The dark and visible matter content of low surface brightness disc galaxies

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Accepted 1997 May 30. Received 1997 May 2; in original form 1996 September 11

Abstract

We present mass models of a sample of 19 low surface brightness (LSB) galaxies and compare the properties of their constituent mass components with those of a sample of high surface brightness (HSB) galaxies. We find that LSB galaxies are dark matter dominated. Their halo parameters are only slightly affected by assumptions on stellar mass-to-light ratios. Comparing LSB and HSB galaxies we find that mass models derived using the maximum disc hypothesis result in the discs of LSB galaxies having systematically higher stellar mass-to-light ratios than HSB galaxies of similar rotation velocity. This is inconsistent with all other available evidence on the evolution of LSB galaxies. We argue therefore that the maximum disc hypothesis does not provide a representative description of the LSB galaxies and their evolution. Mass models with stellar mass-to-light ratios determined by the colours and stellar velocity dispersions of galactic discs imply that LSB galaxies have dark matter haloes that are more extended and less dense than those of HSB galaxies. Surface brightness is thus related to the halo properties. LSB galaxies are slowly evolving, low-density and dark matter dominated galaxies.

Key words: galaxies: fundamental parameters – galaxies: haloes – galaxies: kinematics and dynamics – galaxies: spiral – dark matter.

1 Introduction

The discrepancy between the amount of matter implied by the H\(_{\text{I}}\) rotation curves of spiral galaxies and the amount of matter actually observed (in the form of stars and gas) is usually interpreted as dark matter (DM) haloes that surround the directly observable parts of galaxies, although alternative theories of modified Newtonian dynamics (Milgrom 1983) also provide an efficient description of the observed rotation curves (Sanders 1997). Early studies by e.g. Kallnaj (1983) claimed that, based on the optical rotation curves that were used, there was no need to invoke DM within the optical radius for some galaxies. H\(_{\text{I}}\) observations, however, showed a dramatically different picture (Bosma 1978; Begeman 1987). Substantial amounts of DM were needed to describe the observed flat rotation curves outside the optical disc.

Measurements of the distribution of the DM must be extracted from the rotation curve. These are usually made by computing the rotation curves of the visible matter and subtracting these from the observed total rotation curve. The residuals show the dynamical signature of DM. This procedure typically has the following free parameters: a length scale and density for the halo and a mass-to-light ratio (M/L)\(_{\text{r}}\) of the stellar component. A major problem is that the value of (M/L)\(_{\text{r}}\) is not known a priori, and may differ from galaxy to galaxy. Many different values of (M/L)\(_{\text{r}}\), and therefore many combinations of halo parameters, yield equally good descriptions of the data (van Albada et al. 1985). It is possible for many galaxies to make a good fit to the rotation curve by completely ignoring the contribution of the stellar disc, even though it obviously plays an important role in the inner parts of many galaxies. Without additional knowledge to constrain (M/L)\(_{\text{r}}\) (e.g. information about vertical stellar velocity dispersion or information derived from stellar population synthesis models) it is not possible to unambiguously determine its value. Goodness-of-fit estimators are sensitive to small variations and uncertainties in the data (even when the data is of superior quality) (Lake & Feinwog 1989) and are thus less suited for simultaneously determining all free parameters in a fit. The mathematically ‘best’ fit is therefore not always the physically most meaningful one.

Using the observation that the rotation curves derived for the stellar components of HSB galaxies can usually be scaled so that they almost completely describe the observed total rotation curves in the optical discs of these HSB galaxies (e.g. Kent 1986), van Albada & Sancisi (1986) introduced the maximum disc hypothesis. This hypothesis reduces the number of free parameters in rotation curve fits by maximizing the contribution of the stellar disc [i.e. (M/L)\(_{\text{r}}\) and thus minimizing the amount of DM invoked to explain the observed rotation curves.

It is still unclear whether the maximum disc hypothesis is a realistic one. For some HSB galaxies stellar population synthesis models yield values of (M/L)\(_{\text{r}}\) that are consistent with the maximum disc measurements (van Albada & Sancisi 1986). The presence of disc features such as spiral arms or bars argues for a high (M/L)\(_{\text{r}}\),
2 THE LSB SAMPLE

We refer to BMH96 for an extensive sample description. We have supplemented that sample with the rotation curves of an additional five LSB galaxies as presented in van der Hulst et al. (1993). The rotation curves in general have maximum rotation velocities of between 50 and 120 km s$^{-1}$, and rise more slowly than HSB curves with similar maximum velocity. Only a few of the rotation curves show clear signs of flattening towards the outermost radii. Optical properties of the sample are presented in McGaugh & Bothun (1994), de Blok et al. (1995), and van der Hulst et al. (1993). The properties of the sample galaxies are summarized in the top panel of Table 1. We adopt a Hubble constant of 75 km s$^{-1}$ Mpc$^{-1}$.

As described in BMH96 the rotation curves were derived from major axis position–velocity diagrams. Inclinations were determined from a comparison of optical, H$\alpha$ and kinematical inclinations. The position angle and inclination were both kept fixed during the derivation of the curves. Inspection of the position–velocity diagrams in fig. 2 of BMH96 shows that not all rotation curves are well suited for a disc–halo analysis. A number of observations clearly suffer from low resolution or low inclination.

In order to select only those galaxies where a disc–halo decomposition is a sensible exercise, we restricted the sample by rejecting those galaxies that did not meet one of the following criteria:

(i) inclination $i > 25^\circ$,
(ii) radius $R > 2$ beams,
(iii) asymmetries between approaching and receding sides of rotation curves $< 10$ km s$^{-1}$.

This resulted in the rejection of seven galaxies. These are listed in Table 2 with the reason for rejection.

The choice of a 25$^\circ$ cut-off may seem a rather liberal choice, which in a sense we are forced to apply due to the selection biases against high-inclination LSB galaxies (Davies 1990; Schombert et al. 1992; Dalcanton & Shectman 1996). An analysis of the scatter in the LSB TF relation presented in Zwaan et al. (1995) does show, however, that this scatter does not increase significantly when galaxies with increasingly lower inclination are included until a value of 25$^\circ$.

The effects of lower resolution are more difficult to model, and will be described extensively in the next section.

3 BEAM-SMERRING

Early high-resolution studies of spiral galaxy kinematics using synthesis radio telescopes did mostly target galaxies with large angular sizes (e.g. Bosma 1978; Begeman 1987; Broeils 1992). The rotation curves of these galaxies are in general well-determined. Unfortunately there are only a limited number of galaxies in the sky that have suitably large angular sizes for such detailed investigations. Galaxies with smaller apparent sizes (because of smaller intrinsic sizes and/or larger distances) can only be observed at a smaller linear resolution.

A side-effect of this lower resolution is that the line profiles will be artificially broadened and any sudden change in the velocity gradient within the beam will be smoothed out. The observed change in the velocity gradient will thus appear to be smaller and the rotation curve will appear to rise less steeply. This effect is called ‘beam smearing.’

