24 Lecture 24: Galaxies: Classification and Treatment

"The effort to understand the Universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy."

Steven Weinberg

The Big Picture: Today we define and classify galaxies and outline their main characteristics. We also justify the mean-field approximation in galaxy modeling.

The Hubble Classification of Galaxies

Galaxies are found in a wide range of shapes, sizes and masses, but can be divided into four main types according to *Hubble classification* (see Fig. 42).

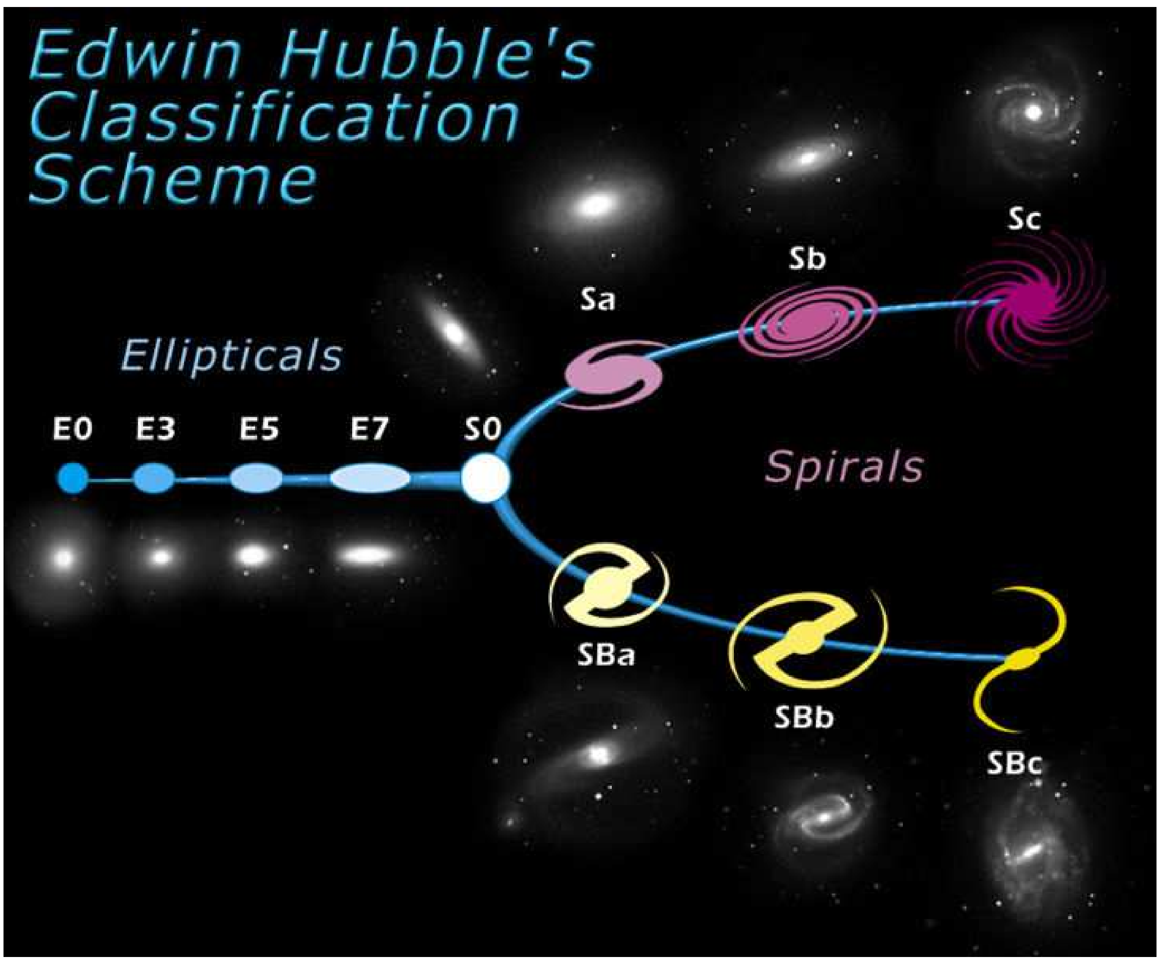


Figure 42: The Hubble classification of galaxies.

