

Homework #3 Solutions

ASTR100: Introduction to Astronomy

Fall 2009: Dr. Stacy McGaugh

Chap. 7, #40

Which of the following is a strong greenhouse gas? A) Nitrogen. B) Water Vapor. C) Oxygen)

The correct answer is: **B) Water Vapor.** Water vapor is actually the greenhouse gas that contributes the most towards the Earth's natural greenhouse effect. Nitrogen and Oxygen are negligible in their contribution.

Chap. 7, #58

Planet Berth! Imagine a planet, which we'll call *Berth*, orbiting a star identical to the sun at a distance of 1 AU. Assume that Berth has eight times as much mass as Earth and is twice as large as Earth in diameter.

A) How does Berth's density compare to Earth's

METHOD #1: Solving using Ratios

First off, the key thing to remember here is that the equation for density is:

$$\text{Density} = \text{Mass} / \text{Volume, or: } \rho = \frac{m}{V}.$$

We can find the density of Berth, in terms of the density of Earth, by using a ratio. This is particularly useful since we're given Berth's mass and diameter in terms of the Earth, and not in SI units. We'll still need to find the volume first though:

$$V = \frac{4}{3}\pi r^3$$

Plug in the radius of Berth, but in "Earth Radii" (keep in mind that this equation wants a radius, not a diameter – which is what was given):

$$V = \frac{4}{3}\pi(2 \text{ Earth Radius})^3$$

This will give you the volume, in terms of the Earth's Volume, if you take a ratio!

$$\frac{V_{\text{Berth}}}{V_{\text{Earth}}} = \frac{\frac{4}{3}\pi(2 \text{ Earth Radius})^3}{\frac{4}{3}\pi(1 \text{ Earth Radius})^3} = \frac{(2)^3}{(1)^3} = 8 \text{ Earth Volumes}$$

Notice how the constants get canceled out! This makes this volume calculation much easier!

Now, we're good to plug and chug with the density equation:

$$\rho = \frac{m}{V} = \frac{8 \text{ Earth Mass}}{8 \text{ Earth Volumes}}$$

Density of Berth = Densities of Earth!

METHOD #2: Explicitly Solving for Density

Recall the density equation from above:

$$\text{Density} = \text{Mass} / \text{Volume, or: } \rho = m/V.$$

The problem gives us the mass of Berth in Earth Masses. Let's convert that into SI units, so we can actually get real densities. The mass of the Earth can be found in the Appendix.

$$\text{Mass of the Earth} = 5.97 \times 10^{24} \text{ kg}$$

$$\text{Mass of Berth} = 8 \text{ Earth Masses} = 8 \times 5.97 \times 10^{24} \text{ kg} = \mathbf{4.78 \times 10^{25} \text{ kg} = \text{Mass of Berth}}$$

Now to calculate the volume of Berth (the second thing needed to calculate the density), we need to remember another key formula for the volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

$$\text{The radius of the Earth} = 6.38 \times 10^6 \text{ m}$$

$$\begin{aligned} \text{The radius of Berth} &= \frac{1}{2} \text{ the Diameter of Berth (which is what's given)} = \frac{1}{2} \times 2 \text{ Earth Diameters} \\ &= 1 \text{ Earth Diameter} = 2 \text{ Earth Radii (remember, diameter} = 2 \times \text{radius)} \\ &= \mathbf{1.28 \times 10^7 \text{ m} = \text{Radius of Berth}} \end{aligned}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi * (1.28 \times 10^7 \text{ m})^3 = 8.78 \times 10^{21} \text{ m}^3$$

$$\mathbf{\text{Volume of Berth} = 8.78 \times 10^{21} \text{ m}^3}$$

Now that we have both "m" and "V", we can calculate out a density!

$$\rho = \frac{m}{V} = \frac{4.78 \times 10^{25} \text{ kg}}{8.78 \times 10^{21} \text{ m}^3} = \mathbf{5444 \text{ kg/m}^3 = \text{Density of Berth}}$$

Finally, let's compare Berth's density to the Earth's. The Earth's density can be found in the appendix or online:

$$\text{The Density of the Earth is } 5500 \text{ kg/m}^3$$

The densities are about the same!

$$\mathbf{\text{Density of Berth} = \text{Density of Earth}}$$

As you can see, this method gave you the same answer as the first method, but it took many more calculations. You should always try to minimize the number of calculations you have to do, so that way you minimize the possible source of errors... thus Method #1 is the best method to use. I include this method only to show you how to possibly compute the individual steps.

