So the rhs of the equation is always negative and therefore we can never have a static solution!

In 1917 the fact that the universe is expanding was not known. Einstein “fixed” this problem by adding a cosmological constant (Λ).

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho_m + \frac{3P}{c^2}\right)a + \frac{\Lambda}{3}a$$

A positive cosmological constant accelerates the expansion

How important is the cosmological constant? For Λ to be important its magnitude must be comparable to the first term

$$\Lambda \approx 4\pi G \rho_m \approx 10^{-36} s^{-1}$$

for 10^{-30} gm/cm^3.

Note: Originally the cosmological constant was just that constant but there is know theoretical reason for that. Models where Λ is a function of time are called quintessence models.

Which terms in this equation drive the evolution of the universe?

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho_m + \frac{3P}{c^2}\right)a + \frac{\Lambda}{3}a$$

At early times the radiation pressure is much larger that the matter density

After that we enter a phase where the expansion is matter dominated

At late times (if $\Lambda \neq 0$) the comoslogical constant will drive the expansion

Lets first solve this for a matter dominated universe, radiation pressure can be ignored and no cosmological constant.

$$\ddot{a} = -\frac{4\pi G}{3}(\rho_m)a$$

This is a 2nd order differential equation so we need some boundary conditions, so lets say that at some time $a=a_0$ and $\rho=\rho_0$ then

$$\rho_m = \rho_0 \frac{a_0^3}{a^3}$$

Then substituting this

$$\ddot{a} = -\frac{4}{3}\pi G\left(\frac{\rho_0 a_0^3}{a^2}\right)$$

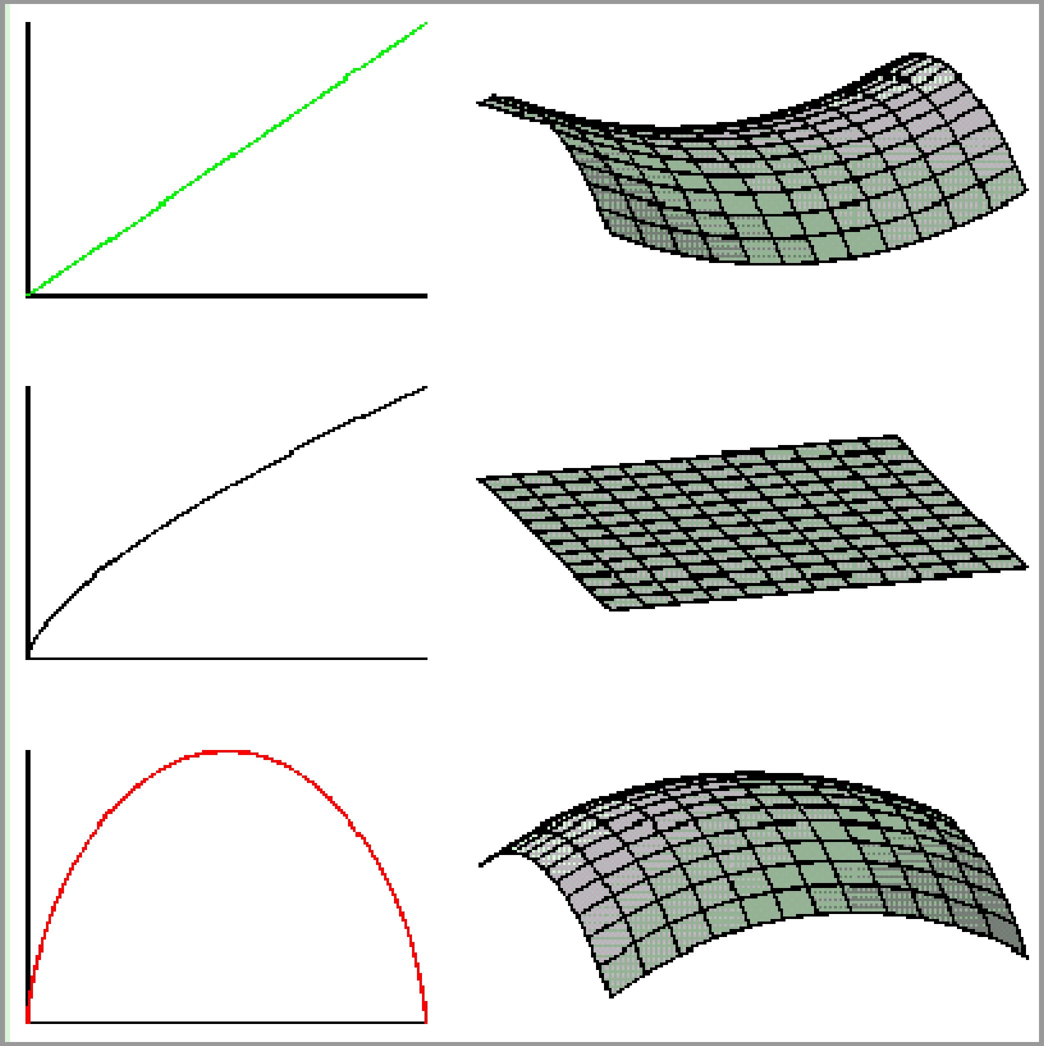
Integrating this once wrt time

$$\dot{a}^2 = -\frac{8}{3}\pi G\left(\frac{\rho_0 a_0^3}{a}\right) - kc^2$$

Where k is just a constant of integration but depending on its value we have three different scenarios...

- If $k>0$ then the universe will expand to some maximum radius turn around and collapse
  - Closed universe
- If $k<0$ then the expansion of the universe will slow but will reach a constant value
  - Open universe
- If $k=0$ then the expansion of the universe will slow but never stop
  - Flat universe

Note: This k is the same k we talked about last time.



Open

Flat

Closed

Data from the WMAP experiment indicates that the Universe is very close to the flat model.

Critical Density

What amount of matter would result in a flat matter dominated universe? Starting with

$$\dot{a}^2 = -\frac{8}{3}\pi G\left(\frac{\rho_0 a_0^3}{a}\right) - kc^2$$

Set a = a_0 ,

$$\frac{kc^2}{{a_0}^2} = -\frac{8}{3}\pi G\rho_0 - \left(\frac{\dot{a}_0}{a_0}\right)^2$$

Critical Density cont.

Recall that

$$H = \frac{\dot{a}}{a}$$

So now we get that

$$\rho_c \equiv \frac{3H_o^2}{8\pi G}$$

Where ρ_c is the critical density.

Critical Density cont.

For a Hubble constant of 71 km/s/Mpc we find that

$$\rho_c \approx 9\text{x}10^{-30}\ \text{gm/cm}^3.$$

We often talk about densities relative to the critical density

$$\Omega = \frac{\rho}{\rho_c}$$

Includes all sources

What about the evolution of the scale factor? If we assume a flat universe we have

$$\dot{a} = -\frac{8}{3}\pi G(\rho_0 a_0^3)\, a^{-\frac{1}{2}}$$

Integrating once

$$\int a^{\frac{1}{2}}\, da = \int A\, dt$$

$$a = \left(\frac{3}{2}A\right)^{2/3} t^{2/3}$$

Age of the Universe

With this in hand we can now find the predicted age of the universe

$$t = \frac{2}{3}\frac{a^{3/2}}{A} = \frac{2}{3}\frac{a^{3/2}}{\dot{a}\,a^{1/2}} = \frac{2}{3}H_0^{-1}$$

Assuming that the Hubble constant is 70 km/s/Mpc we find that the age of the Universe should be 9.3 Gyr.

A cosmological constant does make the solution more complicated lets first look at an “empty” universe. In this case our constant A=0.

$$\dot{a}=\sqrt{\frac{\Lambda}{3}}\,a$$

or

$$\frac{da}{a}=\sqrt{\frac{\Lambda}{3}}\,dt$$

Which has solutions

$$a=a_0 e^{\sqrt{\frac{\Lambda}{3}}\,t}$$

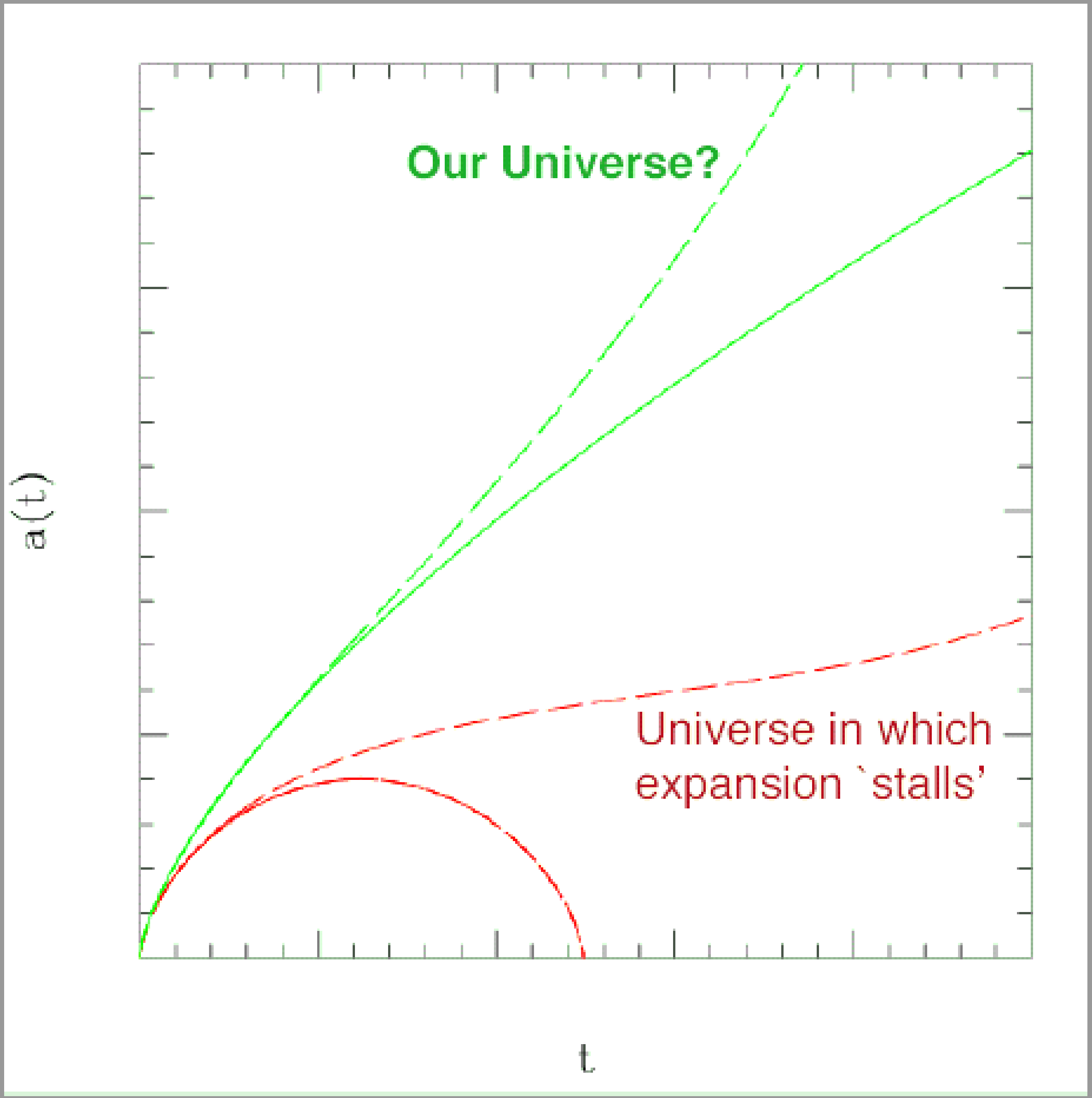
In short there are a considerable number of solutions that lie in between these extremes but we can constrain our solutions by using the WMAP data to say that the universe must be

