Spatial distribution of low-surface-brightness galaxies

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ABSTRACT

The spatial distribution of low-surface-brightness (LSB) galaxies is important both as a test of theories of large-scale structure formation and for the physical understanding of the environmental effects that influence the evolution of galaxies. In this paper we calculate, using redshift samples, the cross-correlation functions $\xi_{AB}(r)$ of LSB galaxies with normal galaxies in complete samples (i.e. CfA and IRAS). This enables us to compare directly the amplitudes and shapes of the correlation functions for LSB galaxies with those for CfA and IRAS galaxies. For pair separations $r \approx 2\ h^{-1}$ Mpc, we find $\xi_{AB}(r) \propto r^{-\gamma}$ with $\gamma \approx 1.7$. This shape of $\xi_{AB}$ is in agreement with that of the correlation functions for other galaxies. The amplitudes ($A$) of $\xi_{AB}(r)$ are lower than those of the autocorrelation functions for the CfA and IRAS samples, with $A_{LSB-CFA} / A_{CFA-CFA} \approx 0.4$ and $A_{LSB-IRAS} / A_{IRAS-IRAS} \approx 0.6$. These results suggest that LSB galaxies are embedded in the same large-scale structure as other galaxies, but are less strongly clustered. This offers the hope that LSB galaxies may be unbiased tracers of the mass density on large scales. For $r \lesssim 2\ h^{-1}$ Mpc, the cross-correlation functions are significantly lower than that expected from the extrapolation of $\xi_{AB}$ on larger scales, showing that the formation and survival of LSB galaxies may be inhibited by interaction with neighbouring galaxies.

We show that a simple hierarchical model, in which LSB galaxies are formed only in haloes lacking close interactions with other haloes, reproduces both the deficit of pairs at small separations and the low amplitude of the correlation function. This model suggests that LSB galaxies should, on average, be younger than normal galaxies, consistent with direct observations. The model also suggests that a strong luminosity (mass) segregation in galaxy clustering is not a necessary consequence of biased galaxy formation, unless the effect of surface brightness (collapse time) is taken into account. It is also possible that a significant fraction of the mass density of the Universe resides in galaxies that have not been observed because of their low surface brightness.

Key words: galaxies: clustering – galaxies: formation – cosmology: observations.

1 INTRODUCTION

According to the theory of biased galaxy formation in an $\Omega = 1$ universe (see e.g. Kaiser 1984; Bardeen et al. 1986, hereafter BBKS), normal and easily detectable high-surface-brightness (HSB) galaxies are more strongly clustered than the underlying mass distribution. Strong selection effects, however, act against the discovery of galaxies of low surface brightness (LSB) (Disney 1976). If LSB galaxies trace the mass distribution in a more representative manner than HSB objects then clearly our perception of galaxy clustering is biased as well. Indeed, it has been suggested that LSB dwarfs corresponding to $\sim 1\sigma$ fluctuations in the initial density field should trace mass and fill the voids (e.g. Dekel & Silk 1986).

Not all LSB galaxies are dwarfs, however (Bothun et al. 1987; Davies et al. 1988; Impey & Bothun 1989; Irwin et al. 1990; Bothun et al. 1990; Knezek 1992; McGaugh 1992; McGaugh & Bothun 1994; van der Hulst & de Blok, in preparation). Indeed, the most common types of objects found in modern field surveys are comparable in size and mass to HSB spirals (Schombert & Bothun 1988; Schombert et al. 1992; Impey et al. 1993). These objects apparently do not fill the voids, and seem to trace the same large-scale structures seen in the same part of the sky by the CfA redshift survey.
For the targets representing the distribution of LSB galaxies, we use a sample constructed from Schombert et al. (1992). This sample (called POSS in the following) consists of 339 galaxies (171 with redshifts) discovered by visual inspection of the plates of the Second Palomar Sky Survey. Like the UGC (Nilson 1973), these objects are selected by a diameter limit, and so probe the range of surface brightness between the isophotal limits of the old (25.3 mag arcsec$^{-2}$; Cornell et al. 1987) and new (26.0 mag arcsec$^{-2}$; Schombert & Bothun 1988) sky survey plates. In general, these newly discovered galaxies are very difficult to discern on the old POSS plates and hence it is not surprising that they were passed over by Nilson. The new POSS sample thus defines a slice in surface brightness beyond the UGC (which already contains some quite low-surface-brightness galaxies) which is well removed from the high-surface-brightness objects typical of the CfA sample. This provides a much cleaner signal in the correlation analysis than can be attained by simply splitting up the UGC (e.g. Davis & Djorgovski 1985; Bothun et al. 1986).

The sky distribution of POSS galaxies with measured redshifts is shown in the middle panel of Fig. 1. The redshift distribution of these galaxies is shown in Fig. 2. These LSB galaxies are virtually never detected by IRAS (Schombert & Bothun 1988), so there is no concern about coincidence between the trace and target galaxies in this case. Coincidence with the CfA sample has been discussed in detail by Bothun et al. (1993).

