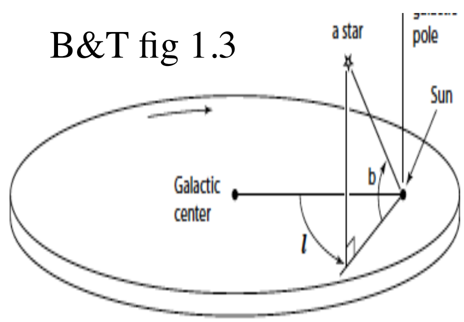
Galactic Rotation S&G Sec 2.3

- The majority of the motions of the stars in the MW is rotational
- Prime way of measuring mass of spiral galaxies
- map out the distribution of galactic gas
- strong evidence for dark matter

Galactic Rotation

- See SG 2.3, BM ch 3 B&T ch 2.6,2.7 and ch 6
- Coordinate system: define velocity vector by π,θ,z
  - π radial velocity wrt galactic center
  - θ motion tangential to GC with positive values in direct of galactic rotation
  - z motion perpendicular to the plane, positive values toward North galactic pole
- origin is the galactic center (center or mass/rotation)
- Local standard of rest (BM pg 536)
- velocity of a test particle moving in the plane of the MW on a closed orbit that passes thru the present position of the sun

B&T fig 1.3



If the galaxy is axisymmetric and in steady state then each pt in the plane has a velocity corresponding to a circular velocity around center of mass of MW

$(\pi,\theta,z)_{LSR}=(0,\theta_0,0)$ with $\theta_0 = \theta_0(R)$

Local Standard of Rest

The Sun (and most stars) are on slightly perturbed orbits that resemble rosettes making it difficult to measure relative motions of stars around the Sun.

Establish a reference frame that is a perfect circular orbit about the Galactic Center.

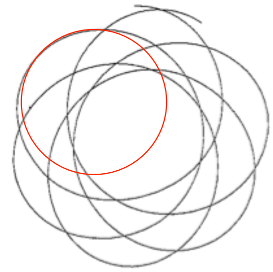
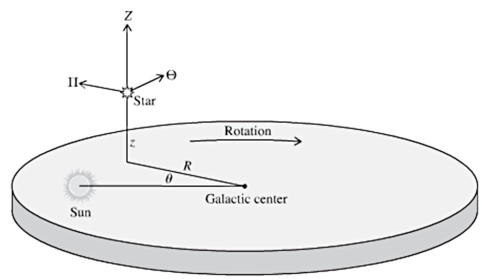


Figure 3-1. A typical orbit in a spherical potential forms a rosette.

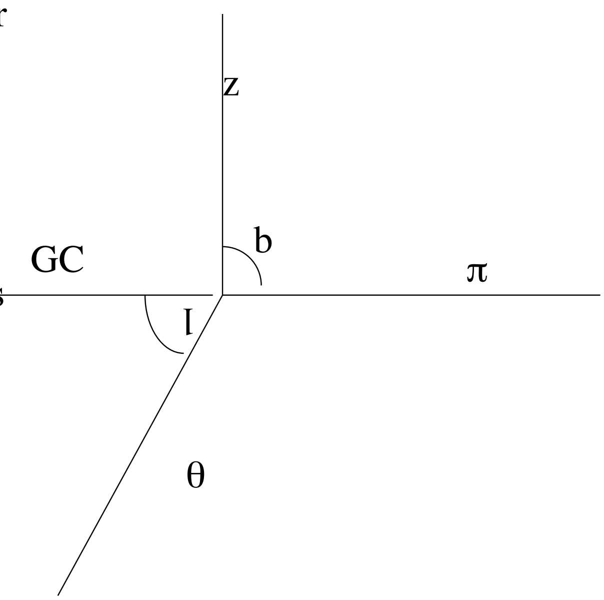
Local Standard of Rest - reference frame for measuring velocities in the Galaxy.
Position of the Sun if its motion were completely governed by circular motion around the Galaxy.

