A Multiwavelength Study of Three Solar Flares

M. R. Kundu, A. Nindos\textsuperscript{1}, S. M. White
Astronomy Department, University of Maryland, College Park, MD 20742

V. V. Grechnev
Institute of Solar Terrestrial Physics, Lermontov St. 126, Irkutsk 664033, Russia

ABSTRACT

In this paper we seek a self consistent model for three strong limb flares observed at 17 and 34 GHz by the Nobeyama radioheliograph (NoRH) and also in soft X rays and hard X rays by the SXT and HXT instruments on board Yohkoh. Additional radio spectral data were provided by the Nobeyama polarimeter. The SXT images showed that the flare geometry is simple with one well defined flaring loop in each event. The HXT images show that the hard X ray sources are located close to the footpoints of the flaring loops above regions of opposite magnetic polarity. The radio maps and time profiles suggest that the 17 and 34 GHz emission is gyrosynchrotron from energetic electrons. The polarimeter data indicate that the 17 and 34 GHz emission is optically thin. The NoRH maps show that radio emission outlines the flaring loops and peaks close to the loop tops. The fact that the radio maxima do not occur above the footpoints implies that the variation of magnetic field along the loops is very small. We try to reproduce the observed microwave morphologies and fluxes using a model gyrosynchrotron loop. The SXT and HXT images constrain the model input parameters: the SXT images provide the shape of the model loops while the HXT images give the distance between the footpoints of the model loops. The high frequency polarimeter data give the energy spectral index of the energetic electrons that produce radio emission. We could not reconcile the observed radio morphologies and fluxes using classic dipole magnetic field models. When the models produce optically thin emission, it is possible to reproduce the observed fluxes but the models show maxima above the footpoints and dips close to the loop tops contrary to the observations. Another serious problem of that set of models is that the reproduction of the observed fluxes requires that the aspect ratios of the model loops is implausibly

\textsuperscript{1}Present address: Section of Astrogeophysics, Physics Department, University of Ioannina, Ioannina, GR-45110, Greece.
small. The $I$ model morphology resembles the NoRH maps if we make the loop optically thick at both frequencies. But the resulting radio spectra are not consistent with the observations. The best fit models (which partly reconcile the observed 17 and 34 GHz morphologies and fluxes) are produced when we invoke a magnetic field with constant strength along the model loop. Such a model loop has uniform thickness. In one of the events we were able to measure the flaring loop thickness directly from the SXT images and confirmed that it was constant. This is probably the first time that there is some evidence for flaring loops of constant thickness in radio data. These model loops are thin with aspect ratios of about 0.1. The derived densities of radio emitting energetic electrons are $1 - 6 \times 10^4$ cm$^{-3}$. The models require the presence of high energy electrons (as high as 5000 keV). The most serious problem of these models is that the resulting $I$ profiles along the loops are flat topped while the observations show more prominent peaks close to the loop tops. Furthermore, the absence of coronal magnetic mirroring in such magnetic configurations may create problems as far as the trapping of the microwave emitting electrons is concerned.

*Subject headings:* Sun: flares    Sun: corona    Sun: radio radiation    Sun: X-rays, gamma rays
1. Introduction

Electromagnetic emission from solar flares can be detected in practically all wavelengths from gamma rays to kilometer-wavelength radio waves. In order to understand the physical processes involved and obtain a complete picture of a solar flare, multiwavelength coverage with good spectral, spatial and temporal resolution is required. Over the last 20 years several multiwavelength studies of solar flares have been published (e.g. Alissandrakis, Schadée, & Kundu 1988; Holman, Kundu, & Kane 1989; Wang et al. 1995; Nishio et al. 1997; Hanaoka 1997; Chiuderi Drago et al. 1998). These studies have helped us to build a unified picture of the flare phenomenon.

Microwave emission from solar flares can provide important diagnostics of acceleration processes in the solar corona because the radio emission is produced by energetic electrons accelerated during the flare. It is well known (e.g. see the reviews by Kundu, & Vlahos 1982; Alissandrakis 1986; Bastian, Benz, & Gary 1998) that the basic emission mechanism of solar microwave bursts is gyrosynchrotron from mildly relativistic electrons (energies of tens to several hundreds of keV) trapped in flaring loops. Gyrosynchrotron emission offers a powerful diagnostic of physical conditions in flaring regions. Unlike X-ray radiation, it is sensitive to magnetic field strength and orientation and can therefore be used to constrain the coronal magnetic field in the flaring source. For a given frequency, the morphology of microwave flare emission depends on the magnetic field configuration (i.e. the geometry of the flaring region and its location on the disk) and the properties of the nonthermal electrons. Usually when the source is optically thick, the maximum intensity occurs close to the loop top. As the source becomes optically thin at high frequencies, emission near the footpoints dominates. However, it will become clear in the following sections of this paper that loop top sources are also possible even in the optically thin case.

The hard X-ray emission of solar flares comes from electrons with energies between 10 and a few hundreds of keV that interact with the ambient protons through the bremsstrahlung mechanism. Using data obtained with the Hard X-ray Telescope (HXT) onboard Yohkoh satellite, Sakao (1994) and Sakao et al. (1994) found that in many cases the HXT imaging data during the impulsive phase of the flares, show two components which are observed on each side of the magnetic neutral line suggesting that the two sources are associated with the footpoints of a single flaring loop. Sakao (1994) also found that hard X-ray emission from the two sources varies simultaneously to within 0.1 sec at the 1σ level, strongly supporting the idea that hard X-rays are emitted from near the footpoints of a flaring loop by energetic electrons streaming downward from near the top of the loop. The beams of energetic electrons that propagate to the chromosphere heat the chromospheric plasma more rapidly than it can radiate the energy away. Therefore the chromospheric
material responds dynamically, expanding mostly along the magnetic field lines. The dense, hot material is expected to move up as a shock hitting the upstream plasma. The upward motion of the heated plasma is known by the misnomer of “chromospheric evaporation”. The chromospheric upflow fills the loops with hot, dense plasma (temperatures of $\sim 10^7$ K, densities of $\sim 10^{11}$ cm$^{-3}$, eg see Aschwanden, & Benz 1997) which emits bright soft X ray radiation.

