Importance of the Interstellar Medium

- Role in the star/gas cycle
  - facilitates ongoing star formation
  - repository for element buildup; integral for chemical evolution
- Gas can cool, so its collapse is dissipational
  - Hot gas $\rightarrow$ cold gas $\rightarrow$ stars
  - Star formation cools spiral disks, leading to arm formation
  - Gas migrates inwards in the gravitational potential
    - Galactic disks are smaller than dark matter halos
    - Galaxies have steep density gradients
    - Galactic nuclei have high densities, including massive black holes
- Gas has important diagnostic properties
  - Doppler effect reveals dynamics of Galaxy
  - Abundances show chemical evolution
  - Physical conditions can be found
  - Some emission lines are seen at cosmological distances
  - High-redshift absorption lines reveal galaxy birth & evolution
- Can dominate the integrated spectral energy distribution
  - Dust: mid-IR to sub-mm
  - Hot ISM phase (and X-ray binaries): soft X-rays
  - HII regions and relativistic plasmas: cm-radio
  - Some emission lines (Ly α, [CII]) are major coolants

Activity in the Interstellar Medium

- ISM energized by stars
  - UV light ionizes atoms, dissociates molecules
  - photoelectric effect heats gas
  - SN shocks heat, ionize and accelerate gas
- ISM is inhomogeneous with phases
  - Hot/warm/cold phases with similar pressures ($P = nkT \sim 1$ eV cm^{-3})
  - Cloud and intercloud media with huge density contrasts ($10^2 - 10^5$)
- Mass and metallicity exchange between phases
  - Cooling: hot $\rightarrow$ warm $\rightarrow$ cold $\rightarrow$ stars
  - SN accelerate gas and rearrange phases (bubbles and fountains out of disk)
  - Tidal encounters and resulting starbursts
    - create bubbles
    - cycle gas into halos
    - convert spirals into ellipticals
- Global distribution of ISM
  - colder phases confined closely to plane
  - hotter and turbulent phases are thicker
  - ISM in disk is thin at small radii and flares at large radii
- ISM is locally complex
  - SN create superbubbles
  - between bubbles are cold, dense sheets
- Equipartition in the ISM
  - Energy densities of all three gas phases, starlight, magnetic fields and cosmic rays are each ~ 1 eV cm^{-3}.

Structures in the Interstellar Medium

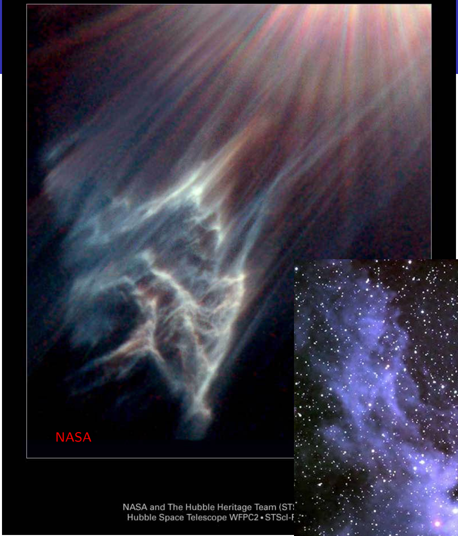
- HII regions
- Reflection nebulae
- Dark nebulae
- Photodissociation regions
- Supernova remnants

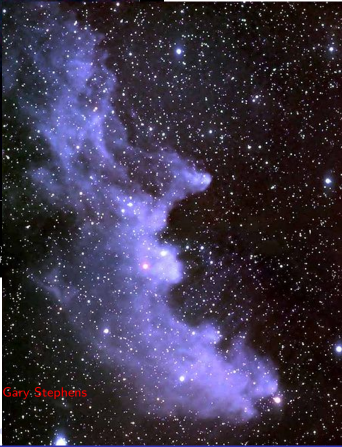
HII Regions

- Ionized H regions formed by O and B0-B1 stars with an abundance of photons with $\lambda < 912\mathring{A}$.
- $n_H \sim 10 - 10^4$ cm^{-3}
- $T \sim 10^4$ K
- Total mass $\sim 5 \times 10^7 \mathcal{M}_\odot$
- $R \sim 0.5 - 10$ pc
- Optical spectra dominated by H and He recombination and [OII], [OIII] and [NII] lines.
- Strong sources of free-free radio emission and thermal emission from warm dust.
- Signposts of massive star formation.

Reflection Nebulae

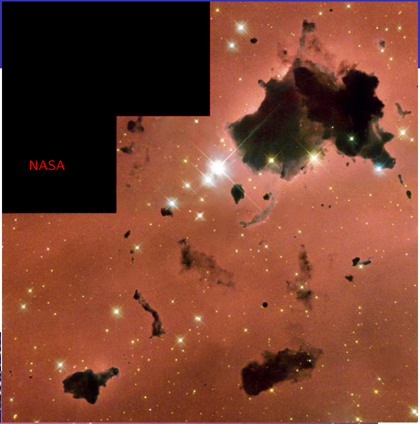
- Bluish dusty nebulae reflect light of nearby stars later than B1.
- $n_H \sim 10^3\ \mathrm{cm}^{-3}$
- Spectrum similar to illuminating star.
- Often seen with HII regions; both are diffuse nebulae.
- Some dust thermal emission.
- Gas from star formation, a chance encounter, or ejecta of late type stars.





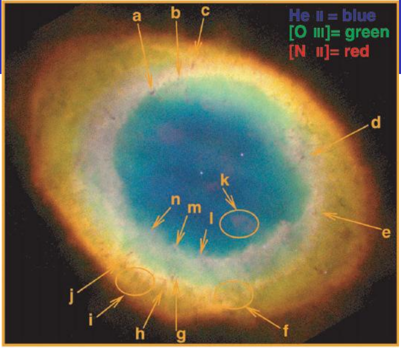
Dark Nebulae

- Made visible by absence of stars or when backlighted.
- $R \sim 0.501 - 100$ pc
- They become bright in the far-infrared.

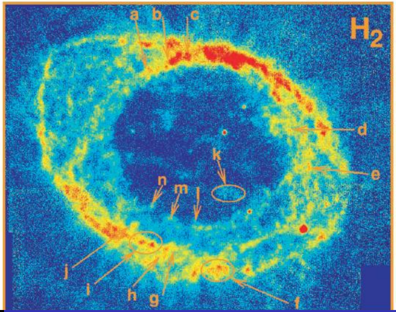


Photodissociation Regions

- Predominately neutral regions in which penetrating far-UV (6 – 13.6 eV) radiation dissociates and ionizes molecules and heats the gas through the photo-electric effect.
- Bright in IR dust continuum and atomic and molecular line emission.
- Includes neutral atomic gas and gas in molecular clouds outside their dense cores.
- Typical examples are the gas at the boundary of a giant molecular cloud or within planetary nebulae.
- Dominate the sky in the infrared.



Speck et al. PASP 115, 170 (2003)



Supernova Remnants

- Ejected material shocks surrounding ISM
- $T \sim 10^6$ K
- Spectrum is that of a high-velocity shock.
- Prominent sources of synchotron radiation and X-radiation.
- Can be compact or wispy.

Components of the Interstellar Medium

Table 2.4 A 'zeroth-order' summary of the Milky Way's interstellar medium (after J. Lequeux)

Component	*Description*	*Density* (cm^{-3})	*Temperature* (K)	*Pressure* (p/k_B)	*Vertical extent*	*Mass* ($\mathcal{M}_\odot$)	*Filling factor*
Dust grains						10^7–10^8	Tiny
large $\lesssim 1\,\mu$m	Silicates, soot		$\sim$20		150 pc		
small ~ 100 Å	Graphitic C		30–100				
PAH < 100 atoms	Big molecules				80 pc		
Cold clumpy gas	Molecular: H_2	> 200	< 100	Big	80 pc	$(2) \times 10^9$	<0.1%
	Atomic: HI	25	50–100	2 500	100 pc	3×10^9	2%–3%
Warm diffuse gas	Atomic: HI	0.3	8 000	2 500	250 pc	2×10^9	35%
	Ionized: HII	0.15	8 000	2 500	1 kpc	10^9	20%
HII regions	Ionized: HII	1–10^4	$\sim$10 000	Big	80 pc	5×10^7	Tiny
Hot diffuse gas	Ionized: HII	$\sim$0.002	$\sim 10^6$	2 500	$\sim$5 kpc	(10^8)	45%
Gas motions	$\frac{3}{2}\langle \rho_{HI} \rangle \sigma_r^2$	$\langle n_H \rangle \sim 0.5$	10 km s^{-1}	8 000			
Cosmic rays	Relativistic	1 eV cm^{-3}		8 000	$\sim$3 kpc	Tiny	
Magnetic field	$B \sim 5\,\mu$G	1 eV cm^{-3}		8 000	$\sim$3 kpc		
Starlight	$\langle \nu h_P \rangle \sim 1$ eV	1 eV cm^{-3}			$\sim$500 pc		
UV starlight	11–13.6 eV	0.01 eV cm^{-3}					

Note: ( ) denotes a very uncertain value. Pressures and filling factors refer to the disk midplane near the Sun; notice that the pressures from cosmic rays, in magnetic fields, and the turbulent motions of gas clouds are roughly equal.

