



## CARMA Memorandum Series #40

### Radio Pointing Measurements for CARMA

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#### ABSTRACT

We describe the process by which accurate radio pointing is achieved for the antennas of the CARMA array. We describe the “pointing models”, i.e., the equations used to convert a source hour angle and declination into the azimuth and elevation measurements on the encoders in the antennas. These equations are different for the OVRO and BIMA dishes due to differences in the hardware. Many of the coefficients in the pointing models are determined using optical pointing measurements with the cameras attached to each antenna: these are described elsewhere. Once optical pointing has been carried out, separate radio pointing observations are obtained on bright sources to measure fixed offsets in azimuth and elevation between the optical camera pointing and the radio pointing, and any difference in the sag of the radio pointing as the antenna elevation is changed and gravity deforms the dish surfaces. The programs and tools used at CARMA for analysis of radio pointing are described.

## 1. Introduction: Antenna Pointing

Radio telescope dishes generally use what is called an *alt/az* mount, in which pointing is achieved by driving the antenna about two axes: a vertical axis, determining the *azimuth* (effectively compass position in the plane of the horizon: the CARMA on-line system uses the convention that zero azimuth is due north and azimuth increases to the east, while the data reduction package MIRIAD uses the convention that zero azimuth is due south and azimuth decreases as you move from east to west), and a horizontal axis (*altitude*) whose position angle rotates as the azimuth changes and about which the dish is rotated to achieve a given elevation on the sky.

Since antennas work in azimuth  $Az$  and elevation  $El$ , measured with respect to the local horizon, while cosmic source positions are identified by right ascension  $\alpha$  and declination  $\delta$ , “pointing” an antenna amounts to determining the current azimuth and elevation corresponding to a given  $\alpha$  and  $\delta$  on the sky. At a given local sidereal time (LST) and observatory latitude  $L$ , the relationship between these coordinate systems is given by the spherical-geometry equations

$$\sin(El) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(H) \quad (1)$$

$$\cos(El)\cos(Az) = \cos(L)\sin(\delta) - \sin(L)\cos(\delta)\cos(H) \quad (2)$$

$$\cos(El)\sin(Az) = -\cos(\delta)\sin(H) \quad (3)$$

where  $H = \text{LST} - \alpha$  is the hour angle of the source. For an idealized rigid antenna which is perfectly level, perfectly symmetric in both reflector, subreflector and feed, and has its axes perfectly aligned, these relationships would be sufficient to achieve pointing.

However, in practice we do not measure the absolute azimuth and elevation, but rather the numbers on encoders attached to both axes of each antenna. The antennas are unlikely to be perfectly level or symmetric, and the readings from the encoders need not match the actual  $Az$  and  $El$ . We deal with these realities by applying corrections to the process of conversion from  $H$  and  $\delta$  to the encoder values for  $Az$  and  $El$ .

## 2. Pointing Models and Constants

The main sources of error in the conversion from  $H$ ,  $\delta$  to the encoder  $Az$ ,  $El$  are due to the following effects:

1. **Tilt:** any difference between the vertical axis of the antenna and the local vertical will produce pointing errors.
2. **Transverse misalignment:** this effect occurs if the radio beam of the antenna is offset from the orthogonal to the elevation axis, and produces a simple azimuth error.

3. **Encoder errors:** simple offsets of the encoder zero points from the true zero points can be determined, as can imperfect alignment of the elevation axis relative to the azimuth axis. However it has to be assumed that, apart from this offset, the encoders are accurate to a precision better than the other sources of error, because additional encoder errors are difficult to determine.
4. **Sag and other gravity effects:** antennas deform as their pointing elevation changes because the stresses on the dish and its support structure depend on the orientation with respect to the vertical. The subreflector and its mount can also exhibit sag. These gravity terms can be quite large.
5. **Global location:** errors in the placement of antennas on the surface of the Earth produce errors in  $H$  and  $L$  and therefore affect both  $El$  and  $Az$  measurements. While baseline solutions for radio data can refine the relative positions of the antennas well, gross errors in the assumed location of the array center may remain. The functional form of the location correction is identical to that for a fixed tilt.
6. **Thermal deformation:** uneven temperature distributions across the dish and backup structure cause uneven expansion and irregular deformation of the dish that is not always reproducible. Thermal effects therefore cannot be removed a priori and are ameliorated by using fans to blow air over the dish and maintain an even temperature distribution. However, thermal offsets are known to be large (up to 1 arcminute) on unshielded BIMA antennas after sunrise in the summer.
7. **Refraction:** this effect depends on the path length of the line of sight through the atmosphere and can be modelled. Due to the very different wavelength regimes and the presence of resonance lines in the atmosphere at radio frequencies, this term is generally different for radio and optical pointing.

