Homework #4 Solutions
ASTR100: Introduction to Astronomy
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Chapter 5: #50

Hotter Sun: Suppose the surface temperature of the Sun were about 12,000K, rather than 6000K.

a. How much more thermal radiation would the Sun emit?
b. What would happen to the Sun's wavelength of peak emission?
c. Do you think it would still be possible to have life on Earth? Explain.

Answer:

a) To calculate the emitted power per square meter we need to use Stefan-Boltzmann's Law, that is,

$$E = \sigma T^4$$

$Where,$

$E = \ Emitted\ Power\ (per\ sqaure\ meter\ of\ surface)$

$T = temperature\ in\ Kelvin\ and$

$\sigma = Stefan - Boltzmann\ Constant = \ 5.7\ \times\ 10^{-8} \frac{watt}{m^2 \times\ K^4}$

At 6000K,

Emitted Power = $\sigma\ \times (6,000)^4$

At 12000K,

Emitted Power = $\sigma\ \times (12,000)^4$

Now calculate the ratio of thermal radiation at the two temperatures to find out how much more thermal radiation would the Sun emit,

$$\frac{E_{12000\,K}}{E_{\ 6000\,K}} = \frac{\sigma\ \times\ (12{,}000)^4}{\sigma\ \times\ (6{,}000)^4} = \frac{(12{,}000)^4}{(6{,}000)^4} = \left[\frac{12{,}000}{6{,}000}\right]^4 = 2^4 = 16$$

$$\therefore\ \frac{E_{12000\,K}}{E_{\ 6000\,K}} = 16$$

Therefore, emitted power at 12000K is 16 times the emitted power at 60000K.

b) To calculate the wavelength of maximum intensity we need to use Wien's Law, that is,

$$\lambda_{max} = \frac{b}{T}$$

Where,

$\lambda_{max} = wavelength\ of\ maximum\ intensity$

$b = Wien's\ constant = 2{,}900{,}000\ nm.K$

$T = temperature\ in\ Kelvin$

$$\lambda_{max} = \frac{2{,}900{,}000}{T} = \frac{2{,}900{,}000}{6{,}000}\ or\ 483.33\ nm$$

$$\lambda_{max} = \frac{2{,}900{,}000}{T} = \frac{2{,}900{,}000}{12{,}000}\ or\ 241.67\ nm$$

$$Ratio = \frac{\lambda_{\max\ \ at\ 12{,}000K}}{\lambda_{\max\ \ at\ 6{,}000K}} = \frac{2{,}900{,}000}{12{,}000} \times \frac{6{,}000}{2{,}900{,}000}$$

$$\therefore \frac{\lambda_{\max\ \ at\ 12{,}000K}}{\lambda_{\max\ \ at\ 6{,}000K}} = \frac{1}{2}$$

Therefore, wavelength of maximum intensity at 12,000K is half the wavelength of maximum intensity at 6,000K.

c) We know that the intensity of light emitted by a given object depends on its wavelength or frequency. Here, the wavelength at 12,000K is reduced by half which means its frequency has increased by half. In the EM spectrum, UV rays have shorter wavelengths. Therefore, at this temperature, the Sun would emit copious ultraviolet light which cannot be suitable for supporting life.

Chapter 5: #52

Doppler Calculations II. In hydrogen, the transition from level 2 to level 1 has a rest wavelength of 121.6 nm. Suppose you see this line at wavelength ay 121.9 nm in Star C and at 122.9 nm in Star D. Calculate each star's speed, and be sure to state whether it is moving toward or away from us.

Answer:

We can calculate an object's radial velocity from its Doppler shift, that is,

$$\frac{v_{rad}}{c} = \frac{\lambda_{shift} - \lambda_{rest}}{\lambda_{rest}}$$

$Where,$

$v_{rad} = object's\ radial\ velocity$

$\lambda_{shift} = rest\ wavelength\ of\ a\ particular\ spectral\ line\ (nm)$

$\lambda_{rest} = shifted\ wavelength\ of\ the\ same\ line\ (nm)$

$c = speed\ of\ light = 3 \times 10^5\ km/s$

For Star C:

$$\frac{v_{rad}}{c} = \frac{121.9 - 121.6}{121.6} = 2.47 \times 10^{-3}$$

$$\therefore\ v_{rad} = 2.47 \times 10^{-3} \times 3 \times 10^5 = 740\ km/s$$

For Star D:

$$\frac{v_{rad}}{c} = \frac{122.9 - 121.6}{121.6} = 1.07 \times 10^{-2}$$

$$\therefore\ v_{rad} = 1.07 \times 10^{-2} \times 3 \times 10^5 = 3207\ km/s$$

Since both the stars have positive values, it implies that they have a red shift. Therefore, both the stars are moving away from us.

Chapter 6: #36

About how old is the solar system? a) 4.5 million years b) 4.5 billion years c) 4.5 trillion years

The correct answer is **b) 4.5 billion years**. The age of a rock can be calculated by using a reliable method called radiometric dating which relies on the careful measurement of the proportions of various atoms and isotopes in the rock. To measure the age of our solar system, we need to do a careful analysis of radioactive isotopes of meteorites because meteorites are rocks that have not melted or vaporized since they first condensed in the solar nebula.

