ASTR100 – Fall 2009 (McGaugh)
Homework #2
(due Tuesday, September 29th)

SOLUTIONS

Problems: Chapter 3 #39, 41; Chapter 4: #34, 35, 47; Extra Credit (2 pts): Ch. 4, #48

Chapter 3

39. What is the circumference of Nearth?

On the equinox, the sun is directly overheard in the city of Nyene, located 1000 km due north of Alectown, and on the equinox in Alectown, the altitude of the sun is 80°.

Angle from the zenith = 90° – altitude = 90° – 80° = 10°
(Angle found above is the latitude difference between Nyene and Alectown)

Nearth's circumference = (360° / latitude difference) * distance between cities
Circumference = (360° / 10°) * 1000 km = (36*1000 km) = **36,000 km**

41. Halley's comet orbits the Sun every 76.0 years and has an orbital eccentricity of 0.97.
(a) Find its average distance by using Kepler's Third Law (applies here because Halley's comet orbits the Sun).

$P^2 = a^3$
P = 76.0 years
$a = P^{2/3} = (76.0 \text{ years})^{2/3} =$ **17.94 AU**

(b) Where does Halley's comet spend most of its time?

Use Kepler's Second Law: As a planet moves around its orbit, it sweeps out equal areas in equal times. So when it is closest to the Sun, the comet will be moving fastest, and conversely when it is furthest from the Sun, the comet will be moving the slowest.

Therefore, Halley's comet must **spend most of its time at aphelion**, when it is furthest from the Sun, because it moves the slowest at that point.

Chapter 4

34. If Earth were twice as far from the Sun, what would the force of gravity between them be?

The equation needed here is the universal law of gravitation, discovered by Newton:
$F = (GM_1M_2)/d^2$ (Note: the force depends on the inverse of distance squared.)

Current force of gravity on Earth: $F1 = (GM_1M_2)/d^2$
"New" force of gravity on Earth: $F2 = (GM_1M_2)/(2d)^2$
$F2 = (F1)/4$, therefore the answer is **(c) one-quarter as strong**.

35. If the Sun were replaced by a black hole of the same mass, what would happen to Earth's orbit?

$F = (GM_1M_2)/d^2$

The law of gravity depends on only the following:
1] the Universal Gravitation constant, G, which will not be affected.
2] the masses of both the Sun/black hole and Earth.
3] the distance between the Sun/black hole and Earth.

Because the mass of the black hole will be the same as the mass of the Sun, and Earth will not start at a different orbital distance when the change occurs, the correct answer must be **(c) Earth's orbit would not change.**

47. What is the orbital period of the planet?

For this problem, we must use Newton's form of Kepler's Third Law.
$P^2 = [(4pi^2)/G(M_1 + M_2)]*a^3$

Mass of star (M_1) is 4 times the mass of the Sun. $M_1 = 4*(1.98892 \times 10^{30}$ kg)
Planet (M_2) is same mass as Earth. $M_2 = 5.9742 \times 10^{24}$ kg
(Mass of planet << Mass of Sun, can be neglected)
The planet is orbiting at 1 AU. $a = 1$ AU $= 1.496 \times 10^{11}$ m
The gravitation constant G is 6.67×10^{-11} m^3 /kg*s^2

$P = sqrt([(4pi^2)/G(M_1)]*a^3)$
$P = sqrt([7.44 \times 10^{-20}\ s^2/m^3] * [1.496 \times 10^{11}\ m]^3)$ = **1.58 x 107 s = 182.9 days**

SIMPLER ANSWER: Think of the mass as $4M_{sun}$. Then $(4pi^2)/G(M_{sun}) = 1$ and the new equation is $P^2 = a^3/4$ where P is in years and a is in AU.
$P = sqrt((1\ AU)^3/4) = sqrt(1/4)$ = **½ year** (note: ½ years = 182.5 days)

EXTRA CREDIT

48. (a) What is the mass of the Earth?

Moon's period of orbit is 27.3 days, and semi-major axis is 384,000 km.
Can neglect mass of Moon (Mass of Moon << Mass of Earth)

$P^2 = [(4pi^2)/G(M_1 + M_2)]*a^3 = [(4pi^2)/G(M)]*a^3$

$P = 27.3$ days $= 2.36 \times 10^6$ seconds
$a = 384{,}000$ km $= 3.84 \times 10^8$ m
The gravitation constant G is 6.67×10^{-11} m^3 /kg*s^2

Solve for M:
$M = [(4pi^2)/(G*P^2)]*a^3 = [0.106 \text{ kg/m}^3] * (3.84 \times 10^8 \text{ m})^3 =$ **6.02×10^{24} kg**

(b) What is the mass of Jupiter?

Io orbits Jupiter every 42.5 hours, at an average distance of 422,000 km.
Can neglect mass of Io (Mass of Io << Mass of Jupiter)

$P^2 = [(4pi^2)/G(M_1 + M_2)]*a^3 = [(4pi^2)/G(M)]*a^3$

$P = 42.5$ hours $= 1.53 \times 10^5$ seconds
$a = 422{,}000$ km $= 4.22 \times 10^8$ m
The gravitation constant G is 6.67×10^{-11} m^3 /kg*s^2

Solve for M:
$M = [(4pi^2)/(G*P^2)]*a^3 = [25.3 \text{ kg/m}^3] * (4.22 \times 10^8 \text{ m})^3 =$ **1.90×10^{27} kg**

(c) What is planet's orbital distance?

Mass of star is the same as Sun, orbital period is 63 days.

Because planet is going around an object that is the same mass as the Sun, the more simple Kepler equation ($P^2 = a^3$) can be used.

$P^2 = a^3$
$P = 63$ days $= 0.172$ years
$a = P^{2/3} = (0.172 \text{ years})^{2/3}$ **= 0.309 AU**