1 Star and Galaxy clusters

1.1 Globular cluster properties

Round, smooth distribution of stars (assume spherical)

Population II (old) stars

10^4 - 10^6 stars in each

Ages $\sim 10^{10}$ years (from stellar evolution models and isochrone fitting).

Traditionally measure surface brightness as a function of R i.e. $\mu(R)$, or (more recently) use high resolution HST images to count stars $N(R)$.

We want star mass density $\rho(r)$ as a function of radius:
Use M/L (~ 2 solar units) or star masses M_* to convert $\mu(R)$ or $N(R)$ to surface mass density $\Sigma(R)$.
Assume spherical symmetry $\Sigma(R) \rightarrow \rho(r)$

From plot of $\mu(R)$ it is clear that the cluster is not homogeneous.
Core radius $\mu(R_c) = \frac{1}{2}\mu(0)$, $R_c \sim 1.5$ pc. ρ constant for $r < R_c$
Median radius, typical radius, characteristic radius: contains half the light (2D). $R_h \sim 10$ pc. NOTE: r_h used by theoreticians is half-mass radius from 3D modelling. Be aware of which definition is being used.
Tidal radius: $\mu \rightarrow 0$, the "edge" of the cluster, is at $r_t \sim 50$ pc.

Mass $M \sim 6 \times 10^5 \mathrm{M}_\odot$

Star masses up to 0.8 $\mathrm{M}_\odot$

Core density $\rho_c = \rho(0)$ 8×10^3 $\mathrm{M}_\odot$ pc^{-3}.

One-dimensional central velocity dispersion $\sigma_r \equiv \sqrt{\bar{v_r^2}} \sim 7$ km s^{-1} (ranges from 2 - 15 km s^{-1}).

1.2 Open clusters

$N \sim 10^2$ - 10^3 stars

Age $\lesssim 10^8$ years $\Rightarrow$ either all formed recently or form and disperse continually.

$R_c \sim 1$ pc

$R_h \sim 2$ pc

$r_t \sim 10$ pc, because of stronger gravity in the disk of the Galaxy, and lower cluster mass.

Mass $\sim 250\ \mathrm{M}_\odot$

$M/L \sim 1$ (solar units)

$\rho_c \sim 100\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$ (cf solar neighbourhood $\bar{\rho} = 0.05\ \mathrm{M}_\odot\ \mathrm{pc}^{-3}$).

$\sigma_r = \sqrt{\bar{v_r^2}} \sim 1\ \mathrm{km\ s}^{-1}$ (system assumed approximately isothermal).

1.3 Clusters of galaxies

Large range of N, and wide spread of M, but typically $N \sim 100$ galaxies, and total masses $\sim 10^{15}\ \mathrm{M}_\odot$ (much of the mass is not visible).

$R_c \sim 250$ kpc

$R_h \sim 3$ Mpc

$\sigma_r \sim 800\ \mathrm{km\ s}^{-1}$

Crossing time

$$t_{\mathrm{cross}} \sim R_h/\sigma_r \sim 10^9 \left(\frac{R_h}{1\ \mathrm{Mpc}}\right) \left(\frac{\sigma_r}{10^3\ \mathrm{km\ s}^-1}\right)^{-1}$$

Age $\lesssim 13.7 \times 10^9$ yr (age of the universe) $\Rightarrow$ dynamically young, often still forming, collapsing for the first time.