In this paper we study three strong flares (GOES classification M3–M6) which were observed with the Nobeyama radioheliograph (NoRH) at 17 GHz and 34 GHz. The flares were also observed in soft X rays and hard X rays with the SXT (soft X ray Telescope) and HXT (hard X ray Telescope) instruments onboard Yohkoh. We try to reproduce the observed microwave morphologies and fluxes using an inhomogeneous loop model of gyrosynchrotron emission (see Nindos et al. 2000). The Yohkoh data provide important constraints on the input parameters of the models. The SXT images give the shape of the flaring loops and the thermal emission of the flares while the hard X ray images provide information about the precipitating population of energetic electrons. The HXT data also give the energy spectral index of the electrons that emit hard X rays with energies between 14 keV and 93 keV. Images of the longitudinal component of the photospheric magnetic field are provided by MDI observations (Michelson Doppler Imager on board SOHO satellite). The gyrosynchrotron models will be compared with the radio observations in an attempt to derive a self consistent picture of the flare site.

2. Data Analysis

The three flares that we present in this paper were selected after we applied the following criteria to an extended database of Nobeyama observations: (1) Both 17 GHz and 34 GHz NoRH data must be available. (2) The radio emission must show a relatively impulsive rise (i.e. the peak radio emission should be reached within 2–3 min from the onset of the flare). (3) Hard X ray data from the HXT must be available. (4) Soft X ray images of the flaring loops must be available, at least for part of the events. The flaring loop configuration that appears in the SXT images must show a simple morphology with one well defined flaring loop (or arcades of loops) in each event. This criterion immediately rules out both compact loop configurations and complex loop systems.

The Nobeyama radioheliograph (NoRH) consists of 84 antennas in a T shaped array (Nakajima et al. 1991). It observes the full solar disk in two circular polarizations at 17 GHz and one linear polarization at 34 GHz, with subsecond time resolution for flare events. The data presented here were processed in the NRAO software package AIPS
using techniques developed by T. S. Bastian, S. M. White, K. Shibasaki and S. Enome and partially described by Nindos et al. (1999). Since in this study our goal is to achieve optimal spatial resolution, the final images were computed using uniform weighting, cleaned and self calibrated. The time resolution of the maps that we computed was 15–20 sec. The sizes of the final restoring beams are 10″ 12″ at 17 GHz and 6″ 8″ at 34 GHz.

All SXT images were processed with the standard calibration software, which does the data decompression, background subtraction, correction for solar rotation and co-registration of images. When possible, we calculated the electron temperatures and emission measures of the soft X-ray emitting plasma of the flares using the filter ratio technique (see Hara et al. 1992). The hard X-ray maps that we present were produced with the maximum entropy method algorithm that has been incorporated into the standard Yohkoh HXT software using data recorded within 12 sec before and after the maximum hard X-ray peak. The same time intervals were used for the computation of the hard X-ray photon spectral indices γ. For the computation of γ we did not take into account the low energy channel of the HXT (energies between 14–23 keV) in order to avoid the thermal emission contribution from possible hot sources. Assuming thick target for the production of hard X-rays, the electron energy spectral index is $\delta_x = \gamma + 1.5$ (for the terminology, see e.g. Hudson, Canfield, & Kane 1978, Tanberg Hanssen, & Emslie 1988, Nitta et al. 1991, Raulin et al. 1999).

Routine MDI observations provide images of the longitudinal component of the photospheric magnetic field. For the 2000 January 12 event the MDI image closest to the radio peak was recorded 6 minutes after the radio peak. For the 1998 May 8 event and the 1999 August 4 event the closest MDI images were recorded 1 min and 110 min after the radio peaks, respectively. The flares that we studied occurred close to the limb, therefore one should treat the information provided by the magnetograms cautiously and try not to overinterpret them.

This research requires very careful co-registration of data obtained at different wavelengths. Our reference time for the co-registrations was the time of the NoRH peaks. We rotated the MDI images to that time and did the overlays using the known pointing information of the instruments. These overlays were checked by comparing the extended preflare thermal emission which is visible in soft X-ray and 17 GHz images. Only in the 2000 January 12 event were the overlays unsatisfactory. In this case, the data co-registration was done by matching the soft X-ray limb with the radio limb (a height of 3000 km was assumed for the soft X-ray limb while the radius of the radio disk was assumed to be 1.0125 times larger than the radius of the optical disk). In all other cases, the agreement was better but in all cases the accuracy of our overlays is not generally better than about 5″.
3. Observational Results

The three flares that we studied are presented in fig 1, 3 and 5. Fig. 1, 3 and 5 show the microwave and hard X-ray images at the time of maximum and also the soft X-ray images and magnetogram images which were observed closest to the radio peak. Fig. 2, 4 and 6 show the time profiles of the flare emission at 17 GHz, 34 GHz, hard X-rays (from the HXT) and soft X-rays (from GOES satellite). The NORH fluxes are taken from the Nobeyama polarimeter data; the time resolution of the polarimeter data presented here is 1 sec. For the 1998 May 8 event, polarimeter data were not available at 34 GHz and we computed the 34 GHz time profile using the AIPS maps. For all events, the AIPS maps were used for the computation of the circular polarization $(V)$ time profiles. The fluxes from the AIPS maps were computed by summing up all pixels in the flare images with brightness temperatures higher than the lowest contours of fig. 1, 3 and 5.

The 17 GHz and 34 GHz $I$ time profiles show a good agreement which suggests that the same populations of energetic electrons are responsible for both the 17 GHz and 34 GHz emission. Furthermore the hard X-ray time profiles show the well known good agreement with the microwave time profiles (e.g. Cornell et al. 1984; Kai, & Nakajima 1987). We note that a detailed timing study of the radio and/or hard X-ray data is beyond the scope of this paper. The 1998 May 8 radio $I$ time profiles (fig. 2) show an impulsive increase and a much slower decay. From about 01:57:12 UT to 01:59:12 UT there are five radio peaks that exceed 90 SFU. These peaks show up in the hard X-ray time profile. However, they do not appear as prominent as they appear in radio because the hard X-ray time profile shows finer structure probably associated with the fact that the time resolution of the HXT (0.1 sec) is better than the time resolution of the radio data (1 sec) presented in fig. 2. We suggest that during the flare the energetic electrons are accelerated more than one time.