The pointing errors, on the sky, can be parametrized in their dependence on  $Az$  and  $El$  in different ways, as discussed in the help file for the MIRIAD program *pnt*. At CARMA, we parametrize the corrections separately for the BIMA and OVRO antennas, based on the antenna properties, as follows:

$  \begin{aligned}  BIMA \ dAz &= a(1) * \cos(El) \\  &+ a(2) * \sin(El) * \sin(Az) \\  &+ a(3) * \sin(El) * \cos(Az) \\  &+ a(4) \\  &+ a(5) * \sin(El) \\  &+ a(6) * \cos(El) * \sin(Az) \\  &+ a(7) * \cos(El) * \cos(Az) \\  &+ a(8) * \cos(El) * \sin(2 * Az) \\  &+ a(9) * \cos(El) * \cos(2 * Az)  \end{aligned}  $	<p><b>Azimuth encoder and mount offset</b></p> <p><b>Tilt of Az axis</b></p> <p><b>Tilt of Az axis</b></p> <p><b>Collimation error</b></p> <p><b>Misalignment of El axis wrt Az axis</b></p> <p><b>“Wobble” residual</b></p> <p><b>“Wobble” residual</b></p> <p><b>“Wobble” residual</b></p> <p><b>“Wobble” residual</b></p>
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$BIMA\ dEl$	$=$	$b(1)$	<b>Elevation encoder offset</b>
	$+$	$b(2) * \sin(Az)$	<b>Tilt of Az axis</b>
	$+$	$b(3) * \cos(Az)$	<b>Tilt of Az axis</b>
	$+$	$b(4) * \sin(El)$	<b>Elevation axis sag</b>
	$+$	$b(5) * \cos(El)$	<b>Elevation axis sag</b>
	$+$	$b(6) * \sin(2 * Az)$	<b>Fourier analysis residual</b>
	$+$	$b(7) * \cos(2 * Az)$	<b>Fourier analysis residual</b>
	$+$	$b(8) * \cot(El)$	<b>Atmospheric refraction</b>
$OVRO\ dAz$	$=$	$o(1)$	<b>Azimuth collimation offset</b>
	$+$	$m(1) * \cos(El)$	<b>Azimuth encoder offset</b>
	$+$	$m(3) * \sin(El)$	<b>Misalignment of El axis wrt Az axis</b>
	$-$	$m(4) * \sin(Az) * \sin(El)$	<b>Tilt of Az axis (North – South)</b>
	$-$	$m(5) * \cos(Az) * \sin(El)$	<b>Tilt of Az axis (East – West)</b>
$OVRO\ dEl$	$=$	$m(2) + o(2)$	<b>Elevation encoder and collimationoffset</b>
	$+$	$o(3) * \cos(El)$	<b>Elevation axis sag</b>
	$-$	$m(4) * \cos(Az)$	<b>Tilt of Az axis (North – South)</b>
	$+$	$m(5) * \sin(Az)$	<b>Tilt of Az axis (East – West)</b>

The BIMA terms are in the order corresponding to “equation 4” of the Mirad *pnt* program, and represent the pointing model used by CARMA for the BIMA antennas. (NB: Not all the terms are independent, e.g. the tilt terms are related,  $a(2) = b(3)$  and  $a(3) = -b(2)$ .) For the OVRO telescopes a smaller set of pointing constants are used but the sets are not disjoint. BIMA  $a(1), a(2), a(3), a(4)$ , and  $a(5)$  correspond to OVRO  $m(1), -m(4), -m(5), o(1)$ , and  $m(3)$  while BIMA  $b(1), b(2), b(3)$ , and  $b(5)$  correspond to OVRO  $m(2) + o(2), m(5), -m(4)$ , and  $o(3)$ . It is important to note that the trigonometric functions are evaluated with the gross offsets of the encoder applied to the  $Az$  and  $El$  values to reduce nonlinear effects.