Galaxies near the start of the sequence (*early-type* galaxies) have little or no cool gas and dust, and consist mostly of old *Population II* stars (old, less luminous and cooler than Population I stars; have fewer heavy elements — "metal-poor"); galaxies near the end (*late-type* galaxies) are rich in gas, dust, and young stars.

Elliptical Galaxies

Elliptical galaxies are smooth, featureless systems containing little or no gas or dust. The fraction of bright galaxies that are elliptical is a function of the local density, ranging from about 10% in low-density regions to 40% in dense clusters of galaxies. The *isophotes* (contours of constant surface brightness) are approximately concentric ellipses, with axis ratio b/a ranging from 1 to about 0.3. Elliptical galaxies are denoted by the symbols E0, E1, etc., where the brightest isophotes of a galaxy of type En have axis ratio $b/a = 1 - n/10$. The *ellipticity* is $\epsilon = 1 - b/a$. Thus the most elongated elliptical galaxies are of type E7. Since we see only the projected brightness distribution, it is impossible to determine directly whether elliptical galaxies are axisymmetric or triaxial.

Surface brightness profiles.
The surface brightness of an elliptical galaxy falls off smoothly with radius. Often the outermost parts of a galaxy are undetectable against the background night-sky brightness. The surface-brightness profiles of most elliptical galaxies can be fit reasonably well by the empirically-motivated $R^{1/4}$ or *de Vaucouleurs'* law

$$I(R) = I(0)e^{-kR^{1/4}}, \tag{496}$$

where the *effective radius* R_e is the radius of the isophote containing half of total luminosity and I_e is the surface brightness R_e. The effective radius is typically $3/h$ kpc for bright ellipticals and is smaller for fainter galaxies.

However, it has been shown that de Vaucouleurs' $R^{1/4}$ law is appropriate only for a subset of elliptical galaxies. Generalizing de Vaucouleurs' law to allow for a varying rate of exponential decay, we arrive at the *Sérsic law* (of which de Vaucouleurs' is a special case when $n = 4$):

$$I(R) = I(0)e^{-kR^{1/n}}. \tag{497}$$

It has been shown that there exists a strong correlation between the observed size of the elliptical galaxy and the best-fit index n: heavier elliptical galaxies have higher values of n.

Central density cusps and supermassive black holes.
With the advent of the Hubble Space Telescope, modeling of elliptical galaxies has undergone a revolution: elliptical galaxies are *not* well-approximated by density profiles with central cores, as once thought, but have logarithmic slopes of the density profiles which increase all the way to the smallest observable radius: the elliptical galaxies have central density cusps. Furthermore, the centers of most elliptical galaxies harbor a supermassive black hole, with mass millions (and sometimes billions) times that of our Sun.

No net rotation.
Most giant elliptical galaxies exhibit little or no rotation, even those with highly elongated isophotes. Their stars have random velocities along the line of sight whose root mean square dispersion σ_p can be measured from the Döppler broadening of spectral lines. The velocity dispersion in the inner few kiloparsecs is correlated with luminosity according to the *Faber-Jackson law*

$$\sigma_p \simeq 220(L/L_\star)^{1/4} \text{ km s}^{-1}. \tag{498}$$

Lenticular Galaxies

Lenticular galaxies have a prominent disk that contains no gas, bright young stars, or spiral arms. Lenticular disks are smooth and featureless, like elliptical galaxies, but obey the exponential surface-brightness law characteristic of spiral galaxies:

$$I(R) = I(0)e^{-R/R_d}, \tag{499}$$

where the disc scale length $R_d = 3.5 \pm 0.5$ kpc. Lenticulars are labeled by the notation S0 in Hubble's classification scheme. They are very rare in low-density regions, comprising less then 10% of all bright galaxies, but up to half of all galaxies in high-density regions are S0's.

The lenticulars form a transition class between elliptical and spirals. The transition is smooth and continuous, so that there are S0 galaxies that might well be classified as E7, and others that sometimes been classified as spirals.

The strong dependence of the fractional abundance of the fractional abundance of S0 galaxies on the local density is obviously an important — but still controversial — clue to the mechanism of galaxy formation.

