Class 6: Powering the stars

- Internal structure of main sequence stars
- Thermonuclear fusion of H \rightarrow He
  - The proton-proton chain
  - The CNO cycle
- The main sequence lifetime of stars

I: Internal structure of main sequence stars

- Stars are gravitationally-confined thermonuclear-fusion reactors
  - In order to support weight of rest of star, core has to be hot and dense (high pressure)
  - Core conditions are sufficient to allow nuclear fusion to occur
- Main sequence stars are in the process of steadily fusing hydrogen to helium
  - Fusion reactions confined to core
  - Energy transported from core to surface of star via radiative diffusion and/or convection
Man’s (unsuccessful attempt) attempt to harness fusion power...
Location of radiative/convective zones changes with stellar mass... complicated interplay of “opacity”, temperature and density.
II : Hydrogen burning

- Already come across this last semester with discussing the Sun
- Basic process $4H \rightarrow He$
  - $0.7\%$ of mass is converted to energy...
  - i.e., the efficiency of this process is $\eta \approx 0.007$

\[
\text{efficiency} = \frac{\text{energy released}}{(\text{total mass processed})c^2}
\]

- About $10^6$ times more efficient that chemical burning

- Actual process by which hydrogen is fused depends upon the mass of the star

For mass $M < 1.3M_{\text{sun}}$,
- Reactions proceed via proton-proton chain
- Reaction rate proportional to approx $T^4$

\[
\begin{align*}
^{1}H + ^{1}H & \rightarrow ^{2}H + e^+ + \nu \\
^{2}H + ^{1}H & \rightarrow ^{3}He + \gamma \\
^{3}He + ^{3}He & \rightarrow ^{4}He + ^{1}H + ^{1}H
\end{align*}
\]
For masses $M > 1.3$ Msun
- Reactions proceed via the **CNO cycle**
- Essentially, carbon acts as a catalyst
- Reaction rate proportional to $T^{20}$

$$\begin{align*}
12^\text{C} + ^1\text{H} &\rightarrow 13^\text{N} + \gamma \\
13^\text{N} &\rightarrow 13^\text{C} + e^+ + \gamma + \nu \\
13^\text{C} + ^1\text{H} &\rightarrow 14^\text{N} + \gamma \\
14^\text{N} + ^1\text{H} &\rightarrow 15^\text{O} + \gamma \\
15^\text{O} &\rightarrow 15^\text{N} + e^+ + \nu \\
15^\text{N} + ^