Bosma (1978) investigated the behaviour of a steeply rising model rotation curve at different resolutions. He showed that if the ratio between the half-power radius of a galaxy and the half-power
\begin{table}
\centering
\caption{Properties of LSB and HSB samples.}
\begin{tabular}{llllllll}
\hline
Name & $D$ (Mpc) & $\mu_0(B)$ (mag arcsec$^{-2}$) & $h$ (kpc) & $B - V$ (mag) & $M_{\text{max}}(B)$ (mag) & $R_{25}$ (kpc) & $R_{\text{max}}$ (kpc) & $V_{\text{max}}$ (km s$^{-1}$) \\
\hline
\textbf{LSB sample} (this work) \hfill \& \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill \\
F561-1 & 63 & 23.3 & 3.6 & 0.55 & -17.8 & 6.4 & 10.1 & 52 \\
F563-1 & 45 & 23.6 & 2.8 & 0.64 & -17.3 & 5.0 & 17.7 & 111 \\
F563-V1 & 51 & 24.3 & 2.4 & 0.59 & -16.3 & 3.2 & 7.4 & 30 \\
F563-V2 & 61 & 22.1 & 2.1 & 0.51 & -18.2 & 6.8 & 9.2 & 111 \\
F564-V3 & 6 & 24.1 & 0.4 & 0.56 & -11.1 & 0.3 & ~2 & ~40 \\
F565-V2 & 48 & 24.7 & 2.7 & 0.51 & -15.4 & 2.3 & 8.4 & 51 \\
F567-2 & 75 & 24.4 & 5.7 & 0.61 & -17.4 & 3.2 & 11.3 & 64 \\
F568-1 & 85 & 23.8 & 5.3 & 0.58 & -18.1 & 7.4 & 14.9 & 119 \\
F568-3 & 77 & 23.1 & 4.0 & 0.61 & -18.3 & 7.9 & 16.5 & 120 \\
F568-V1 & 80 & 23.3 & 3.2 & 0.57 & -17.9 & 6.2 & 19.0 & 124 \\
F571-8 & 48 & 23.9$^a$ & 5.2$^a$ & * & -17.6$^a$ & 7.7 & 15.6 & 133 \\
F571-V1 & 79 & 24.0 & 3.2 & 0.55 & -17.0 & 3.8 & 14.6 & 73 \\
F571-V2 & 16 & * & * & * & * & 3.7 & 45 \\
F574-1 & 96 & 23.3$^a$ & 4.3 & * & -18.4$^a$ & 17.7 & 15.4 & 100 \\
F574-2 & 88 & 24.4 & 6.0 & 0.59 & -17.6 & 3.8 & 10.7 & 40 \\
F577-V1 & 80 & 24.0 & 4.3 & 0.40 & -18.2 & 6.6 & 8.9 & 30 \\
F579-V1 & 85 & 22.8$^a$ & 5.1 & * & -18.8$^a$ & 11.1 & 17.3 & 100 \\
F583-1 & 32 & 24.1 & 1.6 & * & -16.5 & 3.3 & 14.6 & 85 \\
F583-4 & 49 & 23.8$^a$ & 2.7 & * & -16.9$^a$ & 6.4 & 10.0 & 67 \\
U0128 & 60 & 24.2 & 6.8 & 0.60 & -18.8 & 9.0 & 42.3 & 131 \\
U1230 & 51 & 23.3 & 4.5 & 0.47 & -18.3 & 3.2 & 34.7 & 102 \\
U5005 & 52 & 23.8$^a$ & 4.4 & * & -17.8$^a$ & 10.4 & 27.8 & 99 \\
U5750 & 56 & 23.5$^a$ & 5.6 & * & -18.7$^a$ & 9.5 & 21.8 & 75 \\
U5999 & 45 & 23.5$^a$ & 4.3 & * & -17.6$^a$ & 9.8 & 15.3 & 155 \\
\textbf{HSB sample} (Broeils 1992) \hfill \& \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill & \hfill \\
DDO154 & 4 & 23.2 & 0.5 & 0.29 & -13.8 & 1.0 & 7.6 & 48 \\
DDO168 & 3.5 & 23.4 & 0.9 & 0.22 & -15.2 & 1.7 & 3.4 & 55 \\
DDO170 & 12 & 4 & 1.3 & * & -14.5 & 2.5 & 9.6 & 66 \\
N55 & 1.6 & 21.3 & 1.6 & 0.38 & -18.6 & 8.9 & 10.2 & 87 \\
N247 & 2.5 & 23.4 & 2.9 & 0.45 & -18.0 & 7.3 & 9.9 & 108 \\
N300 & 1.8 & 22.2 & 2.1 & 0.56 & -17.8 & 5.1 & 10.6 & 97 \\
N801 & 79.2 & 21.9 & 12 & 0.66 & -21.7 & 36.4 & 58.7 & 222 \\
N1003 & 11.8 & 21.7 & 1.9 & 0.40 & -19.2 & 9.2 & 31.3 & 115 \\
N1560 & 3 & 23.2 & 1.3 & 0.43 & -15.9 & 4.3 & 8.3 & 79 \\
N2403 & 3.3 & 21.4$^b$ & 2.1 & 0.38 & -19.3 & 8.4 & 19.5 & 136 \\
N2841 & 18 & 21.1$^c$ & 4.6 & 0.80 & -21.7 & 22.6 & 81.1 & 323 \\
N2903 & 6.4 & 20.5$^b$ & 2.0 & 0.59 & -21.0 & 11.7 & 24.2 & 201 \\
N2998 & 67.4 & 20.3 & 5.4 & 0.48 & -21.9 & 28.2 & 46.6 & 214 \\
N3109 & 1.7 & 23.2 & 1.6 & * & -16.8 & 4.3 & 8.2 & 67 \\
N3198 & 9.4 & 21.6$^b$ & 2.6 & 0.46 & -19.4 & 11.4 & 29.9 & 157 \\
N5033 & 11.9 & 23.0$^b$ & 5.8 & 0.48 & -20.2 & 18.3 & 35.4 & 222 \\
N5533 & 55.8 & 23.0 & 11.4 & 0.80 & -21.4 & 26.2 & 74.4 & 273 \\
N5585 & 6.2 & 21.9 & 1.4 & 0.42 & -17.5 & 4.7 & 9.6 & 92 \\
N6503 & 5.9 & 21.9 & 1.7 & 0.56 & -18.7 & 5.4 & 22.2 & 121 \\
N6674 & 49.3 & 22.5 & 8.3 & 0.61 & -21.6 & 29.8 & 64.5 & 266 \\
N7331 & 14.9 & 21.5$^b$ & 4.5 & 0.70 & -21.4 & 23.4 & 36.7 & 241 \\
U2259 & 9.8 & 22.3$^b$ & 1.3 & * & -17.0 & 3.7 & 7.6 & 90 \\
U2885 & 78.7 & 22.0$^b$ & 13 & * & -22.8 & 62.8 & 72.5 & 298 \\
\hline
\end{tabular}
\end{table}

$^a$Converted from $R$-band measurement.

$^b$Derived from $r$-band profiles, converted using $B = r + 1$ (Kent 1986).

$^c$Derived from $B$-band profiles, converted assuming $B = r + 1$ (Kent 1986).

$\mu_0$ (face on) of F571-8 was computed assuming $\mu_0 (face \ on) = (z/h) \mu (edge \ on)$ (i.e. optically thin).

$\mu_0$ (face on) was found to be $\sim 1.4 \mu (edge \ on)$ following the method of Peletier et al. (1995).
Table 2. Rejected galaxies.

<table>
<thead>
<tr>
<th>Name</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>F561-1</td>
<td>( R &lt; 2 ) beams; ( i &lt; 25^\circ )</td>
</tr>
<tr>
<td>F563-V1</td>
<td>( R &lt; 2 ) beams; gross asymmetry</td>
</tr>
<tr>
<td>F564-V3</td>
<td>( R &lt; 2 ) beams; low S/N</td>
</tr>
<tr>
<td>F567-2</td>
<td>gross asymmetry; ( i &lt; 25^\circ )</td>
</tr>
<tr>
<td>F574-2</td>
<td>( R &lt; 2 ) beams; V(R) steeply rising</td>
</tr>
<tr>
<td>F577-V1</td>
<td>( R &lt; 2 ) beams; gross asymmetry</td>
</tr>
<tr>
<td>F579-V1</td>
<td>gross asymmetry</td>
</tr>
</tbody>
</table>

beam width is smaller than \( \sim 7 \) to \( \sim 10 \), beam smearing can have serious effects on the steep inner parts of the curve. Rubin et al. (1989) make the point that if one wants to decompose the mass distribution of a galaxy into its disc and halo components one needs rotation curves of high accuracy in their inner portions, as 'the maximum mass which can reside in the disc is constrained principally by the inner rise of the rotation curve'. This statement is based on investigations into a sample of primarily steeply rising rotation curves, but remains true also for slowly rising rotation curves, although, as we will show, to a much lesser extent.

The rotation curves of LSB galaxies presented in BMH96 are not high-resolution rotation curves and beam smearing is a potential problem. The decrease in the slope of the rotation curves would disguise potentially steep rotation curves as gently rising solid-body curves. Here we will present some arguments why this effect is not significant in the BMH96 data, implying that the rotation curves of LSB galaxies are truly slowly rising.

3.1 Evidence from BMH96

The most direct evidence of why beam smearing effects do not dominate the data from BMH96 comes from that data itself. If the data were severely affected by resolution effects, one would not expect to observe steeply rising rotation curves.

Fig. 1 shows the major axis position–velocity diagrams of LSB galaxies F579-V1 and F568-1. Both galaxies are at similar distances (77 versus 85 Mpc, a difference of less than 10 per cent); both were observed with the same telescope with identical beam sizes; both have an inclination of \( 26^\circ \), a comparable H\(_1\) distribution and almost identical scalelengths. The effects of beam smearing are effectively identical in both galaxies.

It is clear that the rotation curve of F579-V1 rises much more steeply than that of F568-1. The fact that we do observe a steeply rising rotation curve in F579-V1 shows that the approximately solid-body rotation observed in F568-1 is not caused by beam smearing. This makes it unlikely that the solid-body rotation observed in other LSB galaxies from BMH96 is caused by observational effects. It should be noted that the solid-body rotation is also seen in those LSB galaxies observed at higher resolutions (e.g. F563-1 and F583-1) and is in general found in other high-resolution observations of very late-type galaxies (see e.g. IC 2574 in Martinseau, Carignan & Roy 1994).

3.2 Modelling beam smearing

Another way to quantify the effects of beam smearing is to construct model galaxies with known properties and 'observe' these at increasingly lower resolutions. As beam smearing affects the full two-dimensional velocity field of a galaxy, it is necessary to construct complete model data cubes, smooth these to lower resolutions using a Gaussian beam and then construct velocity fields and derive rotation curves.

3.2.1 Constructing models

We constructed a number of model data cubes using the task GALMOD in the Groningen Image Processing System GIPSY. This program distributes 'H\(_1\) clouds' in a specified data cube using an input radial H\(_1\) distribution and rotation curve as distribution functions. The galaxy can be given any inclination or position angle. This model can then be 'observed' at any desired spatial and velocity resolution. For simplicity we adopted a uniform, constant density H\(_1\) distribution for all models. We could have used any H\(_1\) distribution, but decided to opt for the simplest distribution to isolate the effects of beam smearing.