B) How does Berth's surface area compare to the Earth's?

Here, we'll only use the ratio method – although you can also explicitly solve for a surface area, and get the same answer.

First, look up the equation for the surface area of a sphere. This can be found in your text, or online:

$$S = \pi d^2$$

Since the problem gives us the diameter of Berth already in terms of the Earth's diameter, we can just take a ratio and solve...

$$\frac{S_{\text{Berth}}}{S_{\text{Earth}}} = \frac{\pi d_{\text{Berth}}^2}{\pi d_{\text{Earth}}^2} = \frac{d_{\text{Berth}}^2}{d_{\text{Earth}}^2} = \frac{2^2}{1^2} = 4$$

Berth has 4 times the surface area of the Earth

C) Based on your answers to a) and b), discuss how Berth's geological history is likely to have differed from Earth's.

Since the density of Berth is equal to the density of Earth, we therefore can assume that Berth and Earth have similar compositions. So Berth is most likely made up of metals and rocks, as opposed to ices and gasses. This also makes sense since we also know Berth orbits a star similar to the Sun, at the same distance – and from the Solar Nebula theory of solar system formation we would expect rocky planets in such a location.

From the large surface area, we can infer that Berth is likely still molten inside. Larger planets tend to retain internal heat more than smaller planets (case in point: Mercury vs. Earth). Because it has a good amount of internal heat, we could expect active volcanism and tectonics and possibly even a magnetic field.

Lastly, because of the high mass, we could also infer that Berth would have a substantial atmosphere. Planets with larger masses can hold on more atmospheric gasses. While this is a weaker deduction, we might guess that Berth could have a more substantial greenhouse effect due to more atmosphere because of its mass.

Chap. 8, #27

Some Jovian planets give off more energy than they receive because: A) fusion is taking place in their cores B) tidal heating is occurring C) They are still slowly contracting after formation.

The correct answer is: C) they are still slowly contracting after formation. While it may seem surprising, this is indeed believed that the source of the internal heat of the Jovian planets is this gravitational release of energy.

Jovian planets, while massive, do not have enough mass to reach the internal pressures and temperatures necessary to ignite nuclear fusion in their cores. If they were massive enough for this, they would no longer be planets – but rather stars in their own right.

While tidal forces are important for heating the satellites of Jovian planets (such as Io and Europa), the Jovian planets themselves are not substantially heated by this effect. To be substantially heated by tides, you need an object of comparable size nearby, tugging on the planet. For example, while Jupiter's moons are massive, they are not nearly large enough to induce such a large effect on Jupiter.

Chap. 8, #28

The main ingredients of most moons of the Jovian planets are: A) rock and metal B) hydrogen compound ices C) hydrogen and helium.

The correct answer is: B) hydrogen compound ices. While you could learn this from the text or lecture, the most conclusive way to show this is to look at the densities of these moons. This information can be found in Appendix E of the text (or online).

While some of Jupiter's moons (notably Io) have high densities around $3,500 \text{ kg/m}^3$, most of the other Jovian satellites (such as Ganymede, Mimas, and Iapetus – just to name a few) have densities closer to $1,200 \text{ kg/m}^3$.

Water ice (which is the main hydrogen compound ice), has a density of $1,000 \text{ kg/m}^3$.

Rocks and Metals have densities closer to $5,000 \text{ kg/m}^3$ or more.

Hydrogen and Helium gas have densities much lower than $1,000 \text{ kg/m}^3$ (our own atmosphere has a density around 1 kg/m^3).

Thus, by looking at these different densities, we can see that the Jovian moons seem to mostly be composed of hydrogen compound ices.

Chap. 9, #17

An object that resembles a comet in size and composition is discovered orbiting in the inner solar system. Does this seem reasonable or surprising? Why?