The 1999 August 4 radio and hard X-ray time profiles (see fig. 4) are simpler while the corresponding 2000 January 12 time profiles (fig. 6) reflect the presence of two flares. An inspection of the radio map movies shows that the two flares occurred at the same location (homologous flares). In all three flares the GOES soft X-ray flux peaks well after the microwave/hard X-ray peaks; this is the well known Neupert effect (Neupert 1968).

The Nobeyama polarimeter records total power fluxes at $1$, $2$, $3.75$, $9.4$, $17$, $34$ and $80$ GHz. For our three events, the $3.75$, $9.4$, $17$, $34$ and $80$ GHz fluxes at the time of maximum are given in Table 1. The turnover frequency is between $3.75$ GHz and $9.4$ GHz for the 1998 May 8 event and the 1999 August 4 event while for the 2000 January 12 event is located between $9.4$ and $17$ GHz. Therefore the bulk of the radio emission at $17$ GHz and $34$ GHz is optically thin. Using the fluxes of the radio emissions alone, we cannot determine quantitatively the optical depths at $17$ and $34$ GHz. However, the radio spectrum in all
three events indicates that both 17 and 34 GHz are on the optically thin side of the spectral peak, hence it is appropriate to assume optically thin emission. The 17 GHz emissions will have more opacity than the corresponding 34 GHz emissions, so if there are any optically thick regions they should be at 17 GHz. However, for each event both 17 and 34 GHz maps appear to be identical in morphology, and since the 34 GHz emission must be optically thin, the similarity between the 17 and 34 GHz morphologies reinforces the spectral result that the 17 GHz emission is also optically thin. We used the Nobeyama polarimeter fluxes from 80 GHz to 17 GHz to compute the slope $\alpha$ of the high frequency part of the flare spectrum (for the 2000 January 12 flare we used the 34 and 80 GHz fluxes only). Once $\alpha$ is known, the computation of the electron energy spectral index $\delta_e$ is straightforward provided that the emission is optically thin ($\alpha = 1.22 - 0.90\delta_e$, Dulk 1985). We find that $\delta_e = 2.2, 2$ and 2.7 for the 1998 May 8 event, the 1999 August 4 event and the 2000 January 12 event, respectively. These values should be taken as lower limits. The energy spectral index of the hard X-ray emitting electrons, $\delta_x$, is 6.0, 5.8 and 5.0 for the 1998 May 8, the 1999 August 4 and the 2000 January 12 flare, respectively. The radio energy spectral indices $\delta_e$ are harder than the hard X-ray spectral indices. Discrepancies between the electron energy spectral indices inferred from the radio and hard X-ray data during the impulsive burst phase have been reported in several previous studies (e.g. Kundu et al. 1994; Silva et al. 1997; Raulin et al. 1999). Recently, Silva, Wang & Gary (1999) studied the statistical properties of 28 impulsive flares and found that $\delta_x > \delta_e$ in 74% of them.

The SXT images in fig. 1, 3 and 5 show the geometry of the flare sites. The 1998 May 8 flare triggered the SXT flare mode and its SXT coverage is excellent. For the 1999 August 4 flare site only full frame SXT images (FFIs) were available. The only useful image is the one presented in fig. 3. This image was obtained about 6 min before the maximum radio emission which occurred at 01:57:37 UT (there are two more FFIs observed 10 and 9 min before the flare peak but they cannot be used in our study because the flare site is covered by saturated emission). However, the low channel HXT map created at the peak of the event shows a loop-like feature which matches the small eastern loop of fig. 3. Therefore no significant changes in terms of the flaring loop geometry are likely to have occurred between the time of the SXT image and the flare peak. The first SXT image of the 2000 January 12 flare was obtained about 9 min after the radio peak which occurred at 01:36:15 UT. However, the shape of the 34 GHz map at the time of maximum delineates fairly well the post-flare loop of fig. 5. Therefore we suggest that that loop represents the flare geometry quite accurately at least to a first approximation.

The SXT images show that both large and smaller loops are present. All three flares occur in the small loops. In the 1998 May 8 flare both the small and the large loop existed at least 2-3 min before the flare onset and no loop motion was observed before or during
the flare. Therefore that flare configuration is different from the so called “double loop” flares (Hanaoka 1997, 1999) where the small flaring loop reflects the emergence of new magnetic flux which interacts with the pre-existing overlying loops and produces the flare. For the other two flares, the available SXT images do not show any loop motions. In any case, Hanaoka’s “double loop” flares show microwave emission at both the small emerging loop as well as at the remote footpoint of the larger loop, which is not the case for the events studied here.

The flare emissions at 17 GHz and 34 GHz cover the flaring loops. The peak of the radio emission is located close to the loop tops. In many cases (e.g. Shevgaonkar, & Kundu 1985; Nindos et al. 2000) such morphologies have been attributed to optically thick sources: when the emission is optically thick usually the microwave peak occurs at the loop top because the magnetic field is lowest there and therefore the effective energy of the electrons emitting there is higher. Low energy electrons (< 100 keV) can produce optically thick emission at 17 and 34 GHz, but only if the magnetic field is very strong (in excess of 2000 G for emission at 34 GHz) and then the resulting spectrum is inconsistent with the spectra we observe for these events: low energy electrons in a strong magnetic field produce a peak in the radio spectrum at a very high frequency with essentially no emission above the spectral peak (as in a thermal gyrosynchrotron model, cf. Dulk 1985). We are therefore unable to find a model in which the radio emission at both 17 and 34 GHz can be optically thick and remain consistent with the observed fluxes. All three events occurred close to the limb and the angle between the magnetic field and the line of sight $\theta$ can be such that radio emission can be produced from the entire loop even if the emission is optically thin. Of course the only problem in this interpretation is that if the magnetic field is much stronger at the footpoints of the flaring loop then the intensity of the optically thin emission should be higher above the footpoints (e.g. see the results from the model computations by Alissandrakis, & Preka Papadema 1984; Preka Papadema, & Alissandrakis 1988, 1992; Nindos et al. 2000). The fact that the maxima do not occur above the footpoints implies that the variation of the magnetic field between the footpoints and the loop tops of the three flaring loops is not large. Therefore the optically thin radio emission peaks close to the loop tops because the angles $\theta$ are larger there. Furthermore even in cases where the bulk of radio emission is optically thin, small regions of optically thick emission may also exist. We shall return to these topics in the next sections of the paper.