- Flat
- With a positive cosmological constant
- a=0 at t =0 (standard Big Bang model

In this case we can show that

$$a=\left(\frac{3A^2}{2\Lambda}\right)^{1/3}\left[\cosh(\sqrt{3\Lambda}t)-1\right]^{1/3}$$

So at early times the universe expands as $t^{2/3}$ and it expands forever!



Now the flat universe is not equivalent to one in which the expansion slows and stops.

Solid lines
$\Lambda = 0$
dashed lines
$\Lambda > 0$

How Old is the Universe?

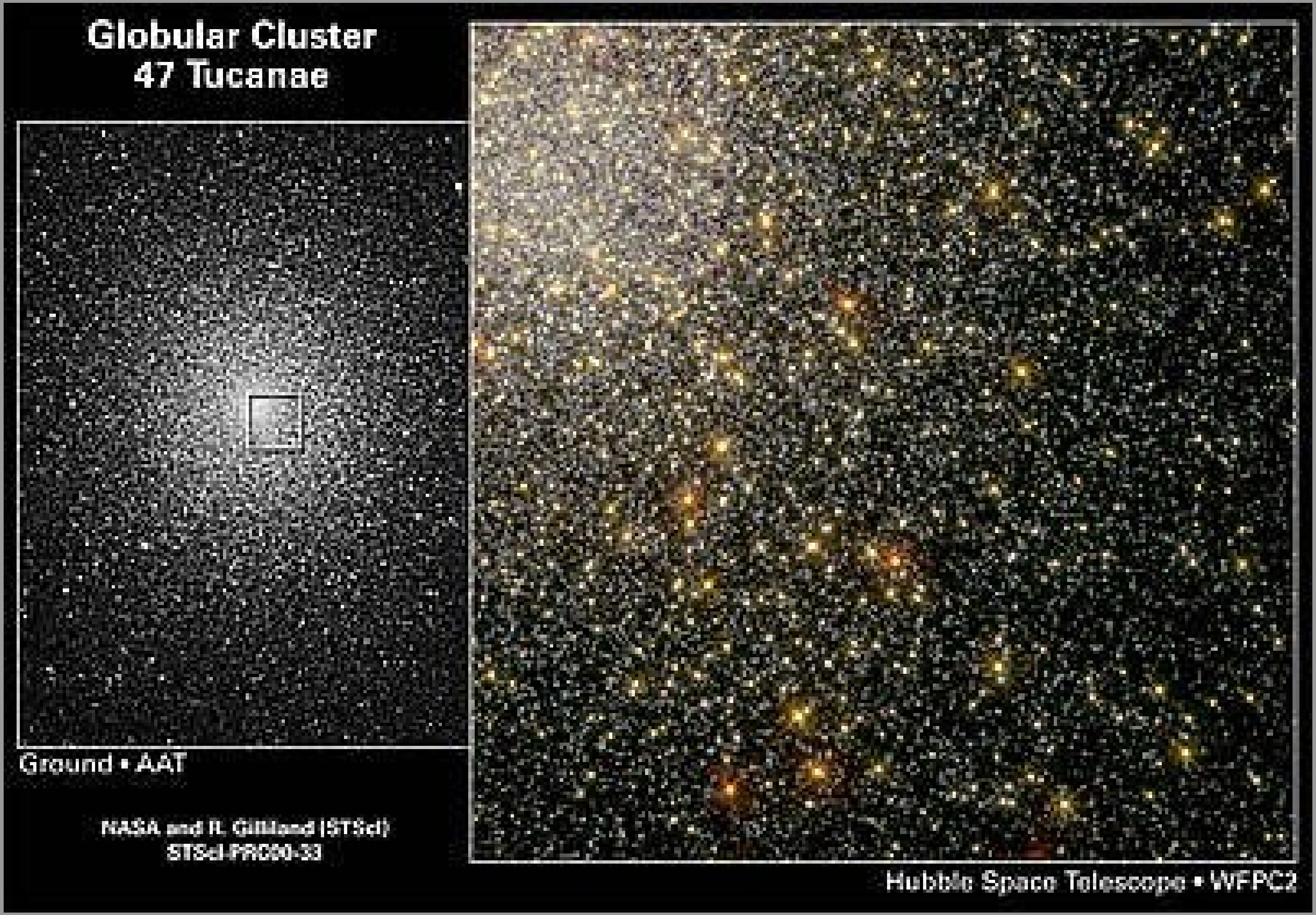
- One of the fundamental questions in cosmology is how old is the universe?
  - Ages of the oldest stars – globular clusters, white dwarfs, nucleocosmochronology
  - Age of galaxies at high redshift
  - Expansion age of the universe derived from cosmological parameters
  - Is there agreement?

Globular Clusters

- They are among the oldest objects in the galaxy, provide a lower limit on the age of the universe
  - Why is it a lower limit?
  - There are a number of uncertainties in these estimates, including errors in measuring the distances and uncertainties in the isochrones used to derive ages (i.e. stellar evolution models)
  - Inputs to stellar evolution models include – oxygen abundance [O/Fe], treatment of convection, helium abundance, reaction rates of $^{14}N + p \rightarrow {}^{15}O + \gamma$, helium diffusion, relating theoretical temperatures and luminosities to observed colors and magnitudes, and stellar opacities

Globular Clusters cont.

- We measure the age of a globular cluster by measuring the magnitude of the main sequence turnoff
- Then compare this to stellar evolutionary models of which estimate the surface temperature and luminosity of a stars as a function of time


Globular Cluster
47 Tucanae
Ground • AAT
NASA and R. Gilliland (STScI)
STScI-PRC00-33
Hubble Space Telescope • WFPC2