Spiral Galaxies

Spiral galaxies, like the Milky Way, contain a prominent disk composed of gas, dust and *Population I* stars (Population I stars include the Sun and tend to be luminous, hot and young, concentrated in the disks of spiral galaxies, and particularly found in the spiral arms). In all these systems the disk contains *spiral arms*, filaments of bright stars, gas, and dust, in which large numbers of stars are currently forming. The spiral arms vary greatly in their length and prominence from one spiral galaxy to another but are almost always present.

In low-density regions of the Universe, almost 80% of all bright galaxies are spirals, but the fraction drops to 10% in dense regions such as cluster cores.

The distribution of surface brightness in spiral galaxy disks obeys the exponential law. The typical disk scale length is $R_d \simeq 3/h$ kpc, and the central surface brightness is remarkably constant at $I_0 \simeq 140 L_\odot$ pc^{-2}.

The circular-speed curves of most spiral galaxies are nearly flat, $v_c(R)$ independent of R, except near the center, where the circular speed drops to zero. Typical circular speeds are between 200 and 300 km s^{-1}. It is a remarkable fact that the circular speed curves still remain flat even at radii well beyond the outer edge of the visible galaxy, thus implying the presence of invisible or dark mass in the outer parts of the galaxy.

Spiral galaxies also contain a spheroid of Population II stars. The luminosity of the spheroid relative to the disk correlates well with a number of other properties of the galaxy, in particular the fraction of the disk mass in gas, the color of the disk, and how tightly the spiral arms are wound. This correlation is the basis of Hubble's classification of spiral galaxies. Hubble divided spiral galaxies into a sequence of four classes or types, called Sa, Sb, Sc, Sd. Along the sequence Sa $\rightarrow$ Sd the relative luminosity of the spheroid decreases, the relative mass of gas increases, and the spiral arms become more loosely wound. The spiral arms also become more clumpy, so that individual patches of young stars and HII regions (a cloud of glowing gas and plasma, sometimes several hundred light-years across, in which star formation is taking place) become visible. Our galaxy appears to be intermediate between Sb and Sc, so its Hubble type is written as Sbc.

Irregular Galaxies

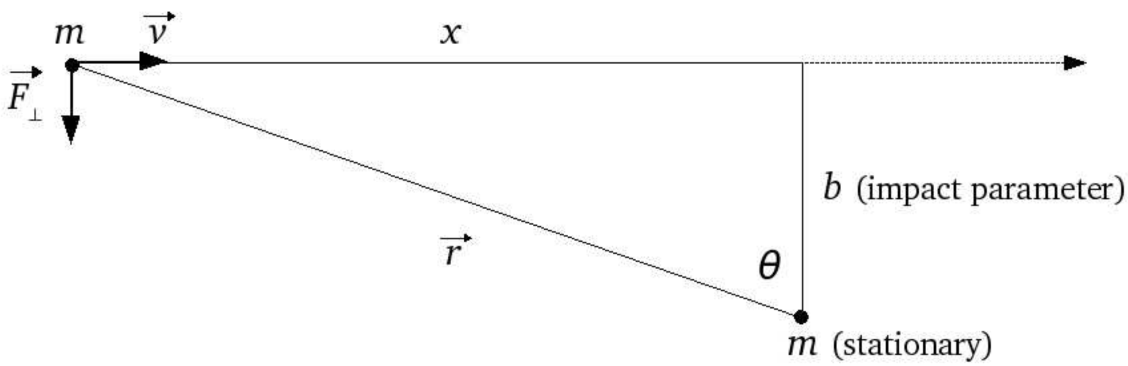
Any classification scheme has to contain an attic – a class into which objects that conform to no particular pattern can be placed. Since the time of Hubble, nonconformist galaxies have been dumped into the irregular class (denoted Irr). A minority of Irr galaxies are spiral or elliptical galaxies that have been violently distorted by a recent encounter with a neighbor. However, the majority of Irr galaxies are simply low-luminosity gas-rich systems. These galaxies are designated Sm or Im.

Galaxies as Collisionless Systems

The mean-field approximation is an effective tool for studying the dynamics of many-body systems when the collisions are rare (*i.e.*, when the collisional time-scales are long compared to the dynamical time of the system studied). When that is the case, the system is said to be collisionless, and the *collisionless* Boltzmann equation can be used. We have already seen the Boltzmann equation in the context of non-equilibrium reactions, where the RHS of the equation represented the non-equilibrium term.