Use cylindrical coordinates for the Galactic plane to define the Sun's motion w.r.t the Local Standard of Rest



Coordinate Systems

The stellar velocity vectors are
z:velocity component perpendicular to plane
θ: motion tangential to GC with positive velocity in the direction of rotation
π: radial velocity wrt to GC
With respect to galactic coordinates
+π= (l=180,b=0)
+θ= (l=30,b=0)
+z= (b=90)



Description of Galactic Rotation (S&G 2.3)

- For circular motion: relative angles and velocities observing a distant point
- T is the tangent point

$V_r = R_0 \sin l (V/R - V_0/R_0)$

Because V/R drops with R (rotation curve is ~flat); for value 0<l<90 or 270<l<360 reaches a maximum at T

So the process is to find V_r^{max} for each l

and deduce $V(R) = V_r + R_0 \sin l$

For $R > R_0$: rotation curve from HI or CO is degenerate ; use masers, young stars with known distances

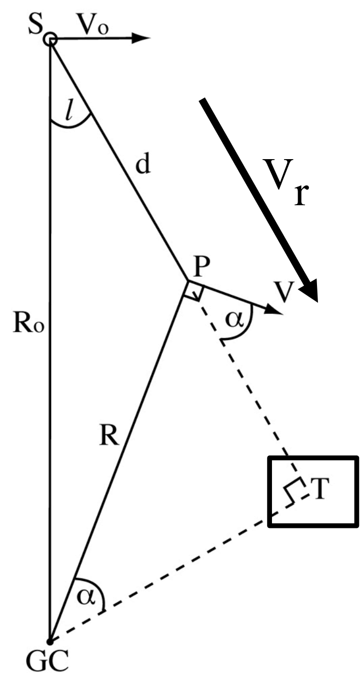


Fig 2.19 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Galactic Rotation

- Then using a bit of trig

$R(\cos \alpha) = R_0 \sin(l)$

$R(\sin \alpha) = R_0 \cos(l) - d$

so

$V_{observered,radial} = (\omega - \omega_0) R_0 \sin(l)$

$V_{observered,tang} = (\omega - \omega_0) R_0 \cos(l) - \omega d$

then following the text expand $(\omega - \omega_0)$ around R_0 and using the fact that most of the velocities are local e.g. $R - R_0$ is small and d is smaller than R or R_0 **(not TRUE for HI)** and some more trig

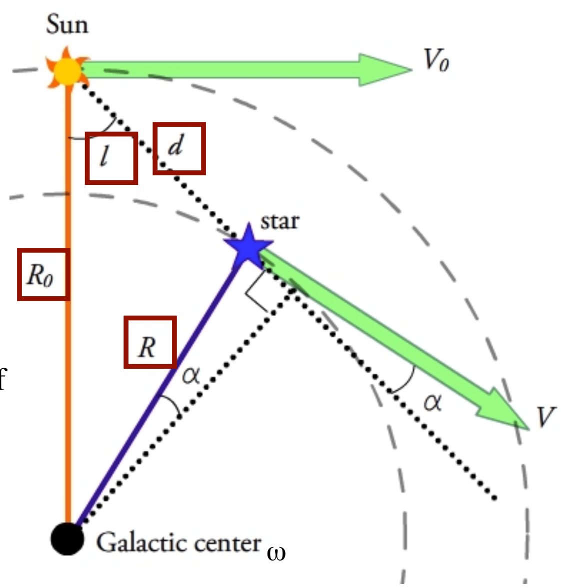
get

$V_{observered,radial} = Ad\sin(2l)$; $V_{obs,tang} = Ad\cos(2l) + Bd$

Where

$A = -1/2\, R_0 d\omega/dr$ at R_0

$B = -1/2\, [(R_0 d\omega/dr) - \omega]$



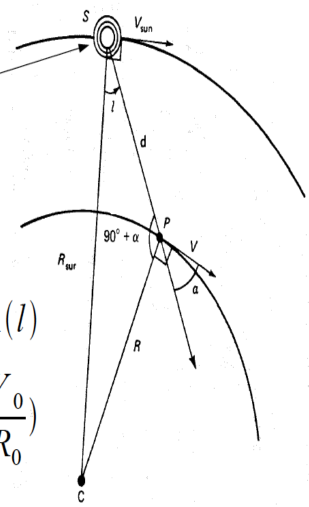
Galactic Rotation Curve- sec 2.3.1 S+G

Assume gas/star has a perfectly circular orbit

At a radius R_0 orbit with velocity V_0; another star/ parcel of gas at radius R has a orbital speed V(R)

since the angular speed V/R drops with radius V(R) is positive for nearby objects with galactic longitude l <l<90 etc etc (pg 91 bottom)

