# **Cosmological Evolution of Galaxies**

Isaac Shlosman

Department of Physics & Astronomy, University of Kentucky Lexington, KY 40506-0055, USA shlosman@pa.uky.edu

## ABSTRACT

I review the subject of the cosmological evolution of galaxies, including different aspects of growth in disk galaxies, by focussing on the angular momentum problem, mergers, and their by-products. I discuss the alternative to mergerdriven growth – cold accretion and related issues. In the follow-up, I review possible feedback mechanisms and their role in galaxy evolution. Special attention is paid to high-redshift galaxies and their properties. In the next step, I discuss a number of processes, gas- and stellar-dynamical, within the central kiloparsec of disk galaxies, and their effect on the larger spatial scales, as well as on the formation and fuelling of the seed black holes in galactic centres at high redshifts.

## 1. Introduction: the paradigm of galaxy evolution

The aim of these lectures is to review the main aspects of secular galaxy evolution, i.e., galaxy evolution on cosmological scales. Two issues stand out in this attempt. First, the subject is immense and cannot be covered within the scope of this chapter. And second, one cannot ignore the fact that galaxy evolution is part of the overall evolution in the Universe – from the largest spatial scales ruled by dark matter (DM) to the smallest ones taken over by dissipative baryons that can form stars and grow supermassive black holes (SMBHs). In other words, the process of galaxy formation can be influenced strongly by a huge range of spatial scales. Hence, our attempt to discuss secular galaxy evolution will be rather modest in depth, only highlighting those issues which appear to lie at the forefront of current research. Lastly, we shall focus on disk galaxy evolution and only briefly mention elliptical galaxies.

Galactic morphology is largely a reflection of underlying dynamical and secular processes on relevant scales. That morphological evolution does indeed take place has been established fairly well, e.g., a recent quantitative analysis comparing galaxy populations at redshifts  $z \sim 0.6$  and z = 0 (Delgado-Serrano *et al.* 2010). While the fraction of ellipticals has barely changed over the last ~5 Gyr, the fraction of peculiar galaxies grew in favour of spirals by a factor of  $\sim 2$ –3. So peculiar morphology increases dramatically with z (e.g., Brinchmann & Ellis 2000).

As the galaxy population does indeed exhibit evolutionary trends, the question is what drives this process and how can we analyse and quantify it. Overall, our goal lies in explaining the origin of the contemporary Hubble sequence and in describing changes in this sequence over z. In this context, two alternative views exist. Firstly, the Hubble fork is determined by the initial conditions, i.e., by Nature. That means, for example, that the massive galaxies form in the highest overdensities, which themselves resulted from initial conditions. Alternatively, it is the environment, i.e., Nurture, not Nature, that determines the galaxy properties. Within this framework, evolution is driven solely by interactions, e.g., between galaxies, between galaxies and the intergalactic medium (IGM), etc.

In the past couple of decades, the issue of structure formation in the Universe, in terms of the dichotomy of top-to-bottom versus bottom-up scenarios, has been resolved in favour of the latter, and of the cold dark matter (CDM) paradigm. Unfortunately, even within the bottom-up framework, it remains unclear when and where the baryons matter. Clearly, on large scales the baryons follow the DM. But where and when do the baryons run  $amok^{1?}$  Inside the DM haloes? In the cold filaments? As the baryons collapse into the haloes, where do stars form – in disks or spheroids? Is gas fragmentation encouraged or suppressed during this infall? To what degree is the angular momentum conserved during collapse?

These questions open a Pandora's box of dissipative baryon dynamics partially decoupled from the DM. They underline pressing problems of structure formation on galactic and subgalactic scales. Most importantly, they emphasise the old/new dichotomy of what is primarily responsible for disk evolution: internal or external factors. Keeping this in mind, it is possible to construct a follow-up list of outstanding problems according to anyone's taste. Why do disks form inside triaxial haloes? (The haloes appear universally triaxial in numerical simulations, e.g., review by Shlosman 2008.) What is the prevailing morphology of the early galaxies: disk, elliptical, or some other unspecified morphology? Can archaeology help to uncover the details of disk formation and evolution? Important issues here are: can disks survive the epoch of major mergers? and do geometrically thick disks come from mergers or from supernova (SN) feedback?

A separate set of problems is related to disk-halo dynamical and secular interactions, which can have profound effects on both components – a kind of a dynamical feedback. To what extent does the disk evolution at high z differ from that at low z? What is the origin of bulges and galactic bars? And what does this tell us about internally versus externally-driven

<sup>&</sup>lt;sup>1</sup>From the Malay meaning 'mad with uncontrollable rage'.

disk evolution?

Beyond star formation, what is the role of baryons in disk evolution? What are the dynamical corollaries of gas presence, e.g., in maintaining the disk spiral structure? More specifically, does the gas (and the baryon fraction) vary systematically with the halo mass,  $M_{\rm h}$ ? When do baryons form central SMBHs? Does feedback from stellar and SMBH evolution encourage or suppress further star formation?

Finally, are disk galaxies doomed, in the sense that ultimately they will fall into highdensity regions, and what does that tell us about the morphology-density relation at high z? When did the current morphology-density relation form, and what regulates star formation in disks, stellar mass loss or cold infall?

Much of the theoretical progress understanding the drivers of galaxy evolution is due to numerical simulations of collisionless and dissipative processes in the Universe. Over the last few decades, astronomy has acquired precious support from an experiment, albeit a virtual one. Because the dominant processes are so nonlinear, the synthesis of theoretical, observational and experimental components has contributed much of our current understanding of structure formation in the Universe on all spatial scales. If our goal is to be able to 'produce' realistic galaxies that can be directly compared with observations, we are well underway.

The challenges in understanding galaxy formation and evolution are amplified by the unknown physics of the dominant processes (e.g., star formation, mechanical and radiative physics, turbulence), supplemented by often counter-intuitive nonlinear dynamics and by insufficient observational constraints. The numerical approach also suffers from the large dynamic range to be addressed by simulations – from  $\sim 10-100$  Mpc down to  $\sim$ AU. Gravity is the source of this difficulty.

The cornerstone of the current galaxy formation paradigm was established and refined in 1970s–1990s. It has been very successful in predicting and explaining disk galaxy properties. A two-stage process has been suggested, based on hierarchical clustering and DM halo formation in the first stage, and gas cooling and collapse into pre-existing potential wells in the second one (e.g., White & Rees 1978; Fall & Efstathiou 1980; Mo *et al.* 1998). Prior to gravitational collapse, the DM haloes acquire angular momentum (J) via gravitational torques, while baryons follow the DM and have the same specific angular momentum, j = J/M. In the next step, while the DM 'warms' up during virialisation, the baryons can cool down and continue the collapse, with j, and hence its distribution, roughly constant. Finally, the low-j baryons accrete onto the inner regions and cause central starbursts, resulting in the formation of galactic bulges in disk galaxies and SMBHs in their centres. Meanwhile high-j baryons form galactic disks with self-regulated star formation. In this framework, j determines the disk size, and the disk surface density fixes the timescale for star formation.

Another milestone has been passed with the understanding that pure DM structure formation leads to a universal density profile in virialised objects (e.g., Navarro *et al.* 1996, hereafter NFW). This density profile has been approximated by a power law with a slope of -2 (in log  $\rho$ -log R) at some characteristic radius  $R_s$ . It becomes shallower and tends to a slope of -1 toward the centre, and steepens to a slope of -3 at large radii. Over a range of radii, the NFW density profile leads to a flat circular velocity curve. The dissipative baryon influx into the halo is accompanied by a substantial adiabatic contraction of the non-dissipative DM in the central region, which is being dragged in, modifying the mass distribution there (e.g., Blumenthal *et al.* 1986). The adiabatic contraction of the DM halo has been invoked in order to explain the 'conspiracy' of so-called maximum disks (e.g., Burstein & Rubin 1985). However, additional processes that may have been omitted in the original estimate of the adiabatic contraction can complicate this picture quite substantially (e.g., Primack 2009).

This galaxy formation paradigm has received substantial support from both observations and high-resolution numerical simulations, although a long list of caveats exists. The theoretical background for DM mass and angular momentum distributions is still unclear and remains at the forefront of astrophysical research.

In this conjecture, the value of the angular momentum, its distribution and conservation, emerge as one of the dominant parameters, if not the dominant one determining galaxy evolution, along with the halo mass. To understand the caveats associated with J, we define the dimensionless angular momentum parameter which characterises the DM haloes (e.g., Bullock *et al.* 2001):

$$\lambda' = \frac{J}{\sqrt{2}M_{\rm h}R_{\rm h}v_{\phi}} \sim 0.03 - 0.05,\tag{1}$$

where  $M_{\rm h}$  and  $R_{\rm h}$  are the halo's virial mass and radius, respectively, and  $v_{\phi}$  is its circular velocity. Numerical simulations point to the universality of the  $\lambda'$  log-normal distribution

$$P(\lambda')d\lambda' = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\ln^2\left(\lambda'/\bar{\lambda'}\right)}{2\sigma^2}\right] \frac{d\lambda'}{\lambda'},\tag{2}$$

where  $\bar{\lambda}' = 0.035 \pm 0.005$  is the best-fit value, and  $\sigma(\lambda') \sim 0.5$  is the width of the lognormal distribution. Equation 1 is a slightly modified version of the original spin parameter  $\lambda$ introduced by Peebles (1969), with  $\lambda \sim 0.01$ –0.1 being the range found for DM haloes, and its median  $\lambda \sim 0.035$ . The simple relation between  $\lambda$  and  $\lambda'$  is given by  $\lambda' = \lambda |\text{ESIS/ENFW}|^{1/2}$ , where ESIS and ENFW are the energies of isothermal and NFW haloes. The collapsing baryons will be stopped by the centrifugal barrier if J is conserved. In the absence of DM,  $\lambda' \sim (R_d/R_h)^{1/2}$ , where  $R_d$  is the radius of a disk embedded in the halo. To reach rotational support,  $\lambda'$  must increase by a factor of ten, to  $\lambda' \sim 0.5$ . This means that a collapse will proceed over two decades in R and the resulting  $R_d/R_h \sim 0.01$ will be uncomfortably small. On the other hand, baryon collapse within a DM halo leads to  $\lambda' \sim R_d/R_h$ . This requires only a collapse by a factor of 10 in R in order to be in agreement with observations, resulting in  $R_d \sim 8 (\lambda'/0.035)(H_0/H_z)(v_{\phi}/200 \,\mathrm{km \, s^{-1}}) \,\mathrm{kpc}$  (e.g., Mo *et al.* 1998). Here  $H_0$  and  $H_z$  are the Hubble constants at z = 0 and at an arbitrary redshift, and  $v_{\phi}$  is the maximum rotational velocity in the disk (which is also the halo circular velocity for maximum disks). The corollaries are that, e.g., haloes with higher  $\lambda'$  lead to lower surface brightness disks, and that the resulting disk size distribution originates in the  $\lambda'$  distribution.

However, the observed spread in  $R_d$  appears to be larger than the spread in  $\lambda'$  (e.g., de Jong & Lacey 2000). Furthermore, an analysis of the Courteau (1997) sample of local massive disks has shown that gas may lose some of its angular momentum (Burkert *et al.* 2009). Comparison of these disks with the half-light radii of  $z \sim 2$  SINS galaxies (e.g., Förster-Schreiber *et al.* 2009; Cresci *et al.* 2009), reveals a similar deficiency of the spin parameter for the latter. We return to this issue in Section 2. While the above discrepancies must be clarified, a number of other caveats threatening the current paradigm require much more serious attention, and will be discussed below.

## 2. Disk growth: the angular momentum problem

Within the galaxy formation paradigm of White & Rees (1978) and Fall & Efstathiou (1980), the DM haloes acquire their spin from tidal interactions and baryons increase their rotational support by falling into the potential wells of these haloes and by conserving their angular momentum. To what degree the baryon angular momentum is preserved during this process has emerged as one of the key issues in our understanding of disk galaxy formation.

## 2.1. The angular momentum catastrophe

So is there a problem with J when forming galactic disks? We start with the so-called angular momentum 'catastrophe' which has emerged from the comparison of observations with numerical simulations of disk formation. While the observed disks have shown a deficiency of specific angular momentum j by a factor of two, modelled disks appear to have radial scalelengths smaller by a factor of 10 compared with observations, resembling bulges rather than disks (e.g., Navarro & Steinmetz 2000). In other words, the actual disks rotate too fast for a given luminosity L. At the same time, the slopes of the modelled disks turned out to be in agreement with the *I*-band Tully-Fisher (TF) relation for late-type galaxies,  $L \sim v_{\text{max}}^{\alpha}$ , where  $\alpha \sim 2.5$ -4, increasing to  $\sim 3$  in *I*. In our context, this is important because TF defines the relation between the dynamical mass and L. Simulations have shown much more compact endproducts of baryonic collapse, where the gas has lost a substantial fraction of its *j*. As we shall see below, the possible remedy to this problem involves a combination of numerical and physical factors, but no universal solution has been found so far.

What are the possible contributors to the angular momentum catastrophe? The first suspect is lack of numerical resolution. Indeed, increasing the resolution by adding smooth particle hydrodynamics (SPH) particles (e.g., Lucy 1977; Monaghan 1982) has improved the situation by helping to resolve the maxima of rotation curves, resulting in flatter and lower curves, thus having an effect on the central and outer regions of modelled galaxies (e.g., Governato *et al.* 2004; Naab *et al.* 2007; Mayer *et al.* 2008).

Numerical non-conservation of j is another suspect. In the outer disks, it can cause an excess of angular momentum transfer to the halo (e.g., Okamoto *et al.* 2005). In the inner disks, this effect could smear the radial gradients of rotation velocity, and artificially stabilise the disks against bar instability. Modelling an isolated galaxy formed from a noncosmological Milky Way-type DM halo has demonstrated that low-resolution disks heat up and lose j to the halo. Colder disks remain larger and have smaller bulge-to-disk ratios. A possible solution lies in the increase of the number of SPH particles and the introduction of a multi-phase interstellar medium (ISM).

Next, the disk ISM cooling below the Toomre (1964) threshold of  $Q \equiv c_s \kappa / \pi G\Sigma = 1$  leads to an excessive fragmentation (we return to this problem when discussing over-cooling). Here  $c_s$  is the sound speed in the gas,  $\kappa$  – the epicyclic frequency, and  $\Sigma$  – the disk (gas) surface density. The multi-phase ISM introduced by Springel & Hernquist (2003) through a modified equation of state clearly alleviated the problem. The gas cooling must be compensated by some kind of feedback to avoid fragmentation and the follow-up runaway star formation. This feedback from stellar evolution, e.g., from SNe and OB stars, has a clear effect on the bulge-to-disk ratio (Robertson *et al.* 2004; Heller *et al.* 2007b).

Finally, it was pointed out that baryons can hitchhike inside DM halo substructures which spiral in to the centre as a result of dynamical friction (e.g., Maller & Dekel 2002). This process is similar to that in a disk, where in the absence of star formation, dense gas clumps would heat up the disk, spiral in to the centre and possibly contribute to the bulge growth (e.g., Shlosman & Noguchi 1993; Noguchi 1999; Bournaud *et al.* 2007). This process, related to overcooling, can be easily over- or underestimated because it depends on a number

of additional processes, e.g., gas-to-stars conversion, ablation, etc. (see Section 6.1). Also, one cannot neglect the contribution of clumpy DM and baryons to the mechanical feedback (i.e., by dynamical friction), and, consequently, to the fate of the central DM cusp and the formation of a flat density core in the halo (e.g., El-Zant *et al.* 2001; Tonini *et al.* 2006; Romano-Díaz *et al.* 2008a).

## 2.2. The cosmological spin distribution and individual haloes

We now turn to the distribution of angular momentum in DM haloes. Early considerations of the baryon collapse assumed an initial solid-body rotation for uniform-density gas spheres. The angular momentum distribution (AMD) can be written as  $M(\langle j \rangle) = M_{\rm h}[1 - (1 - j/j_{\rm max})^{3/2}]$ , where  $j_{\rm max}$  is some maximum j, if baryons preserve j during this process, which results in exponential disks (e.g., Mestel 1963). More generally, what is the shape of an AMD which leads to exponential disks? The initial solid-body rotation law is soon replaced, following assumptions that baryons are mixed well with the DM and share its j. Assuming j conservation,  $M_{\rm d}(\langle j \rangle)/M_{\rm d} = M_{\rm h}(\langle j \rangle)/M_{\rm h}$ , where  $M_{\rm d}$  is the disk mass (e.g., Fall & Efstathiou 1980; Bullock *et al.* 2001; Maller & Dekel 2002), the new AMD can be written as

$$P(j) = \frac{\mu j_0}{(j+j_0)^2} \qquad \text{if } j \ge 0 \tag{3}$$

and P(j) = 0 if j < 0. This leads to a mass distribution with j

$$M(< j) = M_{\rm h} \frac{\mu j}{j+j_0},\tag{4}$$

with two parameters,  $\mu$  – the shape, and  $j_0 = (\mu - 1)j_{\text{max}}$  – whose effect on M(< j) has been displayed in Bullock *et al.* (2001, Fig. 4) for four haloes. When using the best-fit  $j_0$ , and normalising by  $M_{\rm h}$ , all haloes follow the same curve nicely – an indication of the universality of the AMD. The resulting (computed) disk surface density is nearly exponential, but with an over-dense core and an overextended tail.

Using Swaters' (1999) sample of 14 dwarf disk galaxies, van den Bosch *et al.* (2001) have demonstrated that the specific AMDs (i.e., M(< j)) of these disks differ from halo AMDs proposed by Bullock *et al.* (2001). But the total disk j's, i.e.,  $P(\lambda)$ , are similar to those of haloes. Although the pre-collapse baryon and DM AMDs are expected to be similar, the disks formed have been found to lack low and high j – in direct contradiction to the simple theoretical expectations discussed above. Moreover, the mass fraction of Swaters' disks appears to be significantly smaller than the universal baryon fraction in a  $\Lambda$ CDM universe. Does this mean that disks form from a small fraction of baryons and, in spite of

this, attract most of the available angular momentum? One can argue that a redistribution of baryon j should occur (at a stage earlier than anticipated), or that the high-j baryons avoid the disks. What processes are responsible for modifying the baryon AMDs? As argued by van den Bosch *et al.* (2001), a plausible explanation can lie in the selective loss of the low-j gas, leading to essentially bulgeless disks.

The closely-related issue of the existence of bulgeless disk galaxies is discussed in Section 6.

# 2.3. The tidal torque theory and the origin of the universal angular momentum distribution

Rotation of galaxies results from gravitational torques near the time of the maximum expansion (e.g., Hoyle 1949; Doroshkevich 1970; White 1978). If J is calculated using the spherical shell approximation (i.e., linear tidal torque theory, hereafter TTT) further redistribution during the non-linear stage contributes little (Porciani *et al.* 2002a,b). Alternatively, J can be acquired in the subsequent non-linear stage of galaxy evolution via mergers, accounting for the orbital angular momentum and its redistribution (e.g., Barnes & Efstathiou 1987).

According to the TTT, J is gained mostly in the linear regime of growing density perturbations due to the tidal torques from neighbouring galaxies. The maximum contribution to J comes from the times of the maximum cosmological expansion of the shell. Little J is exchanged in the non-linear regime after the decoupling and virialisation of the DM. Baryons are well mixed with the DM and acquire similar j (see Section 1). The TTT follows the evolution of the angular momentum of the DM halo based on the moment of inertia tensor and the tidal tensor. Within this framework, galactic spins can be estimated using the quasilinear theory of gravitational instability. The TTT has been extensively tested by numerical simulations.

So far we have not considered the effect of the mergers on the evolution of J (or  $\lambda$ ). We define mergers as major if their mass ratio is larger than 1 : 3. Wechsler (2001) has argued that major mergers dominate the *j*-balance in galaxies. Do they? The crucial point here is to account for the redistribution of the angular momentum during and after the merger. Hetznecker & Burkert (2006) have separated the main contributors to  $\lambda \sim J|E|^{1/2}M^{-5/2}$ , i.e., J, E and  $M_{\rm h}$ , and followed them during and after the merger event. While the DM halo J experiences a jump during a major merger, the subsequent mass and energy redistribution in the halo washes out the gain in J. A similar conclusion has been reached by Romano-Díaz



Fig. 1.— Evolution of DM halo spin  $\lambda$  as a function of a, the cosmological expansion parameter, during major mergers in three models, C–E. In all frames: the blue (upper) curves show the spin  $\lambda_{\rm s}$  within the characteristic NFW radii. The red (lower) curves show the evolution of the spin  $\lambda_{\rm vir}$  within the halo virial radii. The thick (black) line has been applied to show the 'normal' evolution during major mergers (from Romano-Díaz *et al.* 2007).

et al. (2007) and is displayed in Fig. 1. The corollary of high-resolution DM simulations is that there is no steady increase in the halo cosmological spin  $\lambda$  with time due to major mergers.

To summarise, hydrodynamical simulations show that the angular momentum of baryons is not conserved during their collapse. A number of factors can contribute to this process, but there is no definitive answer yet. The proposed solutions often amount to fine tuning. There is also a mismatch in the j-profiles. The distribution of the modelled specific angular momentum does not agree with the observations.



Fig. 2.— Disk growth mechanisms involve mergers and/or gas accretion. Both options depend on the environment, and their effects are gradually quenched in clusters of galaxies. The dependence on redshift is  $(1 + z)^{2\pm 2}$  for mergers, and seems to have a peak around  $z \sim 2$ , but this is yet to be verified for the gas accretion.

#### 3. Disk growth: mergers

In this and the following sections we shall review the details of secular and dynamical growth in disk galaxies which involve merging and/or the accretion of cold gas (Fig. 2). In the simplest (unrealistic) case of an isolated collapsing cloud, the angular momentum is conserved because the escaping radiation is isotropic and carries little J anyway. It is possible that some of the gas will rebound and escape in the equatorial plane taking away some J.