Discovering a comet within the inner solar system (say, within the orbit of Mars), would indeed be very **surprising**. As we've learned in class, comets are primarily icy bodies that originate from the outer solar system (from the Kuiper Belt and the Oort Cloud). The reason they can exist that far out is because it is extremely cold, that far away from the Sun. When a comet does come near the Sun (specifically when it crosses the "Frost Line", where it's too hot for water ice to stay solid), it starts out-gassing massive amounts of gas and dust. This is why comets become so spectacular when in the inner solar system.

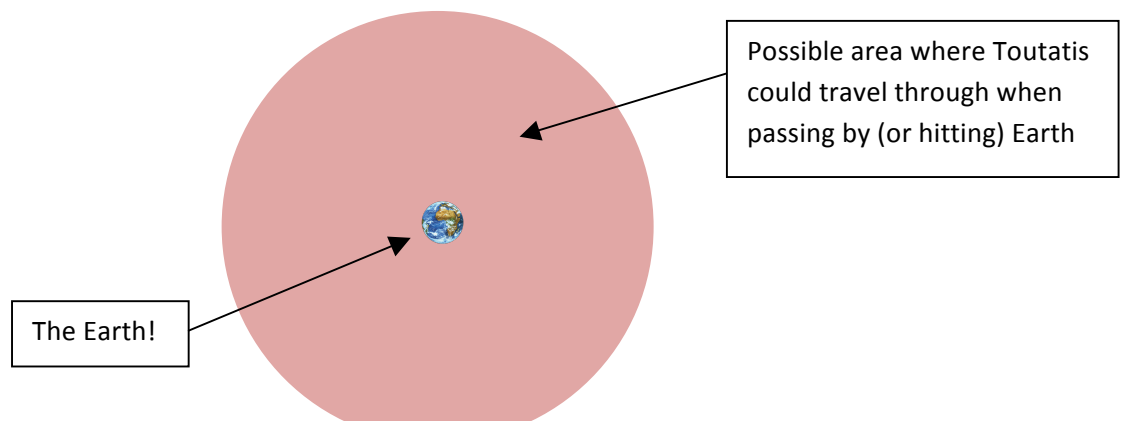
Even though spectacular, these events are ephemeral. If a comet comes down into the inner solar system they will slowly evaporate away – disintegrating due to the heat of the Sun. This is why it'd be surprising to find a comet that orbits entirely within the inner solar system. If the comet has always been there, then it should have been destroyed long ago.

The only possible explanation would be that this comet only recently came to orbit exclusively in the inner solar system. This could be due to the comet originally coming from the outer solar system, and then passing too close to another planet, which could change its orbit. This comet still would have a short lifetime, and be destroyed in a relatively short amount of time.

Extra Credit, Chap. 9, #46

The "Near Miss" of Toutatis! The 5-kilometer asteroid Toutatis passed a mere 1.5 million kilometers from Earth in 2004. Suppose Toutatis were destined to pass somewhere within 3 million kilometers of Earth. Calculate the probability that this "somewhere" would have meant that it slammed into Earth. Based on your result, do you think it is fair to call the 2004 passage a "Near Miss"? Explain.

First, let's draw a picture of this possible future passage of Toutatis:



The area of this possible passage/impact area is just the area of a circle, with a radius of 1.5 million kilometers.

$$\text{Passage Area} = \pi (\text{Radius})^2$$

$$\text{Passage Area} = \pi (1.5 \times 10^6 \text{ km})^2$$

$$\textbf{Passage Area} = \textbf{7 x 10^{12} km^2}$$

Now, we'll need to compare this to the cross sectional area of the Earth, using the same equation:

$$\text{Earth's Area} = \pi (\text{Radius})^2$$

$$\text{Earth's Area} = \pi (6378)^2$$

$$\textbf{Earth's Area} = \textbf{1.3 x 10^8 km^2}$$

Now, to see how likely that Toutatis will hit the Earth, let's take the ratio of these two areas:

$$\text{Probability of Impact} = 1.3 \times 10^8 \text{ km}^2 / 7 \times 10^{12} \text{ km}^2$$

$$\text{Probability of Impact} = 1.9 \times 10^{-5}$$

$$\textbf{Probability of Impact} = \textbf{0.0018 \%}$$

This is an extremely low probability for an impact, so strictly speaking this isn't really a "Near Miss".