Fig. 16 show that the degree of circular polarization $\rho_c$ is small. The absolute value of typical $\rho_c$ is 10\%, 11\% and 5\% for the 1998 May 8 flare, the 1999 August 4 flare and the 2000 January 12 flare, respectively. Both left hand and right hand circularly polarized emission appears in the $V$ maps of the three flares. The MDI neutral lines are not reliable because the active regions under study are close to the limb. Therefore no further study of
the relationship between the observed $V$ emission and the underlying magnetic field can be made.

In all three flares the bulk of the emission of the hard X ray sources in the M1, M2 and H HXT channels is located close to the footpoints of the soft X ray flaring loops above regions of opposite photospheric magnetic field polarity. Fig. 1 and 5 show that the footpoint hard X ray sources are asymmetric in the 1998 May 8 flare and the 2000 January 12 flare. In both events the brighter hard X ray sources are located above the weaker photospheric magnetic field in agreement with the results of Sakao (1994) and Kundu et al. (1995). These properties strongly suggest that the hard X ray emission in these flares comes from thick target bremsstrahlung by nonthermal particles.

4. Model Computations

4.1. Input parameters

The time profiles of the three radio bursts as well as the morphology of the radio images clearly suggest that the 17 and 34 GHz emission is gyrosynchrotron from nonthermal electrons trapped in flaring loops. For the 1998 May 8 event the SXT coverage was excellent and we computed temperatures and emission measures of the soft X ray emitting material. The derived plasma parameters were used for the calculation of the 17 and 34 GHz fluxes from the observed soft X ray emitting material on the basis of thermal free-free emission (see Dulk 1985 for the formula). The resulting fluxes never exceeded 3 SFU. In this section we present model computations of gyrosynchrotron emission to determine the physical parameters of the three flaring loops. The model has been presented in detail by Nindos et al. (2000). The model uses a line dipole magnetic field. The field lines are circles with a common tangent point at the dipole: the magnetic field falls off quadratically with distance from the line dipole. The most important physical parameters of this magnetic field model are the magnetic field strengths at the footpoints and at the loop top. The pitch angle distribution of the trapped electrons is assumed isotropic. One may argue that a loss cone distribution is more appropriate for trapped electrons which produce microwave emission. However, in a loss cone distribution the electrons missing are those with small pitch angles whose gyro acceleration, and hence emissivity, is smallest. Most of the emission is produced by the larger pitch angles, so using an isotropic distribution should not affect the results significantly. The gyrosynchrotron emission from the loop is computed using a code in which the gyrosynchrotron emissivity and opacity are calculated exactly at specified points along the loop and the emitted radio flux is calculated using simple radiative transfer (Nindos et al. 2000; see also Schmahl, Kundu, & Dennis 1986; Nitta et al. 1991). We
shall constrain the input parameters of the models using the observational data that we presented in section 3. The free parameters of the models should be selected so that (1) the models reproduce both the observed morphologies and fluxes of the 17 GHz and 34 GHz emissions and (2) the shape of the model flaring loops is consistent with the shape of the observed flaring loops as inferred from the SXT/HXT images. The reader should refer to the paper by Nindos et al. (2000) for a detailed discussion on how the variation of input parameters affects the models.

The SXT images give the shape of the flaring loops and the hard X-ray sources determine the location of the footpoints of the loops. In our code, the loop can be placed at any heliographic longitude and latitude with an arbitrary orientation (azimuth) with respect to the local north. The model loop lies on a vertical plane above the solar surface whereas the real loop may be tilted. If necessary we can simulate this by changing the latitude of the apparent loop (see Nindos et al. 2000 for details). Overall, we choose sets of parameters for the loop geometry (i.e. azimuth, tilt, maximum height and footpoint separation of the model loop) so that the model loops resemble the observed loops. For each event, we kept the longitude of the model loop equal to the longitude of the flare site and we varied simultaneously the azimuth and tilt angle from -90° to 90° with a step of 10°. Then the model geometrical input parameters were selected by comparing the projection on the plane of the sky of the resulting model loops with the observed loop using the chi-square criterion (of course some models can be ruled out simply by visual inspection). Since the SXT images are the convolution of the sky soft X-ray intensity distribution with the SXT’s point spread function, the observed thickness of the SXT loops provides upper limits for the thickness of the model loops.

The model loop is filled with energetic electrons with a power law energy spectral index equal to the power law energy spectral index of the radio-emitting energetic electrons that we computed in section 3 using the slope of the optically thin part of the Nobeyama polarimeter flux spectra. The photospheric MDI images provide information about the magnetic field strength. However, the 1999 August 4 magnetic field image was not observed the same time when the flare occurred. Furthermore, the flares occurred close to the limb and the longitudinal component of the photospheric field that the MDI observes is lower than the total photospheric magnetic field. Consequently, the photospheric magnetic field strength is ultimately a free parameter in our program. The other free parameters are the low energy cutoff and the high energy cutoff of the energy distribution of the energetic electrons and also the number density of the radio-emitting electrons. We did not follow the chi square criterion for the final selection of the best fit model because there are too many free parameters for such an approach. We only seek “qualitatively” good fits, and varying different free parameters produces changes that we describe here and in Nindos et
al. (2000).

4.2. The 1998 May 8 Flare Model

The orientation (azimuth) of the model loop with respect to the local north is 85° and its tilt angle 0°. The footpoint separation and the maximum height of the model loop are $3.4 \times 10^9$ cm and $7.5 \times 10^8$ cm, respectively. With these input parameters the projection of the model loop on the plane of the sky is similar to the observed flaring loop as it appears in the SXT images. In our model of circular field lines, loop divergence results in a change of magnetic field strength from the footpoints to the loop top. Using the above input parameters, the loop top field strength is a factor of 1.3 lower than the model footpoint field strength. From the Nobeyama polarimeter data we get the energy spectral index of the trapped electrons (see section 3); we find that $\delta_e = 2.2$.