The lowest energy state for a rotating self-gravitating cloud occurs when almost all of the mass is found in the central compact accumulation (i.e., SMBH) and a negligible mass and all of the angular momentum escape to infinity. Such a state is never achieved, however, because the efficiency of J-transfer is declines rapidly as the process goes on. Amusingly, to quote Colgate & Petschek (1986), 'there seems to be too much angular momentum in the Universe to allow the formation of stars [...]. This fundamental problem begs for solution. Since net J appears to be zero over very large scales [...] our problem is restricted to local patches of the Universe where matter collapses to form relatively dense rotating objects [...].' We note that in order to understand how galaxies grow, one needs to resolve the issue of what is the most efficient angular momentum loss mechanism in particular circumstances. Disk galaxies form inside DM haloes, and it seems clear that Nature's choice is the formation of self-gravitating disks rather than a complete and efficient collapse to the very centre. Disks can form slowly from a weakly rotating gas cloud, in an adiabatic process, and their properties in this case will be linked to the initial conditions. Or they can form on a short dynamical timescale, forgetting the initial conditions in the process. In such a process, how much J is transferred to the DM and how much is redistributed among the baryons?

What factors affect the secular growth of galactic disks? The following list of physical factors and associated processes was not compiled in order of significance, and are not mutually exclusive:

- Environment: determined by high/low density fields and their corollary supply of cold gas (from filaments, etc.), major and minor mergers (in the low velocity dispersion field), interactions/harassment (in the high velocity dispersion field), stripping (tidal, ram, strangulation).
- Rate of star formation, feedback from stellar evolution and active galactic nucleus (AGN), galactic winds.
- Intrinsic factors: local and global gravitational instabilities, degree of asymmetry in the host DM halo.

In the following, we shall discuss some of the above factors, but shall avoid going into details about the internal dynamics of stellar bars (see review by Lia Athanassoula, this volume) and general aspects of galactic dynamics (see review by James Binney, this volume). This means that we shall emphasise the external factors driving galaxy evolution, and only occasionally resort to internal factors. Two such exceptions involve the physics of the central kiloparsec and some aspects of the formation of SMBHs (Section 8). We note, however, that this separation is largely artificial, and the action of external triggers of galaxy evolution is frequently associated with 'excitation' of internal degrees of freedom. One such well-known example is the tidal triggering of stellar bars.

In order to grow, galaxies and their haloes must rely on the exterior reservoir of baryons and DM, respectively. These can come in various degrees of virialisation: from smooth to clumpy accretion (see Section 5), from carrying nearly unvirialised to fully virialised baryons and embedded in parent DM haloes. The latter lead to mergers of various mass ratios, from major, to intermediate, to minor. The physics of accretion of course differs profoundly depending on the smoothness of accreting material. Mergers can also include stellar and DM components. In the following, we are going to discuss the main processes which accompany merging: dynamical friction, phase mixing and violent relaxation. Mergers are a diverse phenomenon and their products are equally diverse. We first discuss the criterion for merging, then merger demographics, i.e., their role in forming and growing spheroids and gas-rich disks (wet/dry mergers). We follow this up by reviewing the effect of major mergers on starbursts and AGN, and the effect of minor mergers on disk heating. The relationship between mergers and morphology evolution, stripping, and, briefly, the origin of ellipticals are discussed next. Two questions will be emphasised: whether mergers play an important role in creating spheroids, and whether dry mergers move galaxies along the red sequence.

The process of merging can be defined as an encounter of two galaxies which results in the formation of a single galaxy. Based on the mass ratios of the merging objects, we distinguish between major mergers (>1:3), intermediate mergers (from 1:3 to 1:10) and minor mergers (<1:10). Mergers in gas-rich systems are highly dissipative and denoted as 'wet' mergers, while mergers in gas-poor systems are dissipationless and 'dry'. Finally, mergers can be binary or multiple when more than two galaxies are involved.

Mergers are important because their rate grows rapidly with redshift,  $\propto (1+z)^m$ , although the inferred rate exhibits a substantial scatter. The differences come from matching the rate for rich clusters and for field galaxies, but also because of using different methods, such as close pairs and morphology. Extending the measure to fainter magnitudes generally shows an increase in m. There is also a difference between observational and theoretical estimates. Specifically,  $m=6\pm 2$  in rich clusters (e.g., van Dokkum *et al.* 1999), but it is much smaller in the field ( $m=2.7\pm0.6$ ; e.g., Le Fevre *et al.* 2000). A recent comparison between optical and near-infrared (NIR) bands has revealed a substantial difference in the major merger rate of  $m=3.43\pm0.49$  and  $2.18\pm0.18$ , respectively (Rawat *et al.* 2008). Diverse observations have resulted in an overall range of  $m=2\pm2$ , while numerical simulations result in a narrow range of  $m \sim 3$ , although ignoring the possibility of multiple galaxies per DM halo.

The situation is more confusing with the redshift dependence of the cold gas accretion rate as few observational constraints exist. On the other hand, numerical estimates of accretion growth are possible. For  $z \gtrsim 2$ , accretion rates show a strong increase towards lower z, and a sharp decline thereafter.

In order to delineate the relevant parameter space for mergers, we define two dimensionless parameters, the energy  $\hat{E} \equiv 2E/\sigma^2$  and the specific angular momentum  $\hat{j} \equiv j/\sigma r_{\rm m}$ , where  $\sigma$  is the inner dispersion velocity and  $r_{\rm m}$  is the median (half-mass) radius of an idealised galaxy. With a gross simplification of such spherical, nonrotating equal-mass galaxies it is possible to delineate the  $\hat{j} - \hat{E}$  parameter space of mergers with a characteristic timescale of less than the Hubble time (Fig. 3). Note that merging can occur also from



Fig. 3.— Criterion for merging (modified Figure 7-9 from Binney & Tremaine 1987), which is based on the time required for the merging of two galaxies as a function of the initial position of the binary orbit in the  $\hat{j} - \hat{E}$  plane. Orbits are only possible outside the forbidden region delineated by the solid lane (circular orbits). All elliptical (bound) and some hyperbolic (unbound) orbits below the dashed lane will eventually lead to a merger. However, merging time increases rapidly toward the upper-right corner, and above the dashed lane the merging time exceeds the Hubble time.

unbound orbits. As a rule of thumb, only galaxies with relative velocities less than their internal dispersion velocities,  $v_{\rm rel} \lesssim \sigma$ , will merge. Dispersion velocities inside large galaxies are  $\sigma \sim 200-300 \,\rm km \, s^{-1}$ , while that of clusters of galaxies  $\sim 500-1000 \,\rm km \, s^{-1}$ . The merger orbital angular momentum depends on  $v_{\rm rel}$  and contributes to the spin. A simple recipe to increase the parameter space for mergers is the dynamical friction mechanism.

## 3.1. Mergers and associated processes

#### 3.1.1. Minor mergers and dynamical friction

Minor mergers are those with  $m \ll M$ . When the massive galaxy with mass M moves among the background of low-mass m galaxies (or else) with velocity  $\mathbf{v}$ , dynamical friction is a good approximation. The drag per unit mass resulting from a gravitational wake is given by (Chandrasekhar 1943)

$$\frac{d\mathbf{v}}{dt} = -\frac{4\pi \ln \Lambda G^2 \rho M}{v^2} F\left(\frac{v}{\sqrt{2\sigma}}\right) \left(\frac{\mathbf{v}}{v}\right),\tag{5}$$

where  $\Lambda \approx b_{\text{max}}/\text{max}[r_{\text{h}}; GM/v^2]$  is the Coulomb logarithm,  $b_{\text{max}}$  is the maximum impact parameter,  $\rho$  is the background density of m galaxies, and F is the error function. In the low-v limit, the drag is  $\sim v$ , and in the high-v limit, it is  $\sim v^{-2}$ . Also, the dynamical friction force is  $\sim M^2$ , and the DM appears as an important contributor to the friction during the merging process. Assuming a circular orbit with a radius r for M moving around a bound accumulation of m masses, one can arrive at the analytical solution for r(t) which is a linear function of time. However, in a more realistic case, M plunges in a very elongated orbit, with a small pericentre, leading to a substantial tidal disruption and an associated mass loss very early. This of course complicates the analytical solution (e.g., Diemand *et al.* 2007; Romano-Díaz *et al.* 2010) and tends to decrease the characteristic timescale for the friction (e.g., Boylan-Kolchin *et al.* 2008).

The specifics of the dynamical friction are that it is a local and not a global force in the Chandrasekhar approximation, which assumes that the m particles interact only with M. The situation is more complicated if self-gravitational effects are taken into account, which is most relevant for galactic disks. The most interesting corollary is the introduction of resonances between the orbital motion of M and motion of m in the disk, both azimuthal, radial and vertical (e.g., Lynden-Bell & Kalnajs 1972). Other intricacies appear as well, but are not discussed here (e.g., Mo *et al.* 2010).

The effect of dynamical friction on multiple mergers has been shown explicitly in numerical simulations of Nipoti *et al.* (2003), who ran them with and without friction with the goal of modelling the formation of cD galaxies in the centres of galaxy clusters. The characteristic timescale for merging has been substantially shortened when the Chandrasekhar drag has acted, compared to merging in a fixed background potential.

#### 3.1.2. Mergers: phase mixing and violent relaxation

The two-body relaxation timescale is too long to have an effect on mergers, where the relaxation is dominated by collisionless processes, such as phase mixing and violent relaxation – relaxation in the mean field. Both mechanisms have been introduced by Lynden-Bell (1967). Behind the idea of phase mixing lies the time evolution of a coarse-grained distribution function in a collisionless system. As the classical entropy is conserved in the absence of collisions, the fine-grained function is time-independent, while the coarse-grained function evolves to uniformly cover the available phase space for the system, thus maximising the corresponding coarse-grained entropy. Phase mixing, therefore, tends to destroy coherent phase-space structure.

Violent relaxation is a relaxation in the time-dependent potential of the system, when the specific energy of a particle is not conserved. This process leads to a Maxwellian distribution of velocities in which the temperature is proportional to the mass of the particle. So particle dispersion velocities become independent of the particle mass. Violent relaxation is most relevant at the time of virialisation of the system, so the characteristic time is the crossing time of the system. This leads to a more complete relaxation in the central regions compared to the outskirts, because the dynamical timescale becomes prohibitively long there. On the other hand, galaxy interactions would drive violent relaxation mostly in the outer regions. It is still unclear how efficient this process is overall.

## 3.2. Dry and wet disk mergers: spheroids or rebuilding?

Mergers between systems that include stars and DM only are called dry mergers. Examples: a merger of two elliptical galaxies, or one between an elliptical and a lenticular galaxy. Only limited observational data exists on dry mergers, mostly in clusters of galaxies (e.g., van Dokkum *et al.* 1999). The endproduct of this process is predicted to be an ellipsoidal system (e.g., Toomre & Toomre 1972; Barnes 1992), as inferred from the Sersic law. The mixing appears incomplete, and the metallicity gradient is not fully erased. The emergence of the red sequence of massive galaxies has been linked to the increasing importance of dry mergers after  $z \sim 1$ , because of the seemingly insufficient amount of massive blue galaxies that can serve as their precursors (e.g., Khochfar & Burkert 2003; Faber *et al.* 2007). How important the contribution is of dry mergers in forming the red sequence, however, is unclear.

Dry mergers have been studied using the GEMS (Galaxy Evolution from Morphology and SEDs [Spectral Energy Distributions]) survey in tandem with the COMBO-17 photometric redshift survey in order to constrain their frequency between  $z \sim 0.2-0.7$  (e.g., Bell *et*  al. 2006). Accompanying N-body simulations have been used to explore the morphological signatures of such mergers. An estimated rate of ~0.5–2 major mergers between spheroids over the  $z \sim 0.2$ –0.7 period has been claimed to be consistent with the limit of  $\leq 1$  such event in recent times, estimated using an alternative semi-analytic method. This indicates that dry mergers can indeed be an important factor in driving the evolution towards massive red galaxies at present times, but more work is clearly required.

Scaling relations, such as Faber-Jackson (1976, FJ), Kormendy (1977), the fundamental plane (Djorgovski & Davis 1987), and others can provide, in principle, important information about the formation of massive ellipticals, and constrain it. N-body simulations have shown that multiple dry mergers preserve the FJ relation, but produce lower central dispersion velocities and increase the effective radius, although the fundamental plane of ellipticals remains thin (Nipoti *et al.* 2003). Ciotti *et al.* (2007) confirmed that the FJ, Kormendy and fundamental plane relations are robust against dry merging, although caveats exist. An important question is when are these scaling relations established?

Wet mergers involve gas and are, therefore, dissipative. Here we focus on mergers which involve galactic disks with various gas fractions. When disks are involved, the outcome depends on many more parameters, including the disk plane orientation with respect to the orbital plane, and the alignment of internal spins (rotation) with the orbital spin, i.e., prograde versus retrograde encounters.

The first simulations of disk interactions were performed by Holmberg (1941) using a light-bulb 'supercomputer', with the important result that tidal forces lead to the formation of spiral structure in galaxies. Modern numerical simulations have revealed a rich library of processes involving disk interactions and mergers: stretching, harassment, stripping, strangulation, squelching, threshing, splashback and cannibalism. Additional effects include enhanced star formation rates and quenching the star formation (e.g., recall that spheroids are associated with quenched star formation). The outcome of disk galaxy mergers can be either spheroidal or disk systems.

The early arguments about disk merger remnants were based on N-body simulations, without or with low-resolution gas. The merging proceeded via dynamical friction against the DM component. The DM haloes have been 'soaking up' the internal and orbital angular momenta of merging galaxies, and the collision appeared sticky (e.g., Hernquist 1992). Overall, the simulations have been successful in fitting the properties of the elliptical products, although the remnants appeared too diffuse compared to observed massive ellipticals. The morphology of interacting and merging galaxies has been closely matched (e.g., Hibbard & Mihos 1995). Dubinski *et al.* (1996) have studied the merging of pure stellar disks in live DM haloes, focussing on the shapes of tidal tails, when disks are being stretched, impart-

ing kinetic energy to the stars. The length and mass of tidal tails have been found to be sensitive to the gradient of the gravitational potential. Hence, tails can successfully map the DM potential well and constrain the overall DM mass distribution. The problem was in reproducing the high phase density observed in the centres of elliptical galaxies. This situation changed with the inclusion of a dissipative component in simulations.

Numerical simulations have indicated that the dynamical role of gas is well in excess of its mass fraction. This has been shown for isolated galactic models (e.g., Shlosman & Noguchi 1993; Heller & Shlosman 1994) as well as for fully cosmological models (e.g., Barnes & Hernquist 1996). Due to its dissipative nature, gas is always losing its energy and angular momentum, which leads to a central accumulation, where the gas successfully competes with stellar and DM contributions to the gravitational potential. The deepening of the potential well by the gas resolves one of the outstanding issues we have mentioned above – high phase density in the centres of ellipticals. Barnes & Hernquist (1996) have found that the gas presence shortens the merging timescale and drives a large fraction of gas to the very centre of the remnant. This evolution is relatively insensitive to the detailed physics, given that the gas is able to cool. Simulations of mergers involving disks with ~10% or less gas mass fraction lead to the formation of a spheroidal stellar component with a surface brightness of the de Vaucouleurs 1/4 law, and a central stellar cusp which is not observed in such galaxies. These simulations have shown that wet mergers can be responsible for the formation of some ellipticals. The question is what fraction?

Difficulties with the scenario of ellipticals forming in binary major mergers of disk galaxies include the following: typical ellipticals are more metal-rich than typical present-day disks (see Fig. 4), ellipticals have older stellar populations that seem to form on shorter timescales, and massive ellipticals could not typically have formed from binary mergers of present day disks (while they might form from high-z disks whose descendants no longer exist (e.g., Naab & Ostriker 2009). In addition, binary mergers of any kind are not isotropic, whereas massive ellipticals are.

Probably the most intriguing issue of disk mergers is whether disks can survive mergers. Simulations of disk mergers without gas point to a clear trend of mergers thickening and destroying the disks (e.g., Kazantzidis *et al.* 2008; Robertson & Bullock 2008). The corollary is that the disks must grow rather quiescently. But what about the more relevant situation where disks contain gas, and there are plenty of 'leftovers' from mergers? What is the outcome when the disks not only contain gas but are gas-rich? Is there a critical gas fraction,  $f_{gas}$ , for disk survival?

Indications that disks can reform after *some* major mergers, if sufficient amounts of gas can be maintained, come from simulations of pure-gas, bulgeless disks on prograde parabolic



Fig. 4.— Evolution of metals (in mass) for the merger of two  $M_*$  disks with (solid) and without (dashed) bulge component. The total mass of the progenitor disks used was  $2.9 \times 10^{10} M_{\odot}$ . The metal mass for  $M_*$  ellipticals is indicated by the shaded area. Present-day ellipticals have (at least) a factor of two more metals (from Naab & Ostriker 2009).



Fig. 5.— Disk fraction as a function of its total (baryon+DM) mass at  $z \sim 10.2$  (in  $M_{\odot}$ ) within a high-resolution region of  $7 h^{-1}$  Mpc with binning of 0.25 in total mass. Shown are two defining criteria for the gas disk,  $c/a \leq 0.5$  (blue stars) and  $c/a \leq 0.7$  (red triangles), where a, b and c are the major, intermediate and minor axes of gaseous disks. Haloes in the process of merging have been omitted, overall four objects within this mass range – all of them had more than one disk per halo (from Romano-Díaz *et al.* 2011b).

orbits, in the presence of star formation (e.g., Springel & Hernquist 2005). Moreover, gas-rich disks with  $f_{\text{gas}} \gtrsim 0.5$  (e.g., Robertson *et al.* 2006; Robertson & Bullock 2008), or continuous accretion of the cold gas following a destructive merger (e.g., Steinmetz & Navarro 2002) show a similar trend. Arguments that disk heating has been overestimated in minor mergers have been advanced as well (e.g., Hopkins *et al.* 2008; Romano-Díaz *et al.* 2008b). Recent high-resolution simulations of over-dense regions in the Universe have shown a resilient, disk-dominated population of galaxies (Fig. 5) at  $z \sim 8-10$  (Romano-Díaz *et al.* 2011b). Subsequent analysis of disk growth in such regions reveals that the dominant growth mode is not via major mergers but rather through accretion of cold gas (Romano-Díaz *et al.* 2012), as we discuss in Sections 5 and 7.

Attempts to understand the conditions for disk survival on cosmological timescales have delineated the main contributing factors which can increase the disk endurance: an existing reservoir of cold gas which is able to resupply it on a short dynamical timescale; delayed star formation, in order to avoid a destructive starburst which would quench the growth of the stellar disk; and continued ability of the shocked gas to cool radiatively on a short timescale (more about this in the next section). In other words, the disk rebuilding processes should be more efficient than the destructive processes during the merger event. No 'universal' solution to this problem exists, although with a sufficient amount of fine tuning, progress has been made.

Among the few successful examples of an efficient rebuilding of the disk component is a numerical study of disk galaxy evolution following a 1.6:1 wet merger at  $z \sim 0.8$  (Governato et al. 2009). The environment chosen for this experiment was typical of field haloes and Milky Way parameters for the re-simulated galaxy. SN feedback has been responsible for the delayed star formation. For this purpose, the blastwave approximation has been used (see Section 6.2), where the cooling shuts off over a Sedov crossing time of  $3 \times 10^7$  yr over  $\sim 0.2-0.4$  kpc regions (corresponding also to the resolution limit of the model). During the phase of z < 3,  $f_{\rm gas}$  in the progenitor disks was below 0.25. Over the period of disk rebuilding,  $\sim$  few Gyr, the old stellar population, found in the *thick* disk, has faded considerably. Thus, the formed thin disk dominated the light in the *I*-band, while the thick disk contributed  $\sim 70\%$  of the stellar mass, and the stellar halo component faded by z = 0.

A number of corollaries follow attempts to rebuild and sustain disks over cosmological times. First, it apparently requires the existence of a thick stellar disk component, which represents the population of a pre-merger disk. A beautiful example is that of NGC 4762, which exhibits both thin and thick disks (e.g., Burstein 1979). If the thickening has been abrupt, the radial extent of the thick disk provides the size of the original disk at the merger event. The absence of thick disks in some late-type galaxies, e.g., NGC 4244, can be interpreted as a challenge to numerical simulations. It is, therefore, encouraging that Comerón *et al.* (2011) claim to have identified a sign of the thick disk in this object. On the other hand, it is plausible that in some mergers the stellar disks are destroyed completely.

The other issue lies in the prohibitively long disk-rebuilding timescale at low z, a few Gyr. While this timescale severely limits the number of destructive mergers a galaxy can have at low redshift,  $z \leq 1$ , it is unacceptable at high redshift  $z \geq 6$ . Simulations show, however, that characteristic timescales for similar processes are substantially shorter at high z, by about a factor of ten, which maintains a robust population of disks even in highly over-dense regions (e.g., Romano-Díaz *et al.* 2011b; 2012). The morphology-density relation during the epoch of reionisation, therefore, does not follow the trend it exhibits at low z. At what redshifts does this relation take the form of the observed one?

Among the numerous corollaries of disk mergers is their plausible contribution to classical bulges. The observed frequency and mass fraction of classical bulges in disk galaxies are debatable at present, specifically with respect to other bulge types. For example, the origin of so-called disky bulges is unrelated to galaxy interactions, and they result mostly from stellar bar instabilities (e.g., Combes *et al.* 1990; Pfenniger & Friedli 1991; Raha *et al.* 1991; Berentzen *et al.* 1998; Patsis *et al.* 2002; Martínez-Valpuesta *et al.* 2006; see also review by Kormendy & Kennicutt 2004). This makes it even more difficult to obtain a quantitative estimate of their link to mergers. On top of this, the merger outcome is sensitive to various associated physical processes and kinematical parameters. As numerical simulations themselves depend on subgrid (and sometimes unknown) physics, attempts have focused on semi-analytical models, although their predictive power has not been verified.