The magnitude of the beam smearing effects also depends on inclination. At high inclinations the beam will cover a larger part of the velocity field off the major axis than at lower inclination, thus
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'Diluting' the major axis rotation velocity. For a fixed beam size, the effects of beam smearing will still depend on inclination, with the worst effects occurring in the highest inclination galaxies.

In order to mimic the BMH96 observations, we chose an inclination of 40° for our model galaxies, equal to the average inclination of the 12 F' galaxies in BMH96. To cover the full range of known rotation curve shapes we adopted three different input rotation curves. These are shown in Fig. 2. Radii and velocities are all expressed as fractions of the maximum radius and velocity of the input models $R_{\text{max}}$ and $V_{\text{max}}$. These were identical for all three models.

The first model has a constant rotation velocity at all radii, and is the perfect example of a flat rotation curve. This is an extreme, but not unrealistic, representation of the steeply rising rotation curves found in early-type HSB galaxies (e.g. UGC 2885 or NGC 2841 in the compilation by Rhee 1996).

The second curve is a simplified version of that presented in Fall & Efstathiou (1980):

$$V(r) = V_{\text{max}} \sqrt{\frac{r^2}{r^2 + d^2}}.$$  

The constant $d$ was given a value of $0.2R_{\text{max}}$. This model rotation curve (which we will refer to as the 'FE curve') resembles that of a normal late-type galaxy (see e.g. the high-resolution rotation curves of NGC 1560, 3198 or 2403 in Rhee 1996).

The third curve is another extreme case: pure solid-body rotation with a still increasing rotation velocity at $R_{\text{max}}$. This solid-body rotation is found in dwarf galaxies (see e.g. the well-defined high-resolution curve of IC 2574 in Martin 1994 or NGC 3109 from Rhee 1996).

The full three-dimensional high-resolution model data cubes constructed using these curves were each smoothed with Gaussian beams with FWHM sizes of (1/10, 1/8, 1/6, 1/4, 1/2) × $R_{\text{max}}$. This obviously resulted for each of the three cases in data cubes where the radius of the disc measured 10, 8, 6, 4 and 2 beams respectively. Velocity fields were constructed from these cubes using the GIPSY task MOMENTS. Analogous to the procedure outlined in BMH96, the rotation curves of each of the smoothed data cubes were determined from major axis position–velocity diagrams.

As noted before, all models were computed assuming model galaxy inclinations of 40°. The original and smoothed de-projected curves are presented in Fig. 2. A first glance shows that indeed the effects of beam smearing are most severe for the flat rotation curve which has the steepest inner 'slope'. The beam smearing effects are negligible in the solid-body curve, where the only effect is an apparent flattening at the outermost radii. The FE curve retains its shape and slope down to a resolution of 4 beams. The input and output curves only start to differ significantly at resolutions of 2 beams.

3.2.2 Comparison with observations

We now compare the smoothed models with some of the observations from BMH96. We will use the rotation curves of F568-3 and F568-V1. These galaxies both have inclinations of 40° and are thus directly comparable with the model curves. Both curves have a resolution of slightly less than 4 beams and should thus be compared to the 4-beam models, as is done in Fig. 3. The observed curves were scaled using the $R_{\text{max}}$ and $V_{\text{max}}$ given in BMH96.

It is clear that the observed curves are best matched by the FE curve, although a completely solid-body curve cannot be ruled out at a resolution of 4 beams. The only way the observations can be reconciled with rotation curves that are flat down to a small radius is to use a resolution of 2 beams, that is, a factor of 2 worse than the present observations. At a resolution of 4 beams a flat curve can be ruled out.

The question that remains to be answered, then, is: how likely is it that LSB galaxies have FE-like rotation curves? First, as is shown by the studies of Casertano & van Gorkom (1991) and Broeils (1992) of high-resolution H I rotation curves, only those galaxies with maximum rotation velocities of more than 180 km s⁻¹ ($M_{\odot} < -19$) have steep inner rotation curves. The steep inner curves are usually interpreted as the dynamical signature of a strong bulge component. Galaxies that rotate slower have gradually

![Figure 2. Model rotation curves illustrating the effects of beam smearing. The heavy lines show the input rotation curves: flat (left panel), late-type-like (centre), and solid-body (right). The beam smeared versions of these curves are shown as the dashed and dotted lines. These lower resolution curves were derived using the full three-dimensional data cubes. The resolution expressed in beams is 10 (dotted), 8 (short dash), 6 (long dash), 4 (short dash-dot) and 2 (long dash-dot). It is clear that the effects of beam smearing are most severe in the flat case (left panel), while the effects of beam smearing are only small in the inner parts of the late-type and solid-body curves. In the latter two cases the shape of the curves do not change dramatically down to a resolution of 2 beams. The small bump visible in the FE-curves at $R = 1$ is caused by the limited velocity resolution used in the models, in order to mimic the BMH96 observations.](image-url)
rising rotation curves, resembling the FE model curve. The LSB galaxies from BMH96 have maximum rotation velocities between 50 and 120 km s$^{-1}$, luminosities $M_B$ between $-16$ and $-18$, and have only the faintest trace of a central condensation. So, if LSB galaxies are late-type galaxies, as all of the available evidence suggests, then it is not surprising that they have slowly rising rotation curves.

This is supported by work from Corradi & Capaccioli (1990) who measured the inner slopes of optical rotation curves of 139 galaxies of a large range of Hubble types and luminosities. They found a strong correlation between the magnitude of this slope, Hubble type and luminosity: 75 per cent of the galaxies in their sample with luminosities $19 < M_B \leq -18$ have slowly rising, non-flat rotation curves,\(^1\) while all of the galaxies fainter than $-18$ have slowly rising rotation curves. The result holds for the relation with Hubble types: 80 per cent of the galaxies with Hubble type Sd and later have slowly rising rotation curves.

\section{3.3 Evidence from NGC1560}

As LSB galaxies are late-type galaxies, a third possibility for investigating the effects of beam smearing is by taking a high-resolution observation of a late-type galaxy and smoothing it to lower resolution. We have selected NGC 1560 for this purpose. This galaxy is an LSB dwarf galaxy $\mu_0(B) = 23.2$ mag arcsec$^{-2}$, for other properties see Table 1], for which a high-resolution rotation curve is available (Broeils 1992). This curve is shown in the left panel of Fig. 4. The beam size for this WSRT observation was $13 \times 14$ arcsec$^2$, and the extent of the rotation curve is 38 beams. Combined with the small distance of only 3 Mpc, we can be sure that beam smearing is of no importance in this observation.

We have used one-dimensional smoothing with a Gaussian filter to bring the total rotation curve and those of the stars and gas to lower resolutions. As the effects of one-dimensional smoothing of the rotation curve are less severe than full two-dimensional smoothing of the velocity field, we have smoothed the curves to a resolution of 3 beams. We found by trial and error that this corresponds most closely to a full two-dimensional smoothing with a resolution of 4 beams.

The smoothed curve is shown in the right panel of Fig. 4. The other curves shown in the figure will be discussed in Section 5.1. A comparison of the high- and low-resolution curves shows that the only effects this beam smearing exercise had on the curves is a loss of detail. The shapes and amplitudes of the curves have remained unchanged. This again shows that slowly rising rotation curves can be studied at much lower resolutions than rapidly rising ones without a dramatic loss of information. We conclude that the rotation curves of the LSB galaxies are intrinsically slowly rising. Beam smearing effects do not dominate the BMH96 data.

\section{4 DISK–HALO DECOMPOSITION}

We now return to the disk–halo decompositions and subsequent mass modelling. Before giving the results of the decompositions, we first describe the mass components used.

\textit{Stellar disc.} For the stellar disc the B- and R-band photometry presented in de Blok et al. (1995), and McGaugh & Bohlin (1994) was used. The rotation curve of the disc was computed following Casertano (1983) and Begeman (1987). The disc was assumed to have a vertical sech-squared distribution with a scaleheight $z_0 = h/6$ (van der Kruit & Searle 1981). The exact value of $z_0$ does not influence the amplitude and shape of the rotation curve to any significant degree. No bulge component was included as there is no evidence that bulges contribute significantly to the mass distribution in most LSB galaxies (see de Blok et al. 1995, McGaugh,
Bothun & Schombert 1995). Prior to the decomposition the light distribution curves were resampled to the H\textsc{i} resolution.

Isothermal dark halo. We assume a spherical pseudo-isothermal halo with a density profile

$$\rho(R) = \rho_0 \left[ 1 + \left( \frac{R}{R_C} \right)^2 \right]^{-1},$$

(1)

where $\rho_0$ is the central density of the halo and $R_C$ the core radius of the halo. This density profile results in the rotation curve

$$V_{\text{halo}}(R) = \sqrt{4\pi G \rho_0 R_C^2 \left[ 1 - \frac{R_C}{R} \frac{\arctan \left( \frac{R}{R_C} \right)}{R_C} \right]},$$

(2)

The asymptotic velocity of the halo, $V_\infty$, is given by

$$V_\infty = \sqrt{4\pi G \rho_0 R_C^2}.$$  

(3)

To characterize this halo only two out of the three parameters ($\rho_0$, $R_C$, $V_\infty$) are needed, as equation (3) determines the value of the third parameter. The mass of the halo integrated out to radius $r$ is

$$M_d(r) = \frac{V_r^2}{G} = 4\pi \rho_0 R_C^2 \left[ r - R_C \frac{\arctan \left( \frac{r}{R_C} \right)}{R_C} \right].$$

(4)

Because $(M/L)_e$ is unknown we present disc–halo decompositions using three different assumptions for $(M/L)_e$:

(i) maximum disc (Section 4.2),

(ii) minimum disc (Section 4.3), and

(iii) ‘Bottema disc’ (Section 4.4).