Intercloud/cloud mass is 1/1; intercloud/cloud volume is 49/1

Components of the Interstellar Medium

- Neutral atomic gas Dominated by 21 cm emission. Can be in cold neutral diffuse HI clouds ($n_H \sim 25$ cm^{-3}, $T \sim 80$ K) and warm intercloud gas ($n_H \sim 0.3$ cm^{-3}, $T \sim 8000$ K) mixed with ionized gas. Completely absorbs starlight with $\lambda > 912\AA$ (Lyman edge).
- Ionized gas Traceable through dispersed pulsar signals. optical and UV ionic absorption lines, and Hα recombination line emission. Most Hα emission comes from HII regions, but most mass is in diffuse warm ionized medium. Has a complex structure including filaments of up to 1 kpc in length. Source of ionization is uncertain.
- Molecular gas Dominated by dense *giant molecular clouds* of average size 40 pc, mass $4 \times 10^4 \mathcal{M}_\odot$, density $n_{H_2} \sim 200$ cm^{-3} and temperature 10 K, traceable by $J = 1 \to 0$ CO emission at 2.6 mm. Smaller between spiral arms. Often surrounded by neutral gas forming complexes to 100 pc and $10^7 \mathcal{M}_\odot$. Have high turbulent pressures but are self-gravitating. The site of active star formation, stable for about 30 million years. Many rotational lines from over 200 molecules seen. Show structure on all scales, including dense (10^4 cm^{-3}) cores of 1 pc and $10 - 100$ $\mathcal{M}_\odot$. H_2/CO$\sim 10^4$.

Components of the Interstellar Medium

- Coronal gas Hot (10^6 K) intercloud medium traceable through UV absorption lines (CIV, SVI, NV, OVI). Emit continuum and line radiation in far UV and X-rays. Fills most of halo and some of the disk. Gas heated by stellar winds and supernovae; forms bubbles (in which the Sun is found) and super bubbles from OB associations which pump the gas into the halo; it then cools into clouds and rains back down into the disk.
- Interstellar dust Responsible for most extinction, reddening, scattering and polarization. Dominates IR continuum emission. Typical sizes of 0.1μm, size distribution $n(a) \sim a^{-3.5}$. Contains half the mass of heavy elements and 1% of total gas mass. Larger grains are in radiative equilibrium at ~ 15 K with the stellar radiation field, but up to 75 K near massive stars.
- Large interstellar molecules Visible at mid-IR in broad emission. Dominated by polycyclic aromatic hydrocarbons (PAH) materials containing some 50 C atoms, with densities of $10^{-7} n_H$ and locking up 10% of C. Diffuse interstellar bands, of which more than 200 are known, are attributed to large unsaturated carbon chains. Seem to be the extension of grains into molecular domain; extra-solar nano diamonds and silicates have been extracted from meteorites.

Observational Considerations

- Emission Measure (EM) $= \int < n^2 > dz$ (pc cm^{-6}), proportional to surface brightness
- Column Density (N) $= \int < n > dz$ (cm^{-2}), proportional to absorption

In an ionized gas, n_e is the relevant density.
The ISM is highly opaque in EUV (13.6 – 100) eV, partially transparent in soft X-rays ($\sim$ 0.6 eV), completely transparent by 2 keV.

Medium	n_e cm^{-3}	Size pc	EM pc cm^{-6}	Emission Visibility	N_e cm^{-2}	Absorption Visibility
Young Nova	10^7	10^{-3}	10^{11}	v. bright!	3×10^{22}	thick
PN	10^4	10^{-1}	10^7	bright	3×10^{21}	good
HII Region	10	10^2	10^4	fine	3×10^{21}	good
Diffuse ISM	10^{-1}	10^3	10	difficult	3×10^{20}	good
Halo	10^{-3}	10^4	10^{-2}	invisible	3×10^{19}	fine

From www.astro.virginia.edu/class/whittle/astr553

Gas Distribution

- Molecular gas peaks at 4.5 kpc
- Atomic gas is more uniform.
- Hole at center of Galaxy except for nuclear ring.
- Mass of HI gas is about 5 times that of H_2 gas.
- Disk of molecular gas is very thin, thickness 75 pc.
- Atomic gas has a thickness about 200 pc inside the Sun, and flares to about 1kpc in outer Galaxy.
- The outer disk is warped.

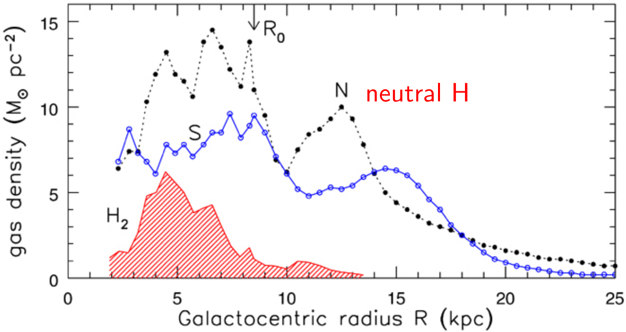


Fig 2.22 (Burton, Dame) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Gas and Dust Budget in the ISM

Table 1.4 *Interstellar gas and dust budgets*

Source	$\dot{M}^a_H$ ($M_\odot$ kpc^{-2} Myear^{-1})	$\dot{M}^b_c$ ($M_\odot$ kpc^{-2} Myear^{-1})	$\dot{M}^c_{sil}$ ($M_\odot$ kpc^{-2} Myear^{-1})
C-rich giants	750	3.0	—
O-rich giants	750	—	5.0
Novae	6	0.3	0.03
SN type Ia	—	0.3^d	2^d
OB stars	30	—	—
Red supergiants	20	—	0.2
Wolf–Rayet	100^e	0.06^f	—
SN type II	100	2^d	10^d
Star formation	−3000	—	—
Halo circulation[f]	7000		
Infall[g]	150		

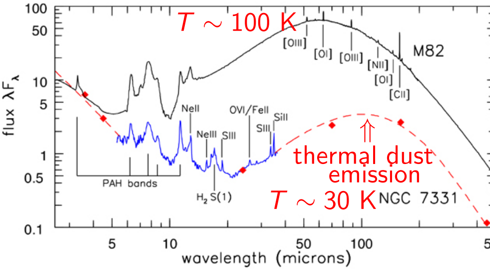
$\dot{\mathcal{M}}_g \sim 1.75 \times 10^{-3} \mathcal{M}_\odot$ kpc^{-2} yr^{-1}

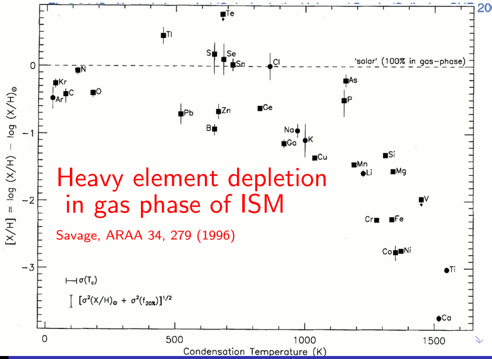
[a] Total gas mass injection rate.
[b] Carbon dust injection rate.
[c] Silicate and metal dust injection rate.
[d] Fraction and composition of dust formed in SN is presently unknown. These values correspond to upper limits.
[e] Dust injection only by carbon-rich WC 8–10 stars.
[f] Mass exchange between the disk and the halo estimated from HI in non-circular orbits and CIV studies.
[g] Estimated infall of material from the intergalactic medium and satellite galaxies.

$\Sigma_g \sim 8 \times 10^6 \mathcal{M}_\odot$ kpc^{-2}
$\tau_g \sim 5$ Gyr
dust/gas ejecta $\sim 1.5\%$

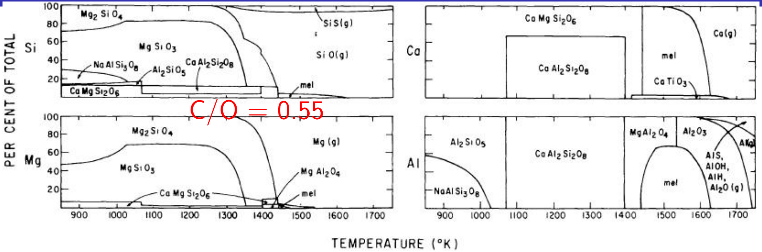
Dust

- Dust absorbs about half the Milky Way star's optical and UV emission
- The typical grain size is 0.1μm.
- Composition are magnesium and iron silicates and soot and graphite.
- In dense clouds, dust has ice (H_2O, CO_2, CH_4, NH_3) mantles.
- About 10% of dust is in small (100 carbon atom) PAH particles; emit from $3-30\mu$m.
- About 1% of the gas mass is in dust; half of heavy element mass in dust.
- About 1 grain per 10^{12} H atoms.
- Dust lifetime ~ 0.5 Gyr.
- Molecules like H_2 primarily form on dust surfaces, $\sim 10^8$ times faster than in gas phase.