3 STATISTICAL METHOD

In order to study the distribution of LSB galaxies in the galaxy density field, we can calculate the mean density of trace galaxies interior to a sphere of radius $r$, around each target (LSB) galaxy. Suppose the two-point cross-correlation function between the target galaxies of type A with trace galaxies B is $\xi_{AB}(r)$ (which is defined as the excess probability of finding a trace galaxy B in a spherical shell of radius $r$ centred on a randomly chosen target galaxy of type A; see Peebles 1980, section 44). The mean density of the trace galaxies inside the sphere is

$$\langle n(r) \rangle = n_0[1 + \xi_{AB}(r)],$$

(1)

where

$$\xi_{AB}(r) = \frac{3}{r^2} \int_0^r x^2 \xi_{AB}(x) \, dx,$$

(2)

and $n_0$ is the overall mean density of trace galaxies. This analysis technique has been widely used to study the relative distributions of two populations (see Mo & Lahav 1993 for a discussion). It is not essential that the target sample be complete (actually one can calculate a cross-correlation function for each target galaxy), provided that the target sample is a fair sample and its incompleteness does not correlate with galaxy clustering. This advantage is important to our analysis because it is virtually impossible to construct a sample that is complete in LSB galaxies. Regardless of the depth of the isophotal limit of a survey, there may always exist objects of such low surface brightness that their apparent diameters do not exceed the survey limit (Disney 1976). While this selection against low-contrast objects is strong, it is an observa-
tional limitation which should in no way be correlated with the spatial distribution of galaxies. That is, there is no evidence that either Nilson (1973) or Schombert & Bothun (1988) and Schombert et al. (1992) looked harder to find LSB objects in underdense regions. The sky coverage of the target sample is fairly small; it is not known whether the clustering of LSB galaxies in this region is similar to that of a fair sample of the Universe. As shown in Section 4, however, the clustering of HSB galaxies in this region is similar to what we know from other samples.

We estimate the cross-correlation function $\xi_{AB}(r)$ using the definition

$$\xi_{AB}(r) = \frac{P_{AB}(r)}{P_{AA}(r)} - 1,$$

where $P_{AB}(r)$ is the number of cross-pairs between target objects (A) and trace objects (B), with separation $r$, in the data; $P_{AA}(r)$ is the corresponding number of cross-pairs between target objects (A) and random points (R) in a random sample that has the same selection function as the trace sample. The average function $\xi_{AB}(r)$ can either be calculated by using equation (2), or estimated from the definition

$$\xi_{AB}(r) = \frac{\Pi_{AB}(r)}{\Pi_{AA}(r)} - 1.$$

Here, $\Pi_{AB}(r)$ is the number of cross-pairs between target objects (A) and trace objects (B), with separation $r$, in the data; $\Pi_{AA}(r)$ is the corresponding expected number of pairs. For our cases, these two definitions of $\xi_{AB}$ give very similar results. The advantage of the second definition is that it is more stable and independent of the choice of bins in $r$ when $r$ is large.

The distances of galaxies are calculated from their redshifts, so these are correlation functions in redshift space. The
redshifts are corrected for a velocity of 300 km s\(^{-1}\) due to the motion in the Local Group (LG), and for the Virgo-centric flow in the way described by Schechter (1980). The model assumes a Virgo-centric infall velocity field varying inversely as the Virgo-centric distance and presumes the mean (LG-frame) velocity of the Virgo cluster to be 1020 km s\(^{-1}\). The Virgo-centric infall of the LG is assumed to be \(v_{\text{LG}} = 220 \text{ km s}\)^{-1}. The median redshift of the LSB sample is over 5000 km s\(^{-1}\), so our results should not be sensitive to the details of local distortions in the Hubble flow. To test this, we have also used infall velocities of \(v_{\text{LG}} = 100\) and 300 km s\(^{-1}\) to bracket the realistic values. Our results do not change significantly for these values of \(v_{\text{LG}}\). Galaxies within 6° of the centre of Virgo and with measured velocities less than 2500 km s\(^{-1}\) are assumed to be at the mean Virgo velocity.

As we can see from equations (3) and (4), an accurate estimate of the cross-correlation functions depends on an accurate estimate of the expected number of pairs \(P_{\text{AR}}\) (or \(\Pi_{\text{AR}}\)) which, in turn, depends on our understanding of the sample selection effects. In our case, an important advantage is that the selection functions of both CfA and IRAS samples are reasonably well known. Since our trace samples are apparent-magnitude or flux limited, the pair counts will be dominated by the contributions from the nearby region if trace galaxies are not weighted by their selection functions. In most cases, we calculate the cross-correlation functions by assigning to each trace galaxy a weight that is proportional to the reciprocal of the selection function at the distance of the galaxy. Such a weighting scheme gives, however, quite noisy results on small scales, because there are not many close pairs of galaxies at large distances. To estimate the correlation function on small scales (e.g. \(r \leq 1 \text{ h}^{-1} \text{ Mpc}\), we use a scheme in which all galaxies are equally weighted. Such a change of weighting scheme on small scales improves the statistics but does not alter the behaviour of the correlation function. The statistical uncertainty in correlation functions is difficult to analyse. To present our results, we will use errors given by the bootstrap resamplings of the target samples (see e.g. Barrow, Bhavsar & Sonoda 1984; Mo, Jing & Börner 1992b).

4 RESULTS

We calculate the cross-correlation functions for the various cases summarized in Table 1. The first column of Table 1 lists the samples that are used to calculate the cross-correlation functions. A case denoted by ‘Sample1–Sample2’ means, for example, that ‘Sample1’ is to be used as the target sample while ‘Sample2’ is the trace sample. The second column indicates the sky region to which the target galaxies are confined. For all cases, we exclude galaxies with corrected redshifts larger than 10 000 km s\(^{-1}\). For the CfA sample, we also exclude galaxies with absolute magnitudes fainter than \(-18.5\). This is done to reduce the weight of local superclusters. This restriction should not influence the results on larger scales, as the correlation function does not depend strongly on luminosity (Phillipps & Shanks 1987; Alimi, Valls-Gabaud & Blanchard 1988; Hamilton 1988; Börner & Mo 1990). The number of galaxies in the target sample is listed in the third column for each case.

