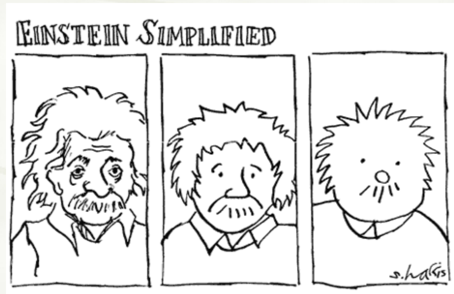
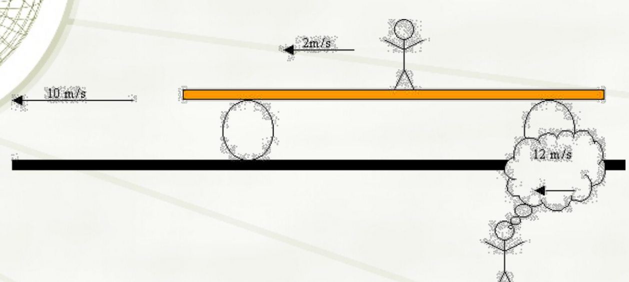
Lecture 8 : Special Theory of Relativity

- The speed of light problem
- Einstein's postulates
- Time dilation



I: THE SPEED OF LIGHT PROBLEM



- Recap
  - "Relativity" tells us how to relate measurements in different frames of reference
  - Galilean relativity
    - Simple velocity addition law : $v_{total}=v_{run}+v_{train}$

Electromagnetic waves

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations).

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / c + (1/c) \partial \mathbf{E} / \partial t$$

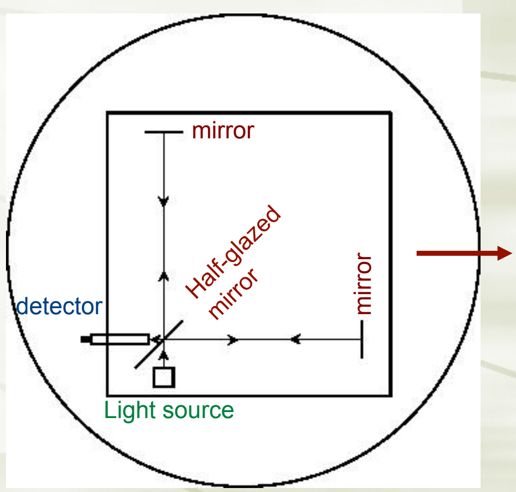
Equations are just written out for fun… you do not need to know them!

- Maxwell's equations:
  - Predict "waves" of electromagnetic energy – quickly realized that these were light waves!
  - The speed of light "c" appears as a fundamental constant in the equations.
  - c=299,792,458 km/s
- **BUT, what frame of reference is this measured relative to???**

- Luminiferous Aether (19th century)
  - Hypothetical substance that fills space - provides a “medium” through which light can travel.
  - Was presumed that “c” should be measured with respect to the rest frame of the Ether.
  - Albert Michelson & Edward Morley attempted to measure motion of Earth through ether…

Michelson-Morley Experiment

- Light leaves source, and is partly reflected 45°/partly transmitted at half-glazed mirror
- Light returning from both paths is collected at detector
- Path length of light along either “arm” of apparatus is the same
- If one arm is along Earth’s motion through ether, and the other arm is perpendicular to motion through ether, then light travel time was expected to be shorter for perpendicular arm



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Michelson-Morley results

- Travel time difference would be measured using interference fringes of light from two paths
- Apparatus could be rotated to make sure no effects from set-up
- Repeated at different times of year, when Earth's motion differs; Earth's speed around the Sun is ~30 km/s
- Experiment performed in 1887
- Results
  - M-M showed that speed of light was same in any direction to within 5 km/s
  - Modern versions of the experiment show constancy to better than 1 micron/s
- So, what's going on??

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Attempts to deal with M-M results

- Maybe the ether "sticks" to the Earth?
  - Gets "dragged" as Earth spins and orbits Sun…
  - Possibility at the time, but no-longer viable.
- Maybe the ether squeezes the arms of the M-M experiment and distorts the result? "Fitzgerald contraction" (1889)?
  - A contraction (in the direction parallel to motion through ether) would change the light travel time to compensate for the difference expected due to different speed of light

$$L = L_0\sqrt{1 - V^2/c^2}$$

- Major mystery ("crisis") in 19th century physics – two highly successful theories seemed incompatible!
  - Mechanics – Galilean Relativity and Newton's laws
  - Electromagnetism – Maxwell's equations

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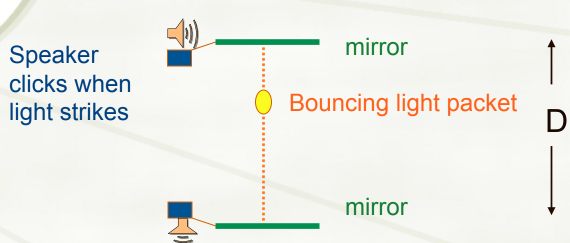
II : Einstein's Postulates of Special Relativity

- Albert Einstein
  - Didn't like idea of Aether
  - Threw away the idea of Galilean Relativity
  - Came up with the two "Postulates of Relativity"

- **Postulate 1** – The laws of nature are the same in all inertial frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all inertial frames of reference.