Schematic C-M diagram for a globular cluster

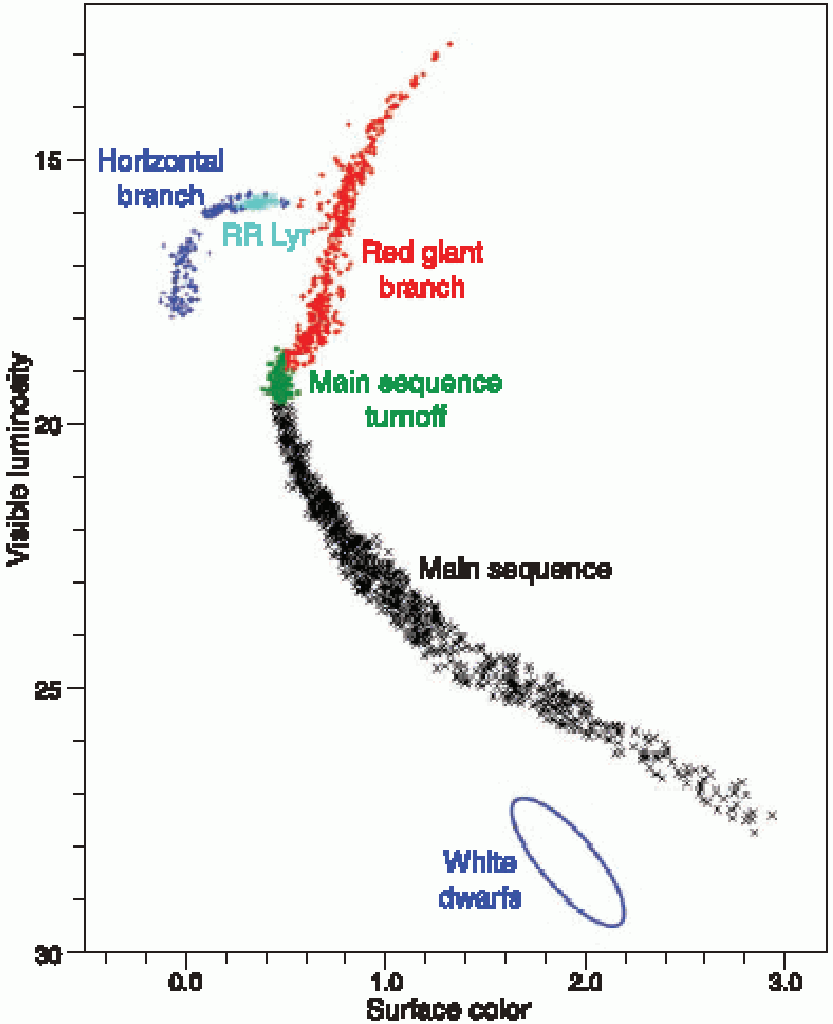
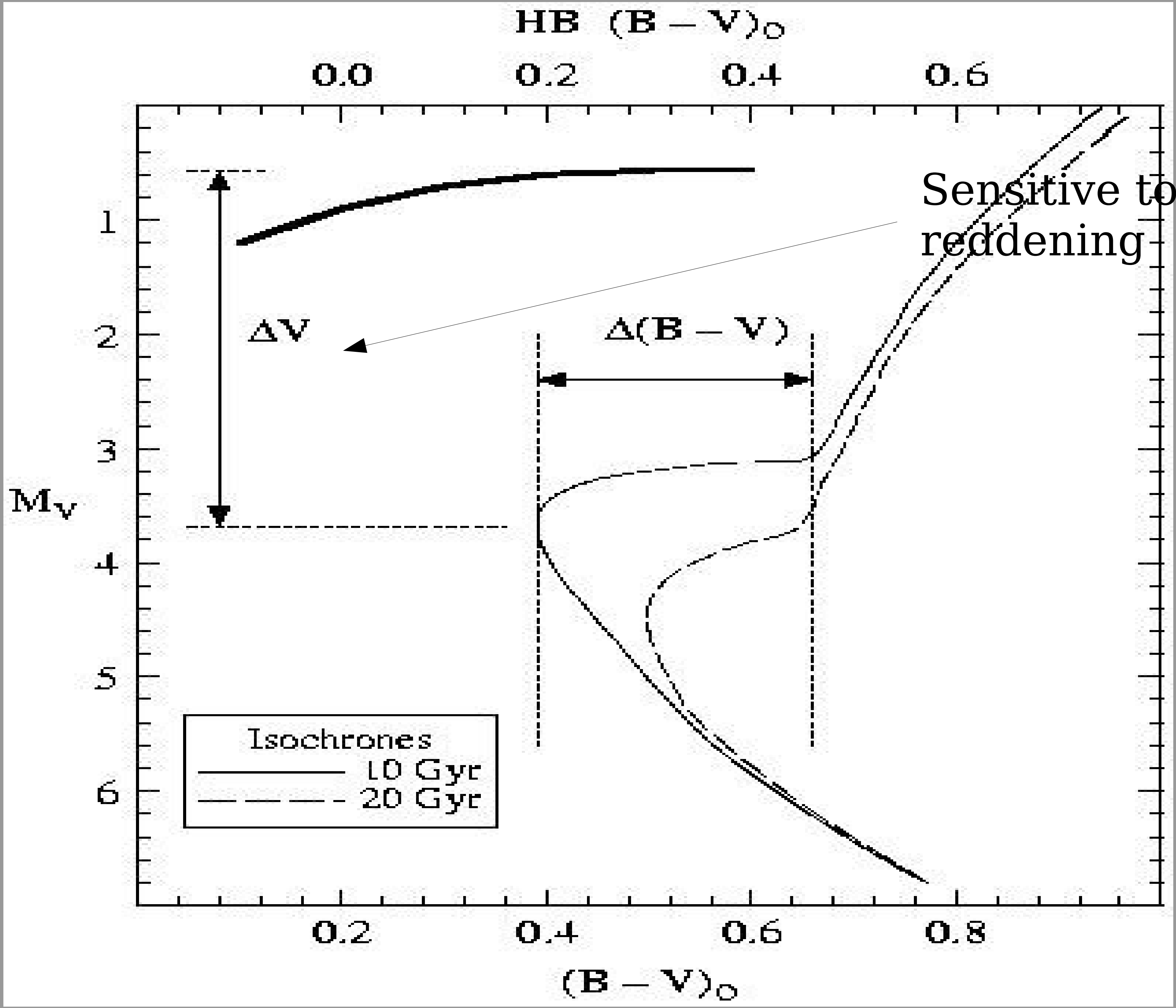
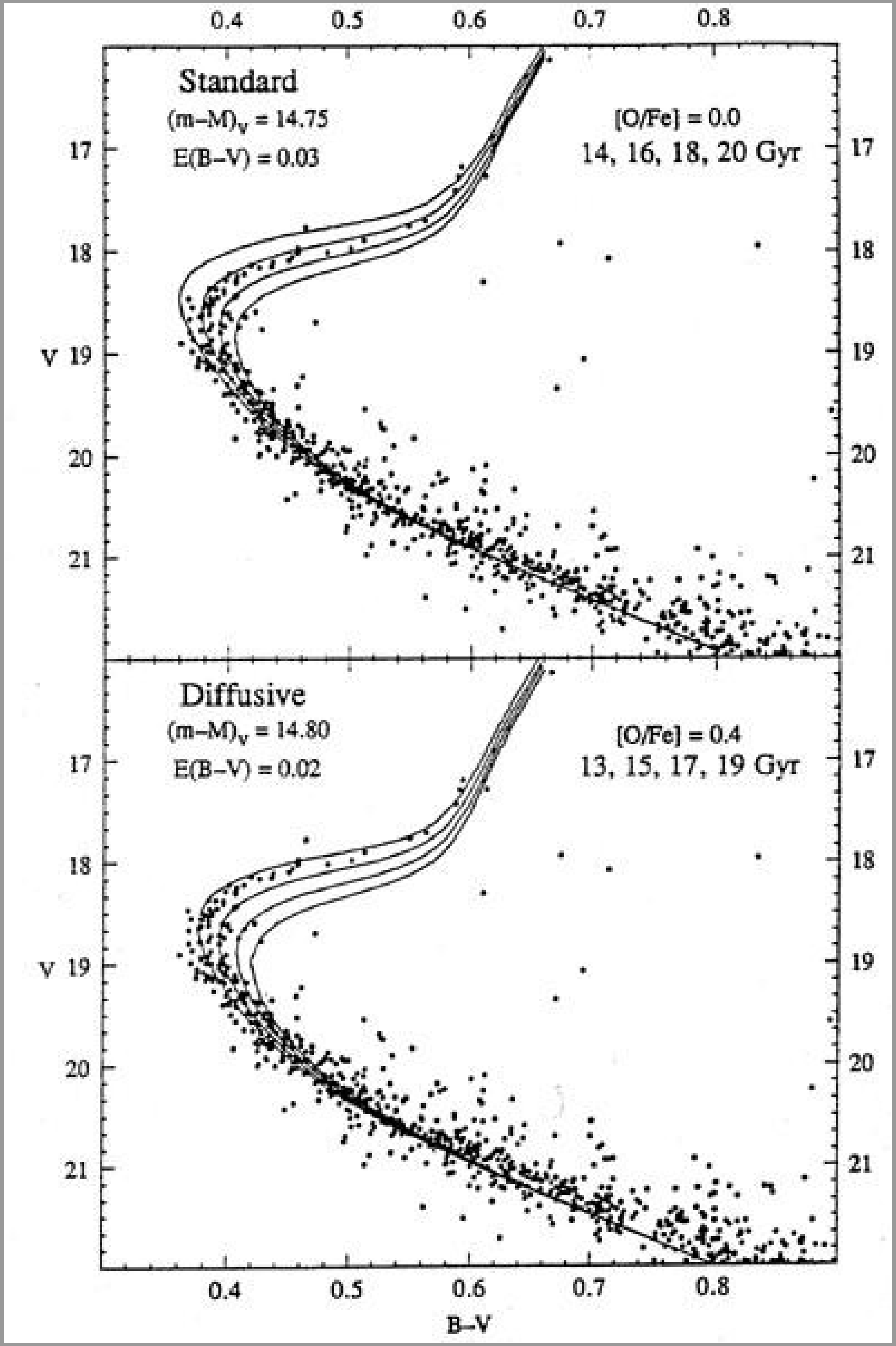
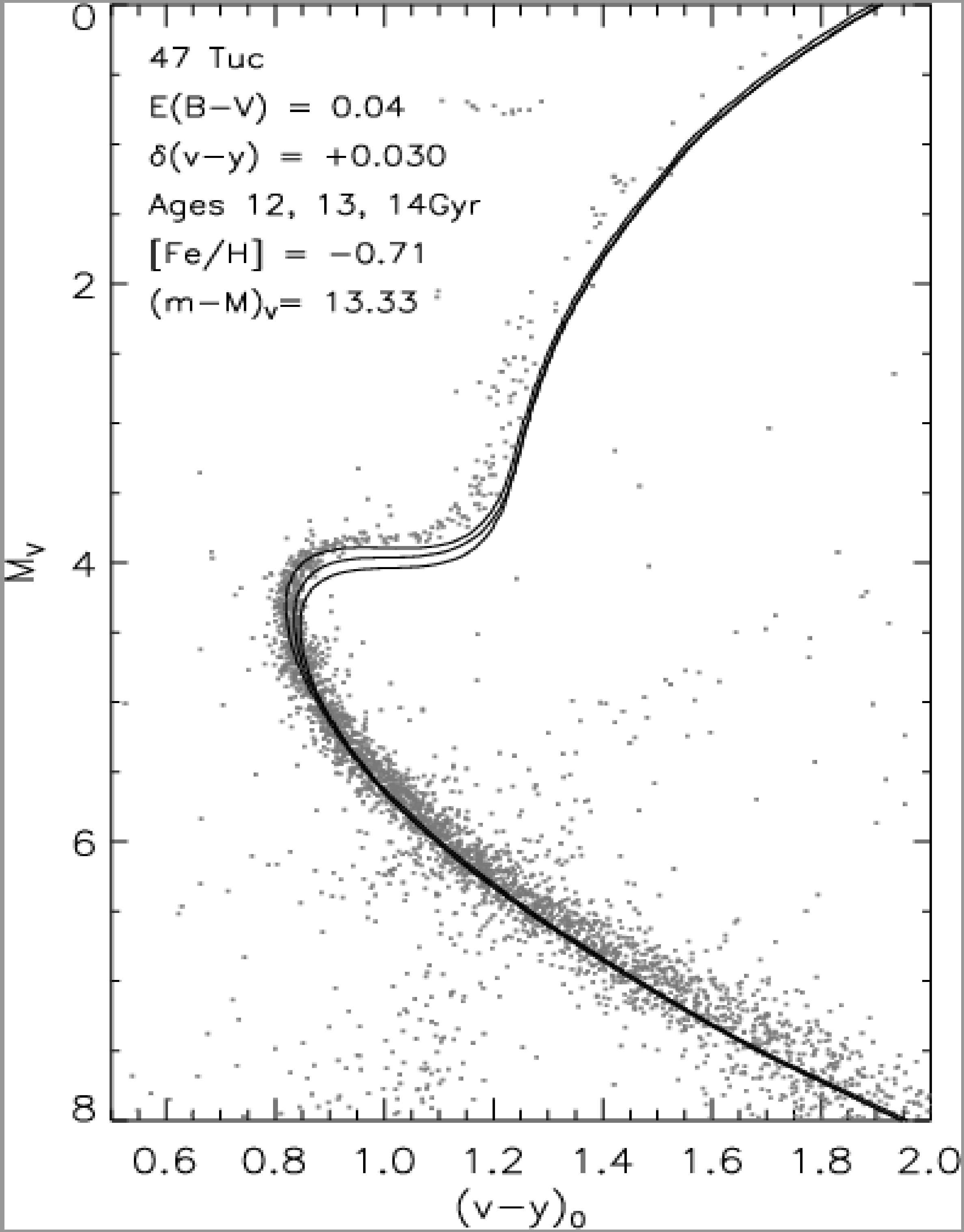


Fig. 2. A schematic color-magnitude diagram for a typical globular cluster (33) showing the location of the principal stellar evolutionary sequences. This diagram plots the visible luminosity of the star (measured in magnitudes) as a function of the surface color of the star (measured in B-V magnitude). Hydrogen-burning stars on the main sequence eventually exhaust the hydrogen in their cores (main sequence turnoff). After this, stars generate energy through hydrogen fusion in a shell surrounding an inert hydrogen core. The surface of the star expands and cools (red giant branch). Eventually the helium core becomes so hot and dense that the star ignites helium fusion in its core (horizontal branch). A subclass is unstable to radial pulsations (RR Lyrae). When a typical globular cluster star exhausts its supply of helium, and fusion processes cease, it evolves to become a white dwarf.