- Galactic Rotation Curve
  - At R_{sun} the lsr has a velocity of V_0
  - A star at P has an apparent velocity of



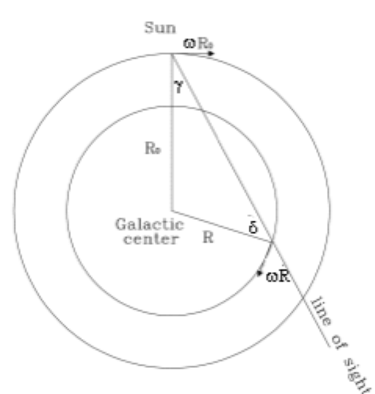
1) $V_r = V\cos(\alpha) - V_0 \sin(l)$

2) $V_r = R_0 \sin(l)\left(\frac{V}{R} - \frac{V_0}{R_0}\right)$

- Convert to angular velocity ω
- $V_{observered,radial} = \omega R(\cos\alpha) - \omega_0 R_0 \sin(l)$
- $V_{observered,tang} = \omega R(\sin\alpha) - \omega_0 R_0 \cos(l)$

In terms of Angular Velocity

- model Galactic motion as circular motion with monotonically decreasing angular rate with distance from center.
- Simplest physics: if the mass of the Galaxy is all at center angular velocity ω at R is $\omega = M^{1/2}G^{1/2}R^{-3/2}$
- If looking through the Galaxy at an angle l from the center, velocity at radius R projected along the line of site minus the velocity of the sun projected on the same line is
- $V = \omega R \sin\delta - \omega_o R_o \sin l$ [l is galactic longitude (in figure this is angle γ)]
- ω = angular velocity at distance R
  ω_o = angular velocity at a distance R_o
  R_o = distance to the Galactic center
  l = Galactic longitude
- Using trigonometric identity $\sin d = R_o \sin l / R$
  and substituting into equation (1)
- $V = (\omega - \omega_o) R_o \sin l$

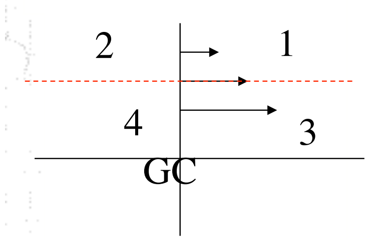


http://www.haystack.mit.edu/edu/undergrad/srt/SRT Projects/rotation.html

Continued

- The tangential velocity v_T= Vsinα-$V_o$cos l
  and Rsinα =$R_o$cos$l$-d
- a little algebra then gives
  V_T= V/R($R_o$cos$l$-d)-$V_o$cos l
- re-writing this in terms of angular velocity
  V_T= (ω-ω_o)$R_o$cos$l$-ωd

- For a reasonable galactic mass distribution we expect that the angular speed ω=V/R is monotonically decreasing at large R (most galaxies have flat rotation curves (const V) at large R) then get a set of radial velocities as a function of where you are in the galaxy
- V_T is positive for 0<l<90 and nearby objects- if R>R_0 it is negative
- For 90<l<180 V_T is always negative
- For 180<l<270 V_T is always positive (S+G sec 2.3.1)