III: TIME DILATION

- Imagine building a clock using mirrors and a light beam.



- One "tick" of the clock is the time it takes for light to travel from one mirror to the other mirror.

$$\Delta T = \frac{D}{c}$$

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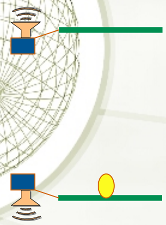
Moving clock



- Now suppose we put the same "clock" on a spaceship that is cruising (at constant velocity, V) past us.
- How long will it take the clock to "tick" when we observe it in the moving spacecraft? Use Einstein's postulates…
- Total distance travelled by light beam is $\Delta s = c \times \Delta t$
- Therefore time $\Delta t = \Delta s/c$
- By Pythagorean theorem, $\Delta s = c\Delta t = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V\Delta t)^2 + D^2}$
- Can solve to obtain $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- Clock appears to run more slowly!!

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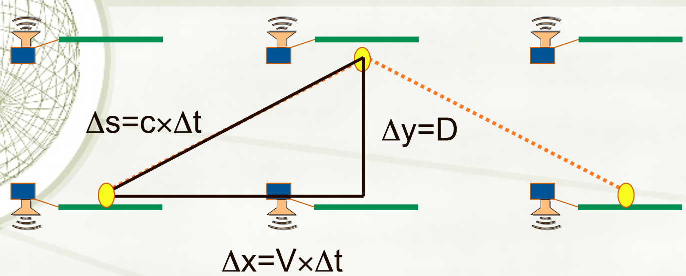
Moving clock



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Now change the point of view…

- For ground-based observer, clock on spaceship takes longer to "tick" than it would if it were on the ground
- But, suppose there's an astronaut in the spacecraft
  - the inside of the spacecraft is also an inertial frame of reference – Einstein's postulates apply…
  - So, the astronaut will measure a "tick" that lasts

$$\Delta T = \frac{D}{c}$$

  - This is just the same time as the "ground" observers measured for the clock their own rest frame
- So, different observers see the clock going at different speeds!

So time is not absolute!!

It depends on your point of view…

Time dilation

- This effect called **Time Dilation**.
- Clock always ticks most rapidly when measured by observer in its own rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

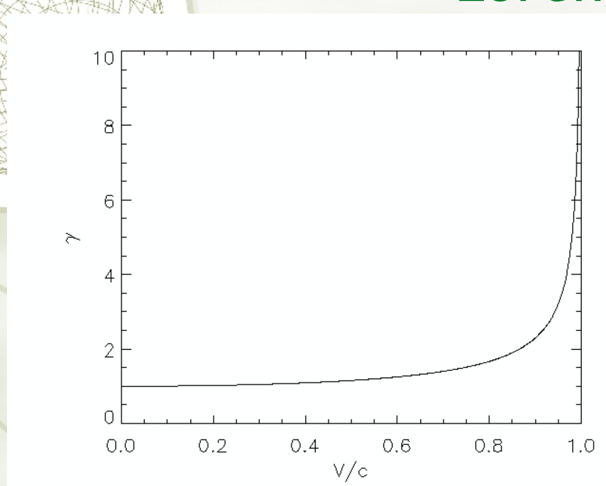
- This slowing factor is called the **Lorentz factor, γ**

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Clocks and time

- Does this “time dilation” effect come about because we used a funny clock?
- No, any device that measures time would give the same effect!
- The time interval of an event as measured in its own rest frame is called the ***proper time***
- Note that if the astronaut observed the same “light clock” (or any clock) that was at rest on Earth, it would appear to run slow by the same factor γ , because the dilation factor depends on *relative speed*
- This is called the ***principle of reciprocity***

Lorentz factor



A 1% effect at v = 0.14 *c*, or about 42,000,000 m/s

Lorentz factor goes to infinity when V→c!

But it is very close to 1 for V/c small

Why don't we ordinarily notice time dilation?

Some examples of speeds in m/s

- 0.0055m/s world record speed of the fastest snail in the Congham,UK
- 0.080 m/s the top speed of a sloth (= 8.0 cm/s)
- 1 m/s a typical human walking speed
- 28 m/s a car travelling at 60 miles per hour (mi/h or mph) or 100 kilometres per hour (km/h); also the speed a cheetah can maintain
- 341 m/s the current land speed record, which was was set by ThrustSSC in 1997.
- 343 m/s the approximate speed of sound under standard conditions, which varies according to air temperature
- 464 m/s Earth's rotation at the equator.
- 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- 1000 m/s the speed of a typical rifle bullet
- 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- 299,792,458 m/s the speed of light (about 300,000 km/s)

Examples of time dilation

- *[We work through some of the examples on the white board during the class]*
- The Muon Experiment
  - Muons are created in upper atmosphere from cosmic ray hits
  - Typical muon travel speeds are $0.99995\times c$, giving $\gamma=100$
  - Half-life of muons in their own rest frame (measured in lab) is $t_h = 2$ microseconds $=0.000002$s
  - Traveling at $0.99995\times c$ for $t_h=0.000002$s, the muons would go only 600 m
  - But traveling for $\gamma\times t_h = 0.0002$s, the muons can go 60 km
  - *They easily reach the Earth's surface, and are detected!*
  - *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with* $\gamma\times t_h$