HB (B – V)o
0.0
0.2
0.4
0.6
Sensitive to reddening
ΔV
Δ(B – V)
1
2
3
4
5
6
MV
Isochrones
10 Gyr
20 Gyr
0.2
0.4
0.6
0.8
(B – V)o

The effects of He diffusion on theoretical isochrones (Chaboyer 1992)





Grundahl. Stetson & Andersen (2002)

Globular Cluster Ages

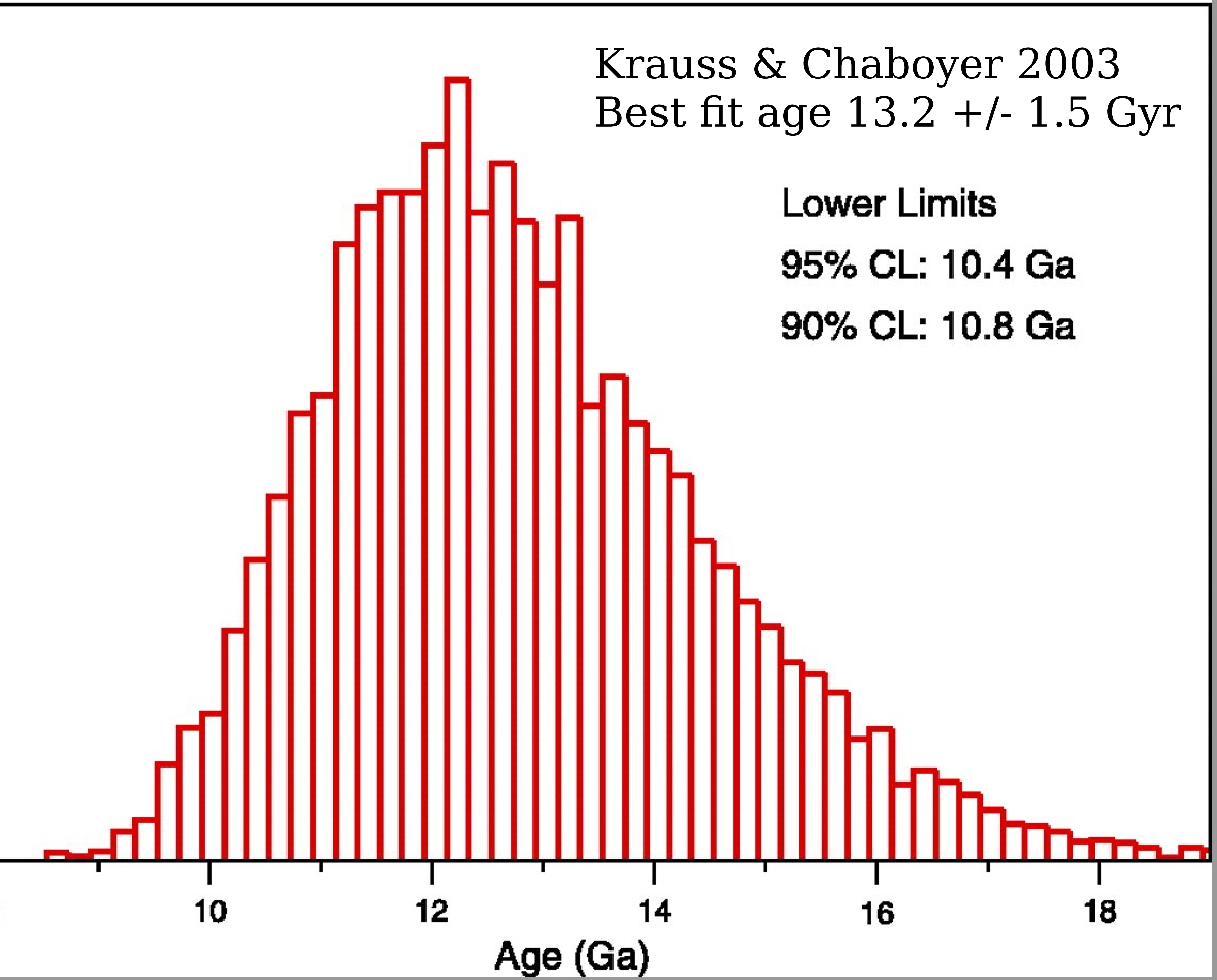
- NGC 6397 is 13.9 +/- 1.1 Gyr old
- NGC 6752 is 13.8 +/- 1.1 Gyr old
  - halo globular clusters
  - No helium diffusion
- 47 Tuc is 11.3 +/- 1.1 Gyr old
  - disk globular cluster
- Adding He diffusion
  - Ages are 13.4 +/- 0.8 (random errors) +/- 0.6 (systematic errors) Gyr for the oldest clusters

Gratton et al (2003)

Error Budget

Parameter	Error %
Mv(RR) (distance)	16
[O/Fe]	7
Convection physics	5
He diffusion	4
$^{14}N \rightarrow {}^{15}O$ rates	3
Color transfomation	3

Krauss & Chaboyer 2003
Best fit age 13.2 +/- 1.5 Gyr



GC results

- So globular clusters are between ~11.7 and 14.7 Gyr old
- Since it probably takes 1-2 Gyr for galaxies to form, we must to add that to the GC ages to get the age of the universe
- Previously estimates were closer to 13-17 Gyr old, what changed?
  - Distances to globular clusters increased by ~10% based on the Hipparcos calibration of the absolute magnitudes of subdwarfs (lowers ages by ~20%)
  - Inputs to stellar evolutionary models
  - Now the younger ages more compatible with ages estimated from expansion of universe

Cooling of White Dwarfs

- White dwarfs are the end stage of stellar evolution for stars with initial masses $<8\ M_{\odot}$
- They are supported by the pressure of degenerate electrons and are slowly cooling and fading as they radiate (no internal heat src)
- We can use the luminosity of the faintest WDs in a cluster to estimate the cluster age by comparing the observed luminosities to theoretical cooling curves
- Theoretical curves are subject to uncertainties related to the core composition of white dwarfs and the detailed radiative transfer calculations which are more uncertain as the star cools
- These stars are often at the limit of detectability and thus require long exposures & in clusters you need HST data to resolve them

Cooling of White Dwarfs cont.

We will assume that WD's have no internal heat sources or sinks so

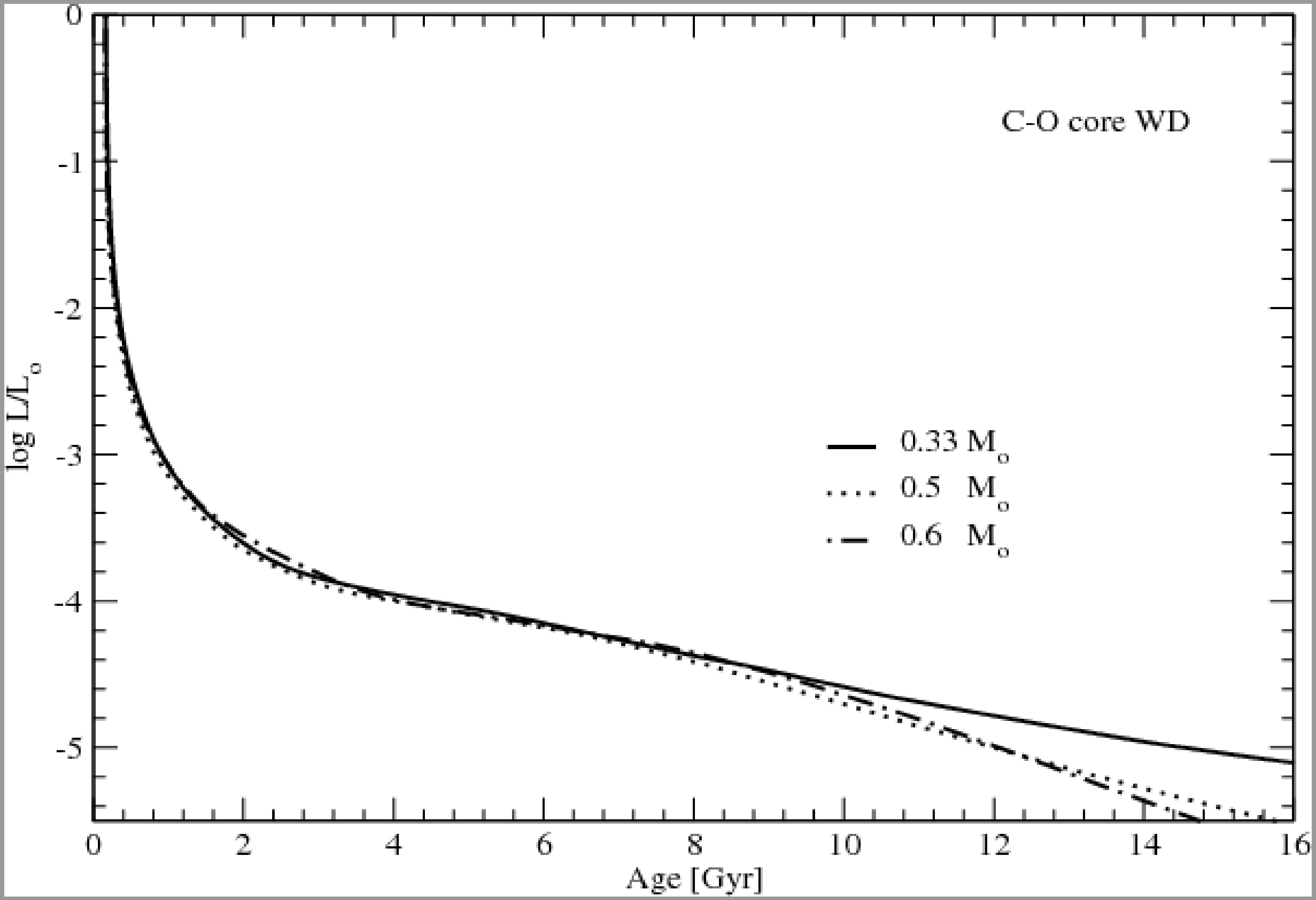
L = - dE(t)/dt, If the energy is just internal heat capacity of a non-relativistic degenerate gas then we can show