Using observational constraints on disk masses and gas fractions,  $f_{\text{gas}}$ , in galaxies, Hopkins *et al.* (2010) have attempted to predict the properties of merger remnants, and, specifically, to quantify the contribution to classical bulge formation from mergers of various mass ratios. The main result was that major mergers dominate the assembly of  $L_*$  bulges, while minor mergers dominate the formation of bulges in low-mass systems. The bulge-to-total mass ratio, B/T, was found to trace the merger mass ratio,  $\mu_{\text{gal}}$ . A simple correlation,  $B/T \sim \mu_{\text{gal}}(1 - f_{\text{gas}})$ , has been identified. The straightforward corollary of this correlation is that increasing the gas fraction tends to suppress the bulge formation, which has been interpreted in terms of a reduced efficiency of gas angular momentum loss with increasing  $f_{\text{gas}}$ . The by-product of this conclusion is that collisionless systems lose angular momentum more efficiently than dissipative ones, something which is difficult to accept.

As a next logical step, one can ask whether gas-rich high-z disks themselves form in mergers. A sample of such massive,  $\sim 10^{11} M_{\odot}$ , disk galaxies at  $z \sim 2$  has been analysed with SINFONI/VLT integral-field spectroscopy (Genzel *et al.* 2008). Large random motions have been detected and interpreted in terms of rapid inflows from cold accretion flows which originated in cosmic web filaments (Section 5). Such turbulence can reduce the viscous accretion timescale to below  $\sim 1 \text{ Gyr}$ . The detailed example of the galaxy BzK 15504 at  $z \sim 2.38$  has been studied in sufficient detail by Genzel *et al.* (2006). This object is characterised by a high star formation rate of  $\sim 140 M_{\odot} \text{ yr}^{-1}$ , a gas mass of  $\sim 4 \times 10^{10} M_{\odot}$ , and a gas-to-star mass ratio of  $\sim 0.5$ . The ratio of circular to dispersion velocity,  $v/\sigma \sim 3$ , points to a geometrically thick disk, and has been interpreted as the formation stage of the thick galactic disk. Interestingly, BzK 15504 shows no obvious signs of a recent or ongoing merger, e.g., no obvious line-of-sight velocity asymmetry. So, is this object a proto-disk caught in the stage of a rapid but secular evolution? Unfortunately, there is no simple answer to this question. The problem lies in that the same kinematic parameters can also characterise a merger remnant, as shown by Robertson & Bullock (2008).

## 4. Mergers and their secondary by-products

#### 4.1. Tidal dwarfs

The definition of a merger process as one which decreases the number of galaxies (see Section 3) has its drawbacks. The deformation and the possible destruction of interacting disks manifests itself also in the creation of clumps of stars and molecular gas – so-called tidal dwarfs, the by-products of mergers. One can argue that tidal dwarfs are not *bona* fide galaxies as they are not expected to contain a significant amount of DM. We shall stay away from this dispute. There is an additional difference between these objects and 'normal' galaxies – they are expected to be made of recycled material with metals and dust, and not have the primordial composition of the first galaxies or of low-metallicity dwarfs. Tidal dwarfs are usually associated with antenna-type tidal tails of their massive parent galaxies, are gas rich, have both old and young stellar populations, and contain both HI and H<sub>2</sub>, as noted by Braine *et al.* (2001) in their survey. Tidal dwarfs are characterised by a much larger (~100×) of CO luminosity compared to other dwarf galaxies of comparable optical luminosity. Because of the relative proximity of these objects, they can serve as testing labs for our understanding of the galaxy formation process, albeit different from that in the early Universe.

## 4.2. Polar ring galaxies and ring galaxies

While the tidal dwarfs represent a transient phenomenon during galaxy mergers, polar rings are expected to describe a rather steady-state situation when the externally acquired material finds stable orbits in the plane orthogonal to the equatorial plane of the galaxy. A number of preferentially early-type disks or ellipticals show such rings lying in their polar planes, and, therefore, kinematically distinct from their parent galaxies. The rings appear younger than their host galaxies, which seem to be depleted of cold gas. Polar rings include young stellar populations, apparently formed after the capture, and are gas-rich (a few times  $10^9 M_{\odot}$ ) and dusty (e.g., van Driel *et al.* 2002).

Two main alternative explanations, based on merger kinematics, include the accretion or capture of satellites from a nearly circular orbit, or the collisional destruction and subsequent capture of a donor from a rather radial orbit. Under special conditions, the accretion of cold gas by the host galaxy can also result in the formation of polar rings. In the accretion scenario (e.g., Schweizer *et al.* 1983; Reshetnikov & Sotnikova 1997), about 10% of the donor disk gas is captured in a polar ring, in less than 1 Gyr. The collision scenario of galactic disks (e.g., Bekki 1998) involves orthogonally oriented disks in a head-on, low-velocity collision.

Bournaud & Combes (2003) have tested both alternatives in numerical simulations and conclude that the accretion scenario is more supported by observations, although one cannot exclude either possibility. We note that numerical simulations of galaxy formation at higher z have demonstrated routinely the formation of polar-ring galaxies as a product of merging and interaction within the computational box (e.g., Romano-Díaz *et al.* 2009; Roskar *et al.* 2010).

An interesting aspect of polar rings is that they can provide information about the DM halo shapes (e.g., Sackett & Sparke 1990). This is possible because the rings are long-lived and, therefore, have sufficient time to settle on regular orbits in the polar plane of the host galaxy, which is determined by the extended DM halo. The measured flat rotation curves of the rings point to the existence of such haloes around parent galaxies. The self-gravity of the gas settling in the rings is also a stabilising factor in their dynamics, otherwise, differential precession would destroy them in a short orbital time. This conclusion is a clear outcome of orbital analysis in a triaxial potential (e.g., Sparke 1986; Arnaboldi & Sparke 1994) and numerical simulations of ring formation and evolution (e.g., Bournaud & Combes 2003). Depending on the mass and orientation of polar rings, a number of stable and unstable equilibria are possible. If the flattening of the DM halo can be constrained independently, e.g., from lensing, one can obtain bounds on the halo (or overall mass) triaxiality, merely assuming that the observed rings are stable.

The Cartwheel galaxy represents another class of rings, most probably originating from head-on collisions involving at least one gas-rich disk. Unlike polar rings, these rings are not stationary, are frequently off-centred, and represent an expanding density wave which triggers star formation (Lynds & Toomre 1976). The relative velocity of these collisions appears to be much higher than those leading to polar rings (e.g., Horellou & Combes 2001). It is characteristic of a galaxy cluster environment, as we discuss below.

## 4.3. 'Mergers' in clusters: galaxy harassment

In a galaxy cluster environment, high-velocity encounters between galaxies have relative velocities  $v_{\rm gal} \sim \sigma_{\rm cl} \gg \sigma_{\rm gal}$ , where  $v_{\rm gal} \sim 10^3 \,\rm km \, s^{-1}$  is the typical relative velocity of galaxies in clusters,  $\sigma_{\rm gal} \sim 100-200 \,\rm km \, s^{-1}$  the intrinsic velocity dispersion in galaxies, and  $\sigma_{\rm cl}$  the velocity dispersion in clusters. The corollary is that high-velocity encounters dominate in clusters and are more frequent than in the field environment, and that direct collisions are rare. Hence, one should expect that galaxies are more morphologically disturbed in clusters, which makes them vulnerable to future encounters, as well as to the effects of cluster tides. The encounter dynamics can be approximated by the impulse approximation. The cumulative effect of the

above processes is called galaxy harassment. One of the main questions is whether galaxies form differently in clusters or whether environmental processes, as described above, make them different.

Observationally, in the local Universe, cluster galaxies appear redder than in the field and are more spheroid-dominated. Tidal disturbances are common from close passages. Local clusters possess no spiral disks, as reflected by their morphology-density relation (e.g., Dressler 1980). In comparison, already at  $z \sim 0.4$ , clusters contain many small disturbed spirals, which are replaced by spheroidals at the faint end of the galaxy luminosity function (LF) at z=0. They contain a substantial population of blue, star-forming and starbursting galaxies – a reflection of the Butcher-Oemler (1978) effect. Star-forming rings are much more frequent in clusters than two-armed spirals (e.g., Oemler *et al.* 1997). Furthermore, in hierarchical models of structure formation, the field galaxy influx into clusters peaks around  $z \sim 0.4$  (e.g., Kauffmann 1995), and the star formation declines abruptly around  $z \sim 0.5$ , resulting in a large population of passive, post-starburst galaxies (e.g., Dressler *et al.* 1999; Poggianti et al. 1999). This quenching of star formation, measured, for example, by a decline in the H $\alpha$  emission, in the cluster environment is a reflection of the overall trend in galaxy evolution which ultimately leads to the formation of the so-called red sequence. In other words, the star formation and morphological evolution in clusters appear to decouple at lower redshifts (e.g., Couch *et al.* 2001).

One way to understand the environmental effects on galaxy evolution is to compare and contrast the evolution of disk galaxies in rich clusters with that of field galaxies. Numerical simulations have revealed the details of galaxy harassment in such over-dense fields (e.g., Moore *et al.* 1998), where disks are subject to interactions with brighter and more massive neighbours – a process that injects energy and makes them vulnerable to the cluster tidal field. Moreover, the gas stripping process acts efficiently in the central regions of clusters (e.g., Tonnesen *et al.* 2007). These processes affect disk galaxies almost exclusively – dense ellipticals are basically immune to the effects of harassment. Furthermore, the  $r^{1/4}$  de Vaucouleurs surface brightness profile appears robust and invariant to harassment, even when a galaxy loses ~40% of its mass during an interaction (e.g., Aguilar & White 1986).

## 4.4. Mergers and star formation rates

Below, we discuss how mergers influence the star formation rates (SFRs). Here we emphasise the morphological evolution these disks experience, which results in loss of the gaseous component, partly ablated and partly falling to the centre, and in a dramatic conversion of disks into spheroidals. Late-type Sc–Sd disks appear to be more affected by this process. In addition, the ram pressure by the intracluster hot gas is  $\sim \rho_{\rm hot} \sigma_{\rm cl}^2$ , while the restoring force is  $\propto 2\pi \Sigma_{\rm tot}$ , where  $\Sigma_{\rm tot}$  is the disk total surface density (gas + stars). The stripping occurs when the ram pressure exceeds the restoring force, leading to the transformation to lenticular galaxies. When the outer hot gas, which is only loosely bound to the DM halo, is stripped, this is called strangulation.

Simulations have also demonstrated an agreement with observations both in accounting for intermediate-age stellar population in these spheroidals and in their shapes – which appear prolate and flattened by velocity dispersion anisotropy.

We now turn to the issue of merger-induced star formation. There is no doubt that mergers are responsible for the largest starbursts, e.g., the ultra-luminous infrared galaxies (ULIRGS). The most intensely star-forming galaxies are in the advanced stage of merging. But is the reverse true? In other words, does the merger rate drive the SFR?

Since  $z \sim 1$ , the cosmic SFR per unit comoving volume appears to decrease by a factor of ten (e.g., Madau *et al.* 1998). Because over this time period most of the star formation is associated with disk galaxies, the inevitable conclusion is that the disks are shutting down their star-forming activity. While in principle a number of processes can contribute to this evolutionary trend, here we focus on the contribution from major mergers to triggering the star formation. Because the LF of galaxies at these redshifts is dominated by 'normal' galaxies (Fig. 6), the effect of mergers cannot be the principal one, as noted by Bell et al. (2005). This conclusion is based on the analysis of the deep  $24 \,\mu m$  survey made by the MIPS (Multiband Imaging Photometer for Spitzer) Team (Rieke et al. 2004), combined with the COMBO-17 redshift and SED survey (Wolf et al. 2004) and the GEMS survey (Rix et al. 2004). The covered redshift interval is 0.65–0.75 and includes about 1500 galaxies in the CDFS (*Chandra* Deep Field South). About 40% of galaxies with stellar masses  $\gtrsim 2 \times 10^{10} M_{\odot}$ have been found to be undergoing a period of elevated, intense star formation at  $z \sim 0.7$ , while only  $\lesssim 1\%$  of similarly massive galaxies exhibit such star-forming activity at z = 0. Moreover, the IR LF and the SFR densities at  $z \sim 0.7$  are dominated by morphologically undisturbed galaxies. More than 50% of the starbursting galaxies are spirals, but  $\leq 30\%$  appear to be strongly interacting. Hence the decline in the SFR is not 'driven' by the decline in the major merger rate. Rather, factors that do not strongly alter the galaxy morphology are at play here, e.g., weak interactions and gas depletion. Bell et al. (2005) argue that the selection procedure used should not introduce any special bias against obscured starburst galaxies.

To quantify the average enhancement in the SFR of major mergers between massive  $\gtrsim 10^{10} M_{\odot}$  galaxies, including pre- and post-mergers, Robaina *et al.* (2009) used COMBO-17 and 24  $\mu$ m SFR from *Spitzer*, in tandem with the GEMS and STAGES *HST* surveys, for  $z \sim 0.4-0.8$ . Major interactions have been defined here as being resolved in *HST* imaging,



Fig. 6.— Estimated 8–1000  $\mu$ m LF, split by morphological type for 397 galaxies at  $z \sim 0.65$ –0.75 (see text for details). Only galaxies detected at 24  $\mu$ m are shown, and no attempt to extrapolate to lower IR luminosities has been made; the sample is grossly incomplete below  $6 \times 10^{10} L_{\odot}$  as denoted by the grey dotted line. In each panel, the grey solid histogram shows the total IR LF. The shaded area shows the IR LF split by galaxy type using GEMS-derived galaxy classifications, where the extent of the shaded area explicitly shows the differences in IR LF given by the three different classifiers. The black histogram shows the IR LF, averaged over the three different classifiers and corrected to reproduce the increased fraction of clearly interacting galaxies seen in GOODS-depth data (from Bell *et al.* 2005).

having a mass ratio of  $\geq 1:4$  based on the luminosity ratio, and exhibiting clear signs of interaction. Major merger remnants have been identified using a highly disturbed 'train wreck' morphology, double nuclei, and tidal tails of similar length, or spheroidal remnants with large-scale tidal debris. Prominent disks with signs of merging, i.e., highly asymmetric spirals or a single tidal tail, have been assumed to be minor mergers.

In addition, the enhancement in the SFR has been evaluated as a function of the projected galaxy separation. While confirming that most starbursting galaxies are in the process of merging, Robaina *et al.* (2009) have found that SFRs in major-merger systems are only elevated by a factor of ~1.8 compared to those in non-interacting ones, when averaged over all interactions and all stages of interaction (see also Li *et al.* 2008; Sommerville *et al.* 2008). The main enhancement is visible for close pairs with projected separations of  $\leq 40$  kpc. Overall, about  $8\% \pm 3\%$  of the total star formation has been estimated to be directly triggered by major interactions. This indeed confirms the conclusion of Bell *et al.* (2005), mentioned above, that major mergers are not the dominant factor in building the stellar mass at  $z \leq 1$ , and, therefore, they are not responsible for the decline in the SFR over this time.

We now discuss the input from numerical simulations which test the above observational results on the relation between major mergers and SFRs. Di Matteo *et al.* (2007) have focussed on this issue by modelling galaxy collisions of all Hubble types while varying both bulge-to-disk mass ratios and  $f_{gas}$ . Direct and retrograde orbits have been used, and star formation in interacting and merging galaxies has been compared to that in isolated galaxies. The main outcome is that the retrograde orbits seem to produce more starbursts, and the star formation efficiency is higher (in the sense of star formation per unit mass). Moreover, these starbursts are essentially nuclear starbursts, from the gas inflow triggered by the gravitational torques from asymmetries induced by the tides.

In a comprehensive study of unequal-mass mergers, Cox *et al.* (2008) have quantified the effect of tidal forces on the star formation. Specifically, they have focused on the effect of mass ratio, merging orbits and galaxy structure on merger-driven starbursts. These kinematical and morphological parameters are of prime importance for the main issue – the relation between mergers and SFRs. It was found that merger-induced star formation is a strong function of the merger mass ratio, which spans over a factor of ~23, being negligible for small mass-ratio mergers – a straightforward dependence on the tide's strength. An additional parameter that is helpful to measure the induced star formation is starburst efficiency – the fraction of gas that is converted into stars over the interaction time. The starburst efficiency was found to be insensitive to the details of the feedback parameterisation from stellar evolution – this is very helpful because the feedback physics is sufficiently uncertain. Overall, the burst efficiencies have been reduced compared to previous studies.

However, while the burst efficiency for equal-mass mergers does not depend on the merger orbital parameters, disk orientation, or the primary galaxy properties, this differs for unequal-mass mergers. Direct coplanar-orbit mergers produce the most significant bursts at close passages. The other important factor appears to be the mass distribution in the primary galaxy. For example, a massive concentrated stellar bulge stabilises the disk and suppresses the induced star formation. More gas driven above the threshold density for star formation reduces the burst efficiency, and so is a more efficient feedback.

In summary, recent studies of mergers agree that the evolution of the merger rates after  $z \leq 1$  is not responsible for the overall decrease in SFR in the Universe. The obvious direction for improving the numerical simulations of merger-induced star formation requires incorporating them into the cosmological context – so far simulations deal with isolated pairs of interacting and merging galaxies. This will allow accounting for the effect of coldgas accretion which is expected to compete with merger-induced galaxy growth, especially at higher redshifts. Lastly, increases in the number of particles, both collisional and collisionless, are important for modelling the disk response during the interaction and merging periods. Small numbers of the particles are known to destabilise the disks owing to increased noise.

## 5. Disk growth: cold accretion

Figure 2 displays two plausible modes of galaxy growth: galaxy mergers and gas accretion. While we know that the frequency of interactions and mergers increases steeply with redshift, the availability of unvirialised gas increases as well. In Sections 3 and 4 we reviewed galaxy growth driven by mergers, mainly the growth of galactic disks. Here we focus on the alternative process of galaxy growth via the accretion of unvirialised baryons and DM.

#### 5.1. The standard view and the new paradigm

The ability of galaxies to grow by means of accretion has been known for some time. The standard view has been that gas falling into a DM halo shocks to a virial temperature,  $T_{\rm vir}$ , at around the halo virial radius,  $R_{\rm vir}$ , and fills it up, remaining in a quasi-hydrostatic equilibrium with  $T_{\rm vir} \sim 10^6 (v_{\rm circ}/167 \,{\rm km \, s^{-1}})^2$  K. Hot, virialised gas cools from the inside out, loses its pressure support and settles into a centrifugally-supported disk (Rees & Ostriker 1977; White & Rees 1978; Fall & Efstathiou 1980).

This view has been substantially modified now in that not all the gas is shocked when it enters the halo. Instead, much of the gas is capable of entering the halo along denser



Fig. 7.— Left: The standard view – gas shocks at  $R_{\rm vir}$ , becomes pressure-supported, then cools down and settles in a disk; Right: The new paradigm – some of the gas shocks, but the majority enters the DM halo in the cold phase along the filaments feeding the disk growth.

filaments and penetrating deeply – this radical shift in understanding has led to a new paradigm (Fig. 7).

## 5.2. Accretion shock?

Birnboim & Dekel (2003) have performed an idealised analytical study of gas accretion on a spherical DM halo, assuming two alternatives: an adiabatic equation of state and radiative cooling. The solution has been tested with a 1-D hydrodynamic code. The incoming gas is not virialised and therefore its motion is supersonic, creating favourable conditions for the virial shock – its existence and stability have been analysed. The crucial support for this shock comes from the post-shock gas. If the virialised gas is adiabatic or its cooling is inefficient, the shock-heated gas becomes subsonic (with respect to the shock) and its support for the shock remains stable, with the shock positioned at  $\sim R_{\rm vir}$ . This is always the case for the adiabatic gas, which is also stable against gravitational collapse (i.e., Jeans instability) if the adiabatic index  $\gamma > 4/3$ . Gravitationally unstable gas will collapse to the centre, thus removing support from the shock, which will rapidly move inwards. The gas can be treated as adiabatic when the radiative cooling timescale is longer than the collapse timescale. The gravitational stability condition is slightly modified for gas with radiative cooling to an effective adiabatic index which includes the time derivatives,  $\gamma_{\text{eff}} \equiv (d \ln P/dt)/(d \ln \rho/dt)$ . Its critical value,  $\gamma_{\rm eff} > \gamma_{\rm crit} \equiv 2\gamma/(\gamma + 2/3) = 10/7$ , is close to the adiabatic case. Here P and  $\rho$  are thermal pressure and density in the gas. For a monatomic gas with  $\gamma = 5/3$ , this stability condition can be rewritten as

$$\frac{\rho_0 R_{\rm s} \Lambda(T_1)}{|u_0|^3} < 0.0126,\tag{6}$$

where  $\rho_0$  is the preshock gas density,  $u_0$  and  $u_1$  are the pre- and post-shock gas velocities,  $T_1 \propto u_0^2$  is the post-shock temperature,  $R_s$  is the shock radius, and  $\Lambda(T)$  is the cooling function.

While the 1-D hydrodynamics is an obvious simplification of both the halo and gas properties, its simplicity has a certain advantage in that it allows one to follow the analytical solution closely. It shows that the adiabatic shock exists always and gradually propagates outwards, coinciding with the virial radius,  $R_{\rm vir}$ . On the other hand, for radiative cooling in the gas with primordial composition, the shock exists only where the inflow encounters the disk, initially. With the halo growth this shock also moves outwards and stabilises around  $R_{\rm vir}$ . In the following, we shall argue that the cold inflow can join the disk smoothly, without being shock-heated – i.e., that 1-D hydrodynamics cannot capture this solution.