The pointing terms have been separated into two subsets based on their physical properties:

- those which depend on the mount and backup structure and should therefore be common to the optical and radio pointing; these are set through the `setBimaMountPointingConstants` arrays for the BIMA antennas (two 9–element arrays for the a and b coefficients);
- and those which depend on the radio beam or aperture, and therefore will not be common to the radio and optical pointing. These are set through the `aperturePointingConstants` arrays, with separate OPTICAL and R3MM sets to be applied for optical and radio pointing, respectively.

In practice, the separation is achieved as follows: in `subarrayInit.py`

- the MountPointingConstant arrays consist of two 9–element arrays representing a(1:9) and b(1:9);
- the following terms in the MountPointingConstant arrays are set to zero:
  - a(4), the collimation term which is in aperture pointing;
  - b(5), the cos(EI) elevation sag term which is in aperture pointing; and
  - b(9), which is an empty element anyway.
- The aperturePointingConstants are 3–element arrays, one for each antenna, and in addition there is one set for optical pointing (“OPTICAL”) and another for radio pointing (“R3MM”).
- The aperturePointingConstant arrays contain three numbers:
  - a(4), the collimation term,
  - b(1), the elevation offset, and
  - b(5), the cos(EI) elevation sag term.

In order to determine all these constants for each antenna by separating out the different dependencies on  $Az$  and  $El$ , we need a large number of measurements of  $dAz$  and  $dEl$  as a function of  $Az$  and  $El$  all over the sky. The control system currently adds corrections as follows:

$$\begin{aligned}(dAz \text{ or } dEl) &= \text{mount model} \\ &+ \text{aperture coefficients} \\ &+ \text{mount offsets} \\ &+ \text{active tilt correction (for OVRO only)} \\ &- \text{on – the – sky offsets (s.offset)}\end{aligned}$$

### 3. Optical Pointing

In practice we determine most of the pointing constants using the optical pointing program. Optical cameras attached to each antenna can be used to measure the positions of bright stars on CCD detectors with a large instantaneous field of view in a very efficient process. In the OVRO dishes, the mount model is specified by the coefficients  $m(X)$ . Aperture coefficients, i.e. terms that depend on which “aperture” you are observing with (the optical cameras, the 3 mm beam or the 1 mm beam), are discussed later and are identified as the  $r(X)$  (radio) or  $o(X)$  (optical) terms.

For the BIMA antennas, the distinction between mount model and aperture coefficients is blurred by the fact that two independent mount models were maintained at Hat Creek. Terms that could be verified to apply for radio and optical pointing (or could not be measured in the radio) were used in common. Also, more

terms were used to describe the mount model at Hat Creek. In light of this, the BIMA dishes have pointing coefficients that are present in the mount model as well as the aperture terms. These two sets of terms are literally added together. Some of the coefficients are left as zero in the mount model, indicating that we believe the pointing constant to be fully dependent on the aperture. For the OVRO antennas, these are added functionally (since, with one exception, the aperture coefficients have no analog in the mount model). In practice, BIMA coefficients  $a(4), b(1)$  and  $b(5)$  are the aperture coefficients. In what follows, given the actual setup of the control system, we will refer to  $a(4), b(1)$  and  $b(5)$  interchangeably with  $r(1), r(2)$  and  $r(3)$  respectively (equivalently  $o(1), o(2)$  and  $o(3)$  for the discussion of optical pointing coefficients). For a more detailed explanation of the optical pointing process and reduction, see the optical pointing memo.

#### 4. Radio Pointing

We can assume that there is an offset between the optical camera pointing and the radio axes (the “aperture collimation offset”). The initial goal, then, of the current radio pointing measurements is to determine the azimuth and elevation offsets,  $\Delta a(4)$  and  $\Delta b(1)$ , that are corrections to the  $a(4)$  and  $b(1)$  terms above resulting from misalignment of the optical camera with respect to the antenna axes.

In addition, the radio pointing changes as the dish reflecting surface flexes, e.g., under gravity as the dish elevation changes. The optical camera is a much smaller feature that is attached locally to just one location on the dish structure, and need not share the same gross deformation that the dish suffers. We therefore expect that there is also a difference between the radio and optical pointing in the elevation dependences, which we fit as an  $r(3) \cos(EI)$  term. In principle, if the antennas are balanced (no tilt) and the optical pointing has been successful, the other terms in the mount model should still apply.