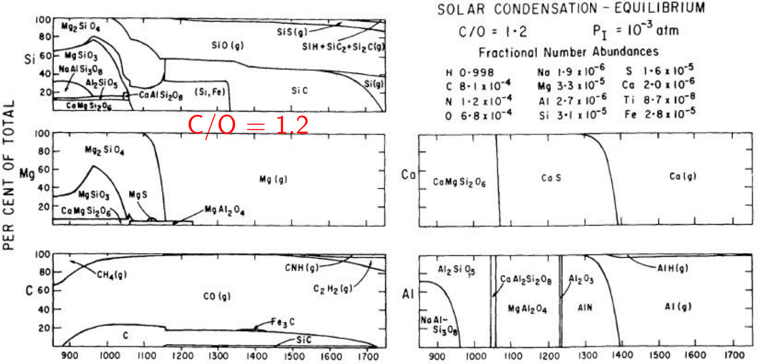




Dust Composition and Condensation Temperatures



Lattimer & Grossman, Moon & Planets 19, 169 (1978)



Energy Sources in the ISM

- Radiation Fields
- Magnetic Fields
- Cosmic Rays
- Kinetic Energy

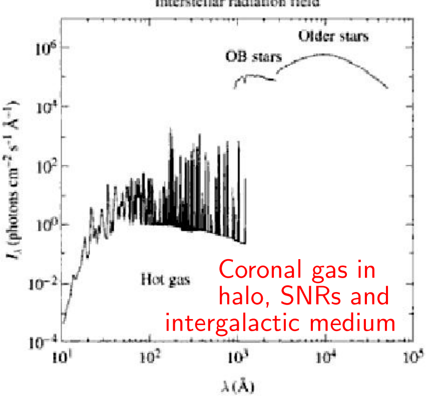
ISM is an open system and needs a continuous energy supply.
The gas layer will cool and dissipate by random motions of clouds without energy.
In HII regions, the recombination time is a few thousand years.
In warm ionized gas, it is 2 Myr.

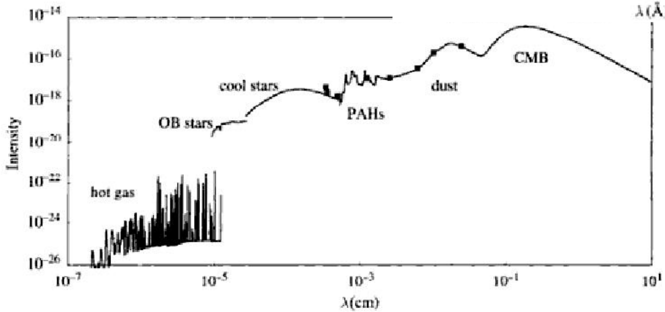
Table 1.2 *Energy balance for diffuse clouds*

Source	P (10^{-12} dyne cm^{-2})	Energy density (eV cm^{-3})	Heating rate (erg s^{-1} H-atom^{-1})
Thermal	0.5	6.0	–5 (–26)[a]
UV	—	0.5	5 (–26)
Cosmic ray	1.0	2.0	3 (–27)
Magnetic fields	1.0	0.6	2 (–27)
Turbulence	0.8	1.5	1 (–27)
2.7 K background	—	0.25	—

[a] Energy loss rate.

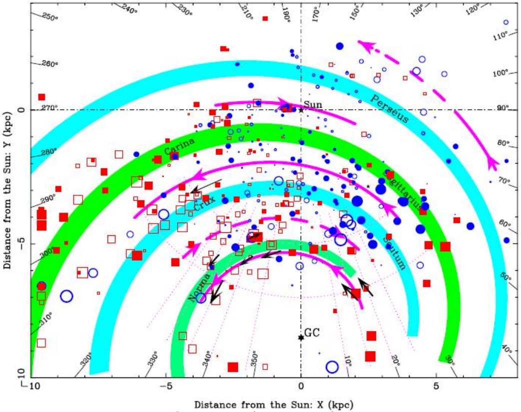
Radiation Fields

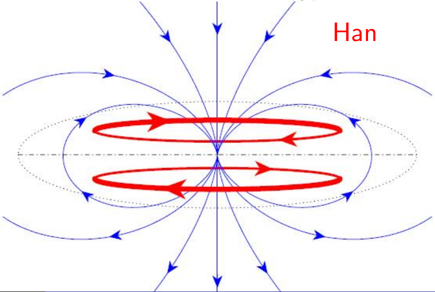




Magnetic Fields

- Important energy and pressure source.
- 5μG near Sun, 8μG around $R = 4$ kpc.
- Traced by synchrotron emission, extragalactic and pulsar rotation measures.
- Circular uniform component of 1.5μ G with two field reversals inside and outside solar circle.
- Non-uniform field connected to superbubbles and shocks.
- In dense clouds, $B \simeq 30\mu$G.





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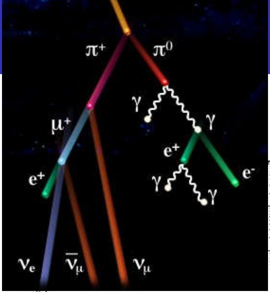
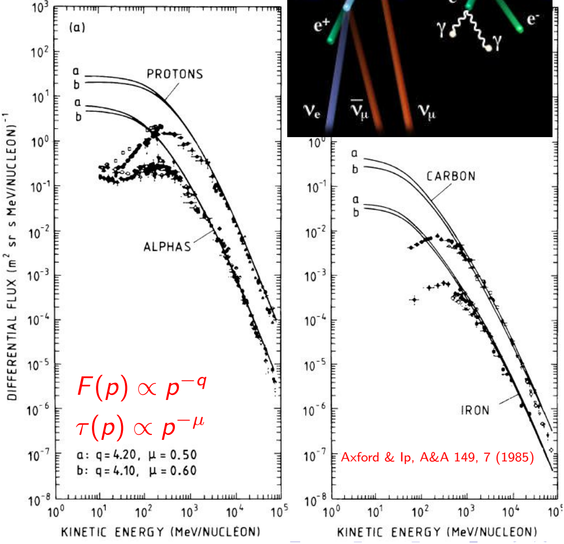
Cosmic Rays

- High energy (≥ 100 MeV/b) particles.
- Mostly p (85%) and He (15%), but also electrons and heavy nuclei up to U
- Contribute 2 eV cm^{-3}.
- Solar have energies up to 10^{10} eV, ejected during solar flares.
- Galactic $10^{10} - 10^{15}$ eV.
- Galactic may originate from SN, 10% of KE of ejecta.
- Extragalactic $< 10^{18}$ eV.
- Extragalactic may originate from supermassive black holes.





Kinetic Energy

- Winds from early-type stars and supernova explosions
- Mechanical energy is just 0.5% of stellar radiation.
- Turbulent energy is $\sim 6 \times 10^{51}$ erg kpc^{-1} near Sun.
- Provides support against gravity for HI gas in Galactic plane.
- HI gas has ordered flows of ~ 5 km s^{-1}.
- Expanding shells from stars and superbubbles from OB associations sweep up and compress ISM, which becomes unstable to Rayleigh-Taylor and Kelvin-Helmholtz instabilities which create turbulence.
- Kinetic energy decays through shock waves when clouds collide, producing line radiation and plasma waves which heat gas.
- Turbulence in molecular clouds supports them against gravitational collapse.

Physics and Chemistry of the ISM

- The ISM is far from thermodynamic equilibrium.
- The velocity distribution of gas well described by a single temperature.
- The excitation, ionization and molecular composition have different characteristic temperatures.
- Collisions cannot compete with fast radiative decay rates of atoms and molecules.
- Cosmic rays and a diluted EUV-FUV stellar radiation field keep chemical compositions from equilibrium.
- The large scale velocity field greatly influenced by turbulence.
- The level populations, ionization degree, chemical composition and temperature are determined by heating and cooling rates.
- In some environments shocks are important.
- Dust grains and large molecules have to be specially treated.

Spectroscopy

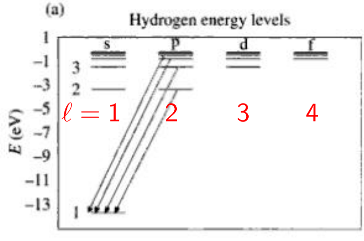
Table 2.1 *Typical transition strength*[a]

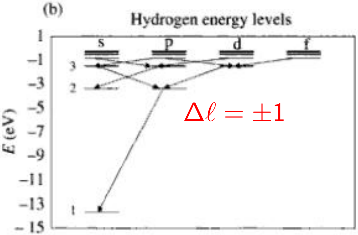
Type of transition	f_{ul}	$A_{ul}(s^{-1})$	Example	λ	$A_{ul}(s^{-1})$
Electric dipole					
UV	1	10^9	Lyα	1216 Å	2.40×10^8
Optical	1	10^7	Hα	6563 Å	6.00×10^6
Vibrational	10^{-5}	10^2	CO	4.67 μm	34.00
Rotational	10^{-6}	3×10^{-6}	CS[b]	6.1 mm	1.70×10^{-6}
Forbidden					
Optical (Electric quadrupole)	10^{-8}	1	[OIII]	4363 Å	1.7
Optical (Magnetic dipole)	2×10^{-5}	2×10^2	[OIII]	5007 Å	2.00×10^{-2}
Far-IR fine structure	$\frac{2\times10^{-7}}{\lambda(\mu m)}$	$\frac{10}{\lambda^3(\mu m)}$	[OIII]	52 μm	9.80×10^{-5}
Hyperfine			HI	21 cm	2.90×10^{-15}

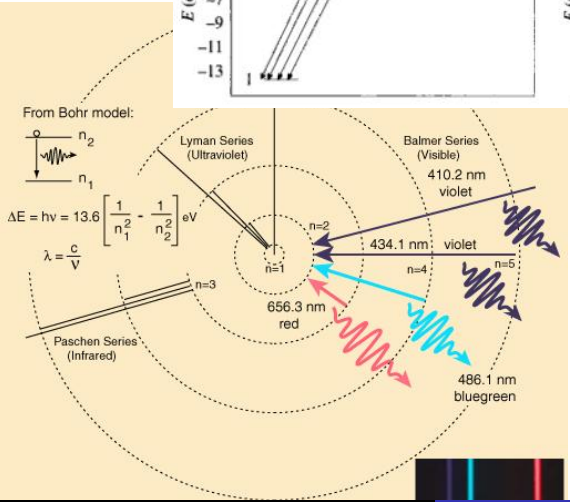
[a] See text for details.
[b] The $J = 1 \rightarrow 0$ transition.

