Lecture 5:

More Newton…

- Newton's Universal Law of Gravity
- Acceleration in circular orbits
- Weak equivalence principle
- Kepler's laws from Newtonian gravity
- The power of Newton's laws

Reference frames and some puzzles…

- Real and fictitious forces

RECAP

- **Newton's 1st law** – ***V*** = *constant* if ***F*** = *0*
- **Newton's 2nd law** – ***F*** = *Ma*
- **Newton's 3rd law** – for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference.
- **Galilean Relativity** – the idea that the laws of nature are the same for a moving observer as for a stationary observer.

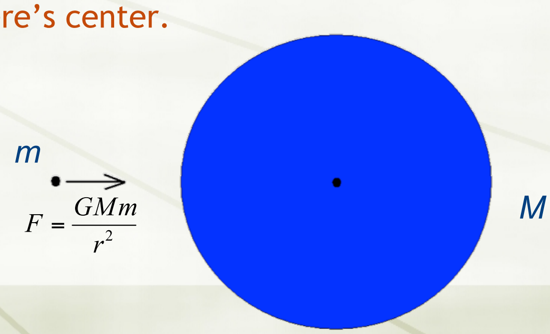
I: NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's law of Gravitation: **A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by**

$$F = \frac{Gm_1m_2}{r^2}$$

- "G" is called the Gravitational constant ($G = 6.67\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$ in mks units)
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings.

- Newton's Law of Gravitation defines the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass M (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass M at the sphere's center.



Inertial and gravitiational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a \qquad m_I\text{=inertial mass}$$

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2} \qquad m_G\text{=gravitational mass}$$

So, acceleration due to gravity is:

$$a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) varies, the rate at which objects fall in a gravitational field will vary…

At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Equivalence of inertial and gravitational mass

- *Experimentally*, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)
- So, m_I/m_G must be the *same* for all bodies
- And we can choose the constant "G" such that $m_I = m_G$, and $a = GM/r^2$
- This is the **weak equivalence principle**

"Weightlessness"

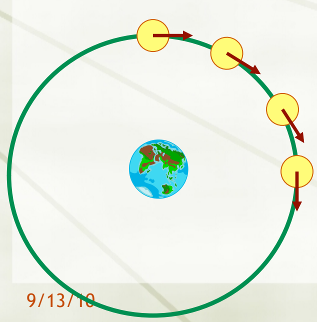
Apollo 10, in orbit (May 18-26, 1969)

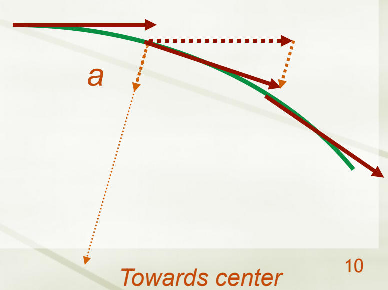
No weight, or free-fall?

- Space Station orbits about 500km above Earth's surface. Radius of Earth is 6300km.
- Newton's inverse square law:
  - Gravitational acceleration at location of space station is 86% of what it is on the Earth's surface!
- So, why do the astronauts feel weightless?
  - The astronauts "fall" toward Earth at the same rate as the space station – another example of the equivalence principle.

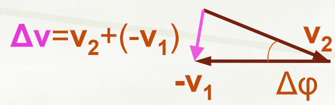
"Falling" in a circular orbit

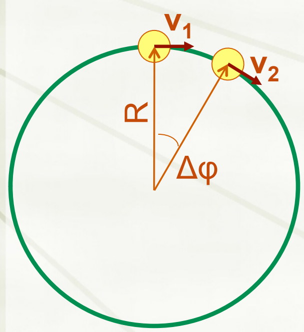
- Newton arrived at the theory of gravity by imagining that the same force causes an apple to fall towards the Earth as the Moon to orbit Earth (continually "falling"), with a decrease in the magnitude of the force with distance
- In what sense is a body in orbit *"falling"?*
- *Direction* of acceleration (= rate of change of vector velocity) is always ***directly towards center of orbit***
- *Acceleration* must be toward center because *gravitational force* is toward center, and **F=ma** is a vector equation





Acceleration in a circular trajectory





For small φ, $\Delta v \cong v\, \Delta\varphi$

For constant speed v, the time it takes to go around is T = perimeter/v = $2\pi R/v$

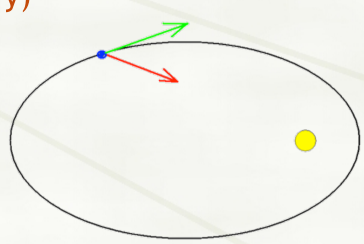
$\Delta\varphi = \Delta t\, 2\pi/T = \Delta t\, v/R$

So $\Delta v/\Delta t = v\, \Delta\varphi/\Delta t = v\, v/R$

$$a = \Delta v/\Delta t = v^2/R$$

II : KEPLER'S LAWS EXPLAINED!

- Kepler's laws of planetary motion
  - Can be derived from Newton's laws
  - Just need to assume that planets are attracted to the Sun by gravity (Newton's breakthrough).
  - Full proof requires calculus (or very involved geometry)



- We're not going to prove this, but…
  - Newton's gravity law ($1/R^2$) is exactly what's needed to make this path be a perfect ellipse – hence Kepler's 1st law.
  - The fact that the force is always towards Sun gives Kepler's 2nd law (equal areas in equal times ⇔ conservation of angular momentum).
  - Newton's 2nd law (F = ma) combined with his gravity law gives Kepler's 3rd law -- the relation between orbit period and semimajor axis

$$P^2 = \frac{4\pi^2}{GM_{sun}} R^3$$

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Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make new predictions about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758
  - Planets have near-elliptical orbits, but they are not exact ellipses due to gravity of *other* planets
  - Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet, leading to discovery of Neptune in 1846
- Newton's laws can be applied to stars in galaxies, galaxies in clusters, etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the typical orbital period and cube of the orbital distance.
- As Newton's physics came to be widely known, there was a huge cultural impact. With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc. Key to shift in thought known as the Enlightenment. The universe is a giant machine! (?)

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III : INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

- Newton's laws were clearly powerful. But they also led to some puzzles, particularly relating to **reference frames**.
- We have already come across idea of **frames of reference that move with constant velocity**. In such frames, Newton's laws (esp. N1) hold. These are called **inertial frames of reference**.
- Suppose you are in an accelerating car looking at a freely moving object (i.e., one with no forces acting on it). You will see its velocity changing because you are accelerating! *In accelerating frames of reference, N1 doesn't hold* – this is a **non-inertial frame of reference**.

Real and fictitious forces

- In non-inertial frames you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces point opposite to the direction of acceleration
- Fictitious forces are always proportional to the inertial mass of the body.