This simple 1-D model predicts a critical value for the DM halo mass above which the shock is supported at  $R_{\rm vir}$ . A weak dependence on the redshift of halo virialisation exists, and a stronger dependence on the gas metallicity as well (because it affects the cooling significantly). The mass range for the critical halo mass appears to be  $\sim 10^{11} M_{\odot}$ for a primordial gas composition, and  $\sim 5 \times 10^{11} M_{\odot}$  for about 0.05 of the Solar metallicity. For this metallicity, Press-Schechter  $M_*$  haloes will generate stable shocks only by  $z \sim 1.6$ . The corollary: virial shocks will form only in the massive haloes mentioned above, at low redshifts. The general condition for shock stability is that the cooling timescale of the shocked gas should be longer than the compression timescale.

A number of issues can complicate these conclusions: arbitrary triaxial halo shapes, the interaction between the supersonic gas infall and the forming disk, and the possible trapping of Ly $\alpha$  photons within the halo gas. The first two issues can be resolved in terms of numerical simulations (see below). The trapping of Ly $\alpha$  photons during gravitational collapse and its effects on the fragmentation and related issues of proto-disk formation are under investigation (e.g., Spaans & Silk 2006; Latif *et al.* 2011).

Following the analytical/1-D hydro approach discussed above, numerical simulations have been performed addressing a number of issues, e.g., what is the maximum temperature of the gas entering the DM haloes? The standard view has been that  $T_{\text{max}} \sim T_{\text{vir}}$ , but as we have already discussed, not all of the gas is shocked to  $T_{\text{vir}}$ . It is helpful to define two modes of accretion – first, a cold mode with a maximum temperature of  $T_{\text{max}} < T_{\text{vir}} \sim 10^5$  K, which was not shock-heated, is distributed anisotropically, and follows filaments into the DM halo. Second, a hot mode with  $T_{\text{max}} > T_{\text{vir}} \sim 10^5 \text{ K}$ , which has been shock-heated at  $\sim R_{\text{vir}}$ , cools down while in a quasi-static gas halo and is accreted quasi-isotropically. The filamentary inflow of the cold gas exhibits much lower entropy,  $\propto T/\rho^{2/3}$ , compared to the shocked halo gas (e.g., Nagai & Kravtsov 2003).

Simulations reveal a more complex picture when some of the gas is accreted via filaments, and some from cooling of the hot halo gas (Keres *et al.* 2005; Dekel & Birnboim 2006). Only about half of the gas follows the expected path of accretion, which is heated to  $T_{\rm vir}$ , then cools down and participates in the star formation. The rest of the gas stays much cooler at all times. The overall emerging picture is that of a bi-modal evolution of accreting gas. Specifically, the cold mode dominates in low-mass galaxies and DM haloes,  $M_{\rm gal} \leq 2 \times 10^{10} M_{\odot}$  and  $M_{\rm h} \leq 2.5 \times 10^{11} M_{\odot}$ , respectively, and the hot mode dominates in the higher-mass objects. As a result the cold mode is expected to dominate at high redshift and in the low-density environment at low redshifts. The hot mode will dominate in the high-density environment at low z, such as in galaxy clusters.

The quoted critical (baryonic) mass,  $M_{\rm gal} \sim 2 \times 10^{10} M_{\odot}$ , obtained from numerical simulations is close to the observed characteristic mass for a shift in galaxy properties,  $M_{\rm gal} \sim 3 \times 10^{10} M_{\odot}$  (e.g., Kauffmann *et al.* 2004; Kannappan 2004), based on a complete sample of SDSS galaxies. These studies focussed on the environmental dependence of various parameters which describe the galaxies, such as morphology, stellar mass, SFR, etc., quantifying the distribution of these properties with respect to galaxy mass. For stellar masses above the critical  $M_{\rm gal}$  no dependence on environmental factors has been found for the distribution of sizes and concentrations at fixed stellar masses, whereas for less massive galaxies, the trend has been detected for galaxies to be somewhat more concentrated and more compact in denser regions. The star formation history has been found to be much more sensitive to the environment: e.g., the relation between the  $\lambda = 4000$  Å break, specific SFR (per unit stellar mass) and  $M_{\rm gal}$  – with the same separator of  $\sim 3 \times 10^{10} M_{\odot}$ . The drop in the specific SFR for less massive galaxies was about a factor of ten over the density interval used in the study, much stronger in comparison with the more massive galaxies.

An interesting corollary is the apparent similarity between the redshift evolution of galaxy properties and their change as a function of a (local) density. In retrospect, this result is almost a common wisdom, reflecting the 'sped up' evolution of galaxies in overdense regions in the Universe.

Hence, compelling observational evidence exists that galaxies below the critical mass of  $M_{\rm gal} \sim 3 \times 10^{10} M_{\odot}$  are much more active in forming stars, have larger gas fractions, lower surface densities, and exhibit late-type morphologies. More massive galaxies have old stellar populations, supplemented with low gas fraction, higher surface densities, and early Hubble

types. This bimodal behaviour can have its origin in the fundamental way the galaxies grow, or rather in the way their growth is limited. If indeed a large fraction of the accreted gas is not heated to the virial temperatures, it can join the disk and be converted into stars. We discuss the associated processes in Section 5.4. The shock-heated gas, on the other hand, can also contribute to the star formation if its cooling time is sufficiently short. So, some of the hot-mode gas in haloes somewhat smaller than the critical one to sustain the shock (e.g., in low-density regions and/or at higher z) will cool down if not subjected to feedback. This gas can contribute to disk or spheroidal buildup over time.

However, as pointed out by Dekel & Birnboim (2006), above the halo mass of  $M_{\rm h} \sim 10^{12} M_{\odot}$ , the cooling timescale for  $\sim 10^{6-7}$  K hot- and low-density gas becomes longer than the Hubble time, and the gas, once heated to the virial temperature, will never cool down and hence will not contribute to the disk growth in any direct way. This hot gas which fills up the halo can also be subject to additional heating by AGN feedback, both mechanical (throughout the halo) and radiative (at the base), because of its high covering factor. For massive haloes, we expect that the only real effect of this feedback can be in generating an outflow of the overheated gas.

While this is only a circumstantial observational argument in favour of the existence of cold filamentary flows, it is nevertheless very intriguing by bringing up the same bimodality in galaxy properties. The prime observational issue of course remains the detection of these flows. Hot gaseous haloes have been detected in X-rays around individual galaxies, groups and clusters of galaxies (e.g., Crain *et al.* 2010a,b; Anderson & Bregman 2011). There is no direct observational evidence in favour of cold accretion flows, although accretion of cold patchy gas has been observed (e.g., Rauch et al. 2011). For higher-redshift galaxies, contradictory claims exist regarding the possibility that diffuse  $Ly\alpha$  emission around them comes from cold accretion flows (e.g., Dijkstra & Loeb 2009) and represents the cooling radiation (Fardal et al. 2001), or, alternatively, is the scattered light coming from the HII regions (e.g., Furlanetto et al. 2005; Rauch et al. 2011). Because of various reasons, including the low emissivity, absorption against bright sources like quasi-stellar objects (QSOs) is the most promising way to detect the cold accreting gas, especially in Ly $\alpha$ . Van de Voort *et al.* (2012) have argued that the high column density HI absorption detected at  $z \sim 3$  originates mostly in accreting gas with  $T \leq 3 \times 10^5$  K, based on numerical simulations. Rakic *et al.* (2012) have interpreted results of the Keck Baryonic Structure Survey of HI Ly $\alpha$  absorption in the vicinity of star-forming galaxies at  $z \sim 2-2.8$  as due to large-scale infall. It is not clear whether the individual HI absorbers can be attributed to cold accretion, based only on their proximity to the galaxy and a low metallicity (e.g., Giavalisco *et al.* 2011).

High-velocity clouds around the Milky Way galaxy (for a recent review see Sancisi

et al. 2008) can be closely related to the cold gas accretion phenomenon discussed here. Sancisi et al. (2008) have detected accretion rates of  $\sim 0.2 M_{\odot}$  in HI clouds, which is possibly a lower limit for our Galaxy, that has a SFR  $\sim 1 M_{\odot} \text{ yr}^{-1}$ . There are numerous ways in which cold gas clouds can form in the accreting matter without being processed by the DM substructure. One such possibility involves Rayleigh-Taylor instabilities in the halopenetrating filaments (Keres & Hernquist 2009). But additional options exist as well. These clouds can subsequently be accreted by the galaxy and contribute to the ongoing steady star formation there.

So why has cold accretion not been detected in a decisive manner so far? One can bring up the similar situation and difficulties in detecting cold accreting gas in AGN. At the same time, outflows are commonly detected both in AGN and in starburst galaxies. The plausible explanation may be in small cross sections, low emissivity and very high column densities along the line-of-sight due to the gas accumulation in the 'equatorial' plane.

## 5.3. Cold flows: redshift evolution

The coexistence of cold and hot modes of accretion can have interesting implications for galaxy growth. These modes depend differently on the environment, as well as on the feedback from stellar evolution and AGN. Additional issues raised so far in the literature involve a plausible difference of the associated initial mass functions (IMFs).

Cosmic star formation exhibits a broad maximum at  $z \gtrsim 1$  and a steep decline below this redshift (e.g., Madau *et al.* 1996). This decline can be associated with the decay of the cold accretion flows (e.g., Keres et al. 2005; Dekel & Birnboim 2006). Below  $z \sim 2$ , massive  $\sim 10^{12} M_{\odot}$  haloes become typical, the shocks are stable around  $R_{\rm vir}$ , and the cooling time of the shocked gas becomes too long, effectively quenching the cold-mode accretion. This defines the critical redshift,  $z_{\rm crit} \sim 2-3$ . After  $z_{\rm crit}$  the star formation will be suppressed in massive haloes and especially in galaxy clusters. Under these conditions, the observed bimodality in galaxy properties can find a natural explanation. In terms of the prevailing colours of stellar populations, which are determined by the stellar ages and SFRs, this shutdown of the cold accretion mode will result in the relatively quick transformation of galaxies with high SFRs. This means that the origin of the red sequence can be traced directly to the switching of the prevailing accretion mode, as noted by Dekel & Birnboim (2006). It would be a strong argument in favour of this picture if a number of bi-modal correlations, such as the colour-magnitude, bulge-to-disk ratio, or morphology-density ones, can be explained as corollaries of the cold gas supply shutdown at various redshifts and environments. However, there is a caveat: the bulge-to-disk ratio can be affected and even dominated strongly by other processes (e.g., Combes *et al.* 1990; Raha *et al.* 1991; Martínez-Valpuesta *et al.* 2006). Other correlations may exhibit similar trends.

Clearly, accretion flows that have been investigated for decades as the mechanism to power AGN are capable of playing a substantial role in growing galaxies embedded in DM haloes. Moreover, within the CDM framework, cold-mode accretion forms naturally because of the low dispersion velocities in the gas that has cooled down in the expanding Universe during the Dark Ages. The large turnover radii, corresponding to the accretion radius, and the substantially sub-Keplerian spin parameter  $\lambda$ , assure a strong dependence on the accretor mass, i.e., DM halo mass. It also means that the cold-mode accretion should dominate at high redshifts, and the hot mode should only pick up at low redshifts, if at all.

Keres *et al.* (2009a) have investigated the cosmological evolution of smooth accretion flows using simulations with sufficient resolution to follow up growth of galaxies in massive haloes only. Cold flows appear to dominate the global gas supply to galaxies basically at all times, especially in small galaxies residing in  $\leq 10^{12} M_{\odot}$  haloes for  $z \geq 1$ . At these redshifts, the galaxy growth was found to be only a function of its mass. At  $z \leq 1$ , the cold accretion on smaller galaxies has decreased sharply. These results have been confirmed by Brooks *et al.* (2009) – for galaxies up to  $L^*$  the cold accretion fuels the star formation. Romano-Díaz *et al.* (2008b) argued that late minor mergers with DM substructure ablate the cold disk gas and quench the star formation there. Cold accretion dominated growth has also been inferred in high-resolution simulations of galaxies at redshifts  $z \geq 6$  (Romano-Díaz *et al.* 2012).

Moreover, the total gas accretion rate has been found to peak at  $z \sim 3$  and to exhibit a broad maximum between  $z \sim 2-4$ , the same as the cold accretion. Hot accretion which consists of a shocked virialised gas that is able to cool down over relative short time has been found to *contribute little* over time, except lately, for  $z \leq 1$ , after peaking at  $z \sim 1.5$ . Mergers become globally important only after  $z \sim 1$ . Finally, the SFR has been estimated to correlate with the smooth accretion rate and to be about a factor two of the Madau diagram.

So, according to Keres *et al.* (2009a), galaxies grow via the accretion of cold and nevershocked gas, while the contribution of the hot mode of cooling shock-heated virialised gas is not important at all. This is a dramatic turnaround and a *paradigm shift* with respect to the standard picture described in Section 5.1. If verified, the implications of this are broad: it is the cold mode of accretion that drives the star formation in galaxies. However, taken at face value, this star formation will lead to fast conversion of gas into stars – an overproduction of the stellar mass already at an early time, if the feedback from stellar evolution and AGN is not efficient enough. In short, a mechanism to suppress the star formation is necessary. (This is discussed in Section 6). Another corollary is the possible shock at the inflow-disk interface. Is it avoidable? (See Section 5.5 for more options.) Is it observable? Lastly, the dominant cold mode of accretion must be incorporated into the prescriptions for semi-analytic models.

## 5.4. Cold flows: between the virial radius and the disk

Understanding the kinematics and dynamics of the cold flow which penetrates the DM halo and is not shocked to virial temperatures is crucial in order to estimate the flow's contribution to disk growth. As the filaments penetrate deep into the halo, their temperature stays approximately constant, because of the efficient cooling, and they are compressed by the surrounding hot halo gas, if it exists. The efficiency of this inflow contribution to the disk growth process is unclear at present. In principle, it can be expected to depend on at least two parameters: the angular momentum in the cold gas and the shape of the background gravitational potential. These will determine the prevailing trajectories within the DM haloes and to some degree the amount of dissipation in the infalling gas. As a result, we shall be able to estimate the infall timescale (which will be longer than the free-fall time within the DM halo) and the way this gas joins the growing disk, by smoothly merging or experiencing a strong shock. In the former case, the infall energy of the gas is transformed into rotational energy. In the latter, part of the infall energy will be radiated away sufficiently close to the shocked interface.

In smaller haloes and especially at higher redshifts, the virial shock is not sustainable at  $\sim R_{\rm vir}$ , and the forming galactic disk can be directly affected by the deposition of matter, linear and angular momenta, and energy by the inflow of the cold gas. How much dissipation is involved in this process? Is the local, i.e., inflow-disk interface, shock-unavoidable?

The 1-D case discussed above is not representative here, as the shock is unavoidable (if the cold inflow exists) and the angular momentum plays no role. Based on 3-D numerical simulations, Dekel & Birnboim (2006) argue that cold streams intersecting among themselves and with the forming disk will trigger starbursts, characterised by the most common mode of star formation in the Universe. The strength of the starburst will determine whether the disk will continue to grow relatively quiescently or whether the process will contribute to the spheroidal component.

Most cosmological simulations lack the necessary resolution to investigate the inflowdisk interface. The simplest way to circumvent this is to reproduce the thermal histories of the gas particles. Brooks *et al.* (2009) found that most of the gas joins the disk unshocked in a smooth accreting component, opposite to the gas accreted with the substructure, i.e., clumpy gas. The only exception is the disk evolution in the most massive halo, well above  $L^*$ . For this halo, the SFR is not exactly balanced by the accretion rate onto the halo, as a



Fig. 8.— 'Cat's cradle' morphology: face-on view of the gas disk (upper frames) and the extended DM regions (lower frames) showing the cold gas inflow joining the disk. The white arrows (right frames) underline the DM filaments and the associated gas inflow. Note that the gas streamlines join the disk at tangent angles which preclude strong shocks from forming and rather assure a smooth unshocked transition flow (from Heller *et al.* 2007b).

substantial delay in star formation occurs due to the prolonged cooling time of the shocked gas.

Heller *et al.* (2007b) have shown that the cold gas filaments can smoothly join the outer disk, being deflected from the disk rotation axis by the centrifugal barrier – no standing shock has been detected there. In this case the infall kinetic energy is converted into rotational energy. Interestingly, the gas filaments are supported by the DM filaments in a configuration which resembles the 'cat's cradle' – a small amorphous disk fuelled via nearly radial string patterns (e.g., Fig. 8).

If the inflow-disk interface shock does not exist or is sufficiently mild, what additional signatures of a recent accretion can be expected deep inside the host haloes? For a number of reasons discussed above and in Section 2, most probably the gas has a non-negligible
amount of angular momentum and will settle in some 'equatorial' plane outside the growing stellar disk. However, the orientation of this plane can differ substantially with respect to the stellar disk plane. This will lead to the formation of inclined and polar rings, warps, etc. Indeed, numerical simulations of such disks in a cosmological setting have demonstrated the formation of rings and warps, as a rule rather than an exception (e.g., Romano-Díaz *et al.* 2009; Roskar *et al.* 2010; Stewart *et al.* 2011). Specifically, Romano-Díaz *et al.* (2009) have demonstrated that the mutual orientation of the rotation axis of the disk, DM halo, and the accreting gas fluctuate dramatically over time, even during the quiescent periods of evolution (see their Fig. 19).

Dekel *et al.* (2009b) have shown that galaxies of  $\sim 10^{11} M_{\odot}$  at  $z \sim 2-3$  – at the peak of SFR in the Universe – have been fed by cold accretion streams, rather than by mergers. About 1/3 of the stream gas mass has been found in clumps, leading to mergers of  $\gtrsim 1/10$ mass ratio; the rest in the smooth streams. The deep penetration of cold streams happened even in DM haloes of  $> 10^{12} M_{\odot}$  which are above the critical mass for virial shock heating (Section 5.2). Dekel & Birnboim (2006) have noted that the cold gas streams are supported by DM streams whose characteristic width is smaller than  $R_{\rm vir}$ , and which are denser than the diffuse halo material. They cross the shock basically staying isothermal because of the short cooling distance. We return to the issue of penetrating streams in Section 7 on high-*z* galaxies.

There is a possibility that the extended XUV disks detected by *GALEX* (*Galaxy Evolution Explorer*), whose population can reach  $\sim 20\%$  locally, have their origin in accretion flows (e.g., Lemonias *et al.* 2011; Stewart *et al.* 2011). A strong argument in favour of such a scenario comes from the observations of such disks around massive early-type galaxies. Moreover, there is no indication that XUV disks prefer tidally-distorted disk galaxies, so they cannot originate as a result of a close passage or a merger event.

In the presence of a disk, one would expect that the gas accretion will join its outer parts, at least the majority of the inflow, as discussed above. The low-j material that can come closer to the rotation axis would be accreted earlier and such orbits would be depopulated quickly.

## 5.5. Cold accretion flows in the phase space

The phase space provides the maximum information about filamentary cold flows. Even 2-D phase space allows for a clear display of the accretion flows. It is especially suitable in order to follow up the phase mixing and violent relaxation processes discussed in Section 3.1.2.

Various complementary presentation options exist here, such as using  $R - v_{\rm R}$ ,  $r - v_{\rm circ}$  and/or  $r - \sigma$ , where R and r correspond to the spherical and cylindrical radii, and  $v_{\rm R}$ ,  $v_{\phi}$  and  $\sigma$  to the radial and azimuthal velocities and to the dispersion velocity, respectively.

Comparison of the evolution of pure DM and DM+baryon models in the  $R - v_{\rm R}$  plane reveals the effect of baryon inflow on the kinematics of the DM halo (Fig. 9 and Romano-Díaz et al. 2009). The mass-averaged radial velocities are negative outside  $R_{\rm vir}$  and lie below the  $v_{\rm R} = 0$  line, and change to positive inside the halo, initially. At later times, the mass-averaged velocity is zero, as the inner halo reaches its virial equilibrium. Both major and minor mergers (substructure) can easily be distinguished by the vertical spikes, and are much more prominent, by a factor of  $\sim 2$ , for models with baryons, before the tidal disruption. Moreover, the smooth accretion can be well separated from the substructure. The width of the inflowing stream, which represents the mass accretion flux, declines with time, while that of the rebounding material increases. In the process of tidal disruption, inclined 'fingers' form, again more prominent in the presence of baryons. The subsequently forming 'shell' structure reveals the insufficient mixing of merger remnants in the form of a  $R-v_{\rm R}$  correlation – 'streamers', which appear to be long-lived. Streamers formed after  $z \sim 1$ largely survive to z=0. The phase space also delineates the kinematical differences of the inner DM haloes in these models: note the outline of  $v_{\rm R}(R)$  at small radii. This effect can be explained in terms of the gravitational potential shape there, which represents a more centrally-concentrated object.

So the phase-space analysis shows that DM haloes, while reaching virial equilibrium, are far from relaxed in other aspects. Streamers are probably the best example of this inefficient relaxation, and are strengthened by the presence of baryons. The degree of relaxation in DM haloes can be further quantified using the smoothing kernel technique (Romano-Díaz *et al.* 2009). This procedure allows us to estimate the contribution of the *excess* DM mass fraction associated with density enhancements (i.e., subhaloes, tidal tails, and streamers) above some smoothed reference density which is time-adjusted. This excess mass fraction in the substructure becomes more prominent with time.