Each of the rotation curves was fitted using a program that determines the best-fitting combination of $R_C$, $V_\infty$, $(M/L)_e$, using a least-squares fitting. Any combination of these three parameters could be fitted, or kept fixed at some initial value. The program uses as input the rotation curve of the gas, the total measured rotation curve, and the rotation curve of the stellar disc computed from the observed light distribution [i.e. with an $(M/L)_e = 1$] (see Begeman 1987).

4.1 HSB and LSB samples

We will compare the properties of our LSB galaxies with those of mostly HSB galaxies with well-defined H\textsc{i} rotation curves. The latter have been taken from the compilation by Broeils (1992) and references therein. This ‘HSB sample’ does not consist of just discs obeying Freeman’s Law (i.e. galaxies with $\mu_0 = 21.6$ mag arcsec$^{-2}$) (Freeman 1970): a large range of central surface brightnesses is found, from Freeman discs to discs with surface brightnesses approaching those of the LSB galaxies. The mean surface brightness of the HSB sample ($\langle \mu_0 \rangle = 22.2$) is still 1.5 mag brighter than that of the LSB sample ($\langle \mu_0 \rangle = 23.7$). Comparing the two samples gives a good impression of the change in properties of spiral galaxies over a large range in surface brightness. Properties of the HSB sample are given in the bottom panel of Table 1 and in Table 3.

4.2 Maximum disc fits

The maximum disc hypothesis attributes as much of the observed rotation velocity to the stellar disc as possible. It therefore always yields an upper limit to the $(M/L)_e$ in a galaxy. For HSB galaxies the maximum disc hypothesis can explain almost all of the observed rotation velocity in the inner parts with just the stellar disc. For LSB galaxies the situation is quite different. Fig. 5 shows the maximum disc decompositions of their rotation curves. The resulting disc and halo parameters are summarized in Table 4, together with the masses of the various components, as measured within the outermost radius $R_{\text{max}}$ of the rotation curve (Table 1). For those galaxies where both $R$ and $B$ photometry were available we have performed separate decompositions for both bands, which are compared in Fig. 6. The core radii and asymptotic velocities correspond very well to each other in both sets, as do the maximum disc $(M/L)_e$ ratios when the $B - R$ colours of the galaxies are taken into account. To compare those LSB galaxies for which only $R$-band data is available with $B$-band literature data, we will convert the measured maximum disc $(M/L)_e$ ratios to $B$-band values assuming an average $B - R$ colour of 0.88 (de Blok et al. 1995, cf. Fig. 6). Our results do not depend on this conversion as the halo is the most dominant component and its parameters are therefore not very sensitive to the precise value of $(M/L)_e$. The maximum disc decompositions for the LSB samples have been taken from Broeils (1992) and are for convenience summarized in Table 3.

4.3 Minimum disc fits

Many of the rotation curves of LSB galaxies are best fitted by a minimum disc, that is, under the assumption that $(M/L)_e = 0$. We have calculated the halo parameters under this assumption (Table 4). We will compare them with the maximum disc parameters in Section 5.4, but already note that there is little change in the structural parameters of the haloes, due to the DM dominance. Although minimum disc is not a realistic assumption for HSB galaxies, we have for comparison also made minimum disc fits to the rotation curves of the LSB sample (Table 5). Not all galaxies could be fitted: especially the earlier types (with bulges), which have rotation curves that rise too steeply to be fitted by a simple isothermal model with a finite core radius, thus clearly showing the need for a compact luminous component.

4.4 Bottema disc

Bottema (1995) has measured the stellar velocity dispersions of a sample of LSB spiral galaxies and found that the dispersion is related to the maximum rotation velocity. For a Freeman disc with a colour $B - V = 0.7$ he derived $(M/L)_e \approx 1.8$. The stellar discs of the galaxies in his sample supply approximately 63 per cent of the total measured velocity at 2.2 $h$ (where the rotation curve of an exponential disc peaks). These results were derived under the explicit assumption of Freeman’s Law (which holds for most of Bottema’s galaxies). It is obvious that Bottema’s original prescription cannot hold in LSB galaxies: 63 per cent of the total rotation velocity is in many cases already more than the maximum disc solutions allow. In Appendix A we give a more general expression for $(M/L)_e$ of a galactic exponential disc, as derived by Bottema (1997). The resulting $(M/L)_e$ only depends on colour, as the effects of luminosity and surface brightness cancel in this model. In the absence of data on stellar velocity dispersions of low or intermediate surface brightness galaxies, this is the best we can currently do taking into account the actual gravitational potential of the disc. In the following we will refer to this prescription as the ‘generalized Bottema disc’. Of course, when applying this generalized scheme to HSB galaxies, the original 63 per cent criterion is retrieved.

The $(M/L)_e$ values given in column (20) of Table 4 show that the range of values implied by the Bottema disc is very small compared with the maximum disc values. This is due to the relatively small range in $B - V$ colours shown by the galaxies. We have used Bottema’s original ‘63 per cent recipe’ (as presented in Rhee 1996).
for galaxies with $\mu_0^g < 23$ mag arcsec$^{-2}$, while the generalized Bottema disc was used for galaxies with $\mu_0^g$ fainter than this value, where 63 per cent of the total rotation velocity is already more than maximum disc allows. In practice the generalized disc was thus used for the LSB galaxies, NGC 247 and the three DDO galaxies. For NGC 55, 3109 and 5585 the Bottema recipes resulted in $(M/L)_b$ values larger than those implied by maximum disc fits and we have thus assumed the maximum disc values. In at least one case (NGC 3109) it can be shown that the discrepancy arises because of a large difference between the optical (Fabry–Perot) and H$\alpha$ rotation curves. If the Fabry–Perot curve is used the maximum disc $(M/L)_b$ value becomes larger than the Bottema value.

5 RESULTS OF DECOMPOSITIONS

5.1 The influence of resolution on disc–halo decompositions

As we have established in Section 3 that beam smearing does not determine the shapes of the rotation curves, we should not expect the results of disc–halo decompositions of the LSB galaxies to be
Figure 6. Comparison of the core radii \( R_c \) (left panel) and asymptotic velocities (middle panel) \( V_e \) of the haloes, and the maximum disc mass-to-light ratio \((M/L)_d\) of the stellar disc (right panel) resulting from separate B- and R-band decomposition of the rotation curves of the subset of LSB galaxies for which both R- and B-band photometry were available. The B- and R-band decompositions are consistent with each other. The full drawn lines are lines of equality. The dotted line in the right panel is the relation between \((M/L)_d\) and \((M/L)_d\) for \(B - R = 0.88\).

Table 3. Maximum disc decompositions of the HSB sample (Broeils 1992).

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<tr>
<th>Name</th>
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<th>( V_e )</th>
<th>( \rho_0 )</th>
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Note: \((M/L)_d\)_\( \bullet \) is in units of \( (M_\odot/L_\odot)_d \); \( R_c \) is in units of kpc, \( V_e \) in units of \( \text{km} \text{s}^{-1} \), \( \rho_0 \) in units of \( 10^{-7} \text{ M}_\odot \text{ pc}^{-3} \); all masses are in units of \( 10^{12} \text{ M}_\odot \). See Broeils (1992) and references therein.

This data, and find that, although again there is some freedom in \((M/L)_d\)_\( \bullet \), the best-fitting value for the low-resolution data is \( = 4 \). We can exclude \((M/L)_d\)_\( \bullet \) = 3 or 5 as possible solutions for this particular case, as the corresponding low-resolution models systematically over- or underestimate the rotation velocities at certain radii. The low-resolution halo parameters are given Table 6, and are comparable with the high-resolution values.

The value \((M/L)_d\)_\( \bullet \) = 4 for the low-resolution case was determined by simply scaling the stellar rotation curve to match the innermost point. This procedure is thus for the low-resolution case equivalent to a formal maximum disc fit in the high-resolution case. Studying high-resolution curves in detail shows that this scaling to match the innermost point is usually what happens in practice when performing a formal maximum disc fit.

5.2 Maximum disc mass-to-light ratios

As is clear from Fig. 5, the rotation curves of the stellar discs in the LSB galaxies (even in the maximum disc case) do not even approach the maximum velocities of the observed rotation curves. This is in sharp contrast to HSB galaxies where the rotation curve of the stellar disc is usually able to explain most of the velocity in the inner parts.

This is also illustrated in the top panel of Fig. 7. This shows the ratio of the peak velocity of the maximum disc at 2.2 \( h \) and the total observed rotation velocity at that radius. At central surface brightnesses brighter than 23 \( B \) mag arcsec\(^{-2}\), the maximum disc can account for 85 \( \pm \) 10 per cent of the total rotation velocity, but this rapidly drops to only 40 per cent or less for LSB galaxies.