Fig. 3 shows the average correlation functions \(\bar{\xi}(r)\) for the cases CfA–CfA (triangles) and POSS–CfA (circles). For comparison, we also show the POSS–CfA cross-correlation functions given by Virgo infall models with \(v_{\text{LG}} = 100\) (three-pointed stars) and \(v_{\text{LG}} = 500 \text{ km s}\)^{-1} (crosses) (see Section 3). The figure shows that our results are not sensitive to the infall model. The correlation functions are normalized by a random CfA catalogue generated by the selection function (SF) given by Strauss et al. (1991). This SF is in good agreement with that derived from the luminosity function (of the same sample) given by Efstathiou, Ellis & Peterson (1988). The straight line corresponds to a differential correlation function, \(\xi(r) = (6 \text{ h}^{-1} \text{ Mpc})^2 r^{-1}\), which is approximately the result of the redshift-space autocorrelation function obtained by Davis & Peebles (1983) for the CfA sample. This line fits reasonably well the result of our CfA–CfA analysis demonstrates that the POSS region is not a peculiar region of galaxy clustering. The error bars show the 1σ standard deviation among the results of 100 bootstrap resamplings of the POSS sample. Clearly, the cross-correlation function for POSS–CfA has a systematically lower amplitude, and much flatter shape on small scales, than that for CfA–CfA. Fig. 4 shows the cross-correlation function in its differential form (equation 3).

Since it is known that spiral galaxies are less strongly correlated than elliptical galaxies (Davis & Geller 1976; Giovanelli, Haynes & Chincarini 1986; Börner, Mo & Zhou 1989; Jing, Mo & Börner 1991; Mo et al. 1992a), and that the LSB galaxies in our samples are disc galaxies, it is interesting to see what happens when spiral galaxies alone are used to trace the galaxy density field. Fig. 5 shows the results when all galaxies with types later than \(T=0\) in the CfA sample are used (this sample is called ‘Spiral’ in our discussion). In this case, we have used the selection function for the total sample, based on the result of Davis & Huchra (1982) that the shape
of the luminosity function does not depend significantly on morphological type. This SF is almost identical to that derived from the luminosity function for all spiral galaxies (with \( T > 1 \)) given by Efstathiou et al. (1988a). For comparison, we also show in Fig. 5 the LSB–Spiral cross-correlation function estimated by using a SF derived from the luminosity function of all spiral galaxies given by Davis & Huchra (1982). Since these two SFs differ substantially from each other, the comparison shows that our results are not sensitive to the SF used. The amplitude of the correlation function for Spiral–Spiral is about 0.7 times that for the total sample, in good agreement with that found by Börner et al. (1989) and Jing et al. (1991). The cross-correlation function for POSS–Spiral has, however, a lower amplitude and a flatter shape than that for Spiral–Spiral, in a similar manner to that shown in Fig. 3.

Fig. 6 shows the average correlation functions \( \langle \xi_{AB} \rangle \) for the cases IRAS–IRAS (triangles) and POSS–IRAS (bullets). The correlation functions are normalized by a random IRAS catalogue generated by using the SF given by Yahil et al. (1991). The straight line corresponds to a differential correlation function, \( \xi(r) = (4 \ h^{-1} \ \text{Mpc}/r)^{1.6} \), which is approximately the result for the redshift-space autocorrelation function obtained by Strauss et al. (1992) for the total IRAS sample. Our correlation function for IRAS–IRAS is in agreement with this result, showing again that the region in consideration is not peculiar in galaxy clustering. Fig. 7 shows the correlation function in differential form. The result is noisy because of the low space density of the IRAS sample. In general, the results show that the cross-correlation function for POSS–IRAS has a systematically lower amplitude and (marginally) an extra deficit at small separations.

4.1 The amplitudes of the cross-correlation functions

As we can see from Figs 1–7, the correlation functions are well described by a power law for \( r \geq 2 \ h^{-1} \ \text{Mpc} \). This enables us to compare the correlation amplitudes for different cases. Such a comparison is important, because it may give the relative bias factors for the different galaxies in consideration, as we will discuss in Section 5. We have performed a formal regression for each of the results in the range \( 2 \ h^{-1} \ \text{Mpc} < r < 10 \ h^{-1} \ \text{Mpc} \) using a power-law model

\[
\xi_{AB}(r) = Ar^{-\gamma},
\]

where the amplitude \( A \) and the power index \( \gamma \) are constants. To compare the correlation amplitude more directly, we have

<table>
<thead>
<tr>
<th>Case</th>
<th>Region</th>
<th>( N_g )</th>
<th>( \gamma )</th>
<th>( A )</th>
<th>( r_0 )</th>
<th>( r_{\beta=2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CfA-CfA</td>
<td>POSS</td>
<td>665</td>
<td>1.78 ± 0.21</td>
<td>49.7 ± 16.7</td>
<td>5.39 ± 0.95</td>
<td>36.9 ± 4.7</td>
</tr>
<tr>
<td></td>
<td>CfA</td>
<td>121</td>
<td>1.50 ± 0.27</td>
<td>13.2 ± 6.0</td>
<td>3.53 ± 1.00</td>
<td>1.93 ± 0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16.2 ± 2.6</td>
<td>3.33 ± 0.31</td>
<td>1.95 ± 0.13</td>
<td></td>
</tr>
<tr>
<td>Spir-Spir</td>
<td>POSS</td>
<td>378</td>
<td>1.69 ± 0.26</td>
<td>29.4 ± 13.0</td>
<td>4.50 ± 1.08</td>
<td>27.2 ± 4.8</td>
</tr>
<tr>
<td></td>
<td>CfA</td>
<td>121</td>
<td>1.58 ± 0.35</td>
<td>8.9 ± 5.6</td>
<td>2.49 ± 0.90</td>
<td>1.51 ± 0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.4 ± 2.6</td>
<td>2.83 ± 0.34</td>
<td>1.78 ± 0.13</td>
<td></td>
</tr>
<tr>
<td>IRAS-IRAS</td>
<td>POSS</td>
<td>388</td>
<td>1.91 ± 0.24</td>
<td>25.5 ± 10.4</td>
<td>3.20 ± 0.63</td>
<td>17.8 ± 2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>17.8 ± 2.1</td>
<td>3.53 ± 0.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POSS-IRAS</td>
<td>POSS</td>
<td>151</td>
<td>1.75 ± 0.28</td>
<td>12.1 ± 6.0</td>
<td>2.52 ± 0.66</td>
<td>1.33 ± 0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.2 ± 1.5</td>
<td>2.52 ± 0.22</td>
<td>1.01 ± 0.25</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( r_0 \) and \( r_{\beta=2} \) are in \( h^{-1} \ \text{Mpc} \); the sample 'Spir' contains all late-type (\( T > 0 \)) galaxies in the CfA sample.
performed another regression by fixing $\gamma$ to be 1.65, approximately the mean value of $\gamma$ for the different cases. The regression results are presented in columns 4 and 5 in Table 1. For comparison we also list (in column 6) the value $r_0 = [A(3-\gamma)/3]^{1/\gamma}$, which is the correlation length at which $\xi = 1$ if the power law (5) is adopted. The errors quoted are derived from the bootstrap errors for individual data points.