A complementary option to study the buildup of DM haloes is in the  $r - v_{\text{circ}}$  plane. In this plane, the halo kinematics is much more symmetric with respect to the  $v_{\text{circ}} = 0$  line. Mergers disrupt this symmetry which is quickly restored. Both major and minor mergers are easily traced in such a diagram. The high degree of symmetry between the number of prograde- and retrograde-circulating particles is very important in order to understand the dynamical state of DM haloes and growing stellar disks, especially the disk-halo resonant and non-resonant interactions which ultimately affect the disk ability to channel baryons toward the centre (e.g., Shlosman 2011). One note of caution: at higher redshifts, the haloes appear



Fig. 9.— Phase space evolution of a DM halo without (left) and with (right) baryons in the  $R - v_{\rm R}$  plane, run from identical initial conditions. The halo has been projected to collapse by  $z \sim 1.3$  with a mass of  $\sim 10^{12} h^{-1} M_{\odot}$ , based on the top-hat model. The epoch shown here corresponds to intensive merger activity and to the cold accretion growth in these simulations. The corresponding redshifts,  $z \sim 4 \rightarrow 1.5$ , are shown in the lower-right corners of each frame. Note the appearance of 'fingers' and 'shell' structure inside and outside of the halo – much more pronounced in the presence of baryons. The inflow containing both substructure and a smooth component is clearly visible as a stream penetrating deeply inside the DM halo, especially in the right-hand frames. The shape of the denser region is 'smashed' against the  $v_{\rm R}$ -axis in the presence of baryons and has a convex shape in the pure DM case. The colours correspond to the DM volume density. The vertical arrows display instantaneous value of  $R_{\rm vir}$ , the dashed white line shows  $v_{\rm R} = 0$ , and the solid blue line – mass-averaged  $v_{\rm R}$  at each R. The velocity axis is normalised by  $v_{\rm circ}$  – the circular velocity at  $R_{\rm vir}$  (from Romano-Díaz *et al.* 2009).

substantially triaxial in the range of radii, and hence  $v_{\text{circ}}$  provides a bad approximation to the mass enclosed within r. The overall symmetry in the  $r-v_{\text{circ}}$  diagram (e.g., Romano-Díaz *et al.* 2009) confirms that there is very little *net* circulation of the DM within the halo. The tumbling of the halo figure is also found to be negligible – the halo appears to be orientated along the main filament which feeds its growth.

#### 6. Disk growth: feedback

Why is it that such a small fraction of baryons has been converted into stars over the Hubble time? Strong arguments exist, as we have discussed in the previous sections, that an additional process which lowers the efficiency of conversion of gas into stars must be taken into account when considering the cosmological evolution of galaxies. High-resolution numerical simulations have demonstrated the necessity for this process. Energy, momentum and mass deposition from stellar and AGN evolution can have a profound effect on the state of the gas within  $R_{\rm vir}$ , when one considers a simple spherical geometry and their isotropic deposition. Whether this indeed happens when the geometry becomes more complex must be verified. This should be performed by including the by-products of any mechanical feedback on various spatial scales: stellar, AGN and galactic winds, as well as radiative feedback from these objects and from the cosmological background radiation.

A compelling example can be found in the comparison study of disk evolution in a cosmological setting, with and without feedback, (Section 2.1; see also Robertson *et al.* 2004), where in the absence of feedback the gas quickly violates the Toomre criterion and fragments. Moreover, without feedback there is an overproduction of metals, especially in small galaxies. Even more revealing is the overcooling problem (Section 2.1) which leads directly to the angular momentum catastrophe and involves gas which is bound to the DM substructures, cools down and falls to the centre of the substructure. The latter spirals in to the inner parts of the parent DM halo losing its J via dynamical friction (e.g., Maller & Dekel 2002, and Fig. 10). The gas, therefore, hitchhikes to the bottom of the potential well without being ablated and is deposited there when the substructure is dissolved by the tidal forces, and possibly contributes to the growth of a (classical) bulge.

Weak feedback has been tested, confirming that such models overproduce the observed baryonic mass function for galaxies, especially for the most and least massive objects (e.g., Keres *et al.* 2009b). Various 'preventive' feedbacks have been used to remove this discrepancy, such as a highly efficient AGN 'radio mode' – this did not improve the fit at the high-mass end. Furthermore, it has been impossible to recover the population of massive quiescent galaxies. The overall conclusion is that the solution should come from a more



Fig. 10.— Sketch of the overcooling problem (from Maller & Dekel 2002). Left: gas cools down and falls toward the centre of a satellite galaxy which is merging with the main DM halo. Dynamical friction accelerates the process. The cold gas hitchhikes with the sinking satellite and is losing its orbital J but is immune to stripping until the satellite falls to the centre of the parent halo and dissolves there. So gas overcooling leads to the J catastrophe (Section 2). Right: The effect of overcooling on the spin distribution of baryons compared to that of the DM. Baryons (filled curve) being tightly bound at the satellite centre spiral in with the satellite toward the inner parent halo and lose most of J. The figure shows that the J deficiency in baryons is almost a factor of ten.

'sophisticated' feedback mechanism which substantially suppresses the star formation in a fraction of galaxies, which increases with mass, while leaving the star formation in the remaining galaxies unchanged.

### 6.1. Bulgeless disks

Galactic bulges may provide a testing ground for our understanding of angular momentum redistribution in a forming disk and for various mechanisms that regulate the star formation. An unexpectedly large fraction, ~76%, of massive,  $\geq 10^{10} M_{\odot}$  galactic disks can be fit with a Sersic index  $n \leq 2$  and ~69% have a B/T mass ratio of  $\leq 0.2$ , both in barred and unbarred galaxies (Weinzirl *et al.* 2009). This result has been obtained from ground-



Fig. 11.— Relative numbers of galaxies with classical bulges and elliptical galaxies (red lines), galaxies with disky bulges (i.e., pseudobulges) (blue line), and all disky bulges (black dotted line) as a function of galaxy stellar mass (from Fisher & Drory 2011).

based imaging of a local sample of 182 galaxies that covers all Hubble types, S0/a–Sm, in the optical and the NIR. Disks inclined by  $i \ge 70^{\circ}$  have been excluded. Two-dimensional bulge-disk-bar decomposition has been performed on *H*-band images. This result is in stark contrast with predictions based on the numerical modelling of such objects – only disks that did not experience a major merger event since  $z \sim 2$  are expected to have such low B/Tratios. This number is more than an order of magnitude smaller than the observed fraction of low B/T disks. Taken at face value, this result points to a serious contradiction between the observationally detected trend and our theoretical understanding, and specifically to the role of major mergers in bulge formation.

This contradiction becomes even stronger in the recent study of the bulge population within a sphere of 11 Mpc radius using *Spitzer*  $3.6 \,\mu\text{m}$  and *HST* data (Fisher & Drory

2011). The dominant galaxy type in the local Universe has been found to possess pure disk properties, i.e., having a disky bulge (Section 3.2) or being bulgeless. The fraction of these galaxies exceeds 80% of the number of galaxies with a stellar mass of  $\geq 10^9 M_{\odot}$ . Classical bulges and elliptical galaxies account for about 25% and disks for about 75% of the total stellar mass within 11 Mpc. Moreover,  $\sim 2/3$  of the star formation in these galaxies occurs in galaxies with disky bulges. Figure 11 delineates the fractions of various bulge types found as a function of stellar mass. Below  $\sim 3 \times 10^9 M_{\odot}$ , galaxies are most likely to be bulgeless. If disky bulges do not originate in mergers (as we have discussed in Section 3.2), the results of this analysis reinforce those of Weinzirl *et al.* (2009) in the most dramatic way.

These results reinforce the opinion that additional physical processes are needed to explain much less massive spheroidal components in disk galaxies.

# 6.2. What type of feedback?

Feedback, or 'feed back', is typically defined as a cause-and-response chain of events that forms a (causal) loop. The loop can be stable or unstable. In simple terms, in the former case, the process repeats itself and we define it as a self-regulated one, and the feedback is negative. In the latter case, the initial conditions for the process are modified and the system must find a new stable loop or disintegrate. In this case the feedback is positive.

In the context of disk evolution, positive or negative feedback can increase or decrease the SFR in a disk. The list of relevant mechanisms is not short and involves radiation (stellar and AGN), winds (stellar, AGN, galactic), AGN jets and backflow cocoons of largescale relativistic jets, turbulence, SN, bubbles and superbubbles, spiral density waves, stellar and gaseous bars, and cold accretion along cosmological filaments. All the above mechanisms can induce both positive and negative feedback on the SFR. The disk response points to a self-regulation achieved with respect to the star formation process.

As we have discussed in Section 2, the absence of any feedback in disk evolution leads to an overestimate of the spheroidal stellar component. Hence, additional processes must regulate the fragmentation and star formation in disk galaxies. The cause for this can lie beyond the disk itself but fully understanding the disk response is crucial for predicting the feedback loop.

The physics of the various types of feedback is very complex and in many cases, e.g., AGN feedback, not yet fully understood. In some cases, e.g., turbulence, our understanding is mostly empirical. Moreover, many relevant processes, like star formation and turbulence, are well below the resolution limit of current numerical models of galaxy evolution in the cosmological context, and are therefore treated on the subgrid level, i.e., purely phenomenologically. We first discuss the main feedback mechanisms, and then, in Section 6.3, survey the star formation and feedback algorithms used in numerical simulations.

### 6.2.1. Supernova feedback

The simplest to model is *gravitational feedback*, which involves collisional heating of the gas. It occurs during gas-rich mergers. The collision-induced shocks help to virialise gas – a process whose efficiency depends on the orbits of the merger components. Most affected appears to be the low-density gas in the outer parts of the galaxy. The fate of the gas, however, is to remain largely bound to the merger product.

The SN feedback or yield is probably the most analysed in the literature. We follow estimates by Dekel & Woo (2003). Kinetic energies are similar for type Ia and type II SNe, although Ia are irrelevant here as they deposit their energy away from the star formation sites. A number of SN explosions resulting from a steady SFR of  $\dot{M}_*$  from gas mass  $M_{\rm gas}$ embedded in the DM halo of  $M_{\rm vir}$  can be estimated as

$$E_{\rm SN} \sim \epsilon \nu M_* t_{\rm rad},$$
 (7)

where  $\epsilon \sim 10^{51}$  erg is the initial energy released by a typical SN,  $\nu \sim 5 \times 10^{-36}$  g<sup>-1</sup> is the number of SNe per unit mass of stars (for the Salpeter IMF,  $\nu \sim 1$  per 100  $M_{\odot}$  of stars), and  $t_{\rm rad}$  marks the end of the adiabatic phase and the onset of radiative phase in the SN expansion. The SFR can be written in terms of the free-fall time as  $\dot{M}_* \sim M_*/t_{\rm ff}$ . So  $E_{\rm SN} \sim \epsilon \nu M_*(t_{\rm rad}/t_{\rm ff})$ . In the relevant range of gas temperatures, a few  $\times 10^4$  to a few  $\times 10^5$  K, the ratio  $t_{\rm rad}/t_{\rm ff} \sim 10^{-2}$  is constant. In order to expel (unbind) the amount of gas,  $M_{\rm gas}$ , from a galaxy which has virial velocity  $v_{\rm vir}$ , the energy released by the SNe should be  $E_{\rm SN} \gtrsim 0.5 M_{\rm gas} v_{\rm vir}^2$  – the binding energy of this gas. For this to happen, the velocity of the gas must exceed the critical velocity of  $v_{\rm crit} \sim 0.14 (\epsilon \nu M_*/M_{\rm gas})^{1/2} \sim 100 (M_*/M_{\rm gas})^{1/2} \, {\rm km \, s^{-1}}$ . In other words, the gas can be removed from DM haloes with virial velocities  $v_{\rm vir} \lesssim v_{\rm crit} \sim 100 \, {\rm km \, s^{-1}}$ , if a large fraction, say  $M_* \sim M_{\rm gas}$ , of the gas is turned into stars.

The associated DM virial mass which supports the formation of  $M_*$  of stellar mass for such  $v_{\rm vir}$  is  $M_{\rm vir} \sim 2 \times 10^{11} M_{\odot}$ , and the corresponding stellar mass  $M_* \sim 3-4 \times 10^{10} M_{\odot}$ , in apparent agreement with the bifurcation mass for bimodal evolution discussed in Section 5.2. So the SN feedback can reheat the ISM, push it into the halo and even expel a large fraction of the gas from smaller galaxies. One SN can in principle unbind a giant molecular cloud. The above estimate makes it plausible that the gas can be driven out of a galaxy by the SN feedback, *if a large fraction of the original gas has been converted into stars*. But SNe are not very efficient in converting the stellar rest mass into kinetic energy because  $f_{\rm SN} \equiv \epsilon/100 \, M_{\odot}c^2 \sim 2.8 \times 10^{-6}$ . While progress has been made in high-resolution hydrody-namical simulations of SN shells sweeping up the ISM, the details are not clear yet, especially how efficiently the energy is distributed among the baryons.

#### 6.2.2. OB stellar winds and AGN feedback

Stellar winds from OB stars are driven by radiation pressure in UV resonance lines of CNO elements and inject about the same amount of kinetic energy over their lifetime as SNe. The asymptotic velocities of these winds are  $\sim 2000-3000 \,\mathrm{km \, s^{-1}}$ . On the subgrid level, these winds from individual stars have been incorporated into numerical simulations (Heller & Shlosman 1994). Understanding the formation of galactic winds under a combined action of OB stars and SNe is more complicated.

AGN winds originate within the central pc – kpc region from the SMBH and are energyand/or momentum-driven. We limit our discussion here to sub-relativistic winds in radioquiet QSOs only and exclude the jets. The mechanisms responsible for these winds are based on radiation power, or are hydromagnetically (MHD) driven from the underlying accretion disk (e.g., Blandford & Payne 1982; Shlosman et al. 1985; Emmering et al. 1992; Königl & Kartje 1994). The former can be driven by absorption (scattering) in the resonance lines similar to those in OB stars, and by the dust opacity. The MHD winds can feed on the rotational energy of the disk. Note that the MHD winds are much more efficient in extracting the angular momentum from the underlying accretion disks. Unlike the radiationdriven winds, they can reduce the mass accretion rate substantially – the mass outflow rate in an MHD wind can exceed the inflow rate for some period of time. At the same time, magnetic torques can extract the angular momentum from the disk *without* much of the outflow at all. This cannot be achieved with the radiatively-driven winds – in order to extract angular momentum from the disk, they must maintain a very high mass flux in the wind. Because this is beyond the scope of our review, we avoid discussing here the magnetorotational instability (MRI) which can generate turbulence and transport J within the disk.

Outflows with clear signatures in UV and optical emission and absorption lines, and in the radio, have been known for a long time. But reliable measurements of a mass outflow rate has been obtained only recently, e.g., for a broad absorption line QSO, SDSS J0318-0600, at ~120  $M_{\odot}$  yr<sup>-1</sup> (Dunn *et al.* 2010). The kinetic luminosity for this object has been estimated at ~0.001 of its bolometric luminosity, at the lower end of assessments available in the literature. Powerful molecular outflows from AGN have been detected with a mechanical luminosity estimated from a few to 100% of the AGN bolometric luminosity (e.g., Reeves *et al.* 2009). This means that they cannot be driven by SNe but require a higher efficiency associated with radiatively-driven winds, or most probably with MHD winds. It also means that such outflows will deplete the *in situ* reservoir of molecular gas in a relatively short time. The molecular tori observed in type 2 AGN in the NIR can represent the base of such molecular outflows (Elitzur & Shlosman 2006).

What is the AGN analogue of the SN feedback efficiency or yield,  $f_{\rm SN}$ , estimated above? The total energy,  $E_{\rm AGN}$ , produced by an AGN depends on the conversion factor,  $\epsilon_{\rm AGN} \sim 0.1$ , of its accretion energy, i.e.,  $E_{\rm AGN} \sim \epsilon_{\rm AGN} \eta M_{\bullet} c^2$ , where  $\eta \sim 10^{-3} - 1$  is the conversion factor of the bolometric luminosity,  $L_{\rm bol}$ , of the AGN into mechanical luminosity, and  $M_{\bullet}$  is the mass of the central SMBH. The exact value of  $\eta$  is not known, and it is expected to depend on the nature of the major contributor to  $L_{\rm bol}$ , on the ratio of  $L_{\rm bol}$  to the Eddington luminosity, and possibly on additional parameters of the flow. We therefore leave  $\eta$  unconstrained and note that in the epoch of galaxy formation at high redshift, which can be characterised by plentiful fuel supply and high Eddington ratios for the AGN, the value of  $\eta$  can be as high as ~1. Moreover, if we assume for brevity that the  $M_{\bullet} - \sigma$  relation is maintained at these redshifts (quite unlikely!),  $M_{\bullet}/M_* \sim 10^{-3}$  and

$$E_{\rm AGN} \sim 10^{-4} \left(\frac{\epsilon_{\rm AGN}}{0.1}\right) \eta M_* c^2, \tag{8}$$

which means that the efficiency of the high-z AGN is  $f_{AGN} \sim 10^{-4} \eta$ . This estimate can be about 10–50 times higher than  $f_{SN}$ , but remains speculative.

The first serious attempt to account for AGN feedback in high-resolution numerical simulations used purely thermal feedback, and no delivery or coupling mechanisms were specified (Springel 2005). A fraction of the isotropic bolometric luminosity of the AGN has been assumed to be deposited locally. Momentum transfer has been ignored. Application of this feedback resulted in limiting the growth of the SMBH and expulsion of the ISM from the galaxy. Less realistic but more detailed simulations have shown that the momentum transfer dominates because of the very short cooling timescales and the inability to retain thermal energy in the dense gas found in galactic centres (e.g., Ostriker *et al.* 2010). This is also symptomatic of OB stellar winds and winds from accretion disks with effective temperatures in the UV.

So AGN can have a potentially dramatic effect on the ISM and IGM based on their energy output. If this energy, momentum and mass around the SMBHs were distributed in a highly symmetric fashion, this could ensure an efficient coupling with baryons. However, besides direct evidence in the form of collimated jet-ISM/IGM interaction in radio galaxies (both in X-rays and radio bands) and in clusters of galaxies, additional evidence is scarce. Furthermore, the relevant physics of AGN output deposition in baryons is still poorly understood. *How exactly is the coupling to baryons achieved?* Even when the feedback energy exceeds the gas binding energy, the gas can escape along the preferential directions or via the fluidised bed-type phase transition, dramatically reducing the feedback.

While models based on energy- and momentum-driven outflows have been proposed, these are mostly phenomenological models. The detailed physics of energy and momentum deposition is under investigation now, as well as attempts to translate it into subgrid physics. The important issues involve the driving mechanisms for the wind, their dependence on mass accretion flows, on Eddington ratio, geometric beaming, and additional factors.

## 6.2.3. Galactic winds

While at present there are still theoretical difficulties with almost any type of wind from any object (e.g., star, accretion disk), these winds nevertheless exist and there are clear incentives in working out the corollaries of this existence. The presence of galactic outflows is supported by numerous observations (e.g., Heckman 1994, and references therein). In most cases they are driven by the SNe and winds from OB stars in galaxies that experience starbursts. The contribution of AGN is debatable at present. The driving by the SNe and stellar winds occurs when ejecta of individual sources form a bubble of hot gas,  $\sim 10^{7-8}$  K, which expands due to the strong overpressure down the steepest pressure gradient and enters the 'blow-out' stage. Besides the mass, energy and momentum injected by such winds, this is probably the main way the highly-enriched material can be injected into the halo and further out into the IGM. The winds appear inhomogeneous and carry embedded cold,  $\sim 10^4$  K clouds with them, so they represent a multiphase ISM. Their velocities appear correlated with the galaxy stellar mass or its SFR (e.g., Martin 2005).

The overall energetics of these bubbles and superbubbles points to the important and even crucial role galactic winds play in galaxy evolution, from determining the size of galactic disks to regulating the star formation process in galaxies, and removing the overcooling problem discussed earlier. In this context, the overcooling results from under-resolved ISM which will radiate away all the thermal energy deposited by the feedback. The proper resolution must correctly treat the merging of individual SN bubbles producing low-density superbubbles which will remain hot for a prolonged period of time – this resolution is not yet achievable in cosmological or individual-galaxy numerical simulations. Only when small volumes representing a region in the disk are modelled, these processes can be followed. To summarise, any modelling of galaxy formation and evolution must include the development of galactic outflows triggered either by stellar or AGN feedback. To circumvent this lack of numerical resolution, the necessary physics can be introduced on the subgrid level. We discuss four such algorithms: the constant velocity, the delayed cooling, the blastwave, and the variable wind models.

- Constant wind model (Springel & Hernquist 2003; Springel 2005)
  - This wind model is based on a multiphase ISM treatment. Two phases, cold and hot, are not directly resolved but coexist in a single SPH particle. So dynamically they cannot be separated. The two phases are followed above the critical gas density which corresponds to the threshold,  $\rho_{\rm SF}$ , above which the star formation is allowed to occur. The SNe directly heat the ambient hot phase whose cooling timescale is long. This effectively modifies the equation of state for the ISM. The cold phase is heated and evaporated via thermal conduction from the hot phase. Mass-transfer equations between the two ISM phases are solved analytically. The galactic wind is triggered by modifying the behaviour of some gas particles into the 'wind' particles. The wind particles are not subject to hydrodynamical forces and experience the initial kick from the SN. All wind particles have the same constant velocity and the same mass-loading factor,  $\beta_{\rm w} \equiv \dot{M}_{\rm w}/\dot{M}_{\rm SF}$ , where  $\dot{M}_{\rm w}$  is the mass loss in the wind and  $\dot{M}_{\rm SF}$  is the SFR. The observational constraints on the value of the loading factor  $\beta_{\rm w}$  are weak.
- Delayed cooling wind model (Thacker & Couchman 2000; Heller et al. 2007b) The energy injection by the SN and OB stellar winds affects the fixed number of neighbouring SPH particles. This process is resolved by at least five timesteps and extends to  $t_{\rm in} \sim 3 \times 10^7$  yr – the feedback timescale. Radiative cooling is disabled for these neighbours. The SN energy is deposited in the form of thermal energy and converted into kinetic energy via an equation of motion and using the energy thermalisation parameter. The affected particles do not interact hydrodynamically over the time period, which depends on the minimum of  $t_{\rm in}$  and the time it takes for the wind particle to move to an ambient density below some threshold density.
- Blastwave wind model (Stinson et al. 2006)

This model is based on the adiabatic (Sedov-Taylor) and snowplough phases in the SN expansion. The blastwave is generated by the collective explosion of many SNe of type II. The maximum radius of the blastwave is given by the Chevalier (1974) and McKee & Ostriker (1977) formalism. Radiative cooling is disabled for  $R \leq R_{\text{blast}}$ . The timescale for the blastwave to reach this radius is of the order of a timestep resolution. The blastwave wind model has been efficient in moving the peak of the star formation to lower redshifts, well beyond the last major merger, and has significantly reduced the SFR during mergers due to the feedback in progenitors.

