ASTR 601 - Radiative Processes

Midterm (Oct. 15, 2009)

1) Radiation transfer (15 pts: 5 points each question)

1) Calculate the flux from a uniformly bright sphere of radius R as a function of the distance, d, from the center of the sphere. What is the flux at the surface ($d = R$)?

2) Write down the formal solution of the radiative transfer equation neglecting scattering processes. Define the source function and opacity. Provide a qualitative explanation for the formation of emission lines in stars. Assuming that the temperature of stars decreases monotonically from the center to the surface, give a qualitative explanation for the formation of absorption lines in stellar atmospheres.

3) What is the necessary condition to describe the atmosphere of a star as a plane-parallel atmosphere? Write the formal solution of the transfer equation in this case. Which other assumptions we made in order to derive the emerging specific intensity $I(\mu, 0) = 3F/4\pi(\mu + 2/3)$? Explain the physical significance of the assumptions as well as how they simplify the calculation of the flux, energy density and specific intensity.

2) Thermal statistics (15 pts: 5 points each question)

1) A non degenerate, non relativistic thermal population of electrons has

$$-\frac{\mu}{kT} = -\ln\frac{n\lambda_T^2}{2} \gg 0,$$

where μ is the chemical potential and $\lambda_T = h/(2\pi m_e kT)^{1/2}$. Explain the physical significance of λ_T from the Heisenberg uncertainty principle, and explain why a gas with $n\lambda_T^3/2 \gg 1$ is said to have low occupation number.

2) Explain the sense in which the phrase, "it's easier to ionize than to excite", may be true, referring to the Saha and Boltzmann equations as needed.

3) Show the steps needed to derive the grand potential for systems of fermions and bosons starting from the definition of grand partition function

$$Q = \sum_{N=0}^{\infty} \sum_{States(N)} \exp\left[(N\mu - E)/kT\right].$$

How can you derive the equation of state $P(n, T)$ of the gas from the grand potential?

Some (possibly) useful numbers:

Astronomical constants

1 yr $= 3.16 \times 10^7$ s
1 pc $= 3.086 \times 10^{18}$ cm
1 AU $= 1.50 \times 10^{13}$ cm
1 $M_\odot = 1.99 \times 10^{33}$ g
1 $L_\odot = 3.85 \times 10^{33}$ erg s^{-1}
1 $R_\odot = 6.96 \times 10^{10}$ cm
G $= 1.33 \times 10^{11}$ km^3 s^{-2} M$_\odot^{-1}$

Physical constants

G $= 6.673 \times 10^{-8}$dyn cm^{2}g^{-2}
c $= 2.998 \times 10^{10}$ cm s^{-1}
h $= 6.626 \times 10^{-27}$ erg s
k $= 1.38 \times 10^{-16}$ erg K^{-1}
$\sigma = $ ac/4 $= 5.67 \times 10^{-5}$ dyn cm^{-2} K^{-4}
$N_0 = 6.02 \times 10^{23}$ mol^{-1}
1 eV $= 1.602 \times 10^{-12}$ erg
e $= 4.803 \times 10^{-10}$ esu
$m_e = 9.109 \times 10^{-28}$ g
$m_p = 1.673 \times 10^{-24}$ g

Units

1 arcsec ($1''$) $= 4.84814 \times 10^{-6}$ radian
1 Angstrom (Å) $= 10^{-8}$ cm
1 Micron (μ) $= 10^{-4}$ cm
1 Jansky (Jy) $= 10^{-26}$ W m^{-2} Hz^{-1} $=$ 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}