We tried all possible combinations for the remaining input parameters but we were not able to reproduce both the observed structures and fluxes at 17 GHz and 34 GHz. In the cases that the model fluxes agree with the observed fluxes, the model morphologies show emission peaks close to the footpoints and a dip close to the loop top. This is not surprising because it is well known (eg Preka Papadema, & Alissandrakis 1992; Bastian, Benz, & Gary 1998; Nindos et al. 2000) that when the emission is optically thin its maxima are located close to the footpoints provided that the magnetic field is strong at the footpoints and weaker at the loop top. Several previous observations have confirmed this picture (eg Shevgaonkar, & Kundu 1985; Bastian, & Kiplinger 1991; Alissandrakis, Nindos, & Kundu 1993; Wang et al. 1995; Nishio et al. 1997). However, this picture clearly disagrees with the appearance of the flare in the NoRH maps (fig. 1). Another important problem of these models is that the resulted aspect ratios (i.e. loop width/loop length) is implausibly small (of about 0.01 or smaller). On the other hand, the dips at both frequencies disappear if we change the input parameters appropriately and make the model loop optically thick. But the resulting fluxes are orders of magnitude higher than the observed polarimeter fluxes. The fact that the radio observations show no emission peaks above the footpoints combined with the polarimeter data which indicate that the emission is optically thin at 17 GHz and 34 GHz suggest that the variation of the magnetic field along the loop must be very small. Therefore we tried a model with a constant magnetic field along the loop. In the models presented by Nindos et al. (2000) the loop thickness was scaled along the loop by $B^{-0.5}$ in order to simulate magnetic flux conservation. For a constant magnetic field model, the loop thickness is constant along the loop. Klimchuk et al. (1992, 2000) have studied non flaring coronal active region loops observed by the SXT, the Extreme Ultraviolet
Telescope (EIT) on board SOHO satellite and the Transition Region and Coronal Explorer (TRACE) satellite and found that the large majority had nearly uniform thickness.

Using the magnetic field model described in section 4.1 but allowing no variations of the magnetic field strength along the model flaring loop, we reached the best fit to the radio fluxes for the following input parameters: the number density of the energetic electrons is $N_e = 10^4$ cm$^{-3}$, the low and high energy cutoffs of their energy distribution are 60 keV and 5000 keV, respectively. The thickness of the loop is $3.8 \times 10^8$ cm and the constant magnetic field is 350 G. In fig. 7 we compare a one dimensional profile of the NoRH maps at the time of maximum flux with the model fluxes as a function of distance along the loop, convolved with the appropriate NoRH beam. The NoRH profiles were computed along the white curve shown in fig. 1 (left panel). The 17 and 34 GHz model emission is optically thin (the $x$ mode optical depth is about $2.6 \times 10^{-3}$ and $2.2 \times 10^{-4}$ at 17 GHz and 34 GHz, respectively). Since the model loop is close to the limb, as expected the variation of the angle $\theta$ between the magnetic field and the line of sight is not large: $74^\circ < \theta < 86^\circ$. This combined with the fact that the magnetic field is constant along the model loop explains why the variation of the model intensity along the loop is small. Therefore the convolution of the model intensities with the NoRH beam gives a flat top profile for a significant part of the model loop. The dip close to the loop top has disappeared. However, the observations show a more prominent peak than the models. The model polarization shows a dip which is associated with the region where the magnetic field is orthogonal to the line of sight. The maximum degree of model circular polarization is about 5% (the observations give maximum values of $\rho_e$ as high as 40% but the typical $\rho_e$ value is about 10%).

4.3. The 1999 August 4 Flare Model

In order to model this event we followed the same procedure as in the study of the 1998 May 8 event. Using the magnetic field model described in section 4.1, we found the appropriate input parameters that make the projection of the model loop on the plane of the sky resemble the observed SXT flaring loop. The problems that we faced using that magnetic field model and trying all possible combinations for the remaining free parameters are the same that we encountered in the modeling of the 1998 May 8 flare and we shall not repeat them here. The best agreement between the models and the observations (both spatial structures and fluxes at 17 and 34 GHz) is reached if we keep the magnetic field strength along the loop constant.

Fig. 8 shows the one dimensional profiles of the NoRH maps at the time of maximum flux and the model fluxes as a function of distance along the loop. The model is a magnetic
loop with an azimuth of 70° and a tilt angle of 20°. The model magnetic field is 320 G along the entire loop. The maximum height of the model loop is $8.8 \times 10^8$ cm, the distance between its footpoints is $2.2 \times 10^9$ cm and its thickness is $3.2 \times 10^8$ cm. The energy spectral index of the radio emitting particles is $\delta_r = 2$. The density of the trapped electrons is $N_e = 4.5 \times 10^4$ cm$^{-3}$ and the low and high energy cutoffs of their energy distribution are 70 keV and 5000 keV, respectively. The 17 and 34 GHz model emission is optically thin (the $x$ mode optical depth is about $1.3 \times 10^{-2}$ and $1.2 \times 10^{-3}$ at 17 GHz and 34 GHz, respectively).

Fig. 8 shows that the model $I$ profiles do not show any dip close to the loop top; the $I$ profiles are flat topped over a significant part of the model loop. The model polarization shows a dip close to the loop top which is associated with the region where the magnetic field is orthogonal to the line of sight. The maximum degree of model circular polarization is about 18\% (the observations give a maximum $\rho_c$ of 23\%).

4.4. The 2000 January 12 Flare Model

Our initial modeling attempts used the magnetic field model described in section 4.1, but the same problems that we described in sections 4.2 and 4.3 appeared again. Therefore, we used the model magnetic field that we described in section 4.1 but we kept the magnetic field constant along the model flaring loop. The best fit model to the radio data is presented in fig. 9. The orientation of the model loop with respect to the local north (azimuth) is 80° and its tilt angle is 0°. The model magnetic field was 370 G along the entire loop. The maximum height of the model loop is $1.3 \times 10^9$ cm, the distance between its footpoints is $2.3 \times 10^9$ cm and its thickness is $4 \times 10^8$ cm. The energy spectral index of the radio emitting particles is $\delta_r = 2.7$. The density of the trapped electrons is $N_e = 6 \times 10^4$ cm$^{-3}$ and the low and high energy cutoffs of their energy distribution are 130 keV and 5000 keV, respectively. The 17 and 34 GHz model emission is optically thin (the $x$ mode optical depth is about $3.8 \times 10^{-2}$ and $2.5 \times 10^{-3}$ at 17 GHz and 34 GHz, respectively). We note that for this event we were able to check the concept of constant loop thickness which is imposed by the use of a constant magnetic field along the model loop. We applied the methodology developed by Klimchuk et al. (1992) and measured the thickness of the SXT flaring loop of fig. 5. Our measurements show that the thickness of the SXT loop is indeed nearly uniform: the standard deviation of the measured widths along the SXT loop is about 0.03 pixels (0.0735'). Unfortunately, we were not able to measure the flaring loop widths in the two other events using Klimchuk et al.'s (1992) technique because part of their soft X ray emission could not be distinguished from the soft X ray emission of nearby loops.
Fig. 9 shows that the $I$ model profiles at both frequencies are flat topped for a significant part of the model loop. The observed profile has a prominent peak close to the loop top. The $V$ models show a dip close to the loop top which is associated with the region where the magnetic field is orthogonal to the line of sight. The typical degree of model circular polarization is about 8% while the observations show a typical $\rho_c$ of about 5%.