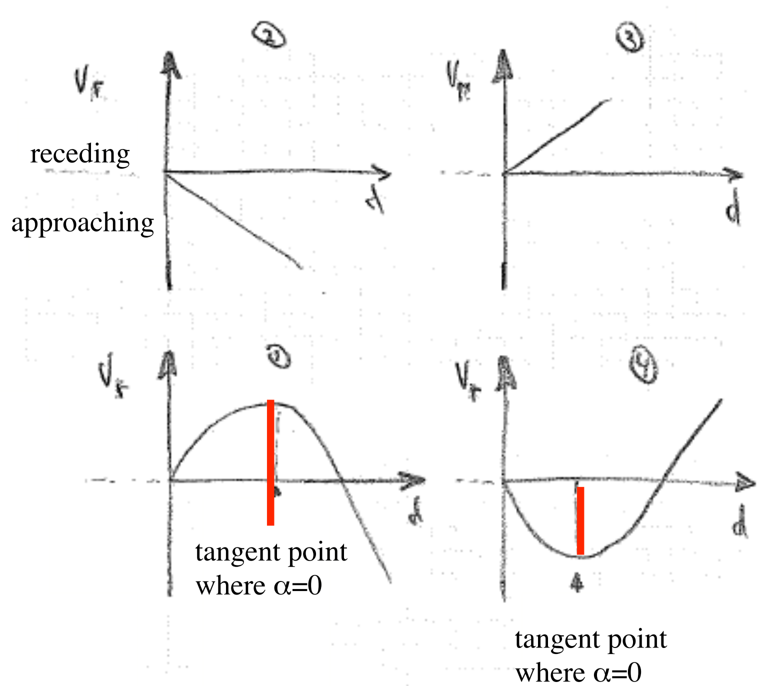
(1) 0<l<90

(2) 90<l<180

(3) 180<l<270

(4) 270<l<360

Veilleux 2010



Oort Constants S&G pg 92-93

Derivation:

- for objects near to sun, use a Taylor series expansion of $\omega-\omega_o$

$\omega-\omega_o = d\omega/dR\ (R-R_o)$

$\omega = V/R$; $d\omega/dR = d/dr(V/R) = (1/R)dV/dr - V/R^2$

then to first order

$V_r = (\omega-\omega_o)R_o \sin l = [dV/dr - V/R](R-R_o)\sin l$; when $d \ll R_o$

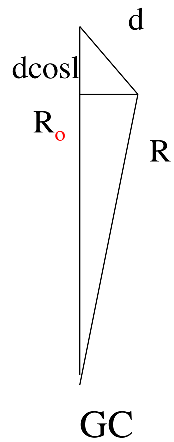
$R-R_o = d\cos l$ which gives

$V_r = (V_o/R_o - dV/dr)d \sin l \cos l$

using trig identity $\sin l \cos l = 1/2 \sin 2l$

one gets the Oort forumla

$V_r = A d \sin 2l$ where



$$A = \frac{1}{2}\left[\frac{V_o}{R_o} - \left(\frac{dV}{dR}\right)_{R_o}\right]$$

One can do the same sort of thing for V_T

Oort Constants (MBW pg 439)

- For nearby objects ($d \ll R$) then (l is the galactic longitude)
  - $V(R) \sim R_0 \sin l\ (d(V/R)/dr)(R-R_0)$
  $\sim d\sin(2l)[-R/2(d(V/R)/dr) \sim d\mathbf{A}\sin(2l)$

- **A is one of 'Oorts constants'**
- The other Oort constant B(pg 93 S+G) is related to the tangential velocity of a object near the sun $V_t = d[A\cos(2l) + B]$
- So, stars at the same distance r will show a systematic pattern in the magnitude of their radial velocities across the sky with Galactic longitude.
- **A is the Oort constant describing the shearing motion and B describes the rotation of the Galaxy**

$$A = \frac{1}{2}\left[\frac{V_o}{R_o} - \left(\frac{dV}{dR}\right)_{R_o}\right]$$