Let us first estimate the collisional relaxation rates for a general self-gravitating N-body system. Then we will particularize the solution to the case of a typical galaxy, and see if a mean field approximation is indeed warranted.

Collisional relaxation time in a general self-gravitating N-body system.
Consider a self-gravitating system, like a galaxy, of identical particles (stars). Consider a two-particle encounter within the framework of the impulse approximation.



From the figure above

$$
\begin{aligned}
F_\perp &= \frac{Gm^2\cos\theta}{x^2+b^2} = \frac{Gm^2 b}{(x^2+b^2)^{3/2}} = \frac{Gm^2}{b^2\left[1+\left(\frac{x}{b}\right)^2\right]^{3/2}} \\
F_\perp &= m\dot{v}_\perp = \frac{Gm^2}{b^2\left[1+\left(\frac{vt}{b}\right)^2\right]^{3/2}}. \qquad (500)
\end{aligned}
$$

Therefore, the change imparted to $v_\perp$ from one collision is (after making a substitution $s \equiv vt/b$):

$$
\delta v_\perp \simeq \frac{Gm}{b^2}\int_{-\infty}^{\infty}\frac{dt}{\left[1+\left(\frac{vt}{b}\right)^2\right]^{3/2}} = \frac{Gm}{bv}\int_{-\infty}^{\infty}\frac{ds}{(1+s^2)^{3/2}} = \frac{2Gm}{bv}. \qquad (501)
$$

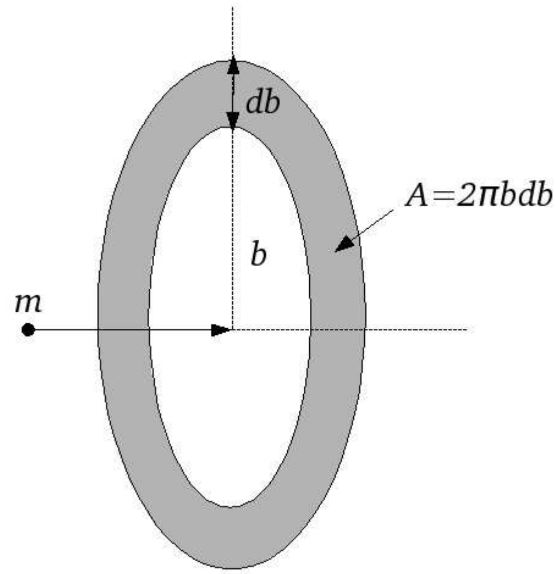
Note that (conceptually):

$$
\delta v_\perp \sim \frac{Gm}{b^2}\frac{2b}{v} \sim \text{(impulsive force)}\times\text{(duration of interaction)}. \qquad (502)
$$

The time it takes a particle to cross the whole system is the "crossing time" τ_{cr}, so $\tau_{cr} \simeq 2R/v$, with R denoting the characteristic size (radius) of the system. The number of collisions this particle encounters in one crossing is, in the range $(b, b+db)$:

$$\delta n_c \sim \frac{\#\text{ of particles}}{\text{cross-sectional area}}\, 2\pi b db \sim \frac{N}{\pi R^2} 2\pi b db \sim 2N\frac{bdb}{R^2}. \tag{503}$$

Therefore, the mean-square change in velocity as the particle "random-walks" through the system



(due to collisions) is

$$\langle \delta v_\perp^2 \rangle \simeq (\delta v_\perp)^2\, \delta n_c \simeq \left(\frac{2Gm}{bv}\right)^2 2N\frac{bdb}{R^2} \simeq 8N\left(\frac{Gm}{Rv}\right)^2 \frac{db}{b}. \tag{504}$$

To get the total change, integrate over all impact parameters:

$$\Delta v_\perp^2 \simeq 8N\left(\frac{Gm}{Rv}\right)^2 \int_{b_{\min}}^{R} \frac{db}{b} \simeq 8N\left(\frac{Gm}{Rv}\right)^2 \ln\left(\frac{R}{b_{\min}}\right). \tag{505}$$

This is the total effect of individual collisions in one crossing time.