H, He have lowest-lying transition energies a large fraction of ionization energies, FUV.
Multi-electron atoms have much smaller transition energies, visible-UV.
Radicals and ions have unpaired electrons in low-lying states, visible.
Molecular vibrational levels lowered by $\sqrt{m_e/M}$, mid-IR.
Molecular rotational levels lowered by m_e/M, mm.

Hydrogen Atom







SELECTION RULES

angular momentum conservation $\Delta J = 0, \pm 1$

electron spin only changed by magnetic field

dipole has odd parity

electric dipole – *allowed*
$\Delta\ell = \pm 1$; Δn arbitrary

magnetic dipole – *forbidden*
$\Delta\ell = 0$; $\Delta n = 0$

elec. quadrupole – *forbidden*
$\Delta\ell = 0, \pm 2$; Δn arbitrary

Statistical Equilibrium in the ISM

Transition strengths are usually expressed in terms of the oscillator strength f_{ji} or the *Einstein coefficients* A_{ij} and B_{ij} and B_{ji}. These are related by

$$B_{ij}/B_{ji} = g_j/g_i\,, \quad A_{ji} = \frac{2h\nu_{ji}}{c^2}B_{ij}\,, \quad B_{ij} = \frac{4\pi}{h\nu_{ji}}\frac{\pi e}{m_e c}f_{ji}\,.$$

The oscillator strength f_{ji} is the effective number of classical oscillators involved in the transition. For an electric dipole

$$f_{ji} = \frac{8\pi^2 e m_e}{3h}\nu_{ji}\mu_{ji}^2$$

which is about 1 for the strongest allowed transitions. In this case, $A_{ji} \sim 10^7$ s^{-1}. For forbidden transitions, $A_{ji} \sim 0.1$ s^{-1}.

Consider a two-level (ℓ, u) atom in statistical equilibrium in a gas with a radiation field whose intensity at the transition energy is $J_{u\ell}$. The rate of collisional excitations and de-excitations are $\gamma_{\ell u}$ and $\gamma_{u\ell}$. In statistical equilibrium, including absorption and stimulated emission,

$$n_\ell(n\gamma_{\ell u} + B_{\ell u}J_{u\ell}) = n_u(n\gamma_{u\ell} + B_{u\ell}J_{u\ell}) + n_u A_{u\ell}.$$

Cooling Rates

$$r = \frac{L}{V} = n_u A_{u\ell} h\nu_{u\ell}\,.$$

Two-level system in statistical equilibrium in the optically thin limit

$$n_\ell n \gamma_{\ell u} = n_u n \gamma_{u\ell} + n_u A_{u\ell}$$

Detailed balance:

$$\gamma_{\ell u} = \gamma_{u\ell} \frac{g_u}{g_\ell} e^{-h\nu_{u\ell}/kT}$$

$$\gamma_{u\ell} = \frac{4}{\sqrt{\pi}} \left(\frac{\mu}{2kT}\right)^{3/2} \times$$

$$\times \int_0^\infty \sigma_{u\ell}(v) v^3 e^{-\mu v^2/2dT} dv$$

$$\sigma_{u\ell}(v) \propto v^\alpha \implies \gamma_{u\ell} \propto T^{(1+\alpha)/2}$$

$(M+M, M+e^-, M^+ + e^-) \implies T^{(-1/2, 0, 1/2)}$

$$\frac{n_u}{n_\ell} = \frac{n\gamma_{\ell u}}{n\gamma_{u\ell} + A_{u\ell}} = \frac{\gamma_{\ell u}/\gamma_{u\ell}}{1 + n_{cr}/n}$$

where $n_{cr} = A_{u\ell}/\gamma_{u\ell}$. Collisions dominate when $n >> n_{cr}$.
For a species with abundance $\mathcal{A} = (n_u + n_\ell)/n = n_\ell(1 + n/n_{cr})/n$,

$$r = n_u A_{u\ell} h\nu_{u\ell} = \frac{\mathcal{A} n n_{cr} \gamma_{\ell u} h\nu_{u\ell}}{1 + n_{cr}/n + \gamma_{\ell u}/\gamma_{u\ell}}.$$

When $n << n_{cr}, n >> n_{cr}$,

$$r \to \mathcal{A} n^2 \gamma_{\ell u} h\nu_{u\ell}, \quad r \to A_{u\ell} n_u h\nu_{u\ell}$$

$$I = \frac{1}{2\pi} \int_0^z n^2 \Lambda(z) dz \propto nN, \quad N_u.$$

Cooling Rates

In the case matter is not optically thin, absorption and stimulated emission must be considered.

$$n_\ell(n\gamma_{\ell u} + B_{\ell u}J_{u\ell}) = n_u(n\gamma_{u\ell} + B_{u\ell}J_{u\ell}) + n_u A_{u\ell}.$$

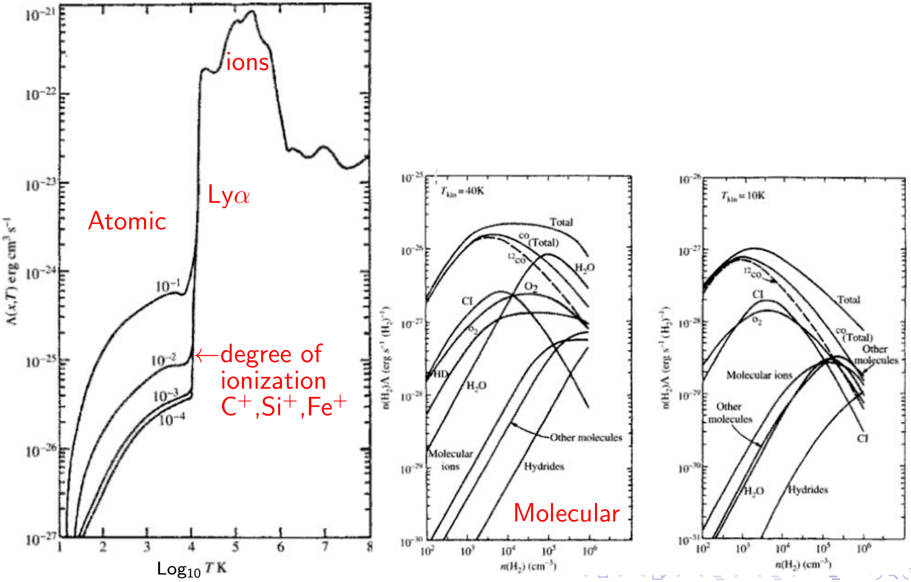
Suppose $\beta(\tau_{u\ell})$ is an escape probability; photons produced locally are only absorbed locally. Also suppose the local optical depth is global. Then

$$(n_\ell B_{\ell u} - n_u B_{u\ell})J_{u\ell} = n_u(1 - \beta(\tau_{u\ell}))A_{u\ell}$$

$$n_\ell n\gamma_{\ell u} = n_u n\gamma_{u\ell} + n_u A_{u\ell}\beta(\tau_{u\ell})$$

$$r = n_u A_{u\ell} h\nu_{u\ell} = \frac{\mathcal{A} n n_{cr}\beta(\tau_{u\ell})\gamma_{\ell u} h\nu_{u\ell}}{1 + n_{cr}\beta(\tau_{u\ell})/n + \gamma_{\ell u}/\gamma_{u\ell}}.$$

In the two limits $n << \beta n_{cr}, n >> \beta n_{cr}$ we recover the same results as in the optically thin case.



ions
Lyα
Atomic
←degree of ionization C+,Si+,Fe+
10^-1
10^-2
10^-3
10^-4
Λ(x,T) erg cm^3 s^-1
Log10 T K
Molecular
T_kin = 40K
T_kin = 10K
Total
CO (Total)
12CO
H2O
O2
CI
HD
Molecular ions
Other molecules
Hydrides
n(H2)Λ (erg s^-1 (H2)^-1)
n(H2) (cm^-3)