#### 5. Observing Programs

The radio pointing measurement (offset of the center of the radio primary beam from the pointing center) is generally carried out in two different ways. To do a full two-dimensional fit of the primary beam and its offset and width, we can do a square grid of pointings offset from the nominal position of a source. One set of antennas is held fixed on the source position while a second set of antennas is offset, and cross-correlation visibilities are generated in the usual fashion so that the amplitudes are affected by the primary beam pattern of the offset antennas. In practice, we separate the OVRO and BIMA dishes since they have different sizes and therefore the offset step size that produces Nyquist sampling (half the primary beam width) is different for the different antenna sizes.

An alternative is to observe in a cross pattern that samples the beam profile in the horizontal (azimuth) and vertical (elevation) directions. This is a faster approach than the square grid, since the number of pointing positions is smaller for the same step size ( $2m$  rather than  $m^2$ ), and it should work just as well if the beam pattern is circularly symmetric. In practice this assumption of symmetry can be confounded by amplitude noise or random bad points. The grid pattern samples more points within the beam response than the cross

and hence the fits tend to be more robust against such fluctuations. Also, the cross pattern requires the pointing be pretty good before we start, or else the arms of the cross may not intersect the main beam lobe at all, so it is better used as a refinement tool.

As a note, there is another way to proceed in general. We need not hold a given set of antennas fixed as closure relationships allow us to determine unique fringe amplitudes at a given pointing position. However, use of reference antennas allows pointing to be done on fainter sources as the fringe amplitude drops to only  $\sqrt{2}$  at the standard half-power offset points (instead of dropping by a factor of 2). In general, for a given signal to noise, reference antenna use decreases the amount of time needed to point accurately so we proceed using reference antennas.

In the early days of commissioning when antenna pointing was only poorly known, the general plan used was:

- Initially the offsets  $\Delta a(4)$  and  $\Delta b(1)$  are unknown and may be large, so we carry out a relatively large (and therefore slow) grid to be sure that we have a good measurement, e.g., a  $7 \times 7$  grid with step size  $0.5'$  for the OVRO dishes,  $0.85'$  for the BIMA dishes. The integration time at each point needs to be long enough for good signal-to-noise, and thus the stronger calibrators are preferred. For 10 seconds at each point, the  $7 \times 7$  grid (which has to be done for both sets of antennas) takes about 20 minutes.
- When  $\Delta a(4)$  and  $\Delta b(1)$  are reasonably well known, we want to inspect the elevation dependence by using a pattern that allows more frequent measurements. This can be done with a  $7+7$  cross pattern that alternates between multiple bright sources in order to sample a wide range of azimuth and elevation. For 10 seconds integration at each point each pattern takes less than 10 minutes per measurement.

Now that there is considerable experience with the array, usually only large (7- or 9-point) crosses are needed to determine the initial offsets after an antenna move.

## 6. Observing Tools

The core routine used for pointing measurements is the program `multiMap` in the module `white.py` (written by Stuartt Corder). A routine that invokes `multiMap` to do the observations, calculates the offsets on the fly and applies corrections to improve pointing in real time now exists (`refPoint`) and is described in a separate memo. Here our description is mostly relevant to pointing runs on multiple sources after antenna moves when we expect that the radio pointing coefficients may have changed.

## 6.1. Observing scripts

A grid or cross pattern employing offsets of the primary beam from a source position in azimuth and elevation can be observed with the multiMap program: in SAC type, e.g.,

```
from white import *
multiMap(1,['3c279'],set1=[2,4],set2=[8,9],intTime=10,intRep=1,mapPoints=7,\
        set1Step=0.5,set2Step=0.8,type='grid',elevLimit=15.0,waitSource=True,\
        doCenter=True,doSwap=True,doSingle=False,doExtra=False)
```

The arguments are as follows:

Required Parameters:

1: number of repetitions of pattern  
['3c273']: source: can be an array of strings (multiple sources) or a single string (single source)