This increasing inability of the maximum disc to explain the observed rotation velocities is illustrated in the middle panel of Fig. 7, which shows the ratio of the peak velocity of the maximum stellar disc and the maximum observed rotation velocity. This ratio systematically changes from \( \sim 0.9 \) at high surface brightnesses (i.e. the disc defines the maximum rotation velocity and the halo ‘conspires’ to keep the rotation curve flat) to values of \( \sim 0.3 \) for the LSB galaxies (i.e. the halo completely determines the dynamics of the galaxies). The bottom panel of Fig. 7 again shows the ratio of the peak velocity of the maximum disc at 2.2 \( h \) and the total observed rotation velocity at that radius, but as a function of \( V_{\text{max}} \).

The maximum disc \((M/L)_d\)_\( \bullet \) values are shown in Fig. 8 as a function of \( \rho_0 \), \( V_{\text{max}} \) and \( B - V \). The relation between \((M/L)_d\)_\( \bullet \) and \( V_{\text{max}} \) (top left) shows a clear segregation between galaxies of different surface brightnesses: at each value of \( V_{\text{max}} \) the HSB galaxies have the lowest value of \((M/L)_d\)_\( \bullet \), while the LSB galaxies always show high \((M/L)_d\)_\( \bullet \) values. Also shown is the relation \((M/L)_d\)_\( \bullet \) \( \propto L^{1/2} \) which was previously derived for HSB galaxies by

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Table 4. Disc and halo parameters for LSB galaxies.

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<th>$V_*$</th>
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<td>*</td>
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Note: $(M/L)_*$ in units of $(M_{\odot}/L_{\odot})$ in the respective bands; $R_C$ in units of kpc, $V_*$ in units of km s$^{-1}$, $\rho_0$ in units of $10^{-3}$ M$_{\odot}$ pc$^{-3}$; all masses are in units of $10^{10}$ M$_{\odot}$.

$^d$ $(M/L)_{\text{bulge}} = 0$.

The absence of a relation between $(M/L)_*$ and $\mu_0$ (top centre) is a result of a selection effect. The maximum disc $(M/L)_*$ is not a simple function of $L$ (or of $V_{\text{max}}$) alone, but also depends on surface brightness.

The surface brightness segregation effect at fixed $V_{\text{max}}$ cannot be an artificial effect caused by the lower resolution of the LSB observations. We have shown in Section 3 that beam smearing effects are potentially most severe in HSB galaxies. If beam smearing were to have affected the LSB curves this would imply even higher values for $(M/L)_*$, thus making the difference between HSB and LSB galaxies seen in Fig. 8 even more dramatic (see also the discussion in de Blok & McGaugh 1996).

### 5.2.1 The maximum exponential disc

The segregation in $(M/L)_*$ can be explained from the properties of the exponential disc and the systematics of maximum disc fitting. As LSB galaxies obey the same Tully–Fisher relation as HSB galaxies (Zwaan et al. 1995), this requires LSB galaxies to have larger scalelengths at fixed $V_{\text{max}}$, in order to still reach the required value of $L$. The maximum rotation velocity of an exponential disc is given by $\psi \propto \sqrt{\psi_0 h}$, where $\psi_0$ is the central mass surface density. In

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Table 5. Bottema disc and minimum disc decompositions of the HSB sample.

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<th>( V_m )</th>
<th>( \rho_0 )</th>
<th>( M_3 )</th>
<th>( \mu_3 )</th>
<th>( M_4 )</th>
<th>( \mu_4 )</th>
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| N55     | 1.3
| d       | 6.4      | 140.4    | 9.0      | 1.53     | 0.26     | 2.4      | 104.2    | 36.2 1.76 |
| N247    | 1.4      | 3.7      | 93.9     | 12.1     | 1.38     | 0.90     | 1.4      | 107.3    | 114 2.11 |
| N300    | 1.6      | 6.9      | 134.2    | 7.1      | 1.70     | 0.46     | 1.4      | 104.7    | 109 2.19 |
| N801    | 1.8      | 9.3      | 178.2    | 6.8      | 29.35    | 13.62    | –        | –        | –     |
| N1003   | 1.3      | 4.3      | 104.2    | 11.0     | 8.06     | 0.59     | –        | –        | –     |
| N1560   | 1.4      | 2.4      | 87.4     | 24.1     | 1.01     | 0.05     | 1.8      | 84.1     | 40.6 0.96 |
| N2403   | 1.3      | 2.7      | 131.6    | 44.3     | 6.73     | 0.92     | 0.8      | 134.3    | 470 7.95 |
| N2841   | 2.0      | 4.0      | 266.3    | 80.4     | 130.4    | 9.63     | –        | –        | –     |
| N2903   | 1.7      | 0.4      | 175.2    | 3933     | 15.03    | 1.97     | –        | –        | –     |
| N2998   | 1.5      | 1.5      | 177.6    | 262.8    | 34.16    | 6.58     | –        | –        | –     |
| N3109   | 1.5
| d       | 8.7      | 141.0    | 4.9      | 0.76     | 0.04     | 3.5      | 92.5     | 12.6 0.81 |
| N3198   | 1.4      | 1.4      | 138.7    | 189.6    | 12.88    | 1.48     | –        | –        | –     |
| N5033   | 1.5      | 0.04     | 181.0    | 37880    | 24.28    | 3.79     | –        | –        | –     |
| N5518   | 2.0      | 0.2      | 190.9    | 13930    | 69.18    | 13.76    | –        | –        | –     |
| N5855   | 1.4
| d       | 1.8      | 99.0     | 56.0     | 1.63     | 0.06     | 1.3      | 98.6     | 104 1.73 |
| N6503   | 1.6      | 0.6      | 110.8    | 610.4    | 6.24     | 0.34     | –        | –        | –     |
| N6674   | –       | –c       | 23.6     | 247.1    | 2.0      | 59.93    | 14.34    | –        | –     |
| N7133   | –c      | –c       | 9.3      | 233.4    | 11.8     | 38.55    | 6.82     | –        | –     |
| U2259   | –c      | –c       | 1.2      | 81.2     | 89.1     | 1.07     | 0.22     | 0.4      | 88.5 948 1.27 |
| U2885   | –       | –c       | 14.9     | 281.2    | 6.61     | 15.33    | 23.91    | 1.0      | 287.9 1656 136.2 |

Note: (M/L)\(\mu_3\) is in units of \((M_0/L_0)\) in the respective bands; \(R_C\) is in units of kpc, \(V_m\) in units of km s\(^{-1}\), \(\rho_0\) is in units of 10\(^{-3}\) \(M_0\) pc\(^{-3}\); all masses are in units of 10\(^{10}\) \(M_0\).

\(d\) The Bottema disc decompositions were taken from Rhee (1996).

\(c\) A dash indicates that a minimum disc fit was not possible.

\(\mu_3\) not determined because of unknown contribution of bulge.

\(\mu_3\) is larger than maximum disc \((M/L)\)\(\mu_3\).

Table 6. Halo parameters of NGC 1560.

<table>
<thead>
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<th></th>
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<td>((M/L)\mu_3)</td>
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<td>(V_m)</td>
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<td>6.2</td>
</tr>
<tr>
<td>(\rho_0)</td>
<td>7.2</td>
<td>5.9</td>
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</tbody>
</table>

practice the value of \(\rho_0\) is not known, and the distribution of the light is used to compute the rotation curve of the disc. This rotation curve of the light has a maximum rotation velocity of

\[ v_1 \approx \sqrt{\Sigma_0 h}, \]

where \(\Sigma_0\) is the central surface brightness in linear units. (In the following we will denote the rotation velocity of the disc by \(v_1\) while the observed total rotation velocity will be indicated by \(V_1\)) in the rotation curve fitting process \((M/L)\mu_3\) is introduced to convert surface brightness to mass surface density and to compute the disc rotation curve. Under the maximum disc hypothesis \((M/L)\mu_3\) is scaled to its maximum possible value. The peak rotation velocity of the maximum disc is then given by

\[ v_m \approx \sqrt{(M/L)\mu_3 \Sigma_0 h}. \]

From equations (5) and (6) it is clear that

\[ \left( \frac{M}{L} \right) \approx \left( \frac{v_m}{v_1} \right)^2. \]

We will now compare an HSB and an LSB galaxy at identical positions on the Tully–Fisher relation. We find for the peak velocity of the rotation curve of the light in the HSB, \(v_1^H\), that \((v_1^H)^2 \approx \Sigma_0^H h^H\), and for the peak velocity in the LSB, \(v_1^L\), that \((v_1^L)^2 \approx \Sigma_0^L h^L\), where \(\Sigma_0^H\) and \(h^H\) are the central surface brightnesses and scalelengths of the LSB and the HSB, respectively. As their luminosities \(L \approx \Sigma_0 h^2\) are identical we find \(h \approx \Sigma_0^{-1/2}\) and we can write \(v_1^L\) as

\[ (v_1^L)^2 \approx \Sigma_0^L h^L \approx \frac{L}{\Sigma_0^L} \left( \frac{\Sigma_0^H}{\Sigma_0^L} \right)^{1/2}. \]

Using equations (7) and (8) we express \((M/L)^H\) in terms of \((M/L)^L\):

\[ \left( \frac{M}{L} \right)^H \approx \left( \frac{v_m^H}{v_1^L} \right)^2 \left( \frac{\Sigma_0^H}{\Sigma_0^L} \right)^{1/2} \left( \frac{L}{\Sigma_0^L} \right)^{1/2} \left( \frac{\Sigma_0^L}{\Sigma_0^H} \right)^{1/2}. \]