## • Variable wind model (Choi & Nagamine 2011)

This model adopts the subgrid multiphase ISM. All the previous algorithms have loaded galactic winds with star-forming particles and their close neighbours only. In this model particles from low-density/high-temperature and high-density/low-temperature are selected. The main parameters are chosen as the wind load  $\beta_w$  defined above and the wind velocity  $v_w$ , and both are constrained by observations. Typically, observations express these two parameters in terms of the host galaxy stellar mass,  $M_*$ , and the SFR. This requires that simulations compute both parameters on the fly as the model is running and not in the post-processing stage, which is challenging. In the original version, the outer density of baryons in a galaxy is limited by  $0.01 n_{\rm SF}$ , where  $n_{\rm SF} \sim 0.01-0.1 \, {\rm cm}^{-3}$  (defined above) is the threshold for star formation. The value of  $n_{\rm SF}$  is based on the translation of the threshold surface density in the Kennicutt-Schmidt law, SFR  $\sim \Sigma_{\rm gas}^{\alpha}$ , where SFR is the disk surface density of star formation,  $\Sigma_{\rm gas}$  is the surface density of the neutral gas, and  $\alpha \sim 1-2$ , depending on the tracers used and on the relevant linear scales.

In the variable wind model, the wind velocity is calculated as a fraction of the escape speed from the host DM halo,  $v_{\rm w} = \zeta v_{\rm esc}$ , where  $\zeta = 1.5$  for momentum-driven and  $\zeta = 1$  for energy-driven winds is a scaling factor. The SFR is determined from the empirical relation with  $v_{\rm esc}$ ,

SFR = 1.0 
$$\left(\frac{v_{\rm esc}}{130\,{\rm km\,s^{-1}}}\right)^3 \left(\frac{1+z}{4}\right)^{-3/2} M_{\odot}\,{\rm yr}^{-1},$$
 (9)

which is consistent with observations (e.g., Martin 2005). Hence, the wind velocity is an increasing function of redshift and the SFR. The load factor,  $\beta_{\rm w}$ , is assumed to represent the energy-driven wind in the low-density case,  $n < n_{\rm SF}$ , and the momentumdriven wind for  $n > n_{\rm SF}$ .

## 6.3. Feedback and star formation

The physics of isolated stellar, gaseous and gas+stars disks have been analysed extensively over the last few decades by means of analytic methods and numerical simulations. The parallel study of galaxy evolution in a cosmological setting placed emphasis on the initial conditions and on the fact that galaxies are open systems that can exchange mass, momentum and energy with their environment. Under these conditions, the rate of secular evolution can be dramatically accelerated and the direction of this evolution altered significantly. The approximation that galaxies are in dynamical equilibrium remains, except during the merger events. Probably the most important corollary of studying galaxies in cosmology is narrowing the range of initial conditions and requiring that the evolution complies with them. Understanding that early disks have been much more gas-rich than disk galaxies in the local Universe brings about the natural question of what prevented the full conversion of this gas into stars over the Hubble time. This can be achieved either by keeping the gas at low densities or high temperatures, to prevent the Jeans instability from developing. However, it is difficult to maintain low densities when the gas assembles in disks, unless strong deposition of momentum, energy or both expels the gas from the disk.

Most of the prescriptions for star formation can be followed from Katz (1992) with some modifications and involve the Jeans instability. An additional constraint introduces the critical volume density for star formation,  $n_{\rm SF}$ , which corresponds to the total HI + H<sub>2</sub> (atomic and molecular hydrogen) surface density threshold in the Kennicutt-Schmidt (K-S) law,  $\Sigma_{\rm SF} \sim 3-10 M_{\odot} \,{\rm pc}^{-2}$  (e.g., Schaye & Dalla Vecchia 2008). However, new observational evidence points to a considerable dispersion in the values of the threshold surface density,  $\Sigma_{\rm SF}$  and the slope  $\alpha$ , as well as to a substantial steepening of the K-S law above  $z \sim 3$  (e.g., Gnedin & Kravtsov 2011 and references therein). A growing body of evidence points to a dependence of the SFR on the surface density of the molecular hydrogen, rather than on the total surface density of the neutral hydrogen.

An intriguing question is whether the Jeans instability in the neutral gas triggers the formation of  $H_2$ , or whether it is the formation of  $H_2$  that triggers the Jeans instability and subsequent gravitational collapse. Current efforts focus on understanding the various factors which regulate the formation and destruction of  $H_2$ , such as dust abundance and metallicity, UV background, gravitational instabilities facilitating the gas cooling, etc. A fully self-consistent model of the molecular gas balance in the ISM will be developed in the next few years.

We have discussed the mechanisms that are responsible for the feedback from stellar and AGN evolution and pointed out the importance of understanding the ways in which energy and momentum are distributed among the ISM and the IGM. While the sources of energy and momentum operate on very small scales,  $< 1 \,\mathrm{pc}$ , compared to characteristic galactic scalelengths and scaleheights,  $> 1 \,\mathrm{kpc}$ , they are deposited on much larger scales and can have a global effect. In addition, there is broad agreement between observations, theory and numerical simulations that the multiphase ISM is required to treat the feedback properly.

One possibility for progress in this direction lies in a further increase of the spatial resolution of numerical models, to an extent that at least part of the subgrid physics is in fact simulated, e.g., turbulent cells in the ISM. Sub-50 pc resolution has shown the formation of hot SN bubbles, superbubbles and chimneys, in tandem with turbulent flows there (e.g.,



Fig. 12.— Effect of density threshold on star formation. Left: a low-density threshold – a single massive star formation region exists; Right: a high-density threshold. In the latter case, a much smaller region becomes Jeans-unstable and a number of smaller star formation sites exist, resulting in lower star formation efficiencies when averaged over all phases of the ISM.

Ceverino & Klypin 2009). Galactic winds develop naturally under these conditions, without additional ad hoc assumptions. Unfortunately, such resolution is difficult to achieve in present-day fully self-consistent cosmological simulations.

Another by-product of high-resolution modelling is the possibility of using a much higher threshold for star formation than usual, corresponding to molecular gas. The effect of going to higher critical densities for star formation is shown in Fig. 12, and results in lower star formation efficiencies when averaged over all phases of the ISM. A proper treatment of the cold gas phase is also crucial in order to account for dissipation in supersonic flows. In fact, in order to reproduce the observed log-normal probability density function (PDF) in the ISM, it is necessary to resolve the broad density range, and in particular to resolve the critical density of 10<sup>5</sup> times that of the average density in the ISM (e.g., Elmegreen 2002). This requirement in tandem with the ISM heating by gravitational instabilities leads to self-regulation, when turbulence limits the efficiency of the star formation process.

Gas-rich disks are prone to fragmentation – the so-called Toomre (1964) instability. The idea of a marginally gravitationally stable gaseous disk was originally proposed by Paczynski (1978), who specifically considered the case of turbulence driven by gravitational instabilities in a disk that was able to cool below  $Q \sim 1$ . The resulting turbulent gravitational viscosity

was found to be responsible for the disk re-heating, angular momentum transfer and inflow. Analysis and numerical simulations of isolated two-component gas+stars disks indeed have demonstrated the formation of massive clumps that migrate to the centre and heat up the stellar component as a result of dynamical friction, in agreement with analytical estimates (Shlosman & Noguchi 1993; Noguchi 1999). The characteristic timescale for spiraling in toward the central kpc is about a couple of orbital periods. Noguchi (1999) further argued that these clumps contribute to the buildup of galactic bulges.

The developing massive clumps will migrate to the centre over a few dynamical times, while maintained in a marginally stable state with Toomre's  $Q \sim 1$ , and contribute there to the bulge growth (e.g., Dekel *at al.* 2009a). In reality, it is not trivial to stabilise the self-gravitating clumps against runaway star formation. Dekel *et al.* propose that gravitational interactions between the clumps will also induce turbulence inside the clumps, which will stabilise them against gravitational collapse over the spiralling-in timescale – an interesting possibility so far not verified.

In the cosmological context, including star formation, these clumps, in addition to forming in the disk itself, can be supplied by the incoming flows from cosmological filaments (Heller et al. 2007b; Dekel et al. 2009a; Ceverino et al. 2010). The clumps did, however, show vigorous star formation. Rather than being supported by clump-clump interactions, Heller et al. found that the feedback from stellar evolution has provided support for the clumps, while some of them have been sheared and/or collisionally destroyed before they entered the central kpc. Moreover, the energy/momentum feedback parameter has been varied and the clumps have appeared earlier and have been more numerous in models where this feedback has been smaller. The final bulge-to-disk mass ratio has also shown a clear anti-correlation with the amount of the feedback. Hence, as shown by Heller et al. energy/momentum feedback inside the gas-rich disk delays its fragmentation, and, at the same time, decreases the star-forming activity inside these massive clumps. Internal turbulent pressure in massive self-gravitating clumps can indeed play the role of delaying the star formation, but it is not clear whether it can be driven by the outside turbulence in the disk. The possible internal drivers of turbulence can be their gravitational collapse and energy input from newly formed massive stars and SNe.

Dense molecular clouds in the ISM form via supersonic turbulence in the ISM. The supersonic velocities decrease on smaller scales as  $v \sim \text{scale}^{0.5}$  (Larson 1981), and may represent a turbulent field dominated by shocks. This result is supported by numerical simulations of supersonic turbulence and by analytical calculations (e.g., Padoan & Nordlund 2002, and references therein), with the possible extention to include the cloud column density,  $v \sim \Sigma^{0.5}$  scale<sup>0.5</sup> (Ballesteros-Paredes *et al.* 2011). The velocity-scale relation may deter-

mine the characteristic scale for gravitational instabilities and sites for star formation. The turbulent driving comes from the larger scales and dissipation occurs within the clouds. Recent results show that turbulent motions inside molecular clouds are driven by gravitational collapse (e.g., Ballesteros-Paredes *et al.* 2011).

#### 7. High-redshift galaxies

The epoch of galaxy formation follows the end of the Dark Ages when baryons could start to accumulate within the DM haloes and star formation was triggered. The scope of this review does not allow us to go into the details of this fascinating subject. Here we shall focus on galaxy evolution during the reionisation epoch, at redshifts  $z \sim 6-12$ . We shall not discuss the formation and evolution of the Population III stars either, which has been largely completed by the onset of the reionisation process, except maybe in low-density regions. Section 8 will touch upon some aspects of SMBH formation in  $10^8 M_{\odot}$  DM haloes. All the problems discussed in the previous sections remain relevant at these high redshifts.

The rapidly increasing list of objects above  $z \sim 6$  makes it possible to study the population of galaxies during reionisation. Deep imaging in multiband surveys using the Wide Field Camera 3 (WFC3) on the *HST*, as well as some ground-based observations using 8 m telescopes, have revealed galaxies via absorption at wavelengths shorter than Ly $\alpha$  from the intervening neutral hydrogen (e.g., Bouwens *et al.* 2010). In many cases, these photometric redshifts could be verified spectroscopically, up to  $z \sim 7$  (e.g., Pentericci *et al.* 2011). The majority of reionisation-epoch galaxies are faint, but much rarer brighter galaxies have also been identified at  $z \sim 8$  by means of a large-area medium-deep *HST* survey (Brightest of Reionizing Galaxies, BoRG) along random lines of sight, including the candidate for the most distant protocluster (Trenti *et al.* 2011). Even fainter galaxies have been found using gravitational lensing by massive galaxy clusters.

## 7.1. The high-redshift galaxy zoo

One of the most successful methods to search for reionisation-epoch galaxies is the dropout method based on the absorption short of some characteristic wavelength, the 912 Å Lyman break and a smaller break at Ly $\alpha$  1216 Å, which originate in the intervening neutral hydrogen (e.g., Steidel *et al.* 1996). Using multiwavelength imaging and filters, objects 'disappear' (drop out) when a particular and progressively redder filter is applied. The resulting break in the continuum spectrum allows us to determine the photometric redshift

of the object. For  $z \sim 6$ , the break lies at ~8500 Å. This technique has been applied first to U-band dropouts – galaxies that lack flux in the U-band  $(z \sim 3)$ , then to g-band dropouts  $(z \sim 4)$ . The choice of the filter determines the targeted redshift. Additional dropouts have been named according to the relevant bands,  $i_{775}$   $(z \sim 6)$ ,  $z_{850}$   $(z \sim 7)$ , Y  $(z \sim 8-9)$  and J  $(z \sim 10)$ . Existing data from NICMOS, GOODS/ACS and UDF can reveal dropouts up to  $z \sim 10$ . The population of detected galaxies has already provided substantial constraints on the galaxy growth in the Universe at that epoch.

The expanding classification of high-z galaxies has its origin in diverse observational techniques used for their detection and study, resembling the early stages of AGN classification, before unification. Galaxies that exhibit a break in the Lyman continuum redshifted to the UV and other bands have been called Lyman break galaxies (LBGs). Complementary to continuum-selected surveys, the Ly $\alpha$  galaxies, or so-called Ly $\alpha$  emitters (LAEs), have been mostly detected in narrow-band imaging surveys. Such surveys typically miss the LBGs because of the faint continuum. Spectroscopic identification of  $z \gtrsim 6$  LBGs is only possible if they have strong Ly $\alpha$  emission, and are bright (e.g., Vanzella *et al.* 2011).

An important question is what is the relationship between various classes of high-z galaxy populations and what are their low-z counterparts. Especially interesting is their relationship to sub-mm galaxies, found at  $z \sim 1-5$ . These sub-mm galaxies have been detected in the 200  $\mu$ m – 1 mm band, via redshifted dust emission, using the Sub-mm Common-User Bolometer Array (SCUBA) camera. These objects have a negative K-correction<sup>2</sup> because the Rayleigh-Jeans (RJ) tail of the Planck blackbody distribution. Galaxies in the RJ tail become brighter with redshift. They are generally not SBGs because of the weak UV emission. The sub-mm galaxy population consists of very luminous objects with bolometric luminosity  $\sim 10^{12-13} L_{\odot}$ , emitted mostly in the IR. Powered by intense starbursts, their estimated SFRs are  $\sim 10^{2-3} M_{\odot} \text{ yr}^{-1}$ .

## 7.2. Mass and luminosity functions

Observations of  $z \gtrsim 6$  galaxies have shown a rapidly evolving galactic LF which agrees with the predicted DM halo mass function (e.g., Bouwens *et al.* 2011). The UV LF of LBGs has been established with its faint end exhibiting a very steep slope (e.g., Bouwens *et al.* 2007). Using the Schechter function fit,  $\phi(L) = (\phi^*/L^*)(L/L^*)^{\alpha} \exp(-L/L^*)$ , the *faint* end of this LF at  $z \sim 7$  has the slope of  $\alpha = -1.77 \pm 0.20$ , and  $\phi^* = 1.4 \times 10^{-3} \text{ Mpc}^{-3} \text{ mag}^{-1}$ , which is consistent with no evolution over the time span of  $z \sim 2-7$  (e.g., Oesch *et al.* 2010). The

<sup>&</sup>lt;sup>2</sup>The K-correction is the dimming of a source due to the 1 + z shift of the wavelength band and its width.



Fig. 13.— UV LFs for  $z \sim 4$ , 6, 8 and projected LF at  $z \sim 10$  (Oesch *et al.* 2012). The  $z \sim 10$  LF extrapolated from fits to lower-redshift LBG LFs is shown as a dashed red line (see also the text). For comparison the  $z \sim 4$  and  $z \sim 6$  LFs are plotted, showing the dramatic buildup of UV luminosity across  $\sim 1$  Gyr of cosmic time. The light-grey vectors along the lower axis indicate the range of luminosities over which the different data sets dominate the  $z \sim 10$  LF constraints.

bright end of the LF evolves significantly over this time period. An even steeper faint end of the LF,  $\alpha = -1.98 \pm 0.23$ , has been claimed recently (Trenti 2012). The SFR appears to decline rapidly with increasing redshift. So by  $z \sim 6$ , the number of ionising photons is just enough to keep the Universe ionised, and most of them come from objects fainter than the current detection limit of the *HST* (e.g., Oesch *et al.* 2010; Trenti *et al.* 2010; Trenti 2012).

An accelerated evolution of galaxies during reionisation has been predicted and observed (e.g., Bouwens *et al.* 2007, 2010; Trenti *et al.* 2010; Lacey *et al.* 2011; Oesch *et al.* 2010, 2012). Strong evolution is expected for  $z \sim 8-10$ , by about a factor of  $\sim 2-5$ . The estimated number of  $z \sim 10$  galaxies has been derived from the observed LF at  $z \sim 6$  and 8 (Fig. 13). Using this LF, six objects are expected to be present in the field at  $z \sim 10$ , but only one has



Fig. 14.— Evolution of the UV LD above  $M_{1400} = -18 \text{ mag} [> 0.06 \text{ L}^*(z = 3)]$  (Oesch *et al.* 2012). The filled circle at  $z \sim 10.4$  is the LD directly measured for the galaxy candidate. The red line corresponds to the empirical LF evolution. Its extrapolation to z > 8 is shown as a dashed red line.

been detected. Hence, the LF appears to drop even faster than expected from the previous empirical lower-redshift extrapolation. The resulting accelerated LF evolution in the range of  $z \sim 8-10$  has been estimated at  $\gtrsim 94\%$  significance level (Oesch *et al.* 2012).

An important conclusion from the above studies has been the realisation that the UV luminosity density (LD) originating in the high-z galaxy population levels off and gradually falls toward higher z, in the range  $z \sim 3-8$  (Fig. 14). The LD data at  $z \sim 4-8$  are taken from Bouwens *et al.* (2007) and Bouwens *et al.* (2011). As can be seen in Fig. 14, the LD increases by more than an order of magnitude in 170 Myr from  $z \sim 10$  to 8, indicating that the galaxy population at this luminosity range evolves by a factor  $\geq 4$  more than expected from low-redshift extrapolations. The predicted LD evolution of the semi-analytical model of Lacey *et al.* (2011) is shown as a dashed blue line, and the prediction from theoretical modelling (Trenti *et al.* 2010) is shown as a blue solid line. These reproduce the expected LD at  $z \sim 10$  remarkably well.

A strong decline in the LF beyond  $z \sim 8$  has corollaries for the reionisation by the more

luminous galaxies at this epoch, as the number of luminous sources appears insufficient for this process. These data point clearly to a strong evolution of the galaxy population, but what is the cause of this evolution?

Analysis and modelling of the available data point to the underlying cause: the accelerated evolution is driven by changes in the DM halo mass function (HMF), as follows from theoretical considerations (e.g., Trenti *et al.* 2010) and semi-analytical modelling (e.g., Lacey *et al.* 2011), and not by the star formation processes in these galaxies. Interestingly, the rapid assembly of haloes at  $z \sim 8-10$  alone can explain the LF evolution (Trenti *et al.* 2010). However, this assumption has never been put to a self-consistent test using high-resolution simulations with the relevant baryon physics. The possible link between LF and the DM HMF has been studied by means of the conditional LF method (e.g., Trenti *et al.* 2010 and references therein) to understand the processes regulating star formation. The main conclusions can be summarised as (1) a significant redshift evolution of galaxy luminosity vs halo mass,  $L_{gal}(M_h)$ , (2) only a fraction  $\sim 20-30\%$  appear to host LBGs, and (3) the LF for  $z \gtrsim 6$  deviates from the Schechter functional form, in particular, by missing the sharp drop in density of luminous  $M \lesssim -20$  galaxies with L. For example, due to the short timescales  $- \Delta z \sim 1$  corresponds to  $\lesssim 170$  Mpc – it becomes difficult to rely on the fast evolution of  $L_{gal}(M_h)$ , while  $M_h$  evolves rapidly at these redshifts.

Due to the nature of the hierarchical growth of structure, high-z galaxies should appear and grow fastest in the highest overdensities, and therefore are expected to be strongly clustered around the density peaks. For example, Trenti et al. (2012) infer the properties of DM haloes in the BoRG 58 field at  $z \sim 8$  based on the found five  $Y_{098}$ -dropouts, using the Improved Conditional Luminosity Function model. The brightest member of the associated overdensity appears to reside in a halo of  $\sim (4-7\pm 2) \times 10^{11} M_{\odot}$  – a 5  $\sigma$  density peak which corresponds to a comoving space density of  $\sim (9-15) \times 10^{-7} \,\mathrm{Mpc^{-3}}$ . It has  $\sim 20-70\%$ chance of being present within the volume probed by the BoRG survey. Using an extended Press-Schechter function, about 4.8 haloes more massive than  $10^{11} M_{\odot}$  are expected in the associated region with the (comoving) radius of 1.55 Mpc, compared to less than  $10^{-3}$  in the random region. For higher accuracy, a set of 10 cosmological simulations (Romano-Díaz et al. 2011a) has been used, tailored to study high-z galaxy formation in such an over-dense environment. A DM mass resolution of  $3 \times 10^8 M_{\odot}$  has been used, and, therefore, haloes with masses  $\gtrsim 10^{11} M_{\odot}$  have been well resolved. The constrained realisation (CR) method (e.g., Bertschinger 1987; Hoffman & Ribak 1991; Romano-Díaz et al. 2007, 2009, 2011a,b) has been instrumental in modelling these rare over-dense regions. We describe this method below.