Since the errors of different data points may not be independent, the real errors for both $\gamma$ and $A$ may be larger.

The values of the power-law index $\gamma$ and the correlation length $r_0$ that we obtained for CFA-CFA are in good agreement with those obtained by Davis & Peebles (1983) for the total sample. For IRAS-IRAS, the power-law index $\gamma \sim 1.9$ that we obtained is much larger than the fiducial value $\gamma = 1.6$. This discrepancy is due to the fact that there is a shoulder around $r = 4 \, h^{-1} \, \text{Mpc}$ in our correlation function, and that the regression is performed for $r > 2 \, h^{-1} \, \text{Mpc}$. By using all data points in the range $0.3 \, h^{-1} \, \text{Mpc} < r < 10 \, h^{-1} \, \text{Mpc}$, we obtain $\gamma = 1.60$ and $r_0 = 3.5 \, h^{-1} \, \text{Mpc}$. 
From Table 1, we can read off the following ratios between the amplitudes of the correlation functions (with \(\gamma\) fixed to be 1.65):

\[
A_{CC}:A_{SS}:A_{LL} = 1:(0.74 \pm 0.16):(0.48 \pm 0.08),
\]

\[
A_{LC}:A_{LS}:A_{CL} = 1:(0.77 \pm 0.20):(0.63 \pm 0.14),
\]

\[
A_{CC}:A_{CC} = 0.44 \pm 0.09,
\]

\[
A_{LS}:A_{SL} = 0.46 \pm 0.12,
\]

\[
A_{LL}:A_{LL} = 0.57 \pm 0.11,
\]

where C–C denotes CfA–CfA; S–S denotes Spiral–Spiral; L–I denotes IRAS–IRAS, L–C denotes LSB–CfA, and so on. The differences between the values in equation (6c) are only marginally significant. However, the basic result is clear: the amplitudes of the cross-correlation functions between LSB galaxies and trace galaxies are lower than the corresponding amplitudes of the autocorrelation functions of the trace galaxies by a factor of about 2.

### 4.2 The shapes of the cross-correlation functions

A remarkable property in the cross-correlation functions between the LSB galaxies and the trace galaxies is the flattening on small scales \((r \approx 2 \, h^{-1} \, \text{Mpc})\). Such a feature is absent in the autocorrelation functions of the trace galaxies, and corresponds to the strong effect on small scales reported by Bothun et al. (1993). To show this feature, we calculate the ratio, \(R(r)\), between the cross-correlation function and the corresponding autocorrelation function of trace galaxies. These ratios are shown in Fig. 8. Starting at about \(r \approx 2 \, h^{-1} \, \text{Mpc}\), the ratios decrease from the global values given by the correlation amplitudes listed in Table 1 to values that are about 5–10 times smaller at \(r \approx 1 \, h^{-1} \, \text{Mpc}\). In order to quantify this, we have performed a formal fit to the ratios, using a model of the form

\[
R(r) = R_0 \left[1 - \exp\left(-\left(\frac{r}{r_c}\right)^\beta\right)\right].
\]

The coefficient \(R_0\) is fixed to be the values given by the ratios between the amplitudes of the correlation functions (equation 6c). The values of \(\beta\) and \(r_c\), given by a least-squares fit, are listed in Table 1. The values of \(r_c\) given by a similar fit, but with fixed \(\beta = 2\), are also listed in Table 1. The results of \(R(r)\) given by this model (i.e. with \(\beta = 2\)) are shown in Fig. 8 by the solid curves. The decrease of the ratios for small \(r\) is quite rapid, with \(\beta = 2\) giving a reasonable fit to the data. The value of \(r_c\) is about \(2 \, h^{-1} \, \text{Mpc}\) for the CfA case, and about \(1.0 \, h^{-1} \, \text{Mpc}\) for the IRAS case, as can be seen by inspection of Fig. 8.

Our correlation functions are estimated by using (corrected) galaxy redshifts as distances. We should therefore consider how our results are affected by having worked in redshift rather than real space. There are two effects: the amplitudes of the correlation functions are altered on large scales, and the shapes are altered on small scales. The basic principles of redshift-space distortion were given by Kaiser (1987). The first effect should be small, or even absent in our cases, because we are considering fluctuations of the galaxy density field on a scale between 2 and 10 \(h^{-1} \, \text{Mpc}\) and the redshift-space enhancement of the correlation function is small in this regime (Suto & Suginoara 1991). The other effect of redshift-space distortions is that virialized random peculiar velocities tend to damp power at small wavelengths, which also reduces \(\xi\) at small separations. The effect can be modelled as a Gaussian convolution in the radial direction, and depends on the peculiar velocity involved. For a typical galaxy pairwise dispersion of 300 km s\(^{-1}\), the smearing ‘\(\sigma\)’ is about \(2 \, h^{-1} \, \text{Mpc}\). This would appear to be worrisome, because it is comparable to the values of \(r_c\) that we derived for the flattening scales of the cross-correlation functions between LSB and CfA (or IRAS) galaxies. If the smearing had such a big effect, however, we would expect to see a