Cooling Rates

$$\frac{L}{V} = n^2\Lambda(T), \qquad t_{\rm cool} \sim \frac{T}{n\Lambda(T)}$$

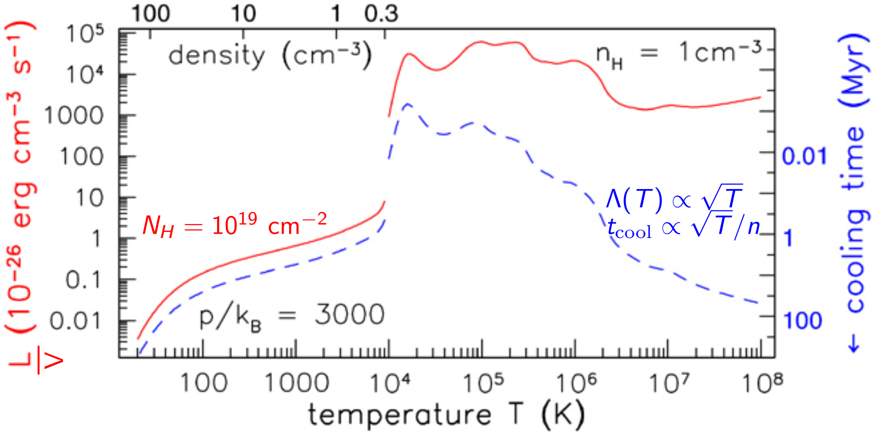


Fig 2.25 (Hensler, Wolfire) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Heating Processes

Processes that couple radiation to gas dominate

- Photoionization The e^- gains kinetic energy from the photon.
  - HII regions: H ionization.
  - HI regions: photoionization of large molecules and small dust particles (photoelectric effect).
  - Molecular regions: photodissociation of molecules (H_2). Collisional de-excitation also heats.
- Dust-gas heating If dust is warmer than gas (protostar envelope).
- Cosmic-ray heating e^- from ionized gases gain kinetic energy and can lead to secondary ionizations.
- X-ray heating e^- from ionization can lead to secondary ionizations. Both galactic and extragalactic contributions.
- Turbulent heating Viscous heating from motions.
- Ambipolar diffusion heating Ions and e^- develop small differential drift velocities from counterplay between magnetic fields and gravity, leading to frictional heating.
- Gravitational heating through compression. Thermal energy $\propto nT$, $\tau_{ff} \propto n^{-1/2}$, heating rate $\propto n^{3/2}T$.

Heating and Cooling Processes

- **Diffuse hot gas**

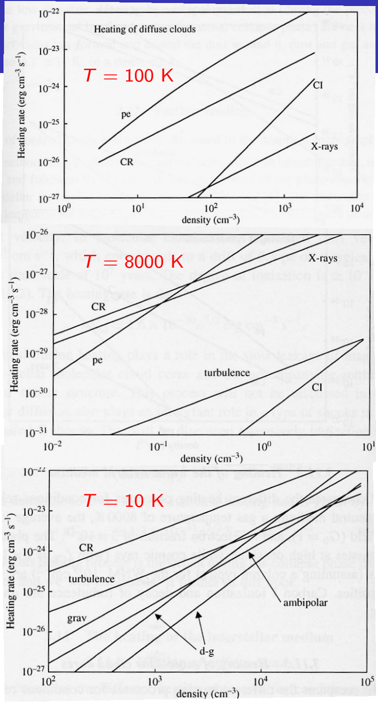
  Heated by supernova shocks, and cools within $10^4 - 10^5$ yr, condensing into cooler clouds. Near the Sun, a supernova shock passes every 1–5 Myr. Heating is primarily by photoionization of HI. Cooled primarily by recombination at highest temperatures

- **Diffuse warm gas**

  Photoelectric heating domnates from stellar UV on small grains. Cooled primarily by e^- collisions which excite low lying electronic states of trace ionized species. Mostly these are forbidden transitions. In neutral regions, cosmic rays and X-rays dominate.

- **Cool molecular gas**

  Cosmic ray ionization heating is important for gas; grains are heated by infrared photons. Cooling occurs through rotational transitions of molecules, such as CO.



Jean's Mass

Consider a uniform density, isothermal cloud. Potential energy

$$\Omega = -\int \frac{G\mathcal{M}}{r} d\mathcal{M} = -\frac{3}{5}\left(\frac{4\pi\rho}{3}\right)^{1/3} G\mathcal{M}^{5/3}$$

Kinetic energy

$$U = \int \frac{3N_o kT}{2\mu} d\mathcal{M} = \frac{3N_o kT}{2\mu}\mathcal{M}$$

Virial Theorem in equilibrium: $\Omega = 2U$

$$\mathcal{M} = \left(\frac{5N_o kT}{G\mu}\right)^{3/2} \sqrt{\frac{3}{4\pi\rho}}$$

Jean's length

$$\lambda_J \equiv c_s\sqrt{\frac{\pi}{G\rho}} = \sqrt{\frac{\pi N_o kT}{G\mu\rho}} \simeq 2.5\,\sqrt{\frac{T}{10\text{ K}}}\sqrt{\frac{100\text{ cm}^{-3}}{n}}\text{ pc}$$

Jean's mass

$$\mathcal{M}_J \equiv \frac{\pi}{6}\rho\lambda_J^3 = \left(\frac{\pi^2}{30}\right)^{3/2}\sqrt{\frac{4}{3}}\mathcal{M} \simeq 39\,\left(\frac{T}{10\text{ K}}\right)^{3/2}\sqrt{\frac{100\text{ cm}^{-3}}{n}}\,\mathcal{M}_\odot.$$

Pressure-Bounded Stable Mass

Hydrostatic equilibrium can be written, equivalent to Virial Theorem,

$$Vdp = -\frac{1}{3}\frac{G\mathcal{M}(r)}{r}d\mathcal{M}(r) = -\frac{1}{3}d\Omega$$

Now suppose the uniform density isothermal cloud of mass $\mathcal{M}$ is bounded by the ISM at a fixed pressure p_o at the radius R.

$$-\Omega = \int_0^{\mathcal{M}} \frac{G\mathcal{M}(r)}{r}d\mathcal{M} = 3\int_0^{\mathcal{M}} \frac{p}{\rho}d\mathcal{M} - 4\pi p_o R^3.$$

For a constant density gas, $\mathcal{M}(r) \propto r^3$

$$-\Omega = \frac{3G\mathcal{M}^2}{5R} = 3N_o kT\mathcal{M} - 4\pi p_o R^3, \quad p_o = \frac{3N_o kT\mathcal{M}}{4\pi R^3} - \frac{3G\mathcal{M}^2}{20\pi R^4}$$

For small R, $p_o < 0$. For large R, $p_o > 0$ but tends to 0 for $R \to \infty$. There must be a maximum of p_o for the radius R_m, $(\partial p_o/\partial R)_{\mathcal{M}} = 0$:

$$R_m = \frac{4G\mathcal{M}}{15N_o kT}, \qquad p_o = \frac{3N_o kT\mathcal{M}}{16\pi}\left(\frac{15N_o kT}{4G\mathcal{M}}\right)^3.$$

$$\mathcal{M} = \left(\frac{15N_o kT}{4G}\right)^{3/2}\sqrt{\frac{3}{4\pi\rho}} = \frac{135}{4\pi^3}\sqrt{\frac{5}{4}}\mathcal{M}_J$$

If the cooling is efficient, a cloud that exceeds the Jean's mass will collapse quickly, on a free-fall time

$$t_{ff} = 1/\sqrt{G\rho} \simeq 10^8 n_H^{-1/2}\ \mathrm{yr}.$$

If the temperature doesn't increa
becomes smaller and the collaps
fragments become optically thicl
Isothermal hydrostatic equilibriu
$dp(r)/dr = -G\mathcal{M}(r)\rho(r)/r^2$,
$d\mathcal{M}(r)/dr = 4\pi\rho(r)r^2$

Let $x = r\sqrt{4\pi G\mu\rho_o/(N_o kT)}$
and $y = \rho/\rho_o$,

$$y'' - (y')^2/y + 2y'/x + y^2 = 0.$$

Note:
$x \to 0, y \to 1 - x^2/6$,
$x \to \infty, y \to 2x^{-2}$

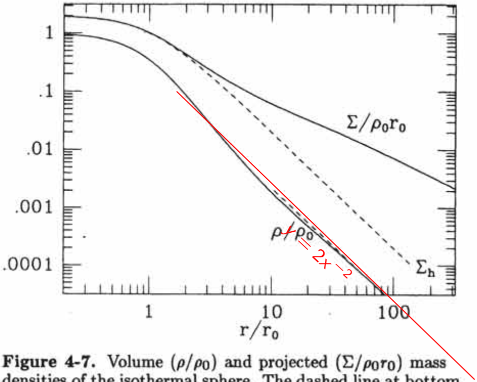


Figure 4-7. Volume (ρ/ρ_0) and projected $(\Sigma/\rho_0 r_0)$ mass densities of the isothermal sphere. The dashed line at bottom right shows the density profile of the singular isothermal sphere. The dashed curve labeled Σ_h shows the surface density of the modified Hubble law (4-128).