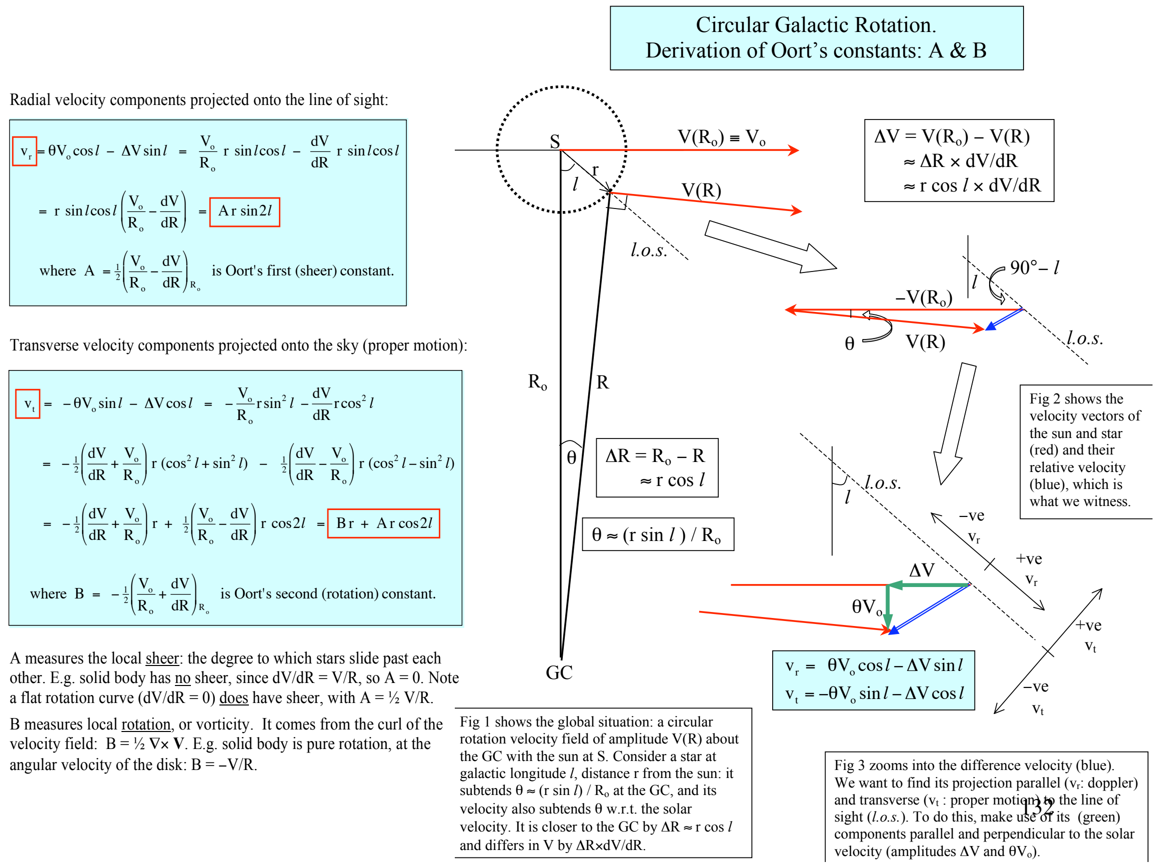
$$B = -\frac{1}{2}\left[\frac{V_o}{R_o} + \left(\frac{dV}{dR}\right)_{R_o}\right]$$

$$A + B = -\left(\frac{dV}{dR}\right)_{R_o} \quad ; \quad A - B = \frac{V_o}{R_o}$$

$A = -1/2[R d\omega/dr]$

<u>Useful since if know A get kinematic estimate of d</u>

Radial velocity $v_r \sim 2AR_0(1-\sin l)$ only valid near $l \sim 90$ measure $AR_0 \sim 115 km/_s$



Oort 'B'

- B measures 'vorticity' B=-(ω=-1/2[Rdω/dr])=-1/2[(V/R)+(dV/dR)] angular momentum gradient

  ω=A-B=V/R; angular speed of Local standard of rest (sun's motion)

  Oort constants are local description of differential rotation

  Values

  A=14.8 km/s/kpc

  B=-12.4 km/s/kpc

  Velocity of sun $V_0=R_0(A-B)$

  I will not cover epicycles (stars not on perfect circular orbits) now (maybe next lecture): : see sec pg 133ff in S&G

Onto Next Part ...

- Chapter 3 of S&G 'the orbits of stars'
- Detailed dynamics of the Milky Way