The CR method consists of a series of linear constraints on the initial density field used

to design prescribed initial conditions. It is not an approximation but an *exact* method. All the constraints are of the same form – the value of the initial density field at different locations, and are evaluated with different Gaussian smoothing kernels, with their width fixed so as to encompass the mass scale on which a constraint is imposed. The set of mass scales and the location at which the constraints are imposed define the numerical experiment. Assuming a cosmological model and power spectrum of the primordial perturbation field, a random realisation of the field is constructed from which a CR is generated. The additional use of the zoom-in technique assures that the high-resolution region of simulations is subject to large-scale gravitational torques. The CRs provide a unique tool to study high-z galaxies at an unprecedented resolution. It allows one to use much smaller cosmological volumes, and, without any loss of generality, accounts for the cosmic variance.

The initial conditions for the test runs described above have been constrained to have a halo of mass  $\sim 10^{12} M_{\odot}$  by  $z \sim 6$ . This halo has reached  $\sim 5 \times 10^{11} M_{\odot}$  by  $z \sim 8$  in compliance with BoRG 58-17871420. Within the field of view of  $70'' \times 70''$  and the redshift depth of  $\Delta z \sim 19$  Mpc about 6.4 haloes more massive than  $\sim 10^{11} M_{\odot}$  have been expected, and the highest number found in the simulations was 10 (Fig. 15). A random (unconstrained) region of the same volume has been estimated to host  $\sim 0.013$  such haloes. The probability of contamination in such a small area is negligible,  $\sim 2.5 \times 10^{-4}$ . In summary, if indeed the brightest member of the BoRG 58 field lives in a massive DM halo, the fainter dropouts detected in this field are part of the overdensity that contributes to the protocluster, depending of course on spectroscopic confirmation. Simulations provide some insight into the fate of this overdensity with a total DM mass of  $\sim (1-2) \times 10^{13} M_{\odot}$  – it has collapsed by  $z \sim 3$ , and is expected to grow to  $\sim (1-2) \times 10^{14} M_{\odot}$  by z = 0.

The evolution of the HMF is very sensitive to the assumed cosmology, because the halo growth rate depends on the average matter density in the Universe. As the DM is not observable directly, numerical simulations are indispensable in studying the halo growth, and analytic techniques provide an additional tool. The process of DM halo formation quickly becomes non-linear which makes an analytical follow-up difficult. Analytically, one relies on modelling the spherical or ellipsoidal collapses, but only N-body simulations reveal the complexity of the process which is hierarchical in Nature. Numerically, the halo growth depends on the force resolution used and on the size of the computational box. The N-body simulations of halo evolution are very accurate,  $\sim 1\%$ , and the analytical methods are  $\sim 10-20\%$  (e.g., Press & Schechter 1974, Bond *et al.* 1991). Nevertheless, the analytical HMF can reproduce the numerical results at least qualitatively, and can be defined<sup>3</sup> as dn/dM, where

<sup>&</sup>lt;sup>3</sup>A variety of definitions of the HMF exist in the literature. We use the differential HMF.



Fig. 15.— The most distant candidate protocluster at  $z \sim 8$  (Trenti *et al.* 2012). Left: DM halo distribution for a simulated protocluster in a comoving volume of  $11 \times 11 \times 19$  Mpc<sup>3</sup> from Romano-Díaz *et al.* (2011a). The largest (blue) circle represents the most massive halo in the simulation,  $\sim 5 \times 10^{11} M_{\odot}$ ; red circles, haloes above  $10^{11} M_{\odot}$ ; green circles, haloes of  $10^{10}-10^{11} M_{\odot}$ . Middle:  $J_{125}$  image of BoRG 58 field, with  $Y_{098}$ -dropouts indicated by (blue) circles. Right: Postage-stamp images ( $3.2'' \times 3.2''$ ) of sources: BoRG 58-17871420, BoRG 58-14061418, BoRG 58-12071332, BoRG 58-15140953, and BoRG 58-14550613 fields (top to bottom).

n(M) is the number density of haloes in the range dM around mass M at redshift z (e.g., Jenkins *et al.* 2001),

$$\frac{dn(M)}{dM} = f(\sigma) \frac{\langle \rho \rangle}{M^2} \frac{d\ln \sigma^{-1}(M)}{d\ln M},\tag{10}$$

where  $\sigma^2$  is the variance of the (linear) density field smoothed on the scale corresponding to M, and  $\langle \rho \rangle$  is the average density in the Universe. In the spherical collapse approximation developed by Press & Schechter (1974),  $f(\sigma) = (2/\pi)^{1/2} (\delta_c/\sigma) \exp(-\delta_c^2/2\sigma^2)$ , where  $\delta_c \approx 1.686$ . Press & Schechter assumed that all the mass is within the DM haloes, i.e.,  $\int_{-\infty}^{+\infty} f(\sigma) d \ln \sigma^{-1} = 1$ . An extension for arbitrary redshift is achieved by taking  $\delta_c = \delta_c(z = 0)/D(z)$ , D(z) being the linear growth factor.

Discrepancies between the analytically derived and numerically obtained HMFs can be sufficient to affect our understanding of galaxy growth during the reionisation epoch, as shown in Fig. 16 (Lukic *et al.* 2007). It is, therefore, important that the shape of the HMF can have a universal character, independent of epoch, cosmological parameters and the initial power spectrum, in particular representations (Jenkins *et al.* 2001), although this must be taken with caution. Violations of universality have been found both at low ( $z \leq 5$  at ~20%



Fig. 16.— The HMFs at four redshifts (z = 0, 5, 10 and 15) compared to different fitting formulae, analytic and numerical (coloured curves). Note that the mass ranges are different at different redshifts. The bottom panels show the ratio with respect to the Warren *et al.* (2006) fit, agreeing at the 10% level for  $z \leq 10$ , and with a systematic offset of 5% at z = 0. At higher redshifts, agreement is at the 20% level. Agreement becomes very close once finite-volume corrections are applied. Press-Schechter is a bad fit at all redshifts, especially at high redshifts,  $z \geq 10$ , where the difference is an order of magnitude. From Lukic *et al.* (2007).

level, Fig. 16), and high  $(z \sim 10-30)$  redshifts, but the issue is still unsettled due a number of numerical concerns (e.g., Lukic *et al.* 2007; Reed *et al.* 2007).

### 8. Disk evolution: the central kpc and the SMBHs

A wealth of issues dominate our understanding of the central regions in galaxies and their role in the overall galaxy evolution on cosmological timescales – the secular evolution. But is there a *dynamically* distinct central kiloparsec region in galaxies? The answer appears to be positive, as a major resonance between the bar and/or spiral arm pattern speed,  $\Omega_{\rm p}$ , on the one hand and the linear combination of the epicyclic frequency,  $\kappa$ , and the angular velocity,  $\Omega$ , on the other is positioned in this area, i.e.,

$$\Omega_{\rm p} = \Omega - \frac{\kappa}{2}.\tag{11}$$

Incidentally, the right-hand side of this relation represents the precession frequency of stellar orbits. The resonance between the orbit precession frequency and the pattern speed is called the inner Lindblad resonance (ILR). A multiple number of ILRs can exist in the neighbourhood but typically their number does not exceed two. The ILR(s), if they are not saturated, dampen the propagation of waves in the stellar 'fluid'. This resonance can trigger various processes in the region, e.g., gas accumulation in the form of nuclear ring(s), nuclear starbursts, nuclear bars, etc. (Shlosman 1999, and references therein). The ILR(s) can pump the kinetic orbital energy into vertical stellar motions. So while there are naturally strong interactions between the inner and outer disks, different processes dominate both regions.

The next question is whether there is a *morphologically* distinct central kiloparsec region in galaxies. The answer is positive again – the ILR(s) act(s) as separators between the inner and outer disk, resulting in detached bars and spiral patterns. The inner region is generally dominated by the bulge and hosts the SMBH.

A number of important issues, which also include the inner kpc directly or indirectly, are discussed by Lia Athanassoula and James Binney (this volume). We shall attempt to avoid unnecessary overlap, although some overlap is actually welcomed. In the discussion below, we shall focus, therefore, on various asymmetries in the mass distribution that drive the evolution, such as disk and halo asymmetries, large-scale stellar bars (briefly), and the dynamics of nested bars. In Section 8.2 we shall touch on the issues related to the formation and evolution of SMBHs at high redshifts. We have already reviewed, to some extent, the feedback from AGN in Section 6. The immediate environment of the SMBHs, e.g., the role of molecular tori, is beyond the scope of this discussion.

Two types of torques can have a dramatic effect on the dynamics within the central kpc,

namely magnetic and gravitational torques. The former can dominate the central 1–10 pc from the SMBHs, while the latter can have an effect outside this region, on scales of  $\gtrsim$  few tens of parsecs. Viscous torques can be important near the major resonances and can be neglected in other regions, in comparison with magnetic and gravitational torques.

#### 8.1. Bars and the morphology of the central kpc

Stellar bars can be formed either as a result of a (spontaneous) break of axial symmetry – the so-called bar instability (e.g., Hohl 1971), or via tidal interaction between galaxies (e.g., Noguchi 1988) or between galaxies and DM subhaloes (e.g., Romano-Díaz *et al.* 2008b). Stellar bars themselves are subject to dynamical instabilities and secular evolution which affect the disk as well. Of these, we shall single out the vertical buckling instability (e.g., Combes *et al.* 1990). This instability has both dynamical and secular aspects. Dynamically, this instability exhibits a spontaneous break of the equatorial symmetry in the *rz* plane (e.g., Pfenniger & Friedli 1991; Raha *et al.* 1991). The action of the vertical oscillations. This results in a vertical thickening of the stellar disk at radii smaller than the position of the vertical ILR, and in the appearance of a characteristic peanut/boxy-shaped bulge. The symmetry is always restored on the dynamical timescale (e.g., Fig. 17, note the flip-flow at ~2.3–2.4 Gyr). Moreover, if the equatorial symmetry is (artificially) imposed, this bulge nevertheless appears, although on a longer timescale and driven by the same resonance.

What the low-resolution simulations have failed to capture, and what has been obtained by Martínez-Valpuesta *et al.* (2006) for the first time, is the *recurrent* break in the equatorial symmetry occurring on a much slower timescale of a few Gyr, around 5.2–7.5 Gyr. This slow buildup of the bar asymmetry long after the first vertical buckling occurred is rooted in the secular evolution of stellar orbits driven by the low-order vertical resonances. Unlike the first buckling, the second phase displays persistent asymmetry. The Fourier amplitude of the symmetry breaking decreases with the next stage of the instability.

About 50% of edge-on disks show peanut/boxy bulges (Martínez-Valpuesta *et al.* 2006 and references therein), which appear to be a clear signature of stellar bars. While we do understand the reasons for the dynamical stage of the buckling instability, we cannot predict the onset of the (second) secular stage of this instability. Models with a gas component show that the amplitude of this instability decreases with increasing gas fraction (Berentzen *et al.* 1998; 2007).



Fig. 17.— The *recurrent* buckling instability. Upper frames (from Martínez-Valpuesta *et al.* 2006): Evolution of the vertical structure in the bar: edge-on view along the bar's minor axis. The length is given in kpc and the values of the projected isodensity contours are kept unchanged. The time in Gyr is given in the upper-right corners. Note the bar flip-flop at 2.3–2.4 Gyr and the persistent vertical asymmetry at 5.2–7.5 Gyr. Lower frames: smoothed version of the above figure at 9.4 Gyr (left), and a matching galaxy from HCG 87 group of galaxies (Hubble Heritage Team), courtesy of J. H. Knapen (right).

## 8.1.1. Nested bars: observational perspective

Getting rid of the angular momentum is a major issue for astrophysical systems (e.g., Sections 2 and 3). Given that a substantial mass is involved, gravitational torques appear as the most efficient mechanism for redistribution of angular momentum on various spatial scales. The formation of disks, therefore, is a reflection of the inefficiency of this process. Gravitational torques are triggered by a non-axisymmetric distribution of matter. The most frequent asymmetry in the disk is in the form of spiral arms, which lead to torques with an amplitude of  $\sim 1\%$  – a quasilinear perturbation, if defined as a ratio of tangential to radial acceleration. On the other hand, bars are strongly non-linear perturbations on the level of  $\sim 10-100\%$ . The importance of bars, at least after  $z \sim 1$ , is reflected in the existence of the branch of barred galaxies in the Hubble fork, although the exact statistics is still being debated (e.g., Jogee *et al.* 2004; Sheth *et al.* 2008).

But the efficiency of bars in extracting angular momentum, for example from the gas, is limited by a decade in radius, due to a strong decay in the gravitational multipole interactions (Shlosman *et al.* 1989, 1990). So it is only natural that bars 'repeat' themselves on progressively smaller scales. In retrospect, it is not surprising that such 'bizarre' systems of *nested* bars exist in Nature.

The first known observation of a system with nested bars has been performed on NGC 1291, classified as an SB0/a galaxy (Evans 1951). An inner twist of the optical isophote has barely been detected, and explained as an inner spiral. A much later observation of this object resulted in a large-scale bar of  $\sim 9.9 \,\mathrm{kpc}$  and a nuclear bar of  $\sim 1.8 \,\mathrm{kpc}$ , misaligned at  $\sim 30^{\circ}$ , with the inner bar leading the outer bar (de Vaucouleurs 1975). A question has been asked on whether the presence of a second bar in NGC 1291 is an 'oddity of Nature' or is a fairly common, perhaps typical, structural feature. De Vaucouleurs concluded that 'the lens-bar-nucleus structure on two different scales in barred lenticular galaxies is probably not rare, and raises an interesting problem in the dynamics of stellar systems' but did not follow up on this issue. A morphological survey of 121 barred galaxies has revealed additional objects with inner structure misaligned with the outer bar, but this has been interpreted as a bulge distorted by the large-scale bar (Kormendy 1979). In a subsequent study, Kormendy (1982) has analysed bulge rotation in barred galaxies and summed up that the kinematics of these triaxial bulges is similar to those of bars while the light distribution is as in elliptical galaxies. All these bars have been stellar in origin. Their mutual interactions have not been discussed and they have not been considered as a dynamically coupled system. Shlosman et al. (1989, 1990) have suggested that nuclear bars can be of multiple types, and that nested bars form a new dynamically coupled system which redistributes angular momentum in galaxies.

In principle, nested bars can be of a few types: stellar/stellar, stellar/gaseous and gaseous/gaseous. The first two types have been observed now, while the third type has not yet been observed<sup>4</sup>. In addition, we do not count as a separate class stellar nested bars which are gas-rich or vice versa.

The first catalogues of double-barred galaxies have been published recently. Laine *et al.* (2002) used an *HST* sample of 112 galaxies in the *H*-band, Erwin & Sparke (2002) analysed a sample of 38 galaxies in the optical, and Erwin (2004) considered 67 galaxies, mostly from Laine *et al.*. The main conclusion of these studies has been that about  $25\% \pm 5\%$  of all disk galaxies host nested bars, and about 1/3 of barred disks possess secondary bars. These numbers show decisively that galaxies with nested stellar bars are not a marginal phenomenon, but form an important class of dynamical systems.

Molecular gas is abundant in galactic centres, but it is not clear what fraction of this gas is in a 'barred' state. While there have been surveys of molecular gas within the central  $\sim$  kpc, the available resolution was insufficient for the detection of nuclear gaseous bars. Some nearby galaxies host gaseous bars, e.g., IC 342, NGC 2273, NGC 2782, Circinus, etc., but their statistical significance is not clear. Surveys with ALMA (the Atacama Large Millimetre/submillimetre Array) will probably resolve this issue.

Detecting gas-dominated nuclear bars requires surveys of gas morphology and kinematics in the central few hundred parsec of disk galaxies at a resolution of  $\sim 10 \text{ pc}$ . An additional detection problem can arise from gaseous bars being very short-lived – an issue related to the stability of these objects, which we discuss below.

An analysis of the nested bars in the Laine *et al.* (2002) sample has clarified some of the basic properties of nested bar systems. Firstly, it has shown that the size distributions of large-scale and nuclear bars differ profoundly and exhibit a bimodal behaviour. Whether bar sizes are taken as physical or normalised by the galaxy size,  $r_{25}$ , there is little overlap between their distributions. In physical units, this division lies at  $r \sim 1.6$  kpc, in normalised – around  $r/r_{25} \sim 0.12$ . This bimodality can be explained in terms of a disk resonance, the ILR, and the above radii can be identified with the position of this resonance. The ILR acts, therefore, as a dynamical separator. The ILRs are expected to form where the gravitational potential of the inner galaxy switches from three-dimensional to two-dimensional. This normally happens at the bulge-disk interface, or alternatively, where the disk thickness becomes comparable to its radial extension. To summarise, while the sizes of large-scale bars in nested bars, as well as those of single bars, exhibit a linear correlation with disk size, nuclear bars do not show

<sup>&</sup>lt;sup>4</sup>One can envisage also the existence of gaseous/stellar bars, where the large-scale bar is gaseous and the inner bar is stellar.

the same behaviour.

Secondly, nuclear bars almost always come in conjunction with large-scale bars – a clear signature that this is a prerequisite for their existence, although one can envisage a scenario where they form separately. Thirdly, the size distribution of nuclear rings in these galaxies peaks at the same radii of  $r/r_{25} \sim 0.12$  (see above), which signals again at the crucial role the ILR plays in the dynamics of these systems. Finally, nuclear bars in nested systems have smaller ellipticities than their large-scale counterparts.

The search for stellar nuclear bars is limited to optical and NIR surface photometry – an insufficient method based on the isophote fitting which can be affected by the presence of nuclear clusters, starbursts and dust. A suggestion to look for the characteristic offset dust lanes delineating shocks in nuclear bars, similar to those observed in large-scale bars, did not work. The reason for this is a different gas-flow response in these systems (see below). Nuclear bars are not scaled-down versions of large-scale bars.

Probably the mostly intriguing aspect of nested bars is that their pattern speeds are different, at least during some particular stages of evolution, as predicted (Shlosman *et al.* 1989, 1990), supported by the detection of random mutual orientations of bars (Friedli *et al.* 1996), and confirmed in a direct measurement of their pattern speeds (Corsini *et al.* 2003).

#### 8.1.2. Theoretical perspective: bars-in-bars mechanism

From a theoretical point of view, nested bar systems provide a great laboratory to study non-linear dynamics in physical systems. How do such systems form? Are they longor short-lived? What is their role in driving the secular evolution of disk galaxies? As in the case of single bars, numerical simulations of such systems are indispensable.

The first attempts to simulate pure stellar nested bar systems have succeeded in forming both bars via the bar instability, but the lifetime of such a system was short (e.g., Friedli & Martinet 1993). The problem with the lifetime has been purely numerical – when the resolution of N-body simulations has been increased, the system of two stellar bars has lived indefinitely long (e.g., Pfenniger 2001). The issue of the initial conditions for such systems, however, is much more fundamental. It is quite revealing that in pure stellar systems one can create nested bars only by *assembling* the stellar disk as bar-unstable on both large and small spatial scales – naturally, because of the shorter timescale of the inner bar, it will develop first. But why would Nature create such a strongly bar-unstable stellar disk? No process known to us can create such a disk by means of a dissipationless 'fluid'. Therefore, strong arguments appear in favour of a dissipational process which naturally involves the gas (Shlosman 2005, and references therein). The initial conditions necessary to create a nested bar system become nearly a trivial matter when dissipation is involved.

The role of stellar bars in angular momentum redistribution is most important when the gaseous component is involved. Bars are very efficient in extracting of the angular momentum from the gas (and during the bar instability also from the stars) and depositing it in the outer disk, beyond the corotation radius. The ability of the DM halo to absorb the angular momentum has been noticed long ago (e.g., Sellwood 1980; Debattista & Sellwood 1998) and quantified in terms of the lower resonances recently (Athanassoula 2002, 2003; Martínez-Valpuesta *et al.* 2006).

The outward flow of the angular momentum in barred galaxies is associated with the inward flow of the gas. Skipping the details, this gas accumulates within the central kpc, where the gravitational torques from the large-scale bar are minimised (e.g., Shlosman 2005). The star formation in the bar is largely dampened because of the substantial shear behind the large-scale shocks. So one should not be concerned with gas depletion before it crosses the ILR. Hence, the action of the large bar leads to a radial inflow – the rate of this inflow depends on a number of factors. In the cosmological setting which is the subject of this review, the outer bar will interact with and can in fact be triggered by the asymmetry (e.g., triaxiality) of the DM halo (Heller *et al.* 2007a), or by interaction with DM substructure (Romano-Díaz *et al.* 2008b).

The accumulation of gas within the ILR can and probably does lead to a nuclear starburst in nuclear rings. If, however, the gas inflow rate across the ILR is high enough, or the star formation is dampened by a local turbulent field, the stellar/gaseous fluid can trigger the formation of a nuclear bar (Shlosman *et al.* 1989), when the gas becomes dynamically important and triggers an additional responce from the stellar fluid, as confirmed by numerical simulations (Friedli & Martinet 1993; Englmaier & Shlosman 2004). In this respect, the large-scale bar is a primary and the nuclear bar is a secondary feature (Fig. 18a). This is the so-called bars-in-bars mechanism.

¿From a dynamical point of view, it is challenging to explain how two gravitational quadrupoles tumble with different pattern speeds,  $\Omega_{\rm p}$  (primary) and  $\Omega_{\rm s}$  (secondary), without exerting a braking effect on each other. Such a system can serve as an astrophysical counterpart of a system of coupled oscillators which is a familiar tool to study non-linear behaviour. Essentially, three dynamical states exist for such a system, but only two of them appear dynamically long-lived. The first state involves two bars with equal pattern speeds – this is a stable state and the bars stay perpendicular to each other, which is energetically advantageous. The second state is made out of two bars which tumble with different pattern speeds, but their ratio is fixed,  $\Omega_{\rm s}/\Omega_{\rm p} \sim \text{constant}$ . In a way, the first state is a special case



Fig. 18.— Formation and evolution of nested bars with dissipation (Englmaier & Shlosman 2004): (a) Schematic structure of a nested bar system; (b) Specific angular momentum, ellipticity, and angle between the primary and secondary bars (top to bottom). Note the correlation between the shape and the angle between the bars; (c) Evolution of ellipticity (dotted line) and semimajor axis (thick dots) of the secondary (nuclear) bar during decoupling; (d) Pattern speed of the primary (dotted line) and secondary (solid line) bars during decoupling, in the inertial frames.

of the more general second one, and is also long-lived. The third case consists of two bars which tumble with different pattern speeds, where their ratio is time-dependent and evolves on a dynamical timescale – this is a transition state between two stable states with the fixed ratio of pattern speeds. Naturally, this state is a short-lived one. We call the first two states *coupled*<sup>5</sup>, and the third one *decoupled*.