Bonner-Ebert Sphere

Mass within a radius X

$$\mathcal{M}(X) = (4\pi\rho_o)^{-1/2}\left(\frac{N_okT}{G\mu}\right)^{3/2} I(X),$$

$$I(X) = \int_0^X yx^2 dx$$

$$p_0(X) = N_okT\rho_o y(X)/\mu$$
$$= \frac{(N_okT/\mu)^4}{4\pi G^3\mathcal{M}^2} I^2(X)y(X)$$

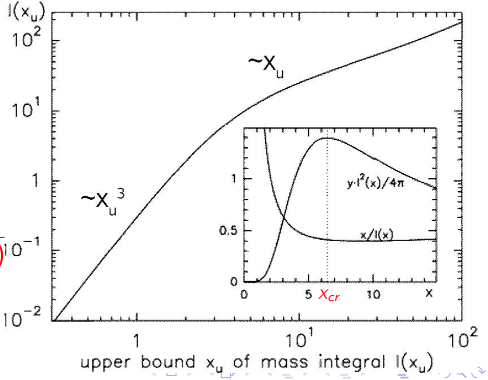
$$R(X) = \sqrt{\frac{N_okT}{4\pi G\rho_o\mu}}X = \frac{G\mathcal{M}\mu}{N_okT}\frac{X}{I(X)}$$

Find $(\partial p_o/\partial X)_{X_{cr}} = 0$: $X_{cr} = 6.5$

$$p_o(X_{cr}) = 1.40\frac{(N_okT/\mu)^4}{G^3\mathcal{M}^2},$$
$$R(X_{cr}) = 0.411\frac{G\mathcal{M}\mu}{N_okT}.$$

Compare to

$\frac{3}{16\pi}\left(\frac{15}{4}\right)^3 \simeq 3.15, \quad 4/15 \simeq 0.267.$



Observing the Density Profile of a Cloud

Extinction is proportional to column density.

$N(R) = \int_{-Z}^{Z} n(r) dz = 2 \int_0^Z n(r) dz$

$R^2 + z^2 = r^2, \qquad R^2 + Z^2 = R_m^2$

Case 1: Isothermal singular truncated sphere

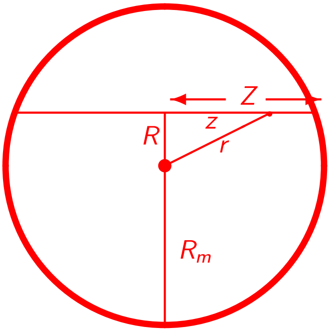
$n(r)/n_o = 2/r^2, \qquad r \le R_m$

$$\Rightarrow N(R) = \frac{4n_o}{R} \tan^{-1}\left(\frac{\sqrt{R_m^2 - R^2}}{R}\right)$$

Case 2: Modified isothermal sphere

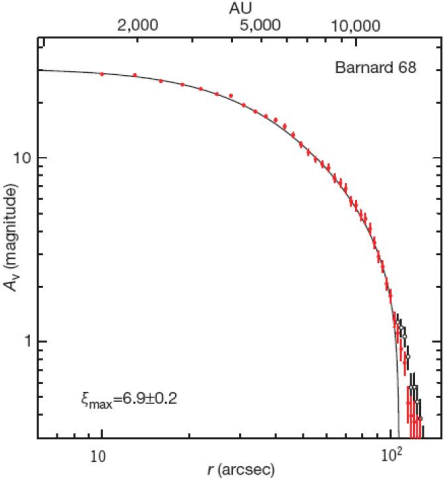
$n(r)/n_o = 2/(2 + r^2)$

$$\Rightarrow N(R) = \frac{4n_o}{\sqrt{2 + R^2}} \tan^{-1} \sqrt{\frac{R_m^2 - R^2}{2 + R^2}}$$



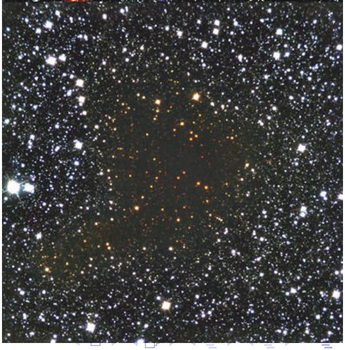
Bonner-Ebert Sphere and B 68

For dense clouds, absorption is sensitive to density. Extinction is wavelength dependent. $A_\nu(\rho) \propto N_H$.



V

IR



Additional Pressure Support

$$\mathcal{M}_{BE} = \left(\frac{1.4}{p_o}\right)^{1/2} \frac{(N_o kT/\mu)^2}{G^{3/2}} \simeq 5.9 \left(\frac{10^{-12}\ \mathrm{erg\ cm^{-3}}}{p_o}\right)^{1/2} \left(\frac{T}{10\ \mathrm{K}}\right)^2 \mathcal{M}_\odot$$

Like the Jean's mass for cloud conditions, additional pressure needed to stabilize massive clouds.

- turbulence, $\mathcal{M}_J \propto \bar{v}^3$
  Consistent with observation that more massive stars are formed in warmer, more turbulent clouds.
- magnetic fields

$$4\pi R^3 p_o = \frac{3N_o kT}{\mu} - \frac{3G\mathcal{M}^2}{5R} + \frac{(BR^2)^2}{3R}$$

If field is frozen in matter, magnetic flux BR^2 is conserved and magnetic term has the same R dependence as gravity term. Thus, if either gravity or magnetism dominates at beginning, they always dominate. Collapse occurs

$$\frac{3G\mathcal{M}^2}{5R} > \frac{B^2 R^3}{3},$$

$$\mathcal{M} > \left(\frac{5}{G}\right)^{3/2} \frac{B^3}{48\pi^2\rho^2}$$

$$\sim \frac{B}{10\ \mu\mathrm{G}} \left(\frac{R}{0.1\ \mathrm{pc}}\right)^2 \mathcal{M}_\odot.$$

Fields generally not frozen due to small ionization fraction ($\sim 10^{-7}$). Ions frozen but neutrals not, leading to drifts. *Ambipolar diffusion* timescale:

$$t_{ad} \approx 2 \times 10^{13} (n_{ion}/n_{neutral})\ \mathrm{yr}.$$

Dynamical Evolution

Euler Equations of Motion in Spherical Symmetry

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{G\mathcal{M}(r)}{r^2} = 0,$$
$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho u\right) = 0,$$
$$\frac{\partial \mathcal{M}}{\partial t} + u\frac{\partial \mathcal{M}}{\partial r} = 0, \qquad \frac{\partial \mathcal{M}(r)}{\partial r} = 4\pi r^2\rho.$$

For $p = c_s^2\rho$, use $X = r/[c_s(\mp t)]$ as a self-similar variable. We will see that $t < 0$ corrsponds to times early in the collapse, $t > 0$ to late times, and $t = 0$ is a *catastrophe* point. Also

$$u(r,t) = c_s V(X), \qquad \rho(r,t) = \frac{D(X)}{4\pi G(\mp t)^2}, \qquad \mathcal{M}(r,t) = \frac{c_s^3(\mp t)}{G} m(X).$$

Find $m = m'(X \pm V) \equiv m' A(X), m' = X^2 D \Rightarrow m = DX^2 A$. Other two Euler equations are

$$\frac{D'}{D} \pm V'A = -DA, \qquad \frac{D'}{D}A \pm V' = -\frac{2A}{X}$$

Self-Similar Solutions

For the static case, $V = 0$, $D = 2/X^2$ and $m = 2X$. This corresponds to the singular isothermal solution.

For $t < 0$, a solution is the *homologous* one

$$D = 2/3, \qquad V = -(2/3)X, \qquad m = (2/9)X^3.$$

For $t \to 0^+$, $X \to \infty$:

$$V' = D - \frac{2}{X^2}, \qquad D' = \frac{D(D-2)}{X}.$$

Asymptotically $D \to \frac{\alpha}{X^2}, V \to \frac{2-\alpha}{X}, m \to \alpha X$ with $\alpha \geq 2$; $V \leq 0$ (infall).

Now consider what happens for $t \to +\infty$, or $X \to 0$. The asymptotic solutions of the Euler equations are equivalent to *free-fall*:

$$D \to \sqrt{\frac{m_0}{2X^3}}, \qquad V = -\sqrt{\frac{2m_0}{X}}, \qquad m_0 = -(X^2 DV)_{X \to 0}.$$

The value of $m_o = 0.975$ can be found by integration. There is a singular point in the flow, however, at the critical point $A(X_c)^2 = 1$, or $X_c - V(X_c) = 1$. The aymptotic ($X \to \infty$) solution $V = 0$ is satisfied at $X = 1$, so $X_c = 1$ and $V(X) = 0$ for $X > 1$.