From the virial theorem for a self-gravitating system $2\bar{T} = \bar{V}$, where bars denote time-averages, so the typical particle speed is

$$2\left(\frac{1}{2}mv^2\right) \simeq \frac{GNmm}{R} \qquad \Longrightarrow \qquad v^2 \simeq \frac{GNm}{R}. \tag{506}$$

We estimate $b_{\min}$ by presuming the virial theorem also applies, in some average sense, to a close encounter (or, in other words, T is sufficiently larger than V so as to avoid forming a bound binary system):

$$v^2 \simeq \frac{Gm}{b_{\min}} \qquad \Longrightarrow \qquad \frac{R}{b_{\min}} \simeq N \qquad \Longrightarrow \qquad b_{\min} \simeq \frac{R}{N} \tag{507}$$

The number of crossings needed for $\Delta v_\perp^2$ to grow to v^2, at which point the particle has completely forgotten its initial conditions is

$$n_{cr} \equiv \frac{v^2}{\Delta v_\perp^2} \simeq \frac{GNm}{R}\frac{1}{8N}\left(\frac{Rv}{Gm}\right)^2 \frac{1}{\ln\left(\frac{R}{b_{\min}}\right)} = \frac{1}{8}\frac{Rv^2}{Gm}\frac{1}{\ln N} = \frac{1}{8}\frac{Rv^2}{Gm}\frac{1}{\ln N} = \frac{1}{8}\frac{N}{\ln N} \simeq \frac{0.1N}{\ln N}, \tag{508}$$

and the corresponding relaxation time is

$$\tau_R = n_{cr}\tau_{cr} \simeq \frac{0.1N}{\ln N}\tau_{cr} \gg \tau_{\rm cr}. \tag{509}$$

Let us now estimate the crossing time for the self-gravitating system τ_{cr}. Consider a particle freely-falling along a diameter of a uniform-density sphere:

$$\begin{aligned}
\ddot{r} &= -\frac{GM(r)}{r^2} = -\frac{G\left(\frac{4\pi}{3}r^3\rho\right)}{r^2} = -\left(\frac{4\pi G}{3}\rho\right)r \\
\ddot{r} &+ \left(\frac{4\pi G}{3}\rho\right)r = 0 \qquad \Longrightarrow \qquad \ddot{r} + \omega^2 r = 0 \\
\omega^2 &= \frac{4\pi}{3}G\rho = \left(\frac{2\pi}{2\tau_{cr}}\right)^2 \qquad \Longrightarrow \qquad \tau_{cr} = \sqrt{\frac{3\pi}{4G\rho}} \\
\Longrightarrow \qquad \tau_{cr} &\simeq \frac{1}{\sqrt{G\rho}}
\end{aligned} \tag{510}$$

Therefore, estimated collisional relaxation time for a typical self-gravitating N-body system is

$$\tau_R \simeq \frac{0.1N}{\ln N}\frac{1}{\sqrt{G\rho}}. \tag{511}$$

Collisional relaxation time for a typical elliptical galaxy.
A typical elliptical galaxy contains about 10^{12} stars of typical mass of $M_\odot$, and has a radius of about $R \approx 100$ kpc, so

$$\begin{aligned}
N &\simeq 10^{12}, \\
R &\simeq 100 \text{ kpc} \simeq 10^5(3.26) \text{ light} - \text{years} \simeq 10^5\ (3.26)\ \left(3\times10^8 \text{ ms}^{-1}\right)\left(\pi\times10^7 \text{ s}\right) \\
&\simeq 3\times10^{21} \text{ m} \\
m &\simeq M_\odot \simeq 2\times10^{30} \text{ kg}, \\
\Longrightarrow \qquad \tau_R &\simeq \frac{0.1}{\ln\left(10^{12}\right)}\left(\frac{10^{12}\left(3\times10^{21}\right)^3}{\left(6.7\times10^{-11}\right)\left(2\times10^{30}\right)}\right)^{1/2} \simeq 5\times10^{25} \text{ s} \simeq \frac{5\times10^{25} \text{ s}}{3\times10^7} \text{ years} \\
\Longrightarrow \qquad \tau_R &\simeq 10^{18} \text{ years} \sim 10^8 t_{\rm Hubble}.
\end{aligned} \tag{512}$$

The relaxation time due to collisions is orders of magnitude longer than the age of the Universe, which means that galaxies are well-approximated by collisionless, mean-field approximation and the collisionless Boltzmann equation.