Chapter 6: #57

Monster Iceballs. The ice-rich planetesimals that formed the cores of the Jovian planets were about 10 times more massive than Earth. Assuming such an iceball has a density of about 2 g/cm^3, what would its radius be? How does this compare to Earth's radius?

Answer:

$$Density, \rho = \frac{Mass}{Volume} = \frac{m}{V}$$

$$Volume\ of\ Sphere, V = \frac{4}{3}\pi r^3$$

Density of the iceball = 2 g/cm^3

Radius of Earth, R_{Earth} = 6.378 x 10^3 km

Mass of Earth, M_{Earth} = 5.97 x 10^{24} kg = 5.97 x 10^{27} g

Mass of Jovian planetesimal, M_{Jovian} = 10 M_{Earth} = 5.97 x 10^{28} g

$$Density = \frac{Mass}{Volume} = \frac{\mathrm{M_{Jovian}}}{\frac{4}{3}\pi(R_{iceball})^3}$$

$$2\frac{g}{cm^3} = \frac{5.97 \times 10^{28} g}{\frac{4}{3}\pi(R_{iceball})^3}$$

$$R_{iceball}{}^3 = \frac{3 \times 5.97 \times 10^{28}}{2 \times 4\pi}\ cm^3$$

$$R_{iceball}{}^3 = 7.126 \times 10^{27}\ cm^3$$

$$\therefore R_{iceball} = 1.924 \times 10^9\ cm$$

$$R_{iceball} = 1.924 \times 10^4\ km$$

To compare the radius of the iceball with the radius of Earth, we need to find out their ratios, that is,

$$\therefore \frac{R_{iceball}}{R_{Earth}} = \frac{1.924 \times 10^4}{6.378 \times 10^3} = 3.02$$

Therefore, the iceball has about 3 times the radius of Earth.

Chapter 10: #47

The color of the Sun: The Sun's average surface temperature is about 5800K. Use Wien's law to calculate the wavelength of peak thermal emission from the Sun. What color does this wavelength correspond to in the visible-light spectrum? Why do you think the Sun appears white or yellow to our eyes?

Answer:

According to Wien's Law,

$$\lambda_{max} = \frac{2,900,000}{T}$$

Therefore, the wavelength of peak thermal emission from the Sun is,

$$\therefore \lambda_{max} = \frac{2,900,000}{5800} = 500\ nm$$

The color at a wavelength of 500nm that corresponds to in the visible-light spectrum is green.

As a thermal emitter, our sun produces lots of light both redder and bluer than the peak. The mixture of these many wavelengths looks yellowish-white to our eyes.

Extra Credit:

Chapter 10: #51

The Lifetime of the Sun. The total mass of the Sun is about 2 x 10^{30} kg, of which about 75% was hydrogen when the Sun formed. However, only about 13% of this hydrogen ever becomes available for fusion in the core. The rest remains in layers of the Sun where the temperature is too low for fusion.

a) Based on the given information, calculate the total mass of hydrogen available for fusion over the lifetime of the Sun.

Answer:

Mass of Sun, M_{Sun} = 2 x 10^{30} kg

But Hydrogen = 75% of M_{Sun} and available hydrogen is 13% of (75% of M_{Sun})

That is,

$$Hydrogen = \frac{75}{100} \times 2 \times 10^{30} = 1.5 \times 10^{30}\ kg$$

$$Available\ Hydrogen\ for\ fusion = \frac{13}{100} \times 1.5 \times 10^{30} = 1.95 \times 10^{29}\ kg$$

Therefore total mass of hydrogen available for fusion = 1.95 x 10^{29} kg

b) Combine your results from part (a) and the fact that the Sun fuses about 600 billion kg of hydrogen each second to calculate how long the Sun's initial supply of hydrogen can last. Give your answer in both seconds and years.

Answer:

Hydrogen that fuses each second in the Sun = 600 x 10^9 kg/sec

Total mass of available hydrogen for fusion = 1.95 x 10^{29} kg

Therefore, the period taken for all the available hydrogen to be exhausted is,

$$Total\ lifetime\ of\ Sun = \frac{1.95 \times 10^{29}}{600 \times 10^{9}} = 3.25 \times 10^{17} sec$$

Also, 1 year = 365 x 24 x 60 x 60 seconds = 3.154 x 10^7 sec

$$Total\ lifetime\ of\ Sun\ in\ years = \frac{3.27 \times 10^{17}}{3.154 \times 10^{7}} = 10.3 \times 10^{9} years$$

which is approximately 10 billion years.

c) Given our solar system is now about 4.6 billion years old, when will we need to start worrying about the Sun running out of hydrogen for fusion?

From the above answer, we know that the total life of the Sun in the main sequence is about 10 billion years. In about another 5 billion years, the Sun will enter a red giant phase where its outer layers will continue to expand as the fuel (hydrogen) in the core is consumed and the core contracts and heats up. Finally, all hydrogen will be exhausted and helium fusion will begin.