ASTR 601 - Radiative Processes

Final (10:30am-12:30am Monday, Dec 18th 2006)

1 Specific intensity from a galaxy [5 pts]

Consider an idealize model of a galaxy where stars are distributed uniformly within a cylinder of radius R and height H. The number density of stars is n [in units of stars per volume]. Model each star as a spherical black body of temperature T_* and radius R_*. The number density of stars n is so low that of all possible lines of sight running through the galaxy, the fraction that intersects a star is $\ll 1$. An observer resolves the galaxy in a face-on viewing geometry, but does not resolve the individual stars making up the galaxy.

a) Write down the specific intensity the observer measures. Neglect terms of the order H/d, where d is the distance to the galaxy.

2 Clouds [10 pts]

Idealize the clouds as a uniform, 1-dimensional slab comprising particles that can only scatter light. A uniform flux of photons irradiates the top of the cloud deck. A photon passing through the cloud gets bounced like a pinball from cloud droplet to cloud droplet, preserving its frequency and never getting absorbed by any droplet. A few photons are lucky enough to make it through the cloud, while most get pinballed back out the way they came. We will calculate the fraction that make it through.

Take the cloud to have a droplet density [droplets per cubic volume] n_{cl}, the droplet radius to be R, and the vertical thickness of the cloud to be z_{max}. Measure vertical distance through the cloud by z, where the top of the cloud is located at $z = 0$ and the base of the cloud is located at $z = z_{max}$.

(a) Write down the optical depth of the cloud, τ.

(b) Incident photons from the sun strike the top of the cloud. The photons have a number flux, F_i [number per time per area]. What is the number density of incident photons at the top of the cloud? Call this photon number density n_i. These incident photons have NOT been scattered yet by any droplet.

(c) These photons random walk through the scattering droplets. Random walks are described by the diffusion equation,

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} \qquad (1)$$

where D is the diffusion coefficient and n is the photon number density. Express D in terms of symbols defined above and whatever fundamental constants you deem appropriate. Hint: dimensional analysis may prove useful.

(d) In steady-state, $\partial n/\partial t = 0$ (the number density of photons everywhere in the cloud does not change with time). Write down the solution to the diffusion equation for $n(z)$. Hint: You should have two, as yet unknown, constants of integration.

(e) To solve for the two constants of integration, you need two boundary conditions. The first condition is that $n_t = n(z = z_{max})$. Here n_t is the number density of photons at the base of the cloud. These photons comprise the transmitted flux. The second condition is that the (net, number) flux, F, of photons at $z = 0$ equals the incident flux, F_i (directed down into the cloud) MINUS the outgoing, reflected flux, F_r (directed up, away from the cloud into space). So we have $F(z = 0) = F_i - F_r$. Recall that the (net) flux $F = -D\partial n/\partial z$. Use the above, and the fact that the incident flux, F_i, must equal the reflected flux, F_r, PLUS the transmitted flux, F_t, to calculate $T = F_t/F_i$, the ratio of the transmitted flux to the incident flux, in terms of τ.

(f) If you clouds did absorb light instead of scattering it, what would be the transmitted flux assuming the same optical depth for scattering and absorption ? Compare the transmitted flux for scattering and absorption for $\tau = 1, 2$ and 3.

3 Cooling time of Supernova remnants at high redshift [10 pts]

We will estimate the cooling time of the hot gas produced by the explosion of a Supernova at high redshift. Let's assume that the gas inside the Supernova remnant is pure hydrogen with a number density $n_H = 0.01$ cm^{-3}, is fully ionized and has a temperature $T = 10^6$ K.

(a) Estimate the cooling time of the gas due to free-free emission. Recall that the total emissivity per unit volume due to free-free emission is approximately

$$\epsilon = (1.4 \times 10^{-27} \text{ ergs}^{-1}\text{cm}^{-3})T^{1/2} n_e n_p.$$

Hint: In order to calculate the cooling time you need to compare the emissivity to the thermal energy density of the gas.

(b) Estimate the cooling time of the gas due to inverse Compton scattering with the radiation of the Cosmic Microwave Background that is a black body with temperature $T = 2.73(1 + z)$ K. Hint: Use the formula for the total Compton power (in the non-relativistic limit) and compare it to the thermal energy of the gas. If you do not remember the formula for the total Compton power try to derive it using dimensional analysis recalling that it depends on U_{ph} and the Thompson cross section $\sigma_T = 6.65 \times 10^{-25}$ cm^2.

(c) At which redshifts Compton cooling dominates over free-free cooling ?

4 Absorption and emission coefficients: [5 pts]

(a) Write down the emissivity and the opacity in terms of the Einstein coefficient.

(b) Write down the opacity in terms of the absorption cross section and the oscillator strength. Hint: if you do not remember the formula for the cross section in terms of the oscillator strength you may be able to derive it using the formula in (c) and recalling why the oscillator strength has such a name.

(c) Recall that the quantum mechanical derivation of the bound-bound cross section can be written in terms of the matrix element $\mathbf{X_{12}}$: $\sigma_\nu = (4\pi^2/3)\alpha\omega\delta(\omega - \omega_{12})|\mathbf{X_{12}}|^2$, where $\alpha = e^2/\hbar c$ is the fine structure constant. Derive the formula for the oscillator strength, f_{12}, in terms of $\mathbf{X_{12}}$. Why f_{12} is called "oscillator strength"?

Some (possibly) useful numbers:

Astronomical constants

1 yr $= 3.16 \times 10^7$ s
1 pc $= 3.086 \times 10^{18}$ cm
1 AU $= 1.50 \times 10^{13}$ cm
1 $M_\odot = 1.99 \times 10^{33}$ g
1 $L_\odot = 3.85 \times 10^{33}$ erg s^{-1}
1 $R_\odot = 6.96 \times 10^{10}$ cm
G $= 1.33 \times 10^{11}$ km^3 s^{-2} M$_\odot^{-1}$

Physical constants

G $= 6.673 \times 10^{-8}$dyn cm^{2}g^{-2}
c $= 2.998 \times 10^{10}$ cm s^{-1}
h $= 6.626 \times 10^{-27}$ erg s
k $= 1.38 \times 10^{-16}$ erg K^{-1}
σ = ac/4 $= 5.67 \times 10^{-5}$ dyn cm^{-2} K^{-4}
$N_0 = 6.02 \times 10^{23}$ mol^{-1}
1 eV $= 1.602 \times 10^{-12}$ erg
e $= 4.803 \times 10^{-10}$ esu
$m_e = 9.109 \times 10^{-28}$ g
$m_p = 1.673 \times 10^{-24}$ g

Units

1 arcsec ($1''$) $= 4.84814 \times 10^{-6}$ radian
1 Angstrom (Å) $= 10^{-8}$ cm
1 Micron (μm) $= 10^{-4}$ cm
1 Jansky (Jy) $= 10^{-26}$ W m^{-2} Hz^{-1} = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}