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**Figure 8.** (a) The average cross-correlation functions between the POSS LSB sample and the CfA sample, divided by the average autocorrelation function of the CfA sample. Bullets show the case (POSS–CfA)/(CfA–CfA); squares show (POSS–Spiral)/(Spiral–Spiral). The error bars are given by 100 bootstrap resamplings of the POSS sample. A horizontal line having value unity is drawn for reference. The solid curve shows the model given by equation (7) with \(\beta = 2\) and \(r_c = 1.95 \, h^{-1} \, \text{Mpc}\). The two dashed curves show the results for models (see Section 5.1) with \(b = 1\) (short-dashed) and \(b = 2\) (long-dashed), and in which both 'LSB galaxies' and 'average galaxies' correspond to haloes with a circular velocity \(V = 200\) km s\(^{-1}\). The dot-dashed curve shows a model with \(b = 1.5\), and in which haloes for both LSB and average galaxies have a mass \(10^{12} \, M_\odot\) at formation. (b) The average cross-correlation functions between the POSS LSB and IRAS samples, divided by the average autocorrelation function of the IRAS sample (bullets). The error bars are given by 100 bootstrap resamplings of the POSS sample. The solid curve shows the model given by equation (7) with \(\beta = 2\) and \(r_c = 1.01 \, h^{-1} \, \text{Mpc}\).
similar or even stronger effect in the autocorrelation functions of CfA and IRAS galaxies, because they are more strongly clustered and presumably have larger pairwise velocity dispersions. That such an effect is not seen in the autocorrelation functions suggests that the smearing does not have a significant effect on their amplitudes. Indeed, if smearing is to be significant, there must be a conspiracy such that the real-space correlation function changes on just the right scale by just the right amount to maintain the smooth power laws observed in redshift space.

As a check we have calculated the projected cross-correlation function, \( w(r_p) \) (where \( r_p \) is the projected separation of galaxy pairs), between the LSB galaxies and CfA galaxies. The projected function is defined and calculated in the same way as in Davis & Peebles (1983). The result is shown in Fig. 9. The dotted line has a slope of \(-0.7\) which corresponds to a real-space power-law correlation function with slope \( y = 1.7 \). As discussed by Davis & Peebles (1983), the projected function should not be affected significantly by the peculiar velocities of galaxies. Clearly, the data have a much more flattened shape on small scales than expected from the power law. Such a flattening is not found in the autocorrelation functions of CfA and IRAS samples (Mo, Jing & Börner 1993b). A more accurate estimate of \( w(r_p) \), however, needs samples with larger sky coverage and higher surface density. We are unable to calculate \( w(r_p) \) for the IRAS case because of the low surface density of the sample. None the less, the behaviour of \( w(r_p) \) must be real and not a result of smearing effects, or Bothun et al. (1993) would not have detected the strong effect on small scales.

5 DISCUSSION

In this section we show how simple assumptions about the formation of LSB galaxies can reproduce the general properties of the cross-correlation functions found in this paper. We consider a simple hierarchical model in which galaxies are associated with regions identified in the initial density field. These regions form haloes within which the visible galaxies form. The haloes are presumably dominated by dark matter, but this is not essential to the model, which follows from the identification of galaxies with density fluctuations in the initial conditions and so depends only on the properties of Gaussian random fields. This simple model must be considered no more than speculative at the present level of our understanding of the physics of galaxy formation, but does help to illuminate the basic principles involved.

Within this framework, we identify LSB galaxies with those haloes that are sufficiently isolated that they have not been incorporated into larger systems by the present epoch. This is motivated by two inferences drawn from the physical properties of LSB galaxies. First, the slow evolution (McGaugh 1993) and young stellar populations (McGaugh & Bothun 1994), together with the low star formation rates (McGaugh 1992) and low gas surface densities (van der Hulst et al. 1993) of LSB galaxies, imply that they collapse late as the result of formation from relatively low (\( \lesssim 1\sigma \)) peaks in the initial density field (McGaugh et al., in preparation). Second, is the apparent lack of tidal interactions experienced by LSB galaxies over a Hubble time (Zaritsky & Lorrimer 1993; Bothun et al. 1993), as evidenced by the lack of nearby neighbours and star formation histories incompatible with burst and fade scenarios (Schombert et al. 1990; McGaugh & Bothun 1994). Our model for the formation of LSB galaxies will satisfy both of these, as galaxies that lack neighbours will not suffer tidal interactions and will, on average, form from fluctuations with lower initial amplitudes (the inverse of the oft-stated property of Gaussian fields that high peaks occur preferentially in regions of high density). This leads naturally to a correlation function (for LSB galaxies) that has a lower overall amplitude and a more flattened shape on small scales than that for average galaxies (average in the sense that no restriction is made on the location of their haloes). Indeed, it will be seen that the low average height of fluctuations corresponds to the low amplitude of the correlation function, and the need to avoid tidal interactions which enhance star formation (and thus surface brightness) corresponds to the flattening seen at small scales.

5.1 Correlation functions

To model the correlation properties of LSB galaxies, we assume that the initial overdensity field, \( \delta(x) \), is Gaussian and described by a power spectrum \( P(k) \). In particular, we consider CDM models using the spectrum given by BBKS. This choice is made so that we can evaluate a specific model; the general results are similar for any hierarchical scenario with Gaussian initial conditions.

