# Lecture 15 : Cosmological Models I

- Standard cosmological models - 3 cases
- Hubble time and other terminology
- The Friedmann equation
- The Critical Density and Ω




*Reading for lectures 15 &16: Chapter 11*



# 0 : Recap

- Use framework of GR to examine Universe as a whole… need to make some simplifying assumptions (Cosmological Principles)
  - Assume **homogeneous** and **isotropic**
  - Observations show this is true on largest scales
- Cosmological Principles imply that there are only three possible geometries for space:
  - **Spherical** ("closed" universe : finite volume)
  - **Flat** ("open" : infinite volume, no curvature)
  - **Hyperbolic** ("open" : infinite volume)

## (Recap continued)

- Expansion of the Universe described by the Scale Factor (R)
  - New way of thinking about Hubble's law… galaxies are "at rest" but the space itself is expanding (i.e. the scale factor is increasing)
- Exactly how the scale factor changes with time is determined by putting the spatial geometries into Einstein's equation
  - Einstein found it was impossible to make the Universe "stand still"… led him to include the Cosmological Constant
  - When Hubble found Universe to be expanding, Einstein considered this to be a huge mistake

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## II : Standard cosmological models

- Let's return to question of how scale factor changes over time
  - Equations of GR relates geometry to dynamics
  - That means curvature must relate to evolution
  - It turns out that there are three possibilities…

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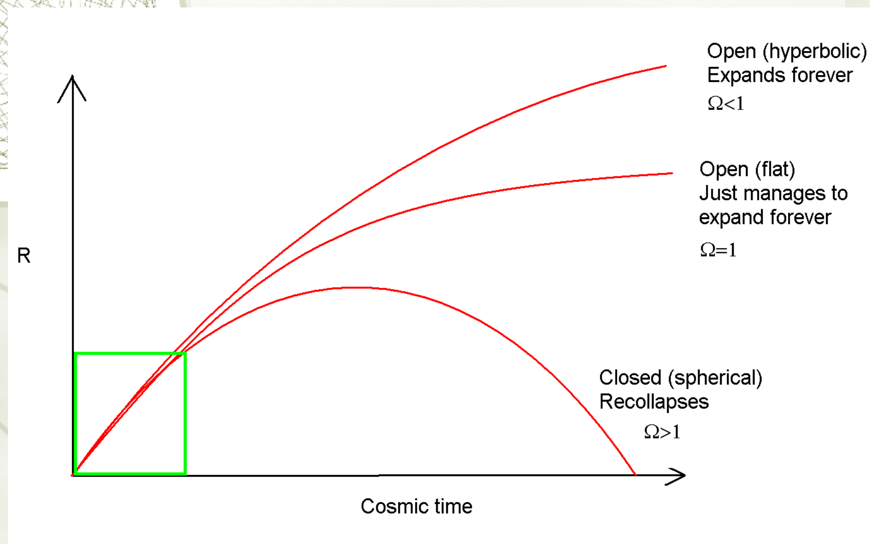





## *Important features of standard models…*

- All models begin with R→0 at a finite time in the past
  - This time is known as the **BIG BANG**
  - Space and time come into existence at this moment… there is no time before the big bang!
  - The big bang happens everywhere in space… not at a point!

- There is a connection between the geometry and the dynamics
  - Closed solutions for universe expand to maximum size then re-collapse
  - Open solutions for universe expand forever
  - Flat solution for universe expands forever (but only just barely… almost grinds to a halt).



## III : The Friedman equation (or "let's get a bit technical!)

- When we go through the GR stuff, we get the **Friedmann Equation…** this is what determines the dynamics of the Universe

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- Here, "k" is the curvature constant…
  - k=+1 for spherical case
  - k=0 for flat case
  - k=-1 for hyperbolic case

- Divide this equation by $R^2$ and we get…

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- Let's examine this equation
  - $H^2$ must be positive (square of a real number), so the RHS of this eq. must also be positive
  - Then, we **must** have negative k if $\rho=0$ (i.e., k=-1)
  - So, empty universes are open and expand forever
  - Flat and spherical Universes can only occur in presence of matter.