Numeric/List Keywords:

set1=[2,4]: Initial moving antennas. The default, 0, makes set1 all available Ovro antennas  
set2=[8,9]: Initial reference antennas. The default, 0, makes set2 all available Bima antennas  
intTime=10: integration time (seconds), 10.0  
intRep=1: number of integrations at each point  
mapPoints=7: width, in points, of the cross or grid (7 by 7, so 49 pointings in all here), default 5  
set1Step=0.5: set1 grid step, arcmin, default 0.9\*Nyquist step  
set2Step=0.8: set2 grid step, arcmin, default 0.9\*Nyquist step  
type='grid': type of pattern: 'grid' (n x n) or 'cross' (n + n)  
elevLimit=15.0: elevation limit  
lstStart='None': LST start time, usually not used in pointing  
lstStop='None': LST stop time, usually not used in pointing  
antwait=-2: controls behavior of antennas when they slew to a new source, default=-2 says don't wait for last 2 antennas

Boolean Keywords:

waitSource= True: waitSource flag: wait for source to rise or go on to next source  
doCenter = True: doCenter flag: for cross pattern, center points are included or not  
doSwap = True: doSwap flag: executes set2 then set1 as reference or only uses set2  
doSingle =False: doSingle flag: assumes single antenna pointing is to be done, True

```
requires doSwap=False  
doExtra =False: doExtra: executes additional center integrations for calibration
```

Focus Keywords: used so far only for testing the relationship between pointing offsets and focus position on the OVRO antennas

```
focusPoints=5:  
focusStep=1.0:  
focusType='grid'  
circleLimit=False
```

The defaults of multiMap are set so that a call to multiMap such as

```
multiMap(1,['3c279'])
```

runs with the defaults described above. As long as keywords are entered in the order given in the full call, you need not actually list the name of the keyword. Keywords given out of this order (or if values are NOT included) must be included explicitly by name in the call. You can list them partially, e.g.:

```
multiMap(1,['3c279'],0,0,10.0,3)
```

gives set1=all available OVRO antennas, set2=all available BIMA antennas, and three 10.0 second integrations at each pointing location in a 5–point cross. All later keywords take their defaults.

The visibility data acquired by multiMap are written to a MIRIAD dataset with the current naming convention of rpnt.{grid,cross}.yyyymmdd.n.mir, where n is the trial number. The multiMap program also creates “offset files” in /home/obs/radioPointing, labelled rpnt.{grid,cross}.yyyymmdd.n.{ovro,bima}-ref.dat, which indicate (first line) which antennas were moving in this pattern and (subsequent lines) what azimuth and elevation offsets were requested at what times. It also creates a rpnt.{grid,cross}.yyyymmdd.-n.splitInfo.txt file which describes the telescopes involved, Miriad readable time stamps for the start of different patterns and the step size. These files are convenient for the IDL analysis programs described next.

## 6.2. Analysis

For those who do not wish to know the details of the analysis routines, it is only necessary to know that the shell script Auto\_point (located in /home/obs/bin, with copies in /home/obs/radioPointData) carries out all the analysis steps and reports the results for a given pointing observation as follows:

```
Auto_point c0067.rpnt.2007apr24.1
```

where the only argument needed is the project identifier. This script works both on refPoint real-time-correction single-source data as well as longer multi-source pointing tracks. Two IDL routines, *pntfit* and *crossfit*, have been written for analysis of the pointing data (*Auto\_point* invokes *crossfit*). These programs organize the data into a map (grid) or two one-dimensional cuts (cross) and fit a gaussian (2D for grid,  $2 \times 1$ D for cross) to the patterns. To use them, we first output the data as an ASCII file. This is accomplished with the C-shell script *To\_rpoint*, which calculates a phase passband solution, applies it to flatten phases across the channels (thus minimizing amplitude decorrelation due to phase slopes across the frequency band), then averages channels 1 to 90 (until late March 2006 only the 15 channels in the LSB window of band 3 were used; now most pointing data is taken in full wide-band mode with 15 channels in each of 6 windows) to form a single channel data set. Then *uvlist* is run with *options=baseline* to output the total amplitudes in the data set to an ASCII file.

```
To_point 2006mar22.rpoint.grid.1.mir 1 1,2,4,5,6,8,9,12,13,14
where
2006mar22.rpoint.grid.1.mir: Miriad data set name
1: antenna to use as reference
1,2,4,5,6,8,9,12,13,14: list of antennas with "good" data
```

In current operations the antenna lists are contained in the offset files. The reference antenna is needed for the bandpass calibration step. The output *uvlist* file should be put in the same directory as the offset pointing files. Then run the IDL routine:

```
idl
crossfit,'2006mar22.rpoint.cross.1.bimaref.dat',[1,2],[9,15]
or
pntfit,'2006mar22.rpoint.grid.1.bimaref.dat',[1,2],[9,15],/norm
```

This version determines the offsets of antennas 1 and 2 with respect to 9 and 15 (hence uses the “bimaref” file in which the BIMA antennas are not offset). The routine *crossfit* with the same arguments fits the cross-pointing data and optionally plots them.

