Astronomy 452 Astrophysics II: Galaxies

Homework II (Due Wednesday, 3/7/2007, in class)

(i) What are the spectral types for stars whose continuum spectra peak at the following wavelength: (a) 50 nm; (b) 300 nm; (c) 600 nm; (d) 900 nm; (e) 1.2 μm; (f) 1.5 μm

(ii) The nearby supergiant star, Betelgeuse has an effective temperature of $T_{\rm eff} \sim 3500$k and an angular diameter of $0.045''$, and a distance $r \sim 140$pc.
(a) What is its physical radius?
(b) What is its luminosity?
(c) What is its expected color?

(iii) Your body temperature is about 310 K. Suppose it can be approximated as a blackbody.
(a) What is the typical wavelength of the radiation from your body.
(b) Is this radiation visible, infrared, X-ray, or radio?
(3) Suppose the surface area of a human body is about $2{\rm m}^2$. How much energy does a human body radiate per second (in Watts)?

(iv) The luminosity of the sun is 3.9×10^{26}W, and the angular diameter of the sun is $\theta = 32$ arc minutes.
(a) What is its physical radius?
(b) What is its effective temperature (assuming the sun is a blackbody)?
(c) What kind of color does it have and what kind of star is it according to color classification?

(v) (a) For a main sequence star of luminosity L, how many kilograms of hydrogen is being converted into helium per second?
(b) Use the formula you derive to estimate the mass of hydrogen atoms that are converted into helium in the interior of the sun ($L_\odot = 3.9 \times 10^{26}$W).
Hint: the mass of an hydrogen atom is $1m_p$ (m_p is the mass of a proton), and the mass of a helium atom is about $3.97m_p$. You need 4 hydrogen nuclei to form one helium nucleus.

(vi) (a) For a star with mass of $1{\rm M}_\odot$, how much energy does it radiate during the main-sequence phase? How much energy does it radiate in the red-giant phase?
(b) Make the same calculation for a mass of $9{\rm M}_\odot$.

(vii) The Salpeter initial mass function (IMF) has the form $\phi(m) = Am^{-1.35}$.
(a)If only stars with masses between $0.1\,\mathrm{M}_\odot$ and $100\,\mathrm{M}_\odot$ are formed, what is the value of A? Suppose we have a galaxy which contain a total of $10^{10}\mathrm{M}_\odot$ of newly-formed stars. What is the total number of these stars? How many of them have initial masses that are between $1\,\mathrm{M}_\odot$ and $2\,\mathrm{M}_\odot$?
(b) What is the total mass in stars with masses between $1\,\mathrm{M}_\odot$ and $2\,\mathrm{M}_\odot$? Make the same estmimates for stars with masses between $10\,\mathrm{M}_\odot$ and $11\,\mathrm{M}_\odot$.
(c) Based on these results, do you think the stellar population in the galaxy dominated by low-mass stars or by high-mass stars?

(viii) We know that the main-sequence age of a star of $1\,\mathrm{M}_\odot$ is about 10^{10} years.
(a) What is the main-sequence age of a star of $10\,\mathrm{M}_\odot$?
(b) Suppose a galaxy of mass $10^{10}\,\mathrm{M}_\odot$ of a coeval stellar population of age 10^7 years. How many stars are still in the main sequence (assuming Salpeter IMF) in the galaxy? How many main-sequence stars with masses between $1\,\mathrm{M}_\odot$ and $2\,\mathrm{M}_\odot$ are there in the galaxy?
(c) The same as above, but for a coeval population of age 10^{10} years.
(d) Discuss the galaxy becomes redder as it ages.

(ix) For gas at a given temperature T, the velocity distribution of atoms in the gas is Maxwellian,

$$P(v)\mathrm{d}v = \frac{1}{\pi^{1/2}b}\exp\left(-\frac{v^2}{b^2}\right)\,\mathrm{d}v\,, \qquad b^2 = 2kT/m\,,$$

where k is Boltzmann's constant, and m is the mass of an atom in the gas. Note that $P(v)\mathrm{d}v$ is proportional to the number of atoms with velocities in the range v to $v + \mathrm{dv}$. Now suppose we are observing emission from the gas that is generated by a transition with an intrinsic (rest-frame) frequency ν_{12}. Find the expression for the line profile to be observed from the gas, i.e. obtain an expression for $\phi(\nu)$ so that $\phi(\nu)\mathrm{d}\nu$ is proportional to the number of photons to be observed with frequencies in the range ν to $\nu + \mathrm{d}\nu$. Observations show that the interstellar medium of the Milky Way has two components of neutral hydrogen, one having a temperature of about 10,000 K and the other having a temperture of about 100 K. Sketch and describe the profile of the 21cm line to be observed from the interstellar medium.