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typical LSB galaxy has $\mu_0 = 24$, we find that condition (10) requires $\sigma_{v(2.2h)}^2 / v_{2h}^2 \geq 0.5$ on average. We find the largest range of $\sigma(2.2h)/v(2.2h)$ at $V_{\text{max}} = 100$ km s$^{-1}$ in the bottom panel of Fig. 7. Taking the ratio of the average value for the LSB galaxies $\sigma(2.2h)/v(2.2h) = 0.45$ and that of the HSB galaxies $\sigma(2.2h)/v(2.2h) = 0.85$, we obtain a value of $\sigma_{v(2h)}^2 / v_{2h}^2 = 0.5$. So although there will be individual cases where condition (10) will not hold (as is shown by the overlap of HSB and LSB points in the top-left panel of Fig. 8), on average condition (10) is always fulfilled, and we find that the intrinsic properties of the exponential disc combined with the maximum disc recipe always yield larger values of $(M/L)_b$ for LSB galaxies than for HSB galaxies. No assumptions have been made on the stellar populations, ages or evolution of the galaxies. The larger fitted maximum disc stellar mass-to-light ratios for LSB galaxies at fixed $V_{\text{max}}$ therefore do not need to reflect the true evolutionary stellar mass-to-light ratio of the disc.

5.2.2 Population effects?

Is it in fact at all possible to explain the systematically higher maximum disc $(M/L)_b$ ratios of the LSB galaxies at fixed $V_{\text{max}}$ in a consistent way by invoking systematic changes in the stellar populations? The systematically larger ratios imply an increasingly more evolved population with decreasing surface brightness: population synthesis models (Larson & Tinsley 1978) show that $(M/L)_b$ of an evolving stellar population is in general an increasing function of time, and hence of increasing $B - V$ colour.

Two examples are shown in in the right-hand panels of Fig. 8, where the upper dotted line represents $(M/L)_b$, as a function of the $B - V$ colour of a stellar population with a declining star formation rate (Larson & Tinsley 1978). The lower dotted line is the track as derived by Jablonka & Arimoto (1992) who have used a model that also takes into account chemical enrichment.

To a certain degree the maximum disc results agree with the trend of increasing $(M/L)_b$ with redder $B - V$: the lower envelope of the distribution of the points increases with redder colour. Uncertainties in the low-mass end of the IMF make it very hard to derive absolute $(M/L)_b$ values from any population synthesis model. This uncertainty may therefore shift the model track up or down, while, for example, metallicity effects may make the slope of the track more shallow (as the Jablonka & Arimoto track shows). However, the observed range at each colour is too large to be explained by models with a constant IMF and where the star formation history is well-behaved and a smoothly varying function of Hubble type.

The high maximum disc $(M/L)_b$ values are furthermore in direct conflict with observational evidence on colours (McGaugh & Bothun 1994; de Blok et al. 1995), metallicities (McGaugh 1994) and gas fractions (McGaugh & de Blok 1997). As the stellar populations of LSB galaxies are in an earlier evolutionary state than those of HSB galaxies (van den Hoek et al. 1997) one would expect them to have lower values of $(M/L)_b$. The values of $(M/L)_b$, as derived under the maximum disc hypothesis are therefore not representative of the stellar population in LSB galaxies.

5.2.3 Disk dark matter?

Trying to explain the maximum disc results in a physical way, while having a stellar population with a lower $(M/L)_b$ than implied by the maximum disc values, requires an additional dark component in the disc that traces the stellar population. This component has to become increasingly important towards lower central surface...
Figure 8. Top row: the maximum disc B-band stellar mass-to-light ratio plotted versus maximum rotation velocity (left panel), central surface brightness (middle panel) and average $B-V$ colour (right panel). The open circles denote galaxies with $\mu_0(B) < 21.9$ mag arcsec$^{-2}$, the triangles represent $21.9 \leq \mu_0(B) < 23.2$ mag arcsec$^{-2}$, and the asterisks galaxies with $\mu_0(B) \geq 23.3$ mag arcsec$^{-2}$. In the left panel the dotted line shows the relation $(M/L_B)_{\lambda} \propto L^{1/2}$. The relation has been arbitrarily shifted to follow the filled HSB data points. In the right panel the steep upper dotted curve shows the colour–mass-to-light ratio track from Larson & Tinsley (1978) while the shallow lower dotted curve shows the track as derived by Jablonka & Arimoto (1992). Both curves have been shifted to go through $\log(M/L_B)_{\lambda} = 0.55$ at $B-V = 0.55$ (see bottom-right panel). Bottom row: idem, but now showing the B-band stellar mass-to-light ratio as derived assuming a Bottema disc.

brightenings. This might be an invisible baryonic component like, for example, cold molecular gas. There has already been much speculation about this in the literature (e.g. Penninger, Combes & Martinet 1994). LSB galaxies would be the ideal test cases for such theories as the major part of their discs should then be made up of this baryonic DM. Again, this component should be of increasing importance towards lower surface brightnesses.

This creates the unattractive picture of two dominant but distinct and differently distributed dark components: one that has to provide the extra mass in the disc to make it satisfy the maximum disc boundary condition, and one that has to provide the surplus velocity in the outer parts to produce the observed rotation curves. These two dominant components then have to ‘conspire’ to produce the observed rotation curve. Either way, DM remains of extreme importance in LSB galaxies.

5.2.4 Non-maximum disc

Another possibility is that discs are simply non-maximal. Part of the halo DM is wrongly attributed to the disc simply because the disc does not dominate the dynamics in the inner parts. This could in principle affect the derived values for the structural parameters of the halo, but this is only of minor importance in LSB galaxies. They are DM dominated, irrespective of the value of $(M/L)_{\lambda}$, so that changing the mass of the disc does not really matter. It is, however, of crucial importance in HSB galaxies where the disc is dominant and the halo parameters therefore sensitive to the precise value of $(M/L)_{\lambda}$.

The maximum disc hypothesis, in combination with the extendedness of LSB galaxies, thus produces a balancing act where LSB galaxies have to have large $(M/L)_{\lambda}$ ratios or extra dark components, which are contrary to what we can derive from other evidence (stellar populations, metallicity, gas fraction). Maximum disc is therefore not the preferred way of making mass models in LSB galaxies.

5.3 Maximum disc–halo parameters

For the LSB galaxies the asymptotic velocity $V_{\infty}$ is well correlated with the observed maximum rotation velocity $V_{\max}$, showing that although many of the LSB curves are still rising in the outermost point, they are close to their true maximum velocity. We will use this observational parameter instead of the fitted $V_{\infty}$. Fig. 9 summarizes the halo parameters for the HSB and LSB galaxies under the assumption of maximum disc. The core radius $R_C$ is rather constant, both as a function of $V_{\max}$ and $\mu_0$. The group of points at large $R_C$ belong to HSB galaxies that are very much dominated by luminous matter, and are maximum disc in the true sense of the word: their inner rotation curve can be explained completely by that of the disc. To avoid having a hollow halo in these galaxies, the maximum disc $(M/L)_{\lambda}$ is usually slightly, but somewhat arbitrarily, lowered. The halo parameters are extremely sensitive to this. In most cases these galaxies have a bulge, which also has to be accommodated in the fitting procedure, with its own uncertainties in its $(M/L)_{\lambda}$. The HSB points at large $R_C$ are thus uncertain.

The central halo densities $\rho_0$ as a function of $\mu_0$ tend to be somewhat higher than those of HSB galaxies, but, again, the uncertainty in the decompositions of the HSB galaxies makes it difficult to say anything more definite.

The ratio between core radius and optical scalelength changes with surface brightness from $R_C \sim 3h$ for the HSB galaxies to $R_C \sim h$ for the LSB galaxies. In the maximum disc picture the optical discs of LSB galaxies are massive and extend further out into the halo.
5.4 Minimum disc

We can illustrate that the halo parameters derived for the LSB galaxies are robust values by comparing the values derived for maximum disc and minimum disc. This is done in Fig. 10, where the core radii and central densities as derived using these two extreme hypotheses are compared. The difference in maximum and minimum disc–halo parameters is clearly a strong function of surface brightness. The largest difference is observed for the HSB galaxies, where the central density $\rho_0$ changes by more than an order of magnitude. A similar conclusion applies to the core radius. This is in sharp contrast to the core radii derived for the LSB galaxies. The minimum disc values differ from the maximum disc values by less than a factor of 2. The slope of the trend with surface brightness changes from negative to positive when going from maximum disc to minimum disc. This is entirely caused by the large change in parameters of the HSB galaxies.

The DM dominance makes the parameters of the LSB galaxy haloes insensitive to the precise $(M/L)_*$ value. The values we derive for the halo structural parameters, but also for the dark mass in these haloes, are therefore likely to be very close to their true values.

5.5 Bottema disc mass-to-light ratios

The most important property that distinguishes the Bottema disc from the maximum disc is its small range of $(M/L)_*$. This is immediately apparent in Fig. 8. The Bottema disc typically implies values of $(M/L)_*$ between 1 and 2. In general the reddest galaxies have the highest mass-to-light ratios.