The probability of finding a spherical region of comoving radius \( R \) with mean overdensity \( \delta \) (linearly extrapolated to present time) is

\[
p(\nu) d\nu = \frac{1}{\sqrt{2\pi}} \exp\left(-\nu^2/2\right) d\nu,
\]

where \( \nu = \delta/\Delta; \Delta = \Delta(R) \) is the rms linear overdensity in a sphere of radius \( R \), extrapolated to the present time. For a spherical perturbation and \( \Omega = 1 \), each shell recollapses to the origin at a time when the mean linear overdensity interior to it extrapolates to the value 1.68 (see, for example, Peebles 1980). It is therefore assumed that a region will be part of a single collapsed structure (halo) at redshift \( z \) if its linear overdensity, extrapolated to the present epoch, is 1.68\((1 + z)\) (see,
for example, Efstathiou et al. 1988b; White & Frenk 1991). The mass and circular velocity of a halo are, assuming the physical overdensity in the virialized sphere to be 178, related to the comoving radius \( R \) (in present units) and redshift \( z \) by

\[
M = \frac{4\pi}{3} \rho_0 R^3, \quad V_c = 1.67(1 + z)^{1/2} H_0 R,
\]

where \( \rho_0 \) and \( H_0 \) are, respectively, the mean density of the Universe and the Hubble constant at the present time. The quantities \( M, V_c, \Delta, \) and \( R \) are therefore equivalent variables for a given redshift.

If we choose \( R \) to be the linear scale (in present units) that gives \( M \) the value of the mass of typical galaxies (e.g. \( R \approx 0.5 \) h\(^{-1}\) Mpc), a region in the density field where \( \delta > 1.68(1 + z) \) will either itself be a halo for galaxy formation, or be a part of a larger halo which may form a bigger galaxy or a system of galaxies. To distinguish between these two different cases requires a complete description of the merging history of haloes. This is a complicated problem and will not be considered here. We will assume instead that, once a region in the density field (smoothed on galactic scales) reaches the critical overdensity \( 1.68(1 + z) \) at redshift \( z \), a galactic halo will form and exist in isolation or in a larger system without losing identification. This is equivalent to assuming that the merger of two or more galactic-size haloes to form a single galactic halo is not frequent. Such an assumption will not change our discussion of the correlation function of LSB galaxies, because in our model we assume that LSB galaxies are formed in haloes that are sufficiently isolated that they have not been incorporated into larger collapsed systems containing other galaxies by the present epoch.

According to Bower (1991) (see also Bond et al. 1991), the fraction of the mass in the Universe that is in haloes with mass in the range \( M \to M + dM \) at redshift \( z = \delta/1.68 - 1 \) is

\[
f(\Delta, \delta) d\Delta^2 = \frac{1}{(2\pi)^{7/2}} \frac{\delta}{\Delta} \exp\left( -\frac{\delta^2}{2\Delta^2}\right) d\Delta^2.
\]

(10)

Similarly, the fraction of the mass \( M_\Delta \) of a spherical region with (linearly extrapolated) overdensity \( \delta_\Delta \) that is in haloes with mass in the range \( M \to M + dM \) (i.e. \( M_\Delta < M_\Delta \)) at redshift \( z_\Delta = \delta_\Delta/1.68 - 1 \) is

\[
f(\Delta_\Delta, \delta_\Delta | \Delta_0, \delta_0, \delta_\Delta_0) d\Delta_\Delta^2 = \frac{1}{(2\pi)^{7/2}} \left( \frac{\Delta_\Delta - \Delta_0}{\Delta_\Delta - \Delta_0} \right)^{\delta_\Delta - \delta_0} \times \exp\left[ -\frac{(\delta_\Delta - \delta_0)^2}{2(\Delta_\Delta - \Delta_\Delta)} \right] d\Delta_\Delta^2,
\]

so the number of \( M_\Delta \) haloes in such a spherical region is

\[
N(1|0) = \frac{M_\Delta}{M_\Delta} f(\Delta_\Delta, \delta_\Delta | \Delta_0, \delta_0, \delta_\Delta_0).
\]

(12)

The average two-point correlation function (in Lagrangian space) at redshift \( z_\Delta \), between mass and haloes with mass \( M \), at redshift \( z_\Delta = \delta_\Delta/1.68 - 1 \) (called haloes of type 1), \( \xi_{1\Delta}(R_\Delta) \), can be formally written as

\[
\xi_{1\Delta}(R_\Delta) = \int \delta_\Delta p(0 | 1) d\delta_\Delta,
\]

(13)

where \( p(0 | 1) \) is the probability of finding a region with (linear) radius \( R_\Delta \) and overdensity \( \delta_\Delta \), given that there is in this region a halo of type 1. According to Bayes' theorem, \( p(0 | 1) \propto p(1 | 0) p(0) \), where \( p(0) \) is the probability of finding a region with radius \( R_\Delta \) and overdensity \( \delta_0 \), and \( p(1 | 0) \) is the probability of finding a halo of type 1 in such a region, so \( p(1 | 0) \propto N(1|0) \). The correlation function \( \xi_{1\Delta}(R_\Delta) \) can now be written as

\[
\xi_{1\Delta}(R_\Delta) = \frac{\int p(1 | 0) p(0) d\delta_\Delta}{\int p(1 | 0) p(0) d\delta_\Delta},
\]

(14)

where \( p(0) \) is given by equation (8) for a fixed \( R_\Delta, z_\Delta < z_\Delta \), and the integration limits are chosen so that the spherical region of radius \( R_\Delta \) has not collapsed by redshift \( z_\Delta \) (see Mo, Miralda-Escudé & Rees 1993a for a discussion).

It should be noted that equation (14) gives only the correlation function in comoving space, without taking into account any dynamical evolution. To treat the dynamical evolution accurately one needs to invoke numerical simulations. Here we calculate the correlation function in physical space, using a simple model of spherical perturbations. The details of this model will be presented elsewhere (Mo et al., in preparation). The general idea is that the physical radius (at a given redshift) of a spherically symmetric perturbation is uniquely determined by the comoving radius \( R_\Delta \) and the extrapolated overdensity \( \delta_\Delta \) (the upper limit in the integration ensures this), and that one can carry out the integration in equation (14) for regions with the same physical radius.