5. Conclusions

We presented radio, soft X-ray and hard X-ray observations of three strong flares that occurred close to the limb. The geometry of the flares, as inferred from the SXT images, is simple with one well-defined flaring loop in each event. The hard X-ray emission in the M1, M2 and H HXT channels shows two sources close to the footpoints of the soft X-ray flaring loops above regions of opposite magnetic polarity. The HXT data are consistent with the view that impulsive phase hard X-ray emission is dominated by thick target bremsstrahlung by nonthermal particles. The radio total intensity ($I$) time profiles in both frequencies show good agreement which suggests that the same population of energetic electrons is responsible for both the 17 and 34 GHz emissions. Data from the Nobeyama polarimeter suggest that the flares are optically thin at 17 and 34 GHz. The 17 and 34 GHz $I$ maps show emission from the entire flaring loops with peaks close to the loop tops. Usually such morphologies are attributed to optically thick gyrosynchrotron emission. However, such an interpretation is not appropriate for our three flares due to the strong support in favor of optically thin emission that the polarimeter data provide. Optically thin radio emission from a limb event can outline the whole loop provided that the flaring loop geometry leads to high angles $\theta$ between the magnetic field and the line of sight along the entire loop. The fact that optically thin emission at such high frequencies does not show maxima above the footpoints of the loops may indicate that the variation of magnetic field strength along the flaring loops is very small. In all flares the typical degree of circular polarization $\rho_c$ is relatively low (about 5% 10%).

The time profiles and morphologies of the three flares at 17 and 34 GHz indicate that their emission in these frequencies is gyrosynchrotron from nonthermal electrons. We tried to reproduce the radio spatial structures and fluxes using a simple inhomogeneous gyrosynchrotron model (see Nindos et al. 2000). The SXT and HXT data constrained some of the model input parameters. The distance between the HXT footpoint sources determines the footpoint separation of the model loop. The model loop maximum height, footpoint separation, azimuth and tilt angle were chosen so that it resembles the SXT flaring loop. The slopes of the high frequency part of the Nobeyama polarimeter flux spectra were
used for the computation of the energy spectral indices of the model nonthermal electrons that produce the radio emission. The other input parameters could be varied. Initially, the magnetic field models we used had magnetic field strengths which were higher at the footpoints and weaker at the loop tops. However, such model could not reconcile the observed radio morphologies and fluxes. When the models produced optically thin emission, it was possible to reproduce the observed fluxes but the $I$ models showed maxima above the footpoints and dips close to the loop tops contrary to the observations. Another serious problem of the latter set of models is that the reproduction of the observed fluxes requires that the aspect ratios (i.e. loop width/loop length) of the model loops is very small (of the order of 0.01 or smaller). The $I$ model morphology would resemble the NoRH maps if we made the loop optically thick at both frequencies. But the resulting radio spectra were not consistent with the observations.

The magnetic field strength of a classic model loop is strong at the footpoints and weaker close to the loop top. Such a classic magnetic field model produces stronger microwave emission close to the footpoints when the emission is optically thin. The ways to suppress/reduce the emission above the footpoints are: (1) if we study a limb event, it is possible that part of the flaring loop is behind the limb. The occultation of the footpoints prevents their strong radio emission from reaching the observer. However in fig. 1 and 5 we have drawn the radio limb on the SXT images and we clearly see that this was not the case for our flares. (2) In the optically thin limit it is very hard to come up with a model in which the loop top appears brightest, because emissivity goes as a high power of $B$ (Dulk 1985) and we were not able to compensate for the lower $B$ at the loop top by a larger line of sight depth at the loop top because the emissivity is proportional to the line of sight depth. That was true of all models that we computed with non constant magnetic field using different azimuth and tilt angles (we varied the azimuth and tilt angles simultaneously from $-90^\circ$ to $90^\circ$ with a step of $10^\circ$). One thing that can be done is make the legs of the loop parallel to the line of sight and the loop top orthogonal to the line of sight, but this is rather implausible for events at the limb. Our models rule out such geometrical configuration because the resulting model loops do not resemble the observed loops. (3) Previous observations and modeling have shown flares where the radio source does not reach all the way down to the footpoints of the flaring loop. Such morphologies have been interpreted within the framework of the classic magnetic field loop model using different arguments. For example Preka Papadema & Alissandrakis (1992) pointed out the importance of gyroresonance absorption which prevents the escape of gyrosynchrotron emission from the limbward side of flaring loops located close to the limb and observed at 1.5 GHz. Nindos et al. (2000) found that if the 5 GHz gyrosynchrotron emission comes from low harmonics of the gyrofrequency then harmonic effects explain why the 5 GHz
emission does not reach the footpoints. (4) Lee, Gary & Shibasaki (2000) pointed out that magnetic trapping can also contribute to restricting the size of radio sources. Fig. 1, 3 and 5 show clearly that the interpretations (3) and (4) are not relevant to our data because the radio emission at both frequencies does reach the footpoints but is simply weaker than the emission from the loop tops. The only exception is probably the 34 GHz source of the 2000 January 12 flare which does not reach the northern HXT source. But the gap is small (< 5") and within the uncertainties of the overlays (see section 2).

The final option, namely to keep the magnetic field constant along the loop, produces the models that best fit the datasets. In these models the emission is optically thin at both frequencies in agreement with the polarimeter data. With such a magnetic field model we are able to reproduce the observed fluxes and the $I$ model morphologies do not show dips close to the loop tops. These models do not in fact match the radio data very well, but we need to make it clear that they are the best fits we can get under the best assumptions we can justify. The most serious disagreement of these models with the data is that the $I$ radio emission along the loops is flat topped whereas the observations show more prominent peaks close to the loop tops (the disagreement is more serious in the 1998 May 8 flare and the 2000 January 12 flare). The $I$ models show flat topped profiles because the magnetic field is constant along the loop and the loops are located close to the limb with such orientation that the angle $\theta$ between the magnetic field and the line of sight does not change significantly along the loop. $I$ peaks close to the loop tops can be produced only if we use the non constant magnetic field model that we tried originally and make the loops optically thick. But then the resulting radio spectra are not consistent with the observations. In the framework of our constant field model such $I$ peaks cannot be reproduced no matter how we vary the remaining free input parameters (i.e. both the parameters that are associated with the loop geometry and the parameters associated with the energetic electrons). We believe that this disagreement may be due to the oversimplified loop geometry that we used.