The striking systematic offset in $(M/L)_*$ at fixed $V_{\max}$ between HSB and LSB galaxies for the maximum disc hypothesis has disappeared (compare with top left panel in Fig. 8). At fixed $V_{\max}$ galaxies now have approximately identical $(M/L)_*$ ratios. The majority of the data points have a much smaller spread and are in much closer agreement with stellar population models (Larson & Tinsley 1978; van den Hoek et al. 1997) than the maximum disc values. The need for an extra dark component in the disc has disappeared.

5.6 Bottema disc–halo parameters

Whereas in the maximum disc case the galaxies with the highest values of $V_{\max}$ have the lowest central halo densities, in the Bottema disc case they have the densest and most compact haloes. The
With option (ii) the properties of the baryonic part of the galaxy are reflected in those of the host halo: dim, diffuse and extended galaxies inhabit extended, low-density haloes. In both cosmologies the dominance of DM increases towards lower surface brightnesses.

5.7.1 Maximum disc cosmology

If, for the moment, we assume the maximum disc results to hold, what are the implications for the evolution of LSB galaxies? In other words, what if the only difference between HSB and LSB galaxies at fixed $V_{\text{max}}$ is that LSB galaxies stick out further into otherwise identical haloes?

The picture that can then be sketched is deceptively simple. For an HSB and an LSB galaxy at identical positions on the TF relation, the LSB galaxy has to be more extended optically (Zwaan et al. 1995), and in order to have the same luminosity as the HSB galaxy, the amount of past and present star formation per area (the surface brightness) has to be lower.$^2$ The LSB optical disc simply encompasses more DM than the HSB optical disc, due to its larger extent in an otherwise identical halo, resulting in an LSB optically extended galaxy with a high total (i.e. DM and luminous matter) M/L. The low star formation rate (as averaged over the lifetime of the galaxy) implies slow evolution in the LSB galaxy and a low (M/L)$_*$

which is difficult to reconcile with the maximum disc hypothesis assumed in the derivation of these conclusions.

Current ideas on the formation and evolution of LSB galaxies (McGaugh 1992; Bothun et al. 1993; Mo, McGaugh & Bothun 1994; Dalcanton, Spergel & Summer 1997) suggest that LSB galaxies form from lower amplitude density peaks than HSB galaxies. As they are also found to be more isolated from their neighbours, and to have a lack of companions with respect to HSB galaxies, they have presumably also undergone less interaction with other galaxies. The extended gas discs also suggest that the collapse of the baryonic matter has been much less efficient than in actively star-forming HSB galaxies.

Although the Bottema disc solution may seem the preferred one, both solutions do have their problems. The maximum disc solution has to explain why galaxies in otherwise identical haloes have different collapse histories as a function of surface brightness, while the other solution has to explain how galaxies with different halo masses but identical luminosities conspire to still end up at the same position on the Tully–Fisher relation.

One conclusion is firm however, the dynamics of LSB galaxies are fundamentally different from those of HSB galaxies. For example, the rotation curves of LSB galaxies can not be explained in $\Omega_0 = 1$ cosmological simulations (Moore 1994; Navarro 1997), which is a problem, certainly in view of the implied numerical richness of LSB galaxies (McGaugh, Bothun & Schombert 1995).

6 MASS RATIOS

6.1 Importance of atomic gas

The increasing dynamical importance of the gas component can readily be seen in Figs 11(a) and (b). Galaxies of lower surface

Option (i) implies that the dimmest and most extended (LSB) galaxies live in the most compact haloes. Presumably the collapse of their massive, dark discs was not as complete as in HSB galaxies (because of the larger angular momenta of LSB galaxies?).

$^2$The bluer colours of LSB galaxies are explained by metallicity effects and the less prominent old population. An equal amount of star formation has a much larger effect on the colors of an LSB galaxy than on an HSB galaxy (de Blok et al. 1995).

$^3$Here we specifically do not include H I and other potentially star-forming material; the low (M/L)$_*$ simply reflects that the LSB has not had enough time to build a large old stellar population as in HSB galaxies.
brightenings have larger values of \( M_{\text{gas}}/M_* \). A similar trend with Hubble type was already noticed by, for example, Broeils (1992). The trend is clear in both the maximum disc and Bottema cases, although of course the gas-to-stellar mass ratios derived using maximum disc stellar masses tend to be larger. The ratio ranges from ~0.2 for the HSB galaxies to ~2 in the LSB galaxies. Clearly, in LSB galaxies the gas component can be of larger dynamical importance than the stellar component, even in the maximum disc case. This effect is more pronounced for the Bottema decompositions because of the limited range in \((M/L)_s\). A fuller discussion of the gas fraction over a large range in surface brightness is presented in McGaugh & de Blok (1997).

\( M_{\text{gas}}/M_{\text{dark}} \) [as computed within the outermost measured point of the rotation curve, \( R_{\text{max}} \) (see Table 1), and assuming Bottema disc] is shown in Figs 11(c) and (e). Panels (d) and (f) show \( M_{\text{gas}}/M_{\text{dark}} \) measured to within 5\( h \). There is clear trend of increasing gas-richness towards lower values of \( V_{\text{max}} \), an effect that is not entirely cancelled when measured within 5\( h \). Slower rotating galaxies contain relatively more gas. The large range in \( \mu_0 \) found at each \( \nu_{\text{max}} \) ensures that no clear trend is visible of \( M_{\text{gas}}/M_{\text{dark}} \) with \( \mu_0 \).

### 6.2 Baryon fraction and dark matter fraction

The total luminous mass \( M_{\text{lum}} \) is defined as the sum of the stellar mass and the gas mass. We here take the stellar mass as implied by the Bottema disc. The ratio of the dark-to-luminous mass ratio is related to the maximum velocity (or luminosity) (Persic & Salucci 1991; Broeils 1992). Galaxies with low maximum velocities and low luminosities are more DM dominated than high-luminosity galaxies. This effect is not very pronounced when \( M_{\text{dark}} \) is computed within the outermost radius (Figs 11g and i); the extended and well-observed rotation curves of LSB galaxies encompass a relatively much larger fraction of the halo. The effect becomes much more pronounced when measured within a fixed number of scalelengths (Figs 11h and j).

The extreme DM domination of the lowest surface brightness galaxies has more general implications. One example is the baryon fraction \( f_b \) of the Universe. This is of direct cosmological significance since \( \Omega_b = \Omega_0 f_b \), where \( \Omega_0 \) is the baryon density of the Universe: a measure of \( f_b \) combined with \( \Omega_0 \) from primordial nucleosynthesis directly yields \( \Omega_b \).

A value of \( f_b \) thought to be representative of the Universal baryon fraction is given by X-ray observations of galaxy clusters (White et
lot, their haloes are very large, worsening the problem. It has been suggested that low-mass galaxies experience significant wind-driven mass loss (e.g. Dekel & Silk 1986) which might rid them of some baryons. However, this clearly has not occurred in the galaxies we are discussing. Their gas has not been swept out; they are very gas-rich and massive. They show no evidence of blow-out events (Bothun, Eriksen & Schombert 1994). It is therefore entirely ad hoc to vary \( f_0 \), and this would add yet another free parameter to the current cosmological models.

Of course, until the nature of the DM is known, it is conceivable that some or all of it is baryonic. Thus, another possibility is that some fraction of haloes is baryonic, and relatively more baryons have been incorporated into the haloes of LSB galaxies, perhaps as MACHOs. How and why this should occur is again not obvious. LSB galaxies show little evidence of an early epoch of star formation that ought to provide the remnants that become MACHOs (McGaugh & Bothun 1994; de Blok et al. 1995), and it is not obvious that these should be distributed in a spherical halo. Completely non-baryonic matter is still required unless primordial nucleosynthesis or large-scale estimates of \( \Omega_0 \) are wrong. The limits derived from the LSB galaxies are, however, hard limits, and they should be reconciled with the cluster values. If that does not prove to be possible, this might, at last resort, even indicate the break down of the DM hypothesis (Sanders 1997).

7 SURFACE AND VOLUME DENSITIES

7.1 Total mass volume densities

A question that remains to be answered is whether the low surface densities now observed in LSB galaxies also imply that they are truly low volume density objects. This question is extensively discussed in de Blok & McGaugh (1996) where the LSB galaxy UGC 128 and the HSB galaxy NGC 2403 (which lie at identical positions on the Tully–Fisher relation) are compared. The conclusion reached there is that the total mass volume density enclosed within a fixed number of scalelengths is a factor \(~10\) lower in UGC 128 than in NGC 2403. We refer to that work for a discussion on the preferred length-scale for measuring densities in galaxies.

We have extended this analysis to our samples of HSB and LSB galaxies. The total mass density within 5\( h \) was computed and shown as a function of surface brightness in Fig. 12. The conclusion reached in de Blok & McGaugh is confirmed: the difference in \( \rho \) between Freeman-like galaxies and LSB galaxies with \( \mu = 24 \) is again a factor of 10.

Although a large range in mass density is inferred at each value of \( \mu_0 \), LSB galaxies are 'diffuse' objects, even in the maximum disc case. The increase in optical scalelength towards lower surface brightnesses is much faster than any change in the DM length-scale. The increase in area spanned by the optical disc can therefore not be compensated by a corresponding increase in total matter enclosed by the optical disc. The enclosed mass increases with \( V^2h \), while the enclosed area (volume) increases with \( h^2 \). The increasing scalelength therefore ensures that for LSB galaxies the average volume or surface density enclosed by the disc is always lower.