In Fig. 8(a) we plot the model prediction for the ratio between the correlation function of the haloes of LSB galaxies and that of average galactic haloes. Here, as discussed above, we identify LSB galaxies with galactic-sized haloes that have not been incorporated into larger structures containing other galaxies by the present epoch. For a moderate bias (e.g. \( b = 1.5 \)), the abundance of galactic-sized haloes (e.g. with circular velocities \( V_c = 200 \) km s\(^{-1}\)) peaks at \( z \approx 3 \); the peak redshift is \( z \approx 1 \) for \( b = 2.5 \) (see White & Frenk 1991). So, as illustration, we take \( z = 2 \) as the formation epoch of average haloes of galaxies. With these assumptions, several examples are presented. The two dashed curves show the results for models with \( b = 1 \) (short-dashed) and \( b = 2 \) (long-dashed), and in which both 'LSB galaxies' and 'average galaxies' correspond to haloes with a circular velocity \( V_c = 200 \) km s\(^{-1}\). Since, for a given circular velocity, the halo mass goes with redshift as \((1+z)^{-3/2}\), the mass of the haloes of 'LSB galaxies' in the above models is about five times that of 'average galaxies' at formation. The dot–dashed curve shows a model with \( b = 1.5 \), and in which haloes for both LSB and average galaxies have a mass of \( 10^{12} \) M\(_\odot\) at formation. To compare model predictions and our results, we also have to assume that the correlation functions of our track galaxies differ from the mass correlation function only by a constant bias factor which is cancelled out in the ratios shown in the figure.

It is gratifying that the simple models give the same trend as the data: the ratios remain relatively flat for separations \( r > 2 h^{-1} \) Mpc, and decrease with decreasing \( r \) for smaller separations. It is also interesting to note that the break scales
and the amplitudes of the ratios at large separations predicted by the models are comparable with those in the data. The agreement may be fortuitous, considering the uncertainties both in assigning galaxies to haloes and in the evolution of the correlation functions, but no fine tuning of parameters is necessary to reproduce the observational trends. The scale of the break at $2\ h^{-1}\ $Mpc corresponds perhaps to groups rather than to individual galaxies, so that tidal interactions between group members may enhance star formation, and hence surface brightness, if the group scale structure has had time to collapse (cf. Lacey & Silk 1991). LSB haloes will by definition not be contained within such structures, and so would not be expected to follow the same trends on small scales. The merging of haloes may be more significant in high-density regions, which would affect the correlation functions for normal galaxies on small scales.

This would, however, reduce the correlation on small scales for normal galaxies rather than enhance it. Our results suggest, therefore, that the observed deficit of pairs of LSB galaxies on small scales and the low amplitude of the correlation function may both be due to the fact that LSB galaxies are formed in haloes lacking close interactions with other haloes.

According to our assumption, the relative amplitudes of the cross-correlation functions at large separations (see equation 6c) can be considered to be the relative bias factor $b$ of LSB galaxies with respect to that of normal galaxies. We therefore have $b_{LSB}; b_{CMB}\approx 0.44$ and $b_{LRAS}\approx 0.57$. If normal galaxies are formed with a bias factor of 1.5–2, as is required to match the observed large-scale clustering and motions of galaxies (see e.g. Jing et al. 1993 for a summary), the bias factor for LSB galaxies on large scales in our sample is about 1 or smaller.

As discussed above, the model that we are considering predicts a lower value of the bias factor for the haloes of LSB galaxies (Fig. 8a). This is because haloes with larger overdensities are more likely to be in high-density regions and to be incorporated into bigger haloes. For a Gaussian density field, one can form a joint probability for the linearity overdensity $\delta_b$ of a galactic-sized region (with radius $R_g$ and mass $M_g$) and the overdensity $\delta_b$ of a larger region (with mass $M_\text{c}$) surrounding it: $P(\delta_b, \delta_\text{c})\ f_\text{c}\ dv_\text{c}$ (see appendix E in BBKS). One can therefore calculate the mean values of $\delta_b$ (for a given $M_g$) for average galactic haloes, and for haloes that have not been incorporated into haloes with mass $> M_\text{c}$. Fig. 10 shows the ratios $\delta_b/\delta_\text{c}$, where $\delta_b$ is the mean value of overdensities for average haloes of galactic size, and $\delta_\text{c}$ is for haloes that satisfy the ‘no-merging’ condition. The results shown are for the cold dark matter (CDM) spectrum (dashed curves), and for a power spectrum $P(k)\propto k^{-1.3}$ (solid curves), for different values of $M_\text{c}$ and $b$. The biased model (i.e. $b=2$ for normal galaxies) gives larger ratios than the unbiased one because the structure in the former case is less evolved, and there are, therefore, more high-density regions that have not yet been incorporated into large collapsed systems. A larger $M_\text{c}$ also gives a larger ratio, as expected. The ratios increase with $R_g$, the linear scale for galactic haloes, because, in a hierarchical clustering scenario, larger haloes form later and hence have less chance to be incorporated into larger systems. It is encouraging that the ratios predicted by models with reasonable values of $R_g$ (e.g. $R_g=0.5\ h^{-1}\ $Mpc) are comparable with the ratios derived from the correlation properties of the LSB galaxy sample and inferred from the physical properties of LSB galaxies (i.e. $\delta_b/\delta_\text{c}\leq1/2$; McGaugh 1992; McGaugh et al., in preparation). That the current simple model adequately reproduces the observational trends lends support to the notion that galaxies did indeed form by gravitational instability from small fluctuations in the primordial density field.

