PROBLEM SET # 2

Problem 1.

The following question is about interstellar reddening in other galaxies. Let’s consider two different situations.

Case 1: Stars are distributed uniformly throughout a thin layer of thickness $D$ with a layer of absorbing material of optical depth $\tau_V$ between the stars and the observer ($\tau_V$ is the optical depth in the visual passband).

Case 2: Same as Case 1, but now the absorbing medium is distributed uniformly with the stars.

Assume $D$ is negligible compared to the distance to the observer, and the absorbing material obeys the so-called Whitford law with $R \equiv A_V / E(B-V) = 3$.

(a) The observer has measured integrated colors in UBV for the slab of stars (seen face on), but these colors are naturally affected by reddening. Calculate the apparent value of $R$ and $S \equiv E(U-B)/E(B-V)$ for Cases 1 and 2 in the limits $\tau_V \to 0$ and $\tau_V \to \infty$.

(b) Mention a couple of realistic situations in which this variation in $R$ and $S$ might actually occur.

*Hint:* Read section 3-11 of Mihalas & Binney (1981) carefully! This book is on reserve at the astronomy library. It will be of some help to solve this problem.

*Note:* The calculation is only academic since real dust grains have an albedo of about 50%, so they scatter much of the light rather than absorb it.

Problem 2.

Refer to Binney & Tremaine for the following questions:
(a) Miyamoto-Nagai potential. Derive equation (2-50b) using equation (2-50a). Assuming $b/a = 1$, make a plot of (a) $\rho_M(R, 0)$ as a function of $R/a$, (b) $\rho_M(0, z)$ as a function of $z/a$. Compare with Figure 2-6.

(b) Logarithmic potential. Derive equation (2-54b) using equation (2-54a). Assuming $q\phi = 0.7$, make a plot of (a) $\rho_L(R, 0)$ as a function of $R/a$, (b) $\rho_L(0, z)$ as a function of $z/a$. Compare with Figure 2-8.

Problem 3.

Here are some back-of-the-envelope calculations to give you a feel for various aspects of the Galaxy. Assume that the Sun is located at $R_0 = 8.0$ kpc, has an orbital velocity $\theta_0 = 220$ km s$^{-1}$, and that the Galaxy has an isothermal dark halo [$\rho = \rho_0(R/R_0)^{-2}$] with cut-off radius 10 $R_0$ which contains altogether 10 times the total mass within the Sun’s orbit. Estimate:

(a) The total mass of the Galaxy

(b) The rotation period of the Sun. How many orbits has an object at $R_0$ completed since the Galaxy formed ($\sim 13$ billion years ago)?

(c) The escape velocity in the Solar neighborhood.

(d) The orbital period of a typical dwarf spheroidal satellite galaxy at 85 kpc. How many orbits has this satellite completed since the Galaxy formed?

Problem 4.

Assume that the Sun is in the midplane of the Galaxy with a vertical component of motion equal to 7 km s$^{-1}$. Assume that the Sun moves through a constant-density region with $\Sigma = 0.13$ $M_\odot$ pc$^{-3}$.

(a) Show that the Sun executes a simple harmonic motion with a force constant $k = 4\pi G\Sigma M_\odot$.

(b) Estimate the maximum height above the plane that the Sun will reach.

(c) In light of the derived value of $z_{\text{max}}$, is the assumption of a constant-density medium realistic? State your reference.

(d) Roughly how many vertical oscillations has the Sun made during its lifetime (4.6 billion years)?