The concept of constant magnetic field along the loop suggests that the loop thickness is also constant due to the principle of magnetic flux conservation. Klimchuk et al. (1992, 2000) have studied several non flaring coronal loops and found that a large majority of them had uniform thickness. But as far as we know, this is the first time that there is some evidence for flaring loops of constant thickness. Furthermore, for the 2000 January 12 event we were able to measure the SXT flaring loop thickness and confirmed that it was constant. As Klimchuk et al. (1992, 2000) note this result implies that the magnetic field in such loops expands with height much less than standard coronal models would predict. Klimchuk et al. (2000) have investigated whether this surprising result can be explained by locally enhanced twist in the field, so that observed loops correspond to twisted coronal flux tubes. However, their model computations show that twist does not seem to be a likely
explanation for the observed minimal expansion with height. Clearly more theoretical work
is needed for a better interpretation of this finding. Furthermore, if the magnetic field is
constant along a flaring loop there is an additional problem that needs to be confronted:
how to keep the microwave emitting electrons trapped in the corona without magnetic
mirroring.

Several different parameters of the three best fit models appear in Table 2. It is
interesting that all models require the presence of high energy electrons (as high as
5000 keV). If we remove the electrons above about 500 keV, the model fluxes decrease
significantly at both frequencies; the 34 GHz fluxes are always affected the most. This
phenomenon has been demonstrated nicely by White & Kundu (2000) in gyrosynchrotron
flux spectra of radio emission from nonthermal electrons (see their fig. 2). We note that in
all models that we computed, no matter whether the magnetic field was constant or not,
such MeV electrons were needed in order to produce fluxes of the order of the observed
fluxes. Our models indicate that the aspect ratios of the flaring loops (i.e. loop width/loop
length) is about 0.1. Such aspect ratios are in the low end of observed flare loop aspect
ratios (see Takahashi 1997 who found that the aspect ratios of flaring loops are usually
0.1 1; on the other hand, the aspect ratio of the non constant magnetic field models whose
fluxes agreed with the observed fluxes was implausibly low). We note that the width of
the model flaring loops are always lower than the width of the corresponding SXT flaring
loops. This may be the result of inadequate SXT spatial resolution. NIXT high resolution
images (Golub et al. 1990) suggest that loop thicknesses may be lower than 1", well below
the SXT’s resolution. An inspection of TRACE movies also shows such thin loops. Another
interpretation may be that the energetic electrons in the coronal part of the loop are confined
in selected field lines whose total volume is only a fraction of the soft X ray flaring loop.
On the other hand the precipitating electrons can move across field lines more easily in the
chromosphere and therefore the process of chromospheric evaporation fills the entire soft
X ray flaring loop with hot thermal plasma.

A comparison between the energy spectral indices inferred from the HXT data and
the energy spectral indices inferred from the polarimeter data shows differences of 2.3 3.8
with the hard X ray indices being larger. Such discrepancies have been reported several
times previously. Using the thick target formulas by Hudson, Canfield, & Kane (1978) (see
also Raulin et al. 1999) we computed the energy distribution of the hard X ray emitting
electrons for the time of hard X ray maximum. We integrated the energy distribution over
the range of 20 100 keV and found rough estimates for the number densities of electrons in
that energy range. The densities we found are about 100 times higher than the densities of
the trapped electrons that produced the radio emission in our models. Since the optically
thin gyrosynchrotron flux increases with increasing density and magnetic field, in principle
we may reproduce the 17 and 34 GHz fluxes by increasing the model input density and appropriately decreasing the model input magnetic field. But such models show problems because (1) the inferred magnetic field strengths are too low (50–100 G) and such low values cannot be supported easily by the MDI data (2) the frequencies of the spectral peaks decrease and are not consistent with the polarimeter data. The lack of hard X-ray data at energies higher than 93 keV combined with the fact that the low energy cutoffs in the gyrosynchrotron models are 60–130 keV does not allow us to investigate whether or not these discrepancies between the derived number densities mean that the hard X-ray emission and the radio emission come from different populations of energetic electrons (e.g., as in the events presented by Kundu et al. 1994).

Our study indicates that the current status of solar flare modeling is not completely satisfactory when we seek self-consistent models that reproduce all the observational aspects of a flare in radio, soft X-rays and hard X-rays. Furthermore, our three dimensional models are static and we do not attempt to follow evolution of flare radio emission. During the HESSI era we will want to follow the 3D evolution from the pre-flare state to the post-flare state. Obviously the complexity of these tasks is great and will require far more sophisticated models.

6. ACKNOWLEDGMENTS

This research at the University of Maryland was carried out with support from NSF grants ATM 96 12738, ATM 99 09809 and INT 98 19917 and NASA grants NAG 5 7370, NAG 5 8192 and NAG 5 7901. We thank the referee for his comments, which led to significant improvement of the paper.
REFERENCES


FIGURE CAPTIONS

Fig. 1. The 1998 May 8 flare. The left and right panels show an SXT image obtained 10 sec before the time of the radio maximum. The middle panel shows an MDI photospheric magnetogram obtained 1 min after the flare. The black solid contours show the 17 and 34 GHz $I$ emission at the time of maximum. The contour levels for both frequencies are (5%, 15%, 25%, 35%, 55%, 75%, 95%) of the maximum brightness temperature which is 9.8 MK and 2.4 MK at 17 and 34 GHz, respectively. In the middle panel the white solid and dotted contours show the 17 GHz left hand and right hand $V$ emission, respectively. The $V$ contours are at (-10%, ±2.5%, ±5%) of the maximum 17 GHz $I$ brightness temperature. In all three panels the dashed contours show the hard X ray emission from the M2 HXT channel at the time of maximum. The HXT contours are at (12.5%, 25%, 50%, 70.7%) of the maximum hard X ray emission. Note that in order to increase the visibility of the HXT contours we have not used the same color for all HXT contours: in the left and right panels we use black and grey for the north and south component respectively; in the middle panel the contours of both HXT components are white. In the left panel, the white curve delineates the flaring loop while the thick black curve shows the radio limb. In this and subsequent solar images, north is up and west to the right. The axes labels denote seconds of arc from disk center. The label in the right top corner of the figure indicates the time of the radio images.