Given the significant environmental dependence on surface brightness detected here, and the lack of such a dependence on luminosity (Alimi et al. 1988; Hamilton 1988; Börner & Mo 1990) or circular velocity (Mo & Lahav 1993), it is worth noting that it is the height of fluctuations (or, equivalently, collapse time) which should be correlated with environment. Indeed, as shown in Fig. 8a, haloes formed at $z=2$ can be more strongly correlated with mass than haloes at $z=0$, even though they have smaller masses or circular velocities. The height of fluctuations is more directly related to surface brightness than to total luminosity or mass (McGaugh 1992), so our results suggest that a strong luminosity segregation in galaxy clustering is not a necessary consequence of biased galaxy formation, unless the effect of surface brightness is also taken into account. It is often stated that LSB galaxies should ‘fill the voids’, but this is misleading. They should have lower correlation amplitudes, but they should still be correlated. If they form from $\sim 1\sigma$ fluctuations and avoid interactions with other galaxies, as seems to be the case, then they should trace structure on scales larger than a few megaparsecs, and possibly in an unbiased way.

5.2 Model predictions

Since our simple model adequately reproduces the correlation function of LSB galaxies, it is interesting to see what pre-
dictions it makes for other properties of LSB galaxies. Because LSB galaxies correspond to haloes with lower initial density in our model, they form late and so we predict that LSB galaxies should be relatively young. Observationally, this appears to be the case, as LSB galaxies are unevolved (Bothun et al. 1990; McGaugh 1993) and have stellar populations with a young mean age (McGaugh & Bothun McGaugh & Bothun 1994; van der Hulst & de Blok, in preparation). The relative formation epoch goes roughly as \((1 + z_{\text{LSB}})/(1 + z_{\text{average}}) \approx \delta_f/\delta_g\). If the average haloes have a mean formation epoch \(z_{\text{average}} \approx 2\), then the average formation epoch for haloes of LSB galaxies is about \(z_{\text{LSB}} \approx 0.5\).

An important issue is the total mass fraction contained in LSB galaxies. From equation (10), one can calculate the mass fraction in LSB haloes in a given mass (or circular velocity) range. The result is shown in Fig. 11. Models with higher bias parameters give higher fractions, because a higher bias parameter corresponds to lower amplitudes of the initial density fluctuations. The fraction depends, however, on the maximum circular velocity (or mass and radius) for galaxy formation, \(V_u\), and the degree of isolation required to prevent the enhancement of surface brightness by tidal interaction. The mass fraction of LSB galactic haloes increases with \(V_u\) because larger haloes form later and so have less chance to be incorporated into larger systems (and are thus considered to be LSB galactic haloes in our simple model). The choice of the correct \(V_u\) requires one to distinguish between the haloes that will form groups of galaxies and those that will form single, giant galaxies. This in turn requires knowledge of the details of the merging history of haloes (see Section 5.1), and \(V_u\) may well be dependent upon environment. In this context, it is worth noting that very massive but very LSB galaxies like Malin 1 (Bothun et al. 1987) do exist. These would carry a large fraction of the mass of the Universe within the framework of the current model. Even for modest \(V_u \approx 250 \, h^{-1} \, \text{Mpc}\) (not large enough to accommodate the existence of Malin 1), the model predicts that up to 30 per cent of the mass can be in LSB galactic haloes. This fraction could, however, be rather lower if weak interactions were sufficient to induce significant star formation. The large scale \((\sim 2 \, h^{-1} \, \text{Mpc})\) of the break in the LSB correlation function would seem to argue against this, but clearly the simple model does not provide a robust prediction of the mass fraction of LSB galaxies. It is suggestive that this could be significant, though.

6 SUMMARY

We have investigated the clustering properties of LSB galaxies in the galaxy density field. We have used a cross-correlation technique to determine the cross-correlation functions between LSB galaxies and (trace) galaxies in complete (CTA and IRAS) samples. This enables us to compare directly the amplitudes and shapes of the correlation functions of LSB galaxies with those for ‘normal’ galaxies. Our main results are summarized as follows.

(i) For pair separations \(r \geq 2 \, h^{-1} \, \text{Mpc}\), the cross-correlation functions between the LSB galaxies and the trace galaxies, \(\xi_{AB}(r)\), have approximately the same shape, with power-law index \(\gamma \approx 1.7\), as the autocorrelation functions for CTA and IRAS galaxies. The amplitudes (\(A\)) of the cross-correlation functions are lower, with \(A_{\text{LSB}-\text{CTA}}: A_{\text{CTA}-\text{CTA}} \approx 0.4\) and \(A_{\text{LSB}-\text{IRAS}}: A_{\text{IRAS}-\text{IRAS}} \approx 0.6\). Hence LSB galaxies are embedded in the same large-scale structures as normal galaxies, but are significantly less strongly clustered. This is the expectation for galaxies that form at the maxima in Gaussian random fields when LSB galaxies correspond to lower initial densities. This lends support to hierarchical models with Gaussian initial conditions, and suggests that LSB galaxies may be unbiased tracers of the mass density.

(ii) For \(r \leq 2 \, h^{-1} \, \text{Mpc}\), the cross-correlation functions are significantly lower than expected from the extrapolation of \(\xi_{AB}\) on larger scales, suggesting that the formation of LSB galaxies (or their survival as such) may be affected by interaction with neighbouring galaxies.

(iii) We have shown that a simple hierarchical model, in which LSB galaxies form only in haloes lacking close interactions with other haloes, successfully reproduces both the observed deficit of LSB galaxy companions and the low amplitude of the correlation function for LSB galaxies.

(iv) This model predicts that LSB galaxies are, on average, younger than normal galaxies, consistent with observations. Such a model also suggests that a significant fraction of the mass in the Universe is associated with LSB galaxies. Many such galaxies may still remain undetected because of the selection effects against low-surface-brightness objects.

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