Self-Similar Isothermal Case

$$\rho(r,t) = \sqrt{\frac{m_o c_s^3}{32\pi^2 G^2}}\, t^{-1/2} r^{-3/2}$$

$$u(r,t) = -\sqrt{2 m_o c_s^3}\, t^{1/2} r^{-1/2}$$

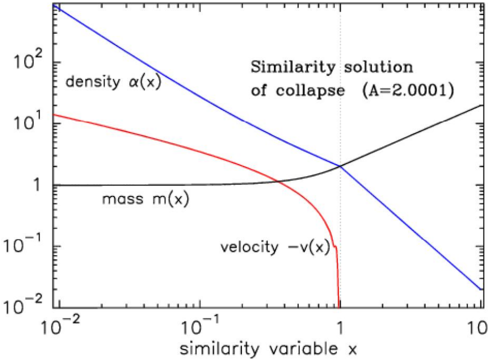
$$= -\sqrt{\frac{2G\mathcal{M}_*(t)}{r}}$$

$$\mathcal{M}_*(t) = \frac{m_o c_s^3}{G} t$$

$$\dot{\mathcal{M}} = 4\pi r^2 \rho u = \frac{m_o c_s^3}{G}$$

$$\sim 2 \times 10^{-6} \mathcal{M}_\odot \ \mathrm{yr}^{-1}$$

$$m(X_c) = 2$$



Pressure-less Case

An analytic collapse solution exists when pressure is negligible compared to gravity, not a bad approximation once collapse begins. Euler equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{G\mathcal{M}}{r^2} = \frac{du}{dt} = \frac{d^2 r}{dt^2}.$$

In spherical collapse or expansion, the mass $\mathcal{M}(r,t)$ internal to a mass point initially at r_0 at $t=0$ does not change.

$$\frac{dr}{dt}\frac{d^2 r}{dt^2} = \frac{1}{2}\frac{d}{dt}\left(\frac{dr}{dt}\right)^2 = -G\mathcal{M}\frac{dr}{dt}\left(\frac{1}{r^2}\right) = G\mathcal{M}\frac{d}{dt}\left(\frac{1}{r}\right).$$

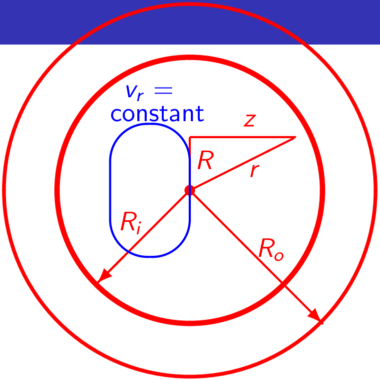
Integrating once and then twice,

$$\left(\frac{dr}{dt}\right)^2 = 2G\mathcal{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\sqrt{2G\mathcal{M}}\ t = r_0^{3/2}\left(\sqrt{\frac{r}{r_0}}\sqrt{1-\frac{r}{r_0}} + \tan^{-1}\sqrt{\frac{r_0}{r}-1}\right)$$

The time to collapse to $r=0$ is $t_{ff} = \sqrt{\pi^2 r_0^3/(8G\mathcal{M})}$. If ρ is assumed uniform, $t_{ff} = \sqrt{3\pi/(32G\rho_0)} = 3.7\times 10^7 (\mathrm{cm}^3/n)^{1/2}$ yr.

Observing Collapsing Clouds



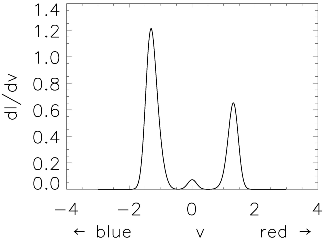
Consequence of $v \propto r^{-1/2}$ collapse is a double peaked line profile. Along a line-of-sight, two parts of cloud contribute at each velocity, but farther point is at greater optical depth and obscured. Blue peak is higher than red peak due to higher temperatures near cloud center. Simple model:

$$z^2 = r^2 - R^2, \qquad Z_o^2 = R_o^2 - R^2$$

$$\left.\begin{array}{rcl} v_r &=& -v_0 z r^{-1}(r+a)^{-1/2} \\ n &=& n_0(r+a)^{-3/2} \end{array}\right\} r < R_i$$

$$\left.\begin{array}{rcl} v_r &=& 0 \\ n &=& n_0(R_i+a)^{-3/2} \end{array}\right\} R_i < r < R_o$$

$$\frac{dI(v)}{dv} \propto \int_{-Z_o}^{Z_o} n(z)^\gamma e^{-\tau(z)} e^{-(v_r(z)-v)^2/\sigma^2} dz$$



Observing Collapsing Clouds

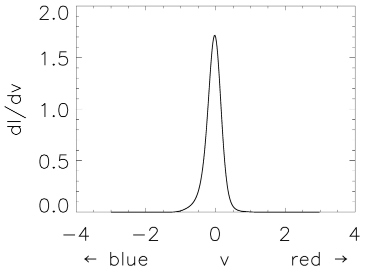
Consequence of $v \propto r$ (homologous) collapse is a single peaked line profile. Along a line-of-sight, only one part of a cloud contributes at each velocity. Simple model:

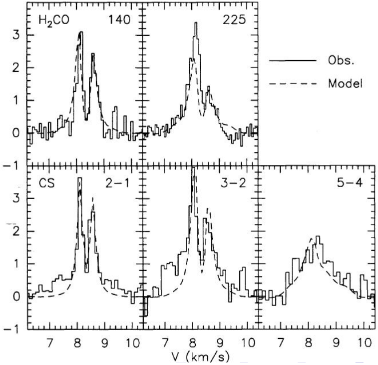
$$z^2 = r^2 - R^2, \qquad Z_o^2 = R_o^2 - R^2$$

$$\left.\begin{array}{rcl} v_r & = & -v_0 z \\ n & = & n_0(r+a)^{-3/2} \end{array}\right\} r < R_i$$

$$\left.\begin{array}{rcl} v_r & = & 0 \\ n & = & n_0(R_i+a)^{-3/2} \end{array}\right\} R_i < r < R_o$$

$$\frac{dI(v)}{dv} \propto \int_{-Z_o}^{Z_o} n(z)^\gamma e^{-\tau(z)} e^{-(v_r(z)-v)^2/\sigma^2} dz$$





Strongren Spheres

High-energy stellar photons ($E > h\nu_1 = 13.6$ eV) ionizes hydrogen in a spherical region surrounding the star. Ionization continues and a balance with recombinations eventually occurs. Recombinations go into both excited states and the ground state. A recombination directly into the ground state can re-ionize. Since not all recombinations go directly to the ground state, the ones that do can't affect the ultimate equilibrium since they are eventually "used up". We can make the on-the-spot approximation – that ionizing photons produced by direct recombination to the gound state are absorbed where they are created and can be ignored.

$\bar{\alpha}_H = 3 \times 10^{-19}$ cm^2 is the average ionization coefficient.
$\beta_A(T_e) \sim 4 \times 10^{-13}$ cm^{-3} s^{-1} is the total recombination coefficient.
$\beta_B(T_e) \sim 2.6 \times 10^{-13}$ cm^{-3} s^{-1} is the net recombination coefficient into excited states.
The flux of ionizing photons is

$$F_\nu = \int_{\nu_1}^{\infty} \mathcal{F}_\nu d\nu = \int_{\nu_1}^{\infty} \frac{L_*(\nu)}{4\pi r^2 h\nu} d\nu$$
$$= \frac{R_*^2}{r^2} \int_{\nu_1}^{\infty} \frac{2\nu^2}{c^2} \frac{1}{e^{h\nu/kT} - 1} d\nu$$
$$\simeq 2 \times 10^{12} \left(\frac{.5\ \mathrm{pc}}{r}\right)^2\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

The last equality is for an O4 star.

Ionization Equilibrium

At a distance r from the star, the optical depth is

$$\tau(\nu, r) = \int_0^r n_H(r)\alpha_H(\nu)dr.$$

The ionization balance condition is

$$n_{H^0}\int_{\nu_1}^{\infty} \mathcal{F}_\nu e^{-\tau}\alpha_H d\nu = n_e n_p \beta_B(T_e).$$

Let $x = n_e/n = n_p/n\,, 1 - x = n_{H^0}/n\,,$ and

$$\bar{\alpha}_H F_\nu = \int_{\nu_1}^{\infty} \mathcal{F}_\nu \alpha_H d\nu.$$

Assuming τ is not sensitive to ν,

$$\frac{1-x}{x^2} \simeq \frac{n\beta_B e^\tau}{\bar{\alpha}_H F_\nu},$$

The production rate of ionizing photons from an O5 star is

$$\mathcal{N}_{Ly\alpha} = \int_{\nu_1}^{\infty} \frac{L_*(\nu)}{h\nu} d\nu \sim 5 \times 10^{49}\ \mathrm{s}^{-1}$$