- Now, suppose the Universe is flat (k=0)
  - Friedmann equation then gives

$$H^2 = \frac{8\pi G}{3}\rho$$

  - So, this case occurs if the density is exactly equal to **the critical density…**

$$\rho = \rho_{crit} = \frac{3H^2}{8\pi G}$$

# *Value of critical density*

- For present best-observed value of the Hubble constant, $H_0$=72 km/s/Mpc
  critical density, $\rho_{critical}=3H_0^2/(8\pi G)$ , is equal to
  $\rho_{critical}=10^{-26}$ kg/m$^3$ ; i.e. 6 H atoms/m$^3$
- Compare to:
  - $\rho_{water}$ = 1000 kg/m$^3$
  - $\rho_{air}$=1.25 kg/m$^3$ (at sea level)
  - $\rho_{interstellar\ gas}$ =2 × 10$^{-21}$ kg/m$^3$



- In general case, we can define the **density parameter…**

$$\Omega = \frac{\rho}{\rho_c}$$

- Can now rewrite Friedmann's equation yet again using this… we get

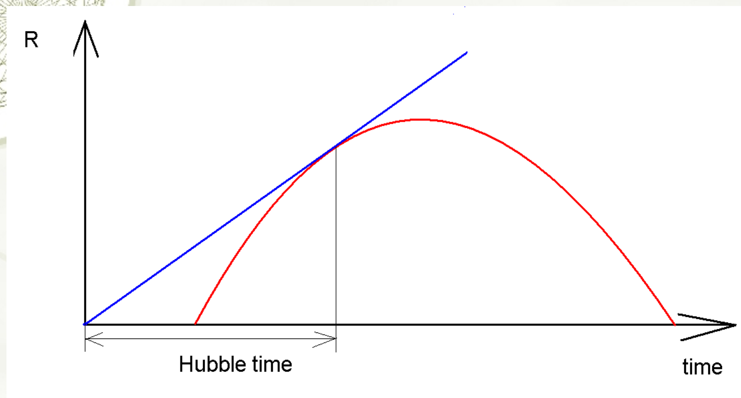
$$\Omega = 1 + \frac{kc^2}{H^2 R^2}$$

*(ok… my brain hurts… what do I need to remember from the past 4 slides???)*

- Important take-home message… within context of the "standard model":
  - $\Omega<1$ means universe is hyperbolic and will expand forever
  - $\Omega=1$ means universe is flat and will (just manage to) expand forever
  - $\Omega>1$ means universe is spherical and will recollapse
- Physical interpretation… if there is more than a certain amount of matter in the universe, the attractive nature of gravity will ensure that the Universe recollapses.

## *III : SOME USEFUL DEFINITIONS*

- Have already come across…
  - **Standard model** (Homogeneous & Isotropic GR-based models, **ignoring Dark Energy**!!)
  - **Critical density** $\rho_c$ (average density needed to just make the Universe flat)
  - **Density parameter** $\Omega=\rho/\rho_c$
- We will also define…
  - **Cosmic time** (time as measured by a clock which is stationary in co-moving coordinates, i.e., stationary with respect to the expanding Universe)




- **Hubble time**, $t_H$=1/H (cosmic time since the big bang if the universe were expanding at a constant rate.)

- **Hubble distance,** D=c$t_H$ (distance that light travels in a Hubble time). This gives an approximate idea of the size of the observable Universe.
- **Age of the Universe**, $t_{age}$ (the amount of cosmic time since the big bang). In standard models, this is always less than the Hubble time.
- **Look-back time**, $t_{lb}$ (amount of cosmic time that passes between the emission of light by a certain galaxy and the observation of that light by us)
- **Particle horizon** (a sphere centered on the Earth with radius c$t_{age}$; i.e., the sphere defined by the distance that light can travel since the big bang). This gives the edge of the actual observable Universe.

## *IV: WHERE HAS RELATIVITY GONE?*

- **Question**: Cosmology is based upon General Relativity, a theory that treats space and time as relative quantities. So, how can we talk about concepts such as…
  - Galaxies being stationary with respect to the expanding Universe? Relativity tells us that there is no such thing as being stationary!
  - The age of the Universe? Surely doesn't time depending upon the observer?
- What do you think??