These programs prompt you whether or not to add the results of each fit to the file *carma\_pointing.fits*, which is in the format needed for Mel Wright’s Miriad program *pnt*. This program fits the offset measurements to pointing equations in various forms; the number of parameters to fit depends on how much data is available and how much of the *Az-El* space it samples. There is now an IDL version of this program, called *fit\_pnt*, which allows you to specify which terms in the pointing equations above should be fit:

```
idl
fit_pnt,'pnt.06mar22'
```

where *pnt.06mar22* is a file containing solutions in *pnt* format.

## 7. Applying Corrections

The pointing terms are defined in the `subarrayInit.py` script, and are therefore loaded whenever this script is applied. Applying offsets to the mount model is done via the optical pointing corrections and is described elsewhere. Application of the radio pointing offsets are done via `s.aperturePointingConstants` (in `subarrayInit.py`). This takes five arguments, `R3MM` (= `carma.control.APERTURE_RADIO3MM`), the three radio pointing corrections (*Az* and *El* offsets and the sag coefficient) and antenna number.

$\Delta a(4)$ ,  $\Delta b(1)$  and  $\Delta b(5)$  are the terms that we fit for with the radio pointing measurements. when we have tracks with sufficient coverage of elevation and azimuth. There is an IDL routine for plotting multiple pointing solutions, `plot_pntfit.pro`, which carries out these fits automatically and reports the results on the plot:

```
idl
plot_pntfit, 'pnt.06mar22'
```

The pointing constants for the OVRO antennas are kept in the `Ovro_m_` (the mount constants, `m(1)`-`m(5)` elements with `m(2)` set to zero), `OvroTilt_` (2 elements), and `OvroOptical_` or `OvroRadio3mm_` arrays (3 elements) with each row of these lists representing an Ovro antenna (1-6). The `OvroTilt_` coefficients are replaced with the most recent measurements if `defaultStart()` is executed at the beginning of data taking.

## 8. Pointing for 1 mm observations

In general, because data taken at 1 mm usually have a higher noise level than 3 mm data, and because radio sources generally have a spectrum falling with frequency so that 1 mm fluxes are smaller than 3 mm fluxes (with a few exceptions, such as the planets and MWC 349, that have optically-thick thermal spectra that rise with frequency), very few sources are strong enough at 1 mm wavelength to be used as reference pointing calibrators. For this reason, in general it is better to retune to 3 mm for reference pointing in a 1 mm track, at the cost of order 1–2 minutes for the retuning process.

Pointing at 3 mm in a 1 mm track relies on accurate knowledge of the offset between the 3 mm and 1 mm radio beams, which can be measured (under good atmospheric conditions using a strong 1 mm source) simply by reference-pointing at 3 mm and then measuring the source offsets at 1 mm. Since the 3 mm and 1 mm receivers are mounted together, it is assumed that that the two beams will have the same dependence on azimuth and elevation, so that only simple constant offsets in *az* and *el* need to be determined. These offsets are stored in `subarrayInit.py` in the arrays `OvroRadioDelta1mm_` and `BimaRadioDelta1mm_`. For all the BIMA receivers, the offsets are very similar (of order -9.0, -5.2 arcminutes in *az*, *el* respectively). For the OVRO receivers, the offsets differ but are relatively small (largest is of order 0.4 arcmin). Corrections determined as described above (measured offsets at 1 mm after pointing at 3 mm) are *added* to these arrays to correct the 1 mm pointing.

## 9. Acknowledgements

This discussion draws heavily on memos on pointing of the BIMA antennas by Rick Forster (dated 1995 Nov 14) and Mel Wright (dated 1992 May 14), available at Mel's pointing web directory <http://astron.berkeley.edu/~wright/carma/pointing/>. Additional discussion may be found in SMA memos 26 and 42 ([http://sma-www.cfa.harvard.edu/memos/tech\\_no.html](http://sma-www.cfa.harvard.edu/memos/tech_no.html)). The help file for the Miriad task *pnt* also contains an extensive discussion.