Strongren Spheres

For an optically thick nebula, the size $\mathcal{R}_s$ of the sphere is determined when the total recombination rate in the sphere equals the ionizing photon luminosity. For constant density and $x \simeq 1$, multiply both sides of balance equation by $4\pi r^2 dr$ and integrate:

$$\int_{\nu_1}^{\infty} (L_*(\nu)/h\nu) \int_0^{\infty} e^{-\tau} d\tau d\nu = \mathcal{N}_{Ly\alpha} = 4\pi n^2 \beta_B \mathcal{R}_s^3/3$$

$$\mathcal{R}_s = 1.2 \left(10^3 \text{ cm}^{-3}/n\right)^{2/3} \left(\mathcal{N}_{Ly\alpha}/5 \times 10^{49} \text{ s}^{-1}\right)^{1/3} \text{ pc}$$

$$\mathcal{M}_s = \frac{\mathcal{N}_{Ly\alpha}}{N_o n \beta_B} \simeq 155 \left(\frac{\mathcal{N}_{Ly\alpha}}{5 \times 10^{49} \text{ s}^{-1}}\right) \left(\frac{10^3 \text{ cm}^{-3}}{n}\right) \mathcal{M}_\odot$$

$$\overline{1-x} = \frac{n\beta_B}{\bar{\alpha}_H F_\nu} \simeq 4 \times 10^{-4} \left(\frac{n}{10^3 \text{ cm}^{-3}}\right) \left(\frac{5 \times 10^{49} \text{ s}^{-1}}{\mathcal{N}_{Ly\alpha}}\right) \left(\frac{r}{1 \text{ pc}}\right)^2$$

The structure within the Stromgren sphere: use $z = r/\mathcal{R}_s$.

$$\tau_s = n\bar{\alpha}_H \mathcal{R}_s \sim 10^3 \left(n/10^3 \text{ cm}^{-3}\right)^{1/3} \left(\mathcal{N}_{Ly\alpha}/5 \times 10^{49} \text{ s}^{-1}\right)^{1/3},$$

$$\frac{1-x}{x^2} = \frac{3z^2 e^{\tau}}{\tau_s}, \qquad \frac{d\tau}{dz} = (1-x)\tau_s$$

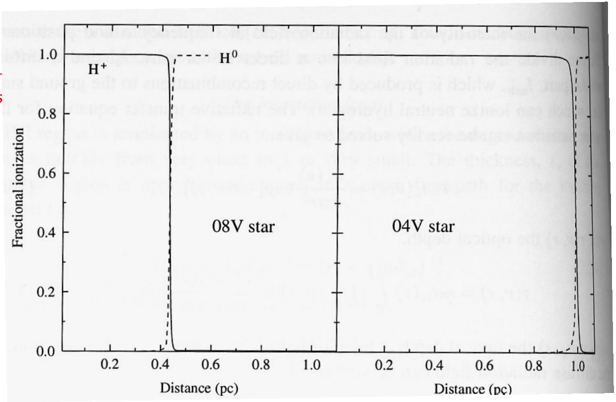
$$\Longrightarrow \tau = -\ln[1-z^3], \qquad (1-x) = \frac{3z^2x^2}{\tau_s(1-z^3)}$$

Ionization Front

The ionization front terminates the HII region and is very narrow as $1-x$ quickly becomes unity. The thickness ℓ of the front is about 1 mean free path for the ionizing photons. Use $x=1/2$:

$$\ell(x=1/2)=(n_H\alpha_H)^{-1}=\frac{2}{n\alpha_H}\simeq 2\times10^{-3}\left(\frac{10^3\ \mathrm{cm}^{-3}}{n}\right)\ \mathrm{pc}$$

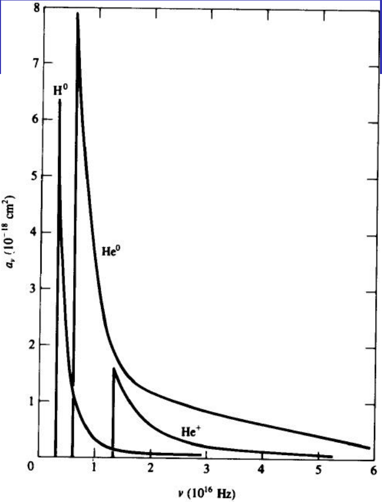
$$\frac{\ell(r)}{\mathcal{R}_s}=\frac{1}{(1-x)n\alpha_H\mathcal{R}_s}\simeq\frac{1}{3}\left(\frac{\mathcal{R}_s}{r}\right)^2.$$



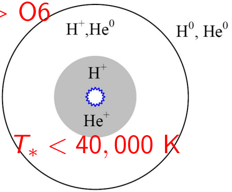
The Effect of Helium

Element/Ion	Ionization Potential
H $\rightarrow$ H$^+$	$h\nu_1 = 13.6$ eV
He $\rightarrow$ He$^+$	$h\nu_2 = 24.6$ eV
He$^+$ $\rightarrow$ He^{++}	$h\nu_3 = 54.4$ eV

Although He is 10% as abundant as H, it must be included for hotter stars since the cross section for ionization of neutral He is 10 times larger than for neutral H at the threshold energy (25 eV). But the ionization energy of He$^+$ is so large that only the hottest stars can ionize it.

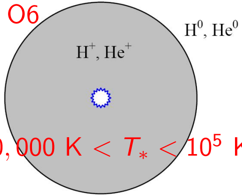


$>$ O6



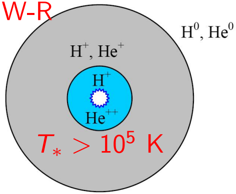
$T_* < 40,000$ K

$<$ O6



$40,000 \text{ K} < T_* < 10^5$ K

W-R



$T_* > 10^5$ K

Treatment of Helium Ionization

Competition for stellar ionizing photons:

$$y = \frac{n_{H^0}\alpha_H}{n_{H^0}\alpha_H + n_{He^0}\alpha_{He}}.$$

Recombinations into He^0 excited states and ground state make photons capable of ionizing H. Use on-the-spot approximation for H and He.

$$\frac{n_{H^0}}{4\pi r^2}\int_{\nu_1}^{\infty}\frac{L_*}{h\nu}\alpha_H e^{-\tau}d\nu + y n_{He^+}n_e(\beta_{H,A}-\beta_{H,B}) = n_{H^+}n_e\beta_{H,B}$$

$$\frac{n_{He^0}}{4\pi r^2}\int_{\nu_2}^{\infty}\frac{L_*}{h\nu}\alpha_{He} e^{-\tau}d\nu + (1-y) n_{He^+}n_e(\beta_{He,A}-\beta_{He,B}) = n_{He^+}n_e\beta_{H,A}$$

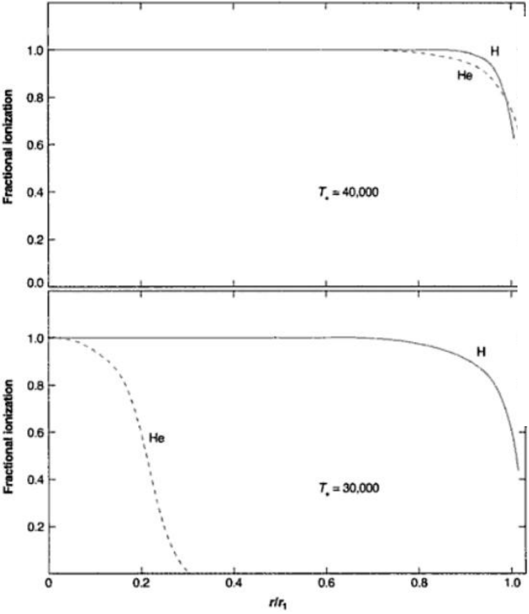
$$d\tau/dr = n_{H^0}\alpha_H\,, \qquad n_e \simeq n_{H^+} \simeq n_H\,, \qquad \mathcal{R}_s < r < \mathcal{R}_2$$

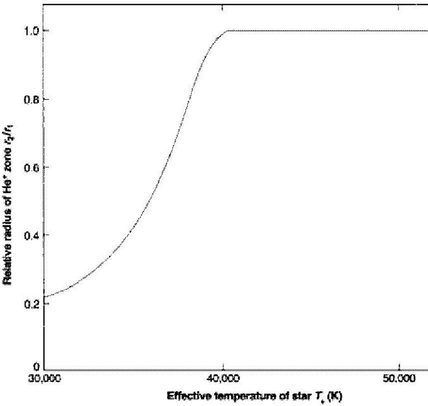
$$d\tau/dr = n_{H^0}\alpha_H + n_{He^0}\alpha_{He}\,, \qquad n_e = n_{H^+} + n_{He^+} \simeq n_H + n_{He}\,, \qquad r > \mathcal{R}_2$$

$$\int_{\nu_2}^{\infty}\frac{L_* d\nu}{h\nu} = \mathcal{N}_2 = \frac{4\pi}{3}\mathcal{R}_2^3 n_{He^+}n_e\beta_{He,B}\,, \int_{\nu_1}^{\infty}\frac{L_* d\nu}{h\nu} = \mathcal{N}_{Ly\alpha} = \frac{4\pi}{3}\mathcal{R}_s^3 n_{H^+}n_e\beta_{H,B}$$

$$\mathcal{R}_s \simeq \left(\frac{3}{4\pi}\frac{\mathcal{N}_{Ly\alpha}}{n_H^2\beta_{H,B}}\right)^{1/3}, \qquad \frac{\mathcal{R}_s^3}{\mathcal{R}_2^3} \simeq \frac{\mathcal{N}_{Ly\alpha}\beta_{He,B}}{\mathcal{N}_2\beta_{H,B}}\frac{n_{He}}{n_H}\frac{n_H+n_{He}}{n_H} \qquad \mathcal{R}_2 < \mathcal{R}_s$$

Structure of H/He Stromgren Spheres





Planetary Nebulae