7.2 Maximum disc baryonic surface densities

The maximum disc results are a good way to demonstrate that LSB galaxies have truly low matter surface density discs. That is, they are not normal surface density discs that happen to have less...
light per square parsec, but the underlying mass surface density distribution is more diffuse too (de Blok & McGaugh 1996).

In Fig. 12 we show the central surface densities calculated by multiplying the central surface brightness with the respective (M/L)_{s} ratios. The resulting surface density is the maximum density the disc can have within the constraints of the observed rotation curve. These values have not been corrected for the presence of H1, as the surface density of the H1 gas becomes progressively lower towards lower surface brightnesses (BMH96). The increase in maximum disc (M/L)_{s} ratio is not fast enough to compensate for the decrease in surface brightness, an effect that will only become more pronounced when hypotheses other than maximum disc are used. Maximum disc is the most conservative scenario, but already here it is clear that the mass surface density in the disc is a function of surface brightness.

8 SUMMARY

We have presented rotation curve decompositions of a sample of LSB galaxies and compared the properties of their constituent mass components with those of a sample of HSB galaxies.

LSB galaxies are very much dominated by DM thus proving to be the ideal test-cases for theories on the structure and formation of DM haloes. The halo parameters derived for these galaxies are insensitive to the assumed mass-to-light ratio of the stellar disc, ensuring that the values we derive are in all probability close to the true values.

This is in sharp contrast to HSB galaxies, where the halo parameters are sensitive to the assumed stellar mass-to-light ratio. Comparison of the HSB and LSB samples shows that the maximum disc solutions imply an increasing stellar mass-to-light ratio for the stellar disc and an increasing compactness for the halo as the central surface brightness decreases, while the Bottema disc solutions imply exactly the opposite. We argue, based on what is known about the evolutionary state of LSB galaxies from other sources, that the maximum disc solution is not representative. To explain the maximum disc results in a physical way, one has to assume large amounts of baryonic (dark) matter in the LSB discs. This would result in massive, evolved discs, which is not consistent with the observed past and present low star formation rates of LSB galaxies. The Bottema disc solutions give lower (M/L)_{s} values, more consistent with what can be derived from other evolutionary probes (metallicity, gas fraction). They imply that LSB galaxies live in extended diffuse haloes. The haloes of LSB galaxies are thus fundamentally different from those of HSB galaxies of the same mass. This is a challenge for cosmological N-body simulations which will have to address this problem, especially considering the numerical richness of LSB galaxies.

LSB galaxies are thus slowly evolving, isolated, diffuse, low-density and extremely DM-dominated galaxies.

ACKNOWLEDGMENTS

We thank Roelof Bottema for allowing us to use his results prior to publication and for his extensive comments on early drafts of this paper. We also would like to thank Theis van der Hulst, Renzo Sancisi and Marc Verheijen for many discussions. We thank Floor Sicking and Kor Begeman for allowing us to use their rotation curve fitting software. We acknowledge remarks made by Dr Kalnajs and the anonymous referee which helped to clarify some aspects of this paper.

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For an infinitely thin stellar disc the maximum rotation velocity is given by
\[ v_{\text{max}} = 0.88 \sqrt{\pi G \sigma_0 h}. \]  
(A5)

Combining equations (A4) and (A5) yields
\[ v_{\text{max}} = 0.88 (v^2_{z, 0})^{1/2} \frac{h}{z_0}. \]  
(A6)

### A1.3 The old disc Tully–Fisher relation

Equation (A5) and the identity \( L = 2 \pi \Sigma_c h^2 \), where \( L \) is the total luminosity, \( \Sigma_c \) the central surface brightness in linear units, and \( h \) the scalelength, imply an old disc Tully–Fisher relation:
\[ v^4_{\text{max,old}} = 0.3 \pi G^2 \Sigma_c^2 \left( \frac{M}{L} \right)_{\text{old}}^2. \]  
(A7)

As the old disc contains all the mass \( v^4_{\text{max,old}} = v^4_{\text{max}} \) \((ML)_{\text{old}}\) is assumed to be the same for all old discs.

### A2 Bottema disc for Freeman galaxies

For Freeman galaxies \( \mu_0 \) is constant. The colour is also assumed to be constant at \( B-V = 0.7 \). Equation (A7) then yields \( M_{\text{old}} = -10 \log(v_{\text{max}}) + C \), or
\[ v_{\text{max}} = 10^{0.1C} 10^{-0.01M_{\text{old}}}. \]  
(A8)

Combining Equations (A6) and (A8) yields
\[ (v^2_{z, 0})^{1/2} = A^{-1} \frac{\sqrt{\Sigma_c}}{r} 10^{-0.1M_{\text{old}}}, \]  
(A9)

where \( A \) is defined as
\[ A = 0.88 \times 10^{-0.1C}. \]  
(A10)

Following Bottema (1995) the empirical relation between luminosity and scalelength ratios is adopted, \( h/z_0 = 0.6 M_{\text{old}} + 17.5 \), together with the corresponding values \( A = 0.75, C = 0.69 \) and \((ML)_{\text{old}} = 1.85\). This results in
\[ v_{\text{max}} = 1.17 \times 10^{-0.1M_{\text{old}}}. \]  
(A11)

### A3 Bottema disc for non-Freeman galaxies

The result derived above strictly speaking only applies to galaxies that obey Freeman’s Law. To generalize to different surface brightnesses we rewrite equation (A7) as
\[ v^4_{\text{max}} = 0.3 \pi G^2 \Sigma_0 F \left( \frac{\Sigma_{\text{0,F}}}{\Sigma_0} \right) \left( \frac{M}{L} \right)_{\text{old}}^2. \]  
(A12)

where \( \Sigma_{0,F} \) is the constant Freeman central surface brightness of 136 \( L_\odot \) pc\(^{-2}\). The scalelengths of the old and young populations are assumed to be equal, so that \( \Sigma_{0,F} / \Sigma_0 = L_{\text{old}} / L_{\text{old}} = 10^{0.01 \delta i} \). This results in
\[ v_{\text{max}} = \text{constant} \times \left( \frac{\Sigma_0}{\Sigma_{\text{0,F}}} \right)^{1/4} 10^{0.1 \delta i} 10^{-0.1M_{\text{old}}}. \]  
(A13)

For Bottema’s sample, with central surface brightnesses approximately equal to the Freeman value and colours close to \( B-V = 0.7 \), an average correction factor \( \langle cf \rangle = -0.5 \) can be
defined. Combination of equations (A6) and (A13) then yields
\[
\langle (a^2_x)^{1/2} \rangle_{R=0} = \frac{\text{constant}}{0.88} \sqrt{\frac{c_0}{h}} \left( \frac{\Sigma_0}{\Sigma_{0,F}} \right)^{1/4} 10^{0.1(c_f-\langle c_f \rangle - M_{\text{od})}} 10^{-0.1 M_{\text{od}}}. \tag{A14}
\]
Again using Bottema’s relation \( h/c_0 = 0.6 M_{\text{od}} + 17.5 \) then implies that
\[
\text{constant} \times 10^{0.1(c_f)} = 1.17.
\]
For the maximum rotation velocity and velocity dispersion of a galactic disc one then finds (using equations A13 and A14)
\[
v_{\text{max}} = 1.17 \left( \frac{\Sigma_0}{\Sigma_{0,F}} \right)^{1/4} 10^{0.1(c_f-\langle c_f \rangle)} 10^{-0.1 M_{\text{od}}}, \tag{A15}
\]
and
\[
\langle (a^2_x) \rangle_{R=0} = 1.33 \left( \frac{c_0}{h} \right)^{1/4} \left( \frac{\Sigma_0}{\Sigma_{0,F}} \right)^{1/4} 10^{0.1(c_f-\langle c_f \rangle - M_{\text{od})}}. \tag{A16}
\]
Conversion to observables using equation (A2) then yields
\[
v_{\text{max}} = 1.17 \left( \frac{\Sigma_0}{\Sigma_{0,F}} \right)^{1/4} 10^{0.2(c_f+0.25)} 10^{-0.1 M_{\text{od}}}. \tag{A17}
\]

### A4 Mass-to-light ratio

From equations A4 and A16 one can derive that
\[
\left( \frac{M}{L} \right)_B = 28.1 \times 10^{-0.2 M_{\odot}} \times 10^{0.4 c_f} 10^{-0.2 \langle c_f \rangle}. \tag{A18}
\]
With \( M_{\odot} = 5.48 \) one finds
\[
\left( \frac{M}{L} \right)_B = 1.79 \times 10^{0.4 c_f} 10^{0.2(c_f-\langle c_f \rangle)} = 2.84 \times 10^{0.4 c_f}
\]
\[
= 1.936 \times 10^{0.4(c_f-\langle c_f \rangle)} - 1.883. \tag{A19}
\]
The mass-to-light ratio does not depend on the surface brightness of the disc, which is due to our assumption that the old disc population has the same mass-to-light ratio for all discs. Discs with identical colours therefore have identical mass-to-light ratios, irrespective of surface brightness.

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