$$L = \frac{-dE_{th}(T)}{dT}\frac{dT}{dt}$$

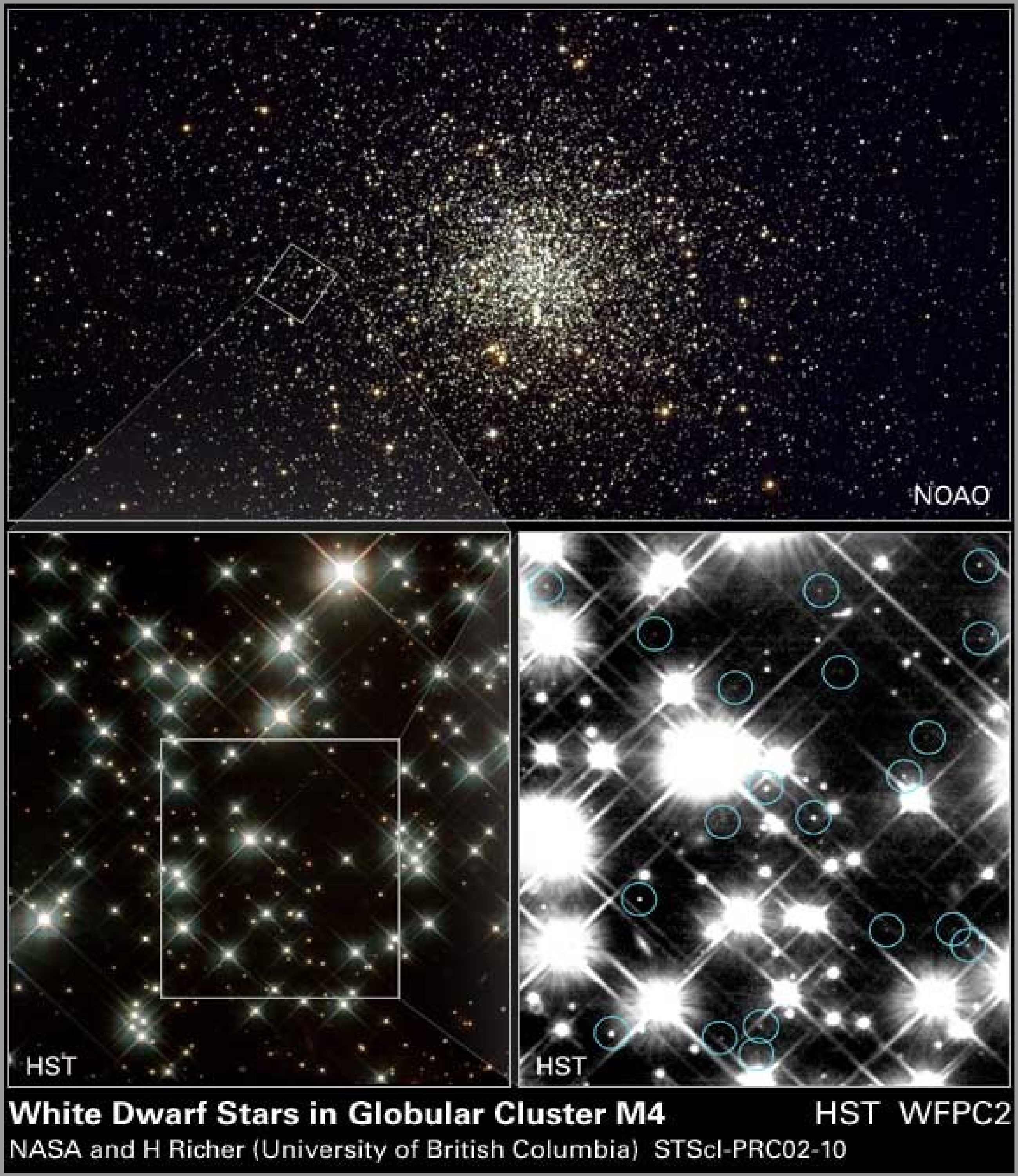
We can rewrite this as

$$L_{cool} \sim t^{-5/7}$$

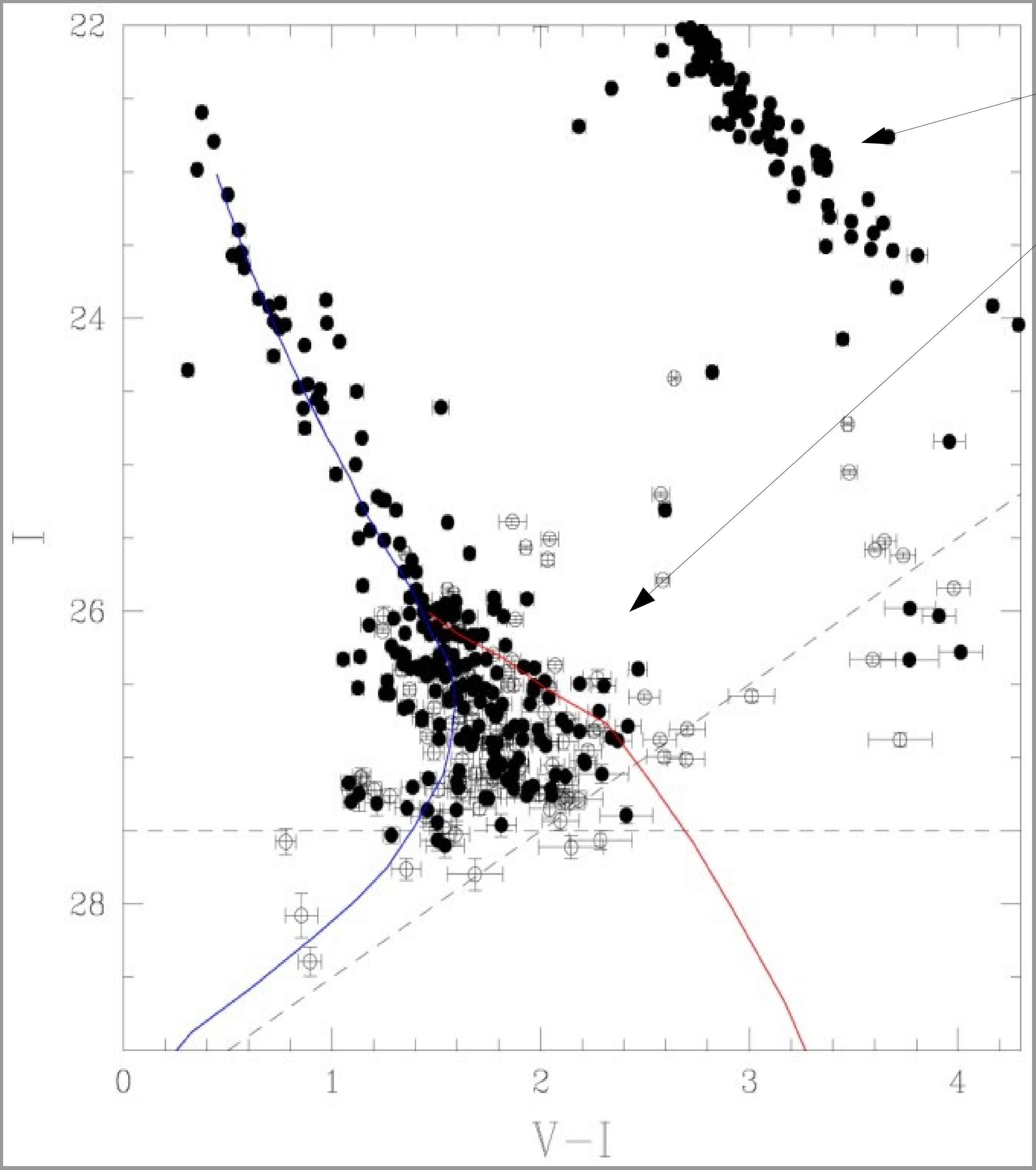
White Dwarf cooling



Prada Moroni&
O. Straniero 2009
A&A



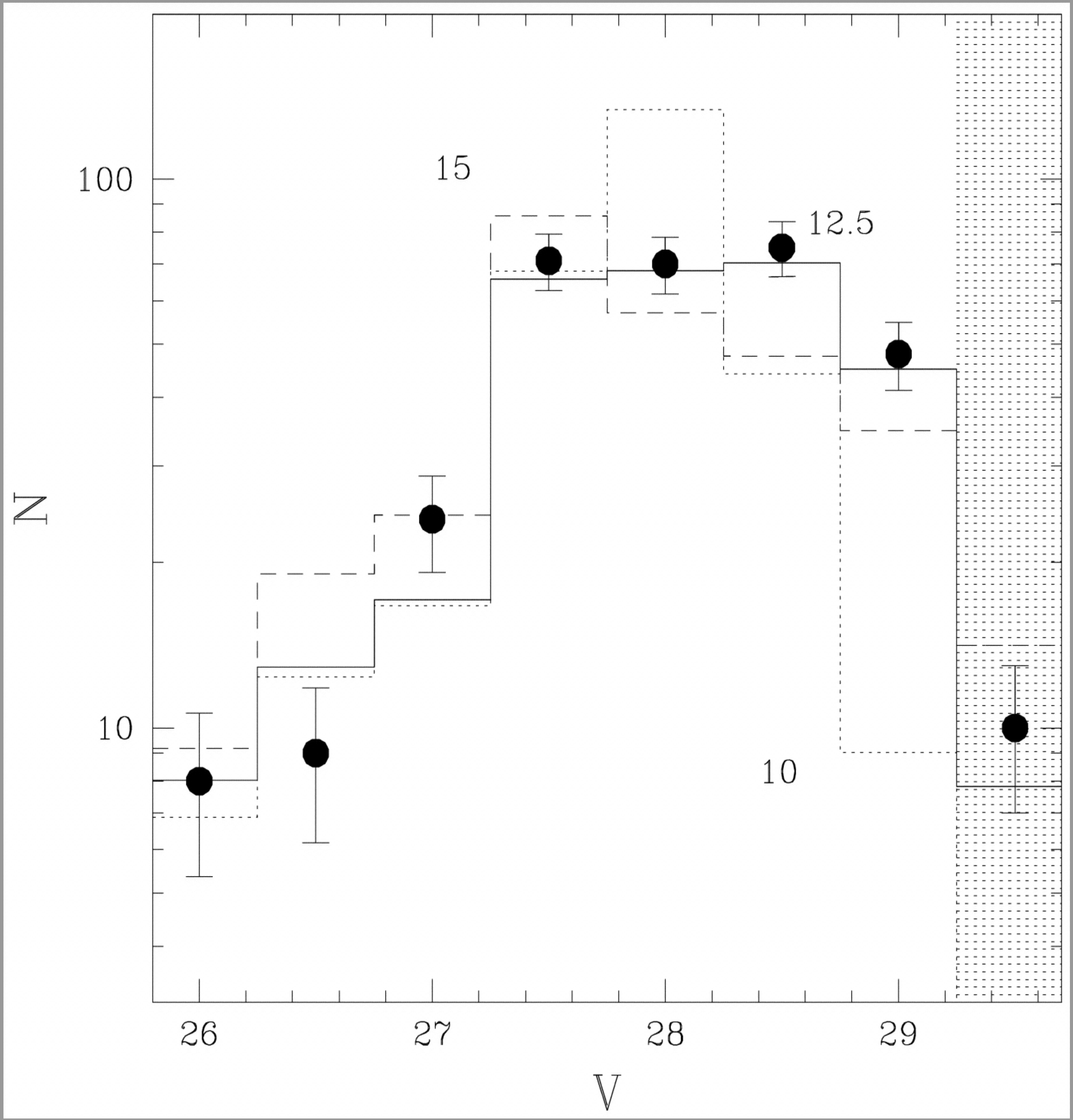