 $<sup>^{5}</sup>$ Note that various definitions of *coupled/decoupled* states exist in the literature, and are refined occasionally by the same authors, including in our own work.

The explanation for the coupled state is based on a non-linear mode coupling: these modes exchange energies and angular momentum and, therefore, support their pattern speeds which otherwise would decay exponentially due to the gravitational torques. When the bars are locked in  $\Omega_s/\Omega_p \sim \text{constant}$ , a local minimum should exist in the efficiency of energy transfer between the bars. A strong resonance which can 'capture' the bar is necessary. One would expect the low resonances to play a major role in the locking process, especially the ILR (e.g., Lichtenberg & Lieberman 1995; Tagger *et al.* 1987; Shlosman 2005).

Numerical simulations enable one to follow the decoupling process in nested bars (Englmaier & Shlosman 2004). Figures 18c,d provide an example of this process when a secondary bar, which forms within the ILR of the primary bar and obeys  $\Omega_s/\Omega_p \sim 1$ , speeds up in a short time, until its corotation radius moves to the ILR position of the primary bar, and  $\Omega_s/\Omega_p \rightarrow 3.6$ . The shape of the nuclear bar depends on their mutual orientation (Fig. 18b), and is dynamically important – the bar axial ratio is one of the measures of its strength and, therefore, determines the fraction of chaotic orbits there, which is a measure of stochasticity within the bar and its possible demise. Another example in the cosmological setting describes the evolution of a nested bar system which is locked in two different coupled states and transits between them (Heller *et al.* 2007a).

Different methods have been developed to quantify nested bar systems, especially to measure the amount and the effect of multi-periodic and chaotic orbits. As we are interested in the secular evolution of these systems, we only mention that the orbital structure associated with long-lived nested bars has been investigated. A counterpart of periodic orbits in single bars is based on the fruitful concepts of a *loop* (Maciejewski & Sparke 2000) and on that of the Liapunov exponents (El-Zant & Shlosman 2003). Orbit analysis based on these two concepts has demonstrated that orbits are dominated by the potentials of single bars with the possibility of migration from bar to bar across the interface between the bars (El-Zant & Shlosman 2003; Maciejewski & Athanassoula 2007).

The gas dynamics at the bar-bar interface is determined by the already irregular gas flow perturbed by the secondary bar with  $\Omega_s/\Omega_p > 1$ . For such bars, the inflow is chaotic and dominated by shocks. Strong dissipation in the gas will not allow it to settle on stable orbits there. Instead the gas will fall to smaller radii (e.g., Shlosman & Heller 2002). The immediate corollary is that one should not expect starbursts throughout secondary bars, except in the central regions. Shocks and shear within the bar would slash molecular clouds reducing the SFRs. In particular, the mode where star formation occurs in massive stellar clusters should be absent in secondary bars, except (maybe) in the central region, although even there it can be dampened if the gas can be maintained in a strongly turbulent state.

The situation is very different for bars with  $\Omega_{\rm s}/\Omega_{\rm p} = 1$ . In this case one should observe

a relaxed flow, and a 'grand-design' shock system, but no random shocks. The dissipation is decreased compared to other cases.

## 8.1.3. Nested bars: evolutionary corollaries

Bars are known to channel the gas to the central kpc. Over a Hubble time, bars are capable of affecting the angular momentum profile in the stellar component as well. This process acts slowly but relentlessly in changing the mass distribution in galaxies. Occasionally, during various instabilities, bars are capable of increasing the central mass concentration in galaxies on short dynamical timescales (e.g., Dubinski *et al.* 2009). Nested bars are the result of this evolution when the amount of gas moved by the primary bar is able to change the stellar dynamics inside the central kpc. Generally, when the gas has reached  $\sim 10\%$  of the mass fraction in the central regions, it can affect the stellar dynamics there. When a secondary bar forms, the local conditions for star formation will be altered as well. The decrease in the SFR will eliminate the ISM sink and facilitate further radial infall of gas. Further surveys of gas kinematics in the central region should answer the question of whether these flows fuel the AGN.

The fuelling of AGN is of course one of the outstanding issues in galaxy evolution. Are they fuelled locally, say by a 'neighbourhood' stellar cluster or by the main body of the host galaxy? Diverging views prevail on this subject. The duty cycle of AGN is not known at present. Does it depend on the class of AGN, say QSOs versus Seyferts? If the AGN are fuelled by the bars-in-bars mechanism, that characteristic timescale of the process can be as short as  $\sim 10^6$  yr, as gaseous bars are short-lived. What fraction of the gas ends up in the accretion disk around the SMBH?

What is clear is that preferring local mechanisms (e.g., star clusters) in fuelling the AGN does not solve the issue, as it begs the question of what fuelled the formation of local stars. Rather, one can argue that star formation in the vicinity of the SMBH is a by-product of the overall gas inflow to the galactic centre. Such an inflow will always be associated with compression and star formation along the AGN fuelling process. The following scenario can actually lead to an anti-correlation of gaseous bars with AGN activity: if the gaseous bar activates the AGN cycle and as a by-product the local star formation, the AGN and stellar feedback would disperse the gas in the next stage.

What is the fate of gaseous bars? The central feedback can drive the local azimuthal mixing of the 'barred' gas component if the energy is deposited in the turbulent motions in the gas, and ultimately contribute to the growth of the stellar bulge, disky or classical.

Strong winds driven by the nuclear starburst can be a by-product. Alternatively, these winds can be driven via hydromagnetic winds, as discussed earlier. Such extensive outflows from the centres of AGN host galaxies have recently been detected.

The study of nuclear bars, especially gaseous ones, will proceed quickly when ALMA comes online. We have omitted interesting options which can in fact be detected by upcoming observations of these objects. One of these is the occasional injection of the gaseous component in the nuclear stellar bars. How does this influence the evolution of the system, and especially the gas inflow toward the central SMBH?

## 8.2. The origin of SMBHs: the by-product of galaxy evolution?

QSOs have been detected so far up to  $z \sim 7$ , when the age of the Universe was substantially less than 1 Gyr. Even more intriguing is the inferred mass of their SMBHs,  $\gtrsim 10^9 M_{\odot}$ . Within the framework of hierarchical buildup of mass, these objects must originate in rare, highly over-dense regions of the Universe.

Indeed, recent studies of high-z QSOs have indicated that they reside in very rare overdense regions at  $z \sim 6$  with a comoving space density of  $\sim (2.2 \pm 0.73) h^3 \,\mathrm{Gpc}^{-3}$ . Numerical simulations give a similar comoving density of massive DM haloes, of a few  $\times 10^{12} \, M_{\odot}$ . The caveat, however, is that this similarity depends on the QSO duty cycle – the fraction of time the QSO is actually active (Romano-Díaz *et al.* 2011a). For a duty cycle which is less than unity, QSOs will appear less rare and reside in less massive haloes. On the other hand, these QSOs are metal-rich and are therefore plausibly located at the centres of massive galaxies.

The causal connection between AGN and their host galaxies is a long-debated issue, and substantial evidence has accumulated in favour of this relation. SMBHs are ubiquitous. If the formation and growth of SMBHs is somehow correlated with their host galaxy growth within the cosmological framework, what are the possible constraints on the formation time of these objects? What are the possible seeds of SMBHs?

## 8.2.1. SMBH seeds: population III versus direct collapse

Population III (Pop III) stars can form BH seeds of  $M_{\bullet} \sim 10^2 M_{\odot}$  (e.g., Bromm & Loeb 2003). An alternative to this is the so-called 'monolithic' (or direct) collapse to a SMBH (Begelman *et al.* 2006). A gas with a primordial composition, which has a cooling floor of  $T_{\rm gas} \sim 10^4$  K, can collapse into DM haloes with a virial temperature of  $T_{\rm vir} \gtrsim T_{\rm gas} \sim 10^4$  K. This corresponds roughly to  $M_{\rm vir} \sim 10^8 M_{\odot}$ . In a WMAP7 (*Wilkinson Microwave Anisotropy*)

*Probe*) universe, this can involve about  $\sim 2 \times 10^7 M_{\odot}$  baryons. If the gas has been enriched previously by Pop III stars, the halo mass can be smaller and the amount of baryons involved can be smaller as well.

The Pop III SMBH seeds will be required to grow from  $\sim 10^2 M_{\odot}$  to  $\gtrsim 10^9 M_{\odot}$  in less than  $\sim 6 \times 10^8$  yr, i.e., from  $z \sim 20$  to  $z \sim 7$ . The e-folding time for SMBH growth is (Salpeter 1964):

$$t_{\rm e} \sim \epsilon \frac{c\sigma_{\rm T}}{4\pi G m_{\rm p}} \left(\frac{L_{\rm E}}{L}\right) \sim 4.4 \times 10^8 \epsilon \left(\frac{L_{\rm E}}{L}\right) \,{\rm yr},$$
 (12)

where  $L_{\rm E}$  is the Eddington luminosity and  $\epsilon \sim 0.1$  is the accretion efficiency. The growth time to the SMBH masses estimated for high-z QSOs is

$$t_{\rm growth} \sim 3.1 \times 10^9 \epsilon \left(\frac{L_{\rm E}}{L}\right) \,{\rm yr},$$
 (13)

which is uncomfortably long and close to the age of the Universe. If the SMBH growth is via mergers, this requires frequent merger events. When the merging rate is too high, SMBHs can be ejected via slingshots, which should limit the efficiency of this process. If the dominant mode is accretion, it will be limited by  $\sim L_{\rm E}$  and maybe by the feedback, which again would limit the efficiency, or even cut off the accretion process.

Alternatively, monolithic (direct) collapse will require a growth from  $\sim 10^{6-7} M_{\odot}$  to  $\sim 10^9 M_{\odot}$ , from  $z \sim 12$ –20 to  $z \sim 6$ –7. This will involve fewer mergers, but growth by accretion may lead to fragmentation of the accretion flows, star formation and depletion of the gas supply for the seed SMBH.

The collapse rates can be estimated from  $\dot{M} \sim v^3/G$  (Shlosman & Begelman 1989), where v is the characteristic infall velocity, which results in mass accretion rates of  $\sim 10^{-4}$  to  $10^{-5} M_{\odot} \text{ yr}^{-1}$  for a stellar collapse with virial temperatures  $\sim 10^{1-2} \text{ K}$ ,  $\sim 10^{-2} \text{ to } 10^{-4} M_{\odot} \text{ yr}^{-1}$ for a Pop III collapse with  $\sim 10^{2-3} \text{ K}$ , and  $\gtrsim 10^{-1} M_{\odot} \text{ yr}^{-1}$  for a direct collapse to the seed SMBH with  $\gtrsim 10^4 \text{ K}$ .

However, a spherical collapse over decades in r is improbable – the angular momentum barrier will stop it sufficiently quickly. Numerical simulations of a baryon collapse into  $10^8 M_{\odot}$  DM haloes have shown explicitly that the angular momentum plays an important role (e.g., Wise *et al.* 2008), as expected of course. The baryon collapse in the presence of a typical halo angular momentum,  $\lambda \sim 0.05$ , will develop virial turbulent velocities, driven by the gravitational potential energy. The turbulence is supersonic. The collapse can proceed until the angular momentum barrier stops it. If the system reaches equilibrium, the turbulent velocities will decay and the disk will fragment forming stars. However, the ability of the flow to fragment depends on its equation of state, and should not be taken for granted (e.g.,
Paczynski 1978; Shlosman *et al.* 1990). Begelman & Shlosman (2009) find that such a decay in the turbulent support will trigger a bar instability in the gaseous disk, *before* it fragments. In other words, the global instability sets in before the local instability develops. The bar instability will create intrinsic shocks in the gas and the collapse will resume – again pumping energy into turbulent motions. This is the same bars-in-bars runaway scenario as discussed in the previous section.

The turbulent support is especially helpful in suppressing fragmentation in  $\sim 10^8 M_{\odot}$  DM haloes when the baryons are metal-rich. Previously, it was suggested that the fragmentation in metal-rich flows in such haloes would suppress the formation of a SMBH, and lead to star formation instead.

Supersonic turbulence requires continuous driving. A reliable diagnostic of this flow is the density PDF, discussed in Section 6. The log-normal PDF depends on the density fluctuations around some average  $\rho_0$ , or  $x \equiv \rho/\rho_0$  when normalised (Fig. 19). The supersonic turbulence developing during the overall collapse will generate gas clumps. Two spatial scales characterise this turbulence – the Jeans scale and the so-called transition scale, below which the flow becomes supersonic, i.e., in essence the clump size.

A fraction of these clumps will be Jeans-unstable. However, only the fraction of these clumps that contract on a timescale shorter than the free-fall time for the overall collapse can actually contribute to the star formation. These 'active' clumps should have  $x \gtrsim x_{\rm crit} \sim (v_{\rm turb}/v_{\rm K})^2 \mathcal{M}^2$ , where  $v_{\rm turb}$  is the typical turbulent velocity,  $v_{\rm K}$  is the Keplerian velocity and  $\mathcal{M}$  is the Mach number of the supersonic flow. Other clumps, even when Jeans-unstable, will be swept away by the next crossing shock. To estimate the fraction of star-forming clumps, one should integrate the PDF from  $x_{\rm crit}$  to  $\infty$ . For  $\mathcal{M} \gtrsim 3$ , this fraction is  $\leq 0.01$ . Hence in a flow with a mildly supersonic turbulence the SFR is heavily dampened.

## 8.2.2. Direct collapse: two alternatives

Present numerical simulations of the direct collapse to the SMBH cannot answer the ultimate question about how and when the SMBH will form. At small spatial scales new physical processes should become important, such as radiative transfer, as the matter becomes opaque. This can be estimated to happen at  $\sim 10^{13} \alpha \sigma_{10}^3 (\kappa/\kappa_{es})$  cm, where  $\sigma_{10} \equiv \sigma/10 \text{ km s}^{-1}$ , and  $\kappa_{es}$  is the electron scattering opacity. At what stage will the bars-in-bars cycle be broken? What are the intermediate configurations that lead to the SMBH?

The fate of the direct collapse has been analysed by Begelman *et al.* (2006) under the assumption that the angular momentum is unimportant. Under these conditions, there is no



Fig. 19.— Simulation of direct collapse to the SMBH (from Choi *et al.* 2012). Left: the projected gas density along the rotation axis at an intermediate time of  $t \sim 4.6$  Myr shows the central disk-like configuration. The frame captures the central runaway collapse forming one of the gaseous bars on a scale of  $\sim 1 \text{ pc}$ . The collapse proceeds from the initial scale of  $\sim 3 \text{ kpc}$  – the DM halo virial radius. The overall gas density profile at this time is  $\propto r^{-2}$ . Right: a representative log(PDF) of the gas density field in the left frame (blue histogram line). The red solid line is the analytical counterpart of the log-normal PDF (see text). The close match between the two distributions confirms the fully developed turbulence field in the collapsing gas.

preferential channel for energy release. The photon trapping in the collapsing matter results in the formation of a single accreting massive object in a pressure equilibrium, termed a *quasistar*. The follow-up evolution leads to thermonuclear reactions within the object for ~10<sup>6</sup> yr. At some point, neutrino cooling becomes important, which defines the quasistar core, ~10  $M_{\odot}$ . Neutrino-cooled core collapse will determine the seed SMBH mass. The subsequent rapid super-Eddington growth of the seed will result in  $M_{\bullet} \sim 10^{4-6} M_{\odot}$ .

What is the possible alternative to this scenario? Choi *et al.* (2012) have assumed that the angular momentum dominates at some spatial scale of the inflow and it does indeed define the preferential channel for energy release. Under these conditions, photon trapping may not be important, and an optically-thick disk/torus will form instead of a quasistar. This is a way to bypass the thermonuclear reaction stage as well, but at the price of additional dynamical instabilities. The SMBH seed which forms will be more massive than in the previous case,  $M_{\bullet} \sim 10^{6-7} M_{\odot}$ .

## 9. Summary and future prospects

The cosmological evolution of galaxies is a fascinating subject which has experienced explosive growth lately due to the incredible rate of new observational data and the development of new methods and codes in this observationally and computationally intensive research field. In this review, we have discussed various aspects of galaxy evolution. The original paradigm of this evolution is being replaced slowly but persistently by a modified view – a process which is driven by a long list of recent discoveries.

Probably, nowhere is this more obvious than in our understanding of the main factor(s) behind galaxy growth. The cold accretion scenario has successfully challenged the mergeronly picture. The new approach raised a number of challenging questions. How important is the accretion shock? What fraction of the gas is actually processed by the shock? How does the accretion flow join the growing disk? Via shocks or smoothly assembling in the outer disk? These outstanding questions will be answered shortly via high-resolution numerical simulations. On the other hand, observations must answer questions about the redshift evolution of cold accretion flows. Furthermore, the issue of the actual detection of flows in cosmological filaments is an open one and will help to resolve at least partly the problem of the missing baryons. And of course the increasing number of detected galaxies at redshifts corresponding to the re-ionisation epoch must answer the fundamental questions about the morphological types of the first galaxies, their mode of growth and the other scaling relations established at low redshifts, e.g., the morphology-density relation.

The subject of stellar and AGN feedback on galaxy evolution is truly a Pandora's box. The inclusion of feedback has clearly solved the overcooling problem. However, the current subgrid physics used to quantify SN feedback, and feedback from OB stellar winds, AGN and galactic winds has too many parameters to be fine-tuned. For AGN feedback, mechanisms more sophisticated than considering the AGN as a giant O star are needed. Does it quench the star formation and clean the galaxies of their ISM, or is this feedback much more anisotropic, producing much less disturbance for the rest of the host galaxy. At high redshifts,  $z \sim 6-10$ , does the QSO feedback induce or dampen galaxy formation?

How do galactic winds form? They are clearly detected in observations, but the underlying theory requires much more work. In fact, almost any model of the wind from any astrophysical object experiences difficulties in describing the physics of wind *initiation*.

There are too many bulgeless disks in the Universe. This comes as a surprise and as another challenge. One approach attempted successfully has related this phenomenon to the feedback problem. Indeed, back-of-the-envelope estimates show convincingly that the SNe can expel the ISM and therefore eliminate the bulge buildup in smaller galaxies. Additional work is required to understand the efficiency of this process. Could it be too efficient? Massive disks provide another challenge. In a number of cases cold gas has been detected in the central regions, but no indication of intensive star formation, at least not in the mode of massive clusters. How can one suppress the star formation while leaving the gas intact? One possibility lies in pumping the energy into turbulent motions in the gas. Observations will resolve this issue soon.

Where do stars form? This simple question has an interesting twist. Do stars form as a result of Jeans instability in the molecular gas? Or does the molecular gas itself form as a result of the Jeans instability in the neutral gas? This issue has immediate consequences for numerical simulations of galaxy evolution. Because we still cannot resolve the star-forming regions, what should be used as a density threshold for star formation?

The growing list of high-redshift galaxies presently extending to  $z \sim 10.4$ , and protoclusters extending to  $z \sim 8$ , will soon increase dramatically. We have already mentioned the importance of measuring the scaling relations at these redshifts, but even more basic parameters, like the galaxy LF, or the rate of star formation, must be measured as well. Is the evolution of the *specific* SFR so flat even at redshifts beyond 7?

Different challenges are expected to resolve the kinematics of cold gas in galactic centres. A high-resolution survey of gas morphology will probably be produced by ALMA. What fraction of this gas is in a relaxed orbital motion, and what fraction is in a 'barred' state? What is the relation between the AGN host galaxy and the central SMBH? We know that Seyfert activity is probably not triggered by galaxy interactions or mergers. We also know that the brightest of AGN, the QSOs, can be fuelled by this process. Overall, a number of factors can fuel the accretion onto the SMBH, but it would be interesting to understand the statistics of local versus non-local sources of fuelling. If both are involved, where is the boundary between them?

The bulge buildup is partly a dissipative process, but it appears now that stellardynamical instabilities play an important role as well. The relation between the origin of peanut/boxy bulges and vertical buckling in stellar bars has been now established. But this instability is *recurrent*, and what determines the timescale of the onset of the next stage in this instability is not yet clear.

The fate of nuclear bars is really 'lost' in the darkness. While purely stellar nested bars can live indefinitely, the gas can still make a difference, especially in gas-rich nuclear bars. What is their relation to the bulge build-up? And to AGN fuelling?

Finally, an outstanding issue is when and where the SMBHs form and how massive their seeds are. Are they formed at very high redshifts from Pop III remnants, or at lower redshifts in a direct collapse inside DM minihaloes of  $\sim 10^8 M_{\odot}$ ? The separate problems in galaxy evolution we have discussed above join together to complicate this process. Does the accretion flow fragment or does the induced turbulence dampen the Jeans instability and does the seed SMBH form more massive? Is the angular momentum problem resolved in this case by the bars-in-bars mechanism? Observers can hope to detect the quasistars – the last stage before the horizon is formed. Or does the energy escape along the preferred channel without interacting and stopping the inflow? In this case, the evolution may bypass the quasistar stage. The detection of intermediate-mass black holes can help here.

I would like to complete these lecture notes with my favourite question asked by Stanislaw Lem in his novel *Ananke*: 'Where is that order and whence came this mocking illusion?'

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