Fig. 2. Time profiles of the 1998 May 8 flare. From top to bottom: 17 GHz $I$, 17 GHz $V$, 34 GHz $I$, hard X ray emission in the 33-53 keV HXT channel and soft X ray emission from GOES 1.5-10 keV data. The 17 GHz $I$ profile has been derived from the Nobeyama polarimeter data at 1 sec time resolution. The 17 GHz $V$ profiles and the 34 GHz $I$ profile have been derived from the AIPS maps at 15 sec time resolution. The dashed and solid $V$ profiles show left hand and right hand circularly polarized emission, respectively.

Fig. 3. The 1999 August 4 flare in the same format as in fig. 1 with the following exceptions. The SXT image was obtained about 6 min before the maximum radio emission. The MDI image was obtained 1 hour and 50 min after the flare peak. In the left and right panels, the white solid contours show the 17 and 34 GHz $I$ emission. The contour levels for both frequencies are (5%, 15%, 25%, 55%, 75%, 85%) of the maximum brightness temperature which is 25.7 MK and 6.9 MK at 17 and 34 GHz, respectively. In the middle panel the dotted and solid white contours show the left hand and right hand circularly polarized emission. The $V$ contour levels are at (±2.5%, ±5%, ±10%) of the maximum 17 GHz $I$ brightness temperature. In all panels the hard X ray emission is denoted by white dashed contours.
Fig. 4. Time profiles of the 1999 August 4 flare in the same format as in fig. 2 with the exception that the 34 GHz time profile was derived from the Nobeyama polarimeter data at 1 sec time resolution.

Fig. 5. The 2000 January 12 flare in the same format as in fig. 3 with the following exceptions. The SXT image was obtained about 10.5 min after the time of the radio maximum and the MDI image was obtained about 6 min after the time of the radio maximum. The maximum brightness temperature is 70.5 MK and 42.5 MK for the 17 and 34 GHz map, respectively. In the middle panel the dashed black and the dashed white contours show the left hand and right hand circularly polarized emission. The $V$ contour levels are at (±0.5%, ±1%, 2.5%) of the maximum 17 GHz $I$ brightness temperature. In all panels the thick solid contours show the the hard X ray emission from the HXT M1 channel. The HXT contours are black in the left and right panel and white in the middle panel.

Fig. 6. Time profiles of the 2000 January 12 flare in the same format as in fig. 4. The only exception is that in this figure the time profile of the hard X ray emission from the 23 33 keV HXT channel is presented.

Fig. 7. Top panel: One-dimensional representation of the 1998 May 8 NoRH flare at the time of maximum. These profiles were computed along the solid white curve shown in the left frame of fig. 1. Bottom panel: spatial profiles of the best fit models to the 1998 May 8 flare as a function of the distance along the loop (see text for information about the model input parameters). For comparison with the observations, each profile has been convolved with the appropriate NoRH beam. In both panels, we present the absolute values of the $V$ profiles. For a better comparison, we have divided the NoRH values with the appropriate beam.

Fig. 8. Same as in fig. 7 for the 1999 August 4 NoRH maps and models (see text for information about the model input parameters).

Fig. 9. Same as in fig. 7 for the 2000 January 12 NoRH maps and models (see text for information about the model input parameters).
<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th>GOES Class</th>
<th>Flux 3.75 GHz (SFU)</th>
<th>Flux 9.4 GHz (SFU)</th>
<th>17 GHz Flux (SFU)</th>
<th>34 GHz Flux (SFU)</th>
<th>80 GHz Flux (SFU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998-05-08</td>
<td>W82S15</td>
<td>M3.1</td>
<td>42</td>
<td>182</td>
<td>94</td>
<td>65^a</td>
<td>-</td>
</tr>
<tr>
<td>1999-08-04</td>
<td>W64S18</td>
<td>M6.0</td>
<td>246</td>
<td>583</td>
<td>428</td>
<td>208</td>
<td>165</td>
</tr>
<tr>
<td>2000-01-12</td>
<td>E79N13</td>
<td>M2.8</td>
<td>300</td>
<td>550</td>
<td>946</td>
<td>582</td>
<td>144</td>
</tr>
</tbody>
</table>

^aDerived from the flare map.
Table 2. Model Parameters

<table>
<thead>
<tr>
<th>Date</th>
<th>$B_{foot}^a$ (G)</th>
<th>$B_{top}^b$ (G)</th>
<th>Tilt$^c$ (deg)</th>
<th>Loop Length (cm)</th>
<th>Loop Thickness (cm)</th>
<th>Density (cm$^{-3}$)</th>
<th>$E_{min}^d$ (keV)</th>
<th>$E_{max}^e$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998-05-08</td>
<td>350</td>
<td>350</td>
<td>0</td>
<td>$3.9 \times 10^9$</td>
<td>$3.8 \times 10^8$</td>
<td>$1 \times 10^4$</td>
<td>60</td>
<td>5000</td>
</tr>
<tr>
<td>1999-08-04</td>
<td>320</td>
<td>320</td>
<td>20</td>
<td>$3.2 \times 10^9$</td>
<td>$4.0 \times 10^8$</td>
<td>$4 \times 10^4$</td>
<td>70</td>
<td>5000</td>
</tr>
<tr>
<td>2000-01-12</td>
<td>370</td>
<td>370</td>
<td>0</td>
<td>$3.7 \times 10^9$</td>
<td>$3.2 \times 10^8$</td>
<td>$6 \times 10^4$</td>
<td>130</td>
<td>5000</td>
</tr>
</tbody>
</table>

$^a$Footpoint magnetic field strength.

$^b$Loop top magnetic field strength.

$^c$Angle between the plane of the loop and the local vertical.

$^d$Low-energy cutoff of the radio-emitting electrons.

$^e$High-energy cutoff of the radio-emitting electrons.
Fig. 1.
Fig. 3.
Fig. 4.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.