White Dwarfs in M4 ~123 orbits of HST time and reaches a limiting magnitude of V=30!



Main sequence

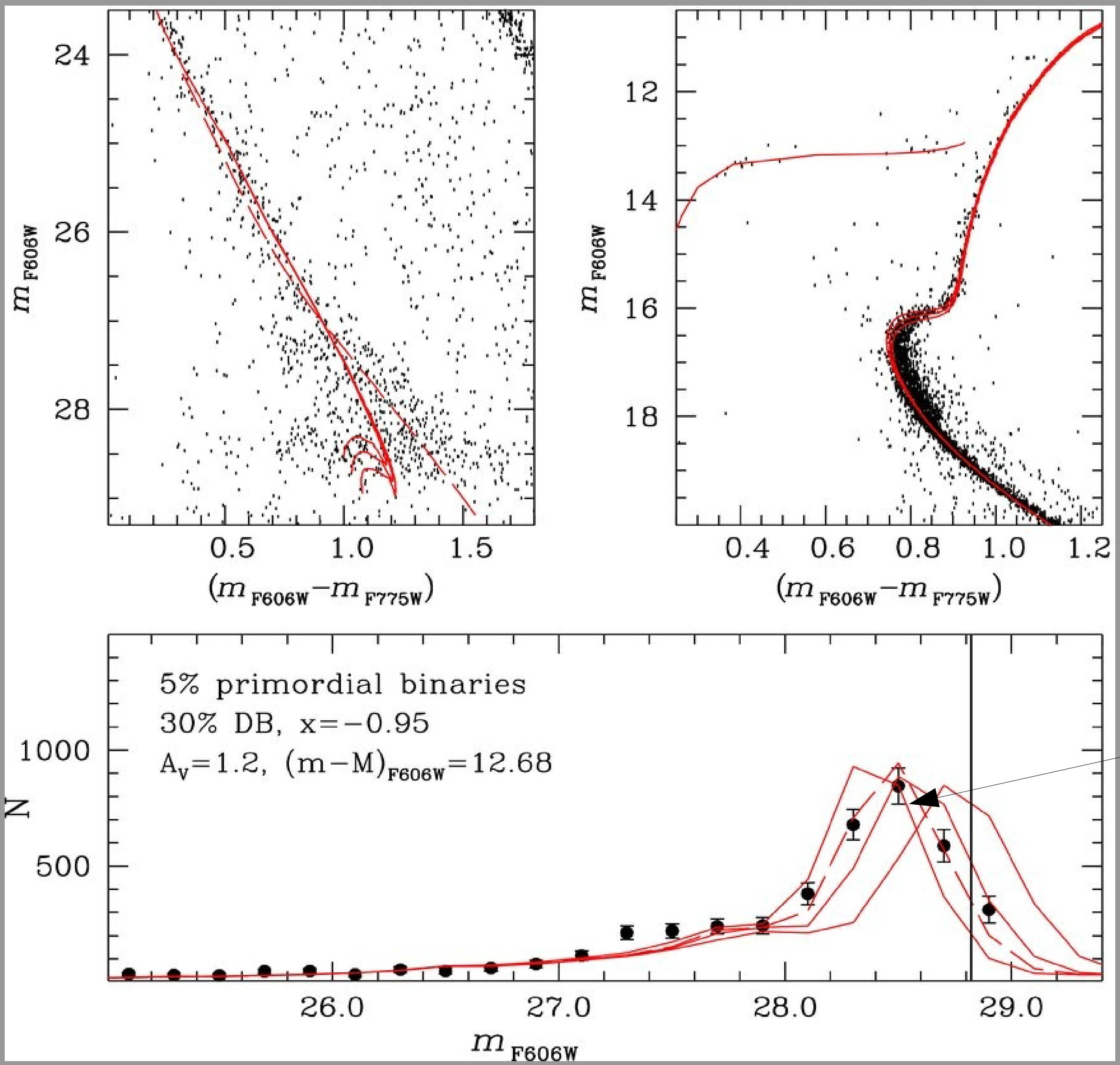
White Dwarfs

WD's can have either a hydrogen outer atmosphere (blue curve) or He outer atmosphere (red curve) Hansen et al. 2002.



WD luminosity function and theoretical predictions for different ages (Hansen et al. 2002)

WD's in M4



Bedin et al. 2009

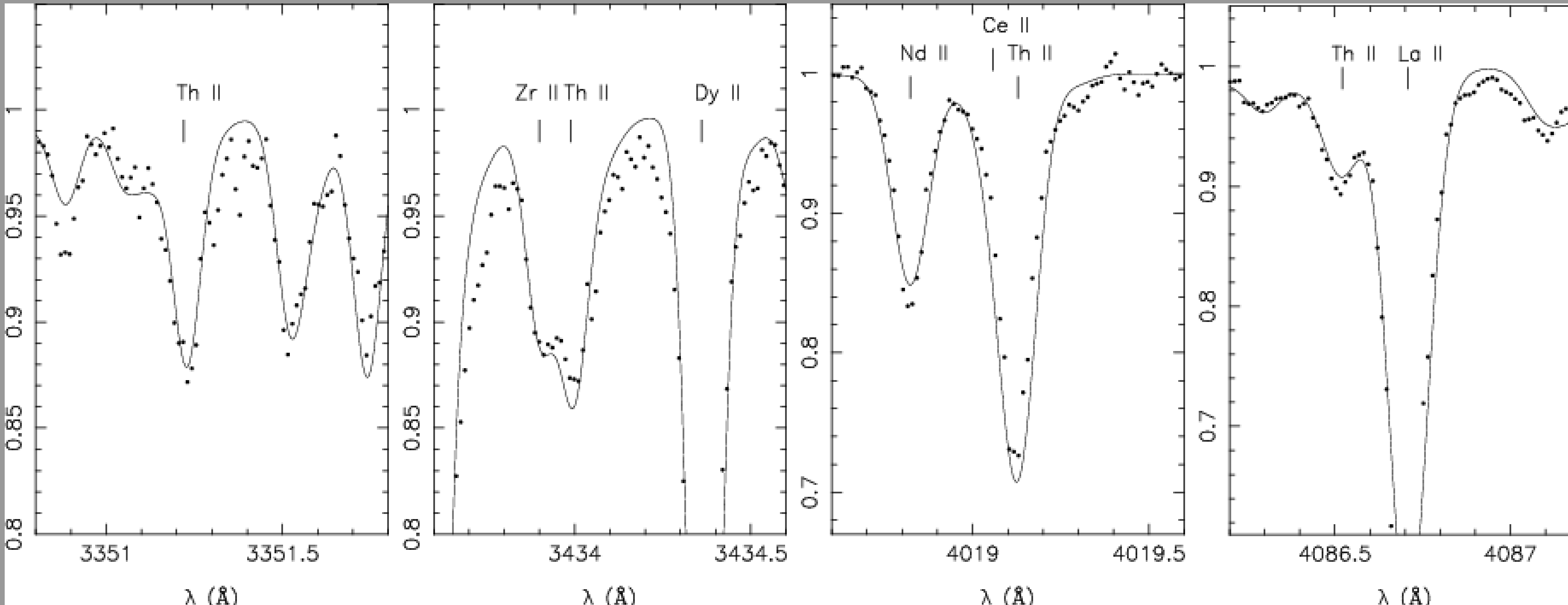
11, 11.6, 12 & 13Gyr

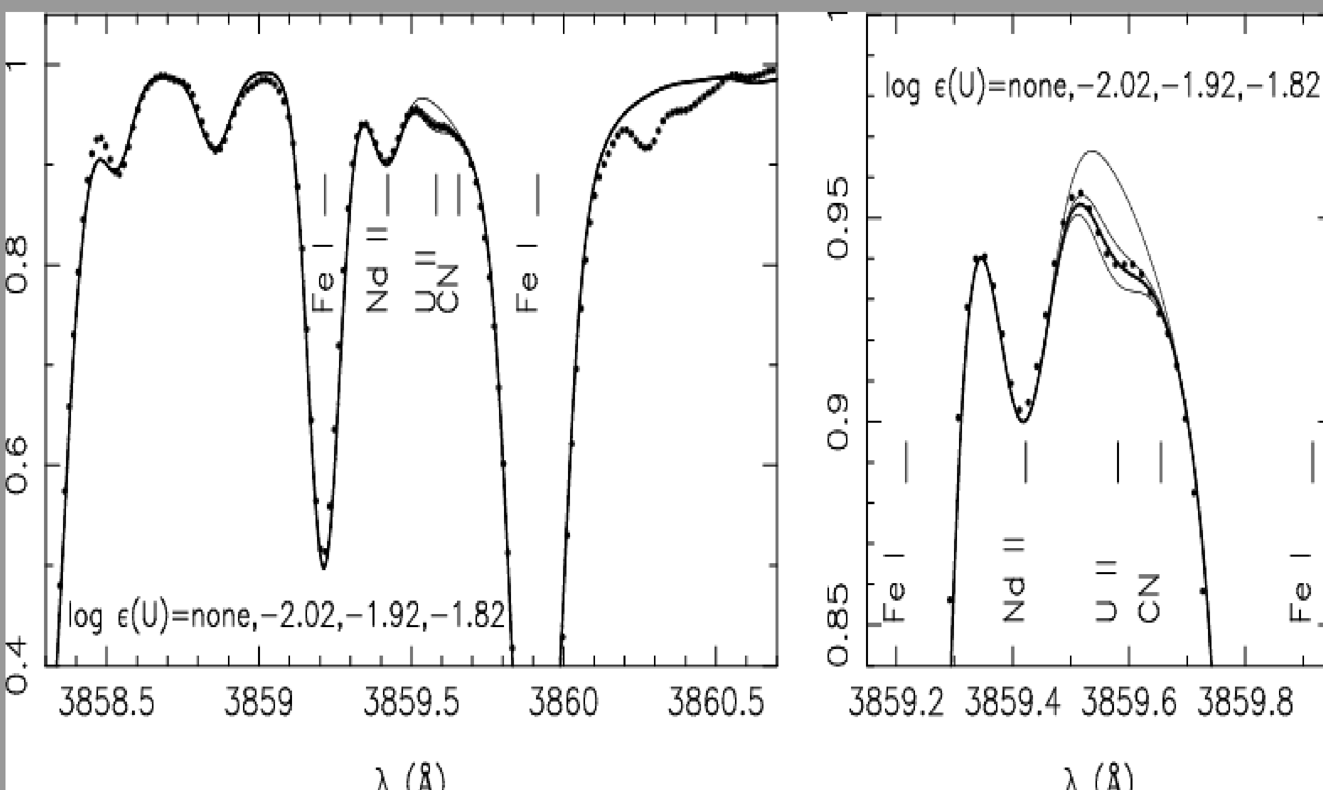
Cooling of White Dwarfs cont.

- The age of the GC M4 is 12.7 +/- 0.7 Gyr from the white dwarf cooling curves (Hansen et al. 2002)
- The errors don’t take into account uncertainties in the theoretical cooling calculations so the true error is larger
- This age is consistent with the ages of GCs found from the main sequence turnoff luminosities
- This has been done for one globular cluster, and more need to be measured, but this is very time consuming.

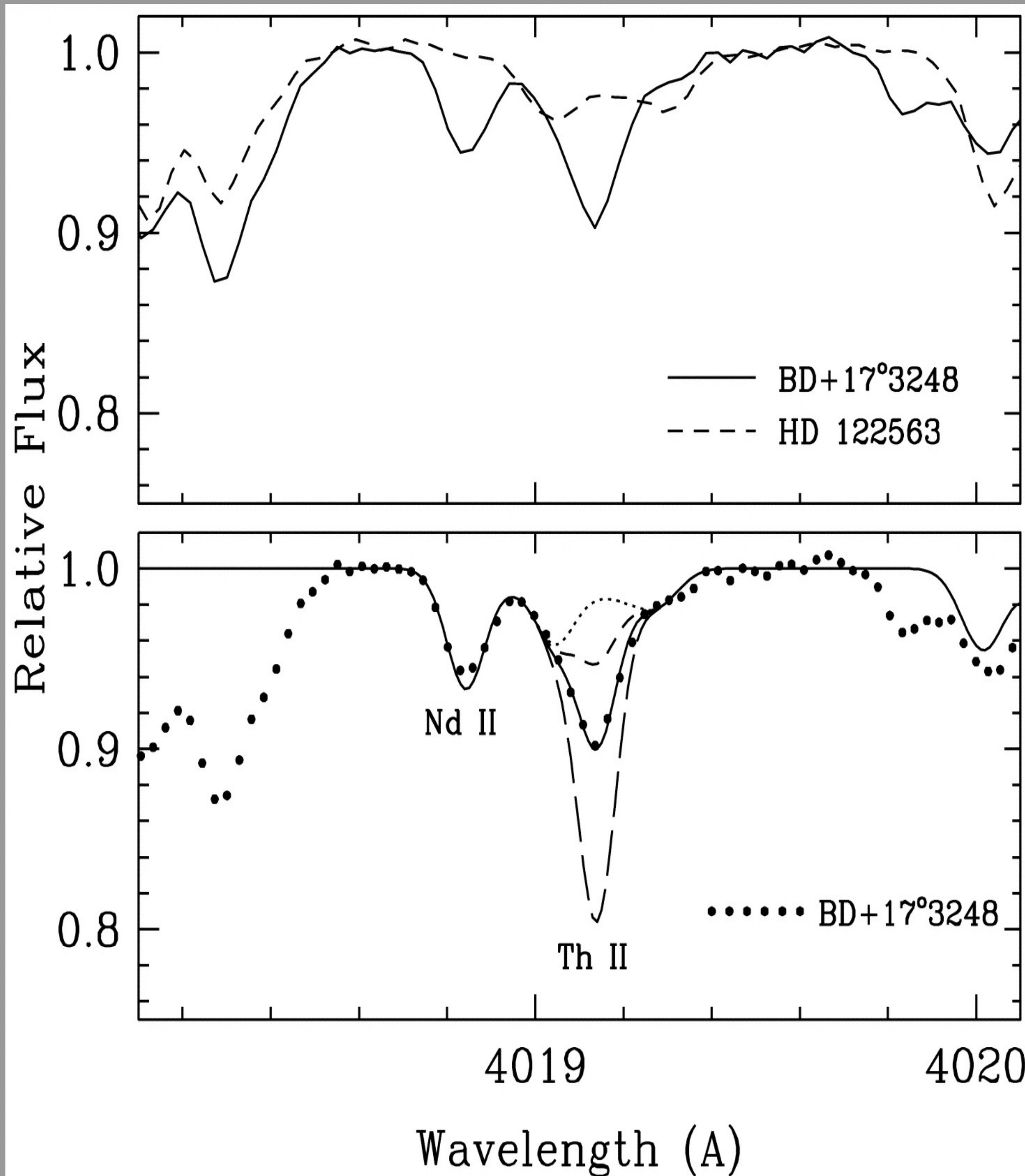
Nucleocosmochronology

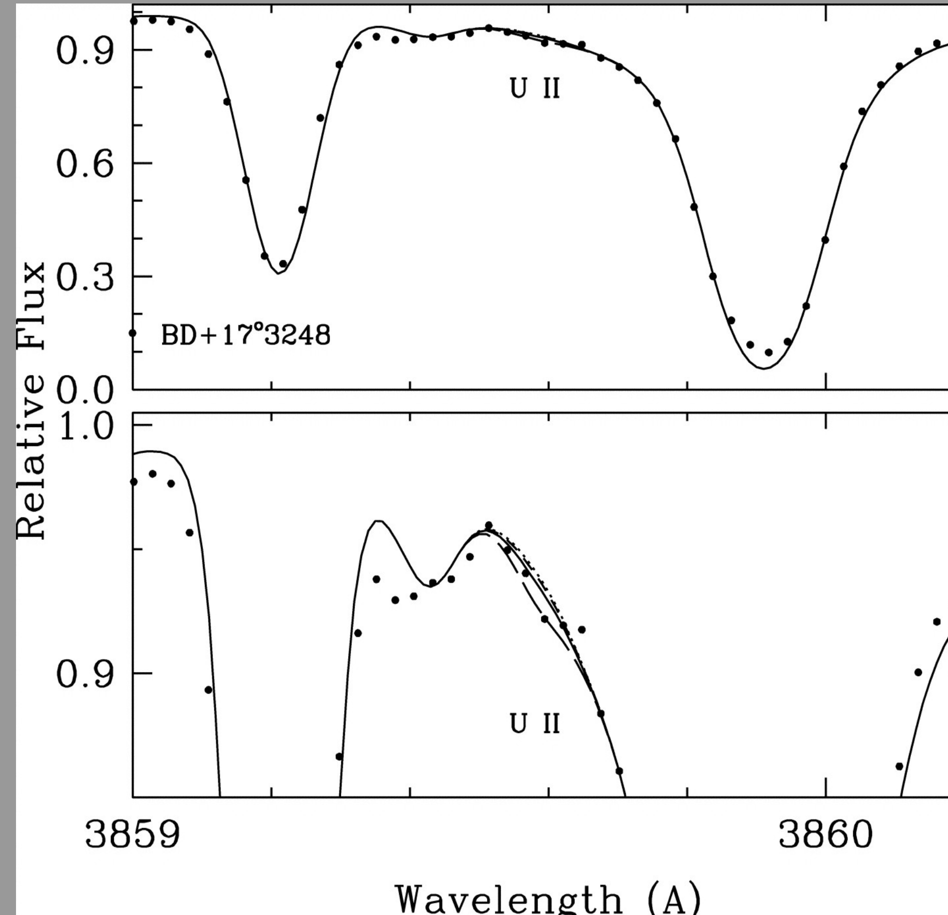
- Can use the radioactive decay of elements to age date the oldest stars in the galaxy
- This has been done with ^{232}Th (half-life = 14 Gyr) and ^{238}U (half-life = 4.5 Gyr) and other elements
- Measuring the ratio of various elements provides an estimate of the age of the universe given theoretical predictions of the initial abundance ratio
- This is difficult because Th and U have weak spectral lines so this can only be done with stars with enhanced Th and U (requires large surveys for metal-poor stars) and unknown theoretical predictions for the production of r-process (rapid neutron capture) elements





Th and U abundances
of CS 31082-001
Hill et al (2002)





Thorium and uranium measurements of BD+17 3248
Cowan et al (2002)

TABLE 4

CHRONOMETRIC AGE ESTIMATES FOR BD +17°3248

Chronometer Pair	Predicted	Observed	Age (Gyr)	Solar[a]	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	≥13.4	2.32	11.0
U/Ir	0.05036	0.0045	≥15.5	0.0369	13.5
U/Pt	0.013	0.0019	≥12.4	0.01846	14.6

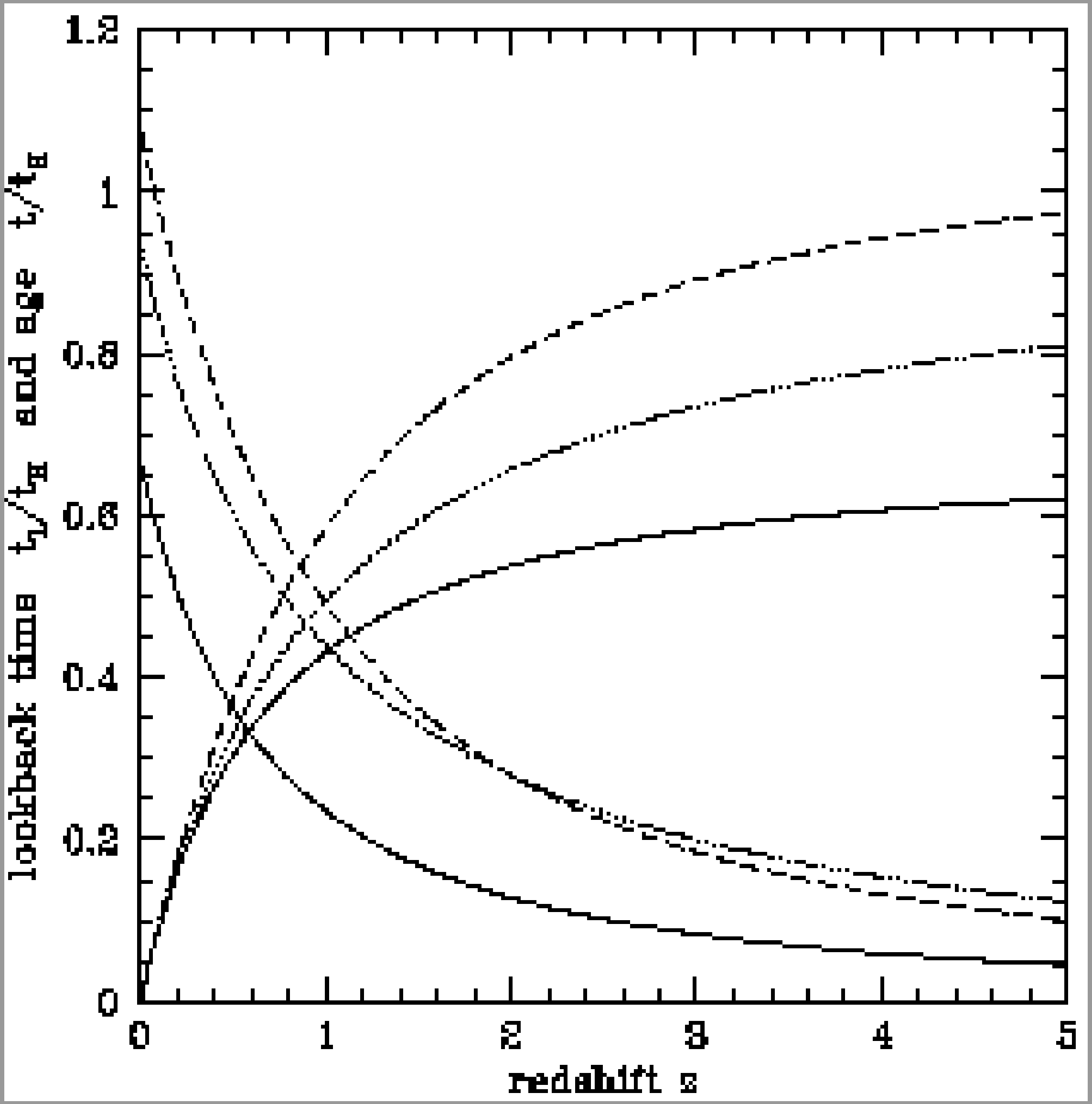
[a] From Burris et al. 2001.

Note the large range of ages 10 – 21.7 Gyrs, the mean is 13.8 but how significant is it? Cowan et al (2002)

Nucleocosmochronology cont.

- Uranium has been detected in two stars
  - BD+17 3248
  - CS 31082-001
- Age of BD+17 3248 13.8 ± 4 Gyr
- For CS 31082-001
  - 12.5 ± 3 Gyr Uranium
  - 14.0 ± 2.4 Gyr U/Th

We can also look at distant galaxies



Ω_M,Ω_Λ=0.05,0

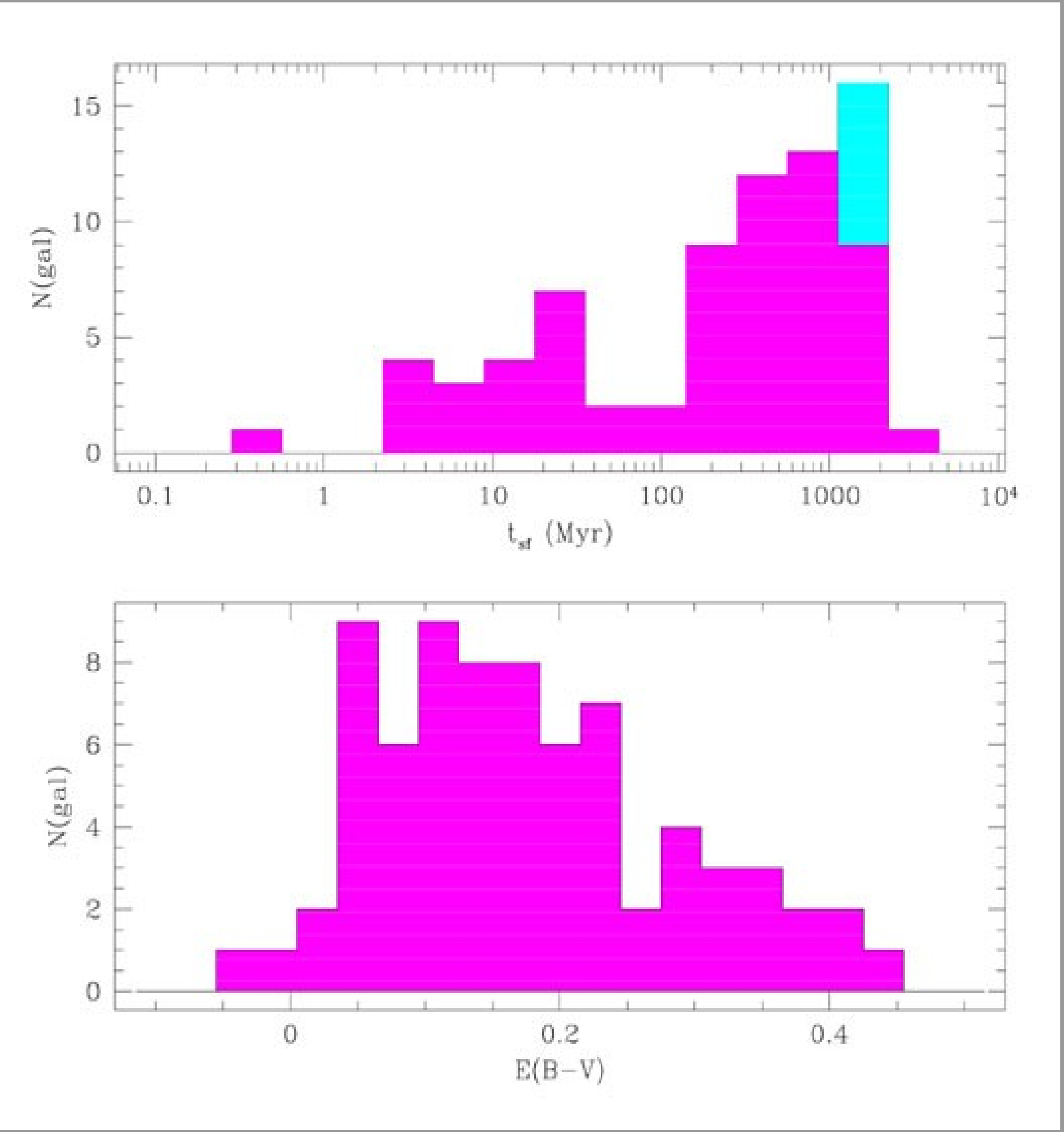
Ω_M,Ω_Λ=0.2,0.8

Ω_M,Ω_Λ=1,0

For Ω_M,Ω_Λ=0.05,0, H_0=70: age=12.5 Gyr
For Ω_M,Ω_Λ=1,0, H_0=70: age=9.3 Gyr
For Ω_M,Ω_Λ=0.2,0.8, H_0=70: age=15.02 Gyr

Ω_M,Ω_Λ=0.3,0.7, H_0=70: age=13.46 Gyr

- If we can measure the age of distant galaxies we can determine a limit on the age of the universe since $t_L = t_0 - t(z)$ so $t_0 = t_L + t(z)$
- An elliptical at z=1.55 and it is ~3.5 Gyr old based on its spectrum
- We also observe large numbers of Lyman-break galaxies at z=3 – 3.5 which appear to be ~320 Myr old. They also span a range of ages ...
- If $\Omega_M,\Omega_\Lambda=0.3,0.7$ and $H_0=70$ then $t_L(z=1.5)=9.3$ Gyr and $t_L(z=3)=11.4$ Gyr. So $t_0 > 12.8$ or 11.7 Gyr.



Stellar population ages of Lyman Break Galaxies Shapley et al (2001)

Median age=320Myr
20% > 1 Gyr

Age of the Universe

- Observed ages of the oldest objects in the universe are between 12-15 Gyr old
- Expansion age of the universe is 13.46 Gyr old for current best model with a cosmological constant (Ω_M,Ω_Λ=0.3,0.7 and H_0=70).
- These ages did NOT agree ~10 years ago when the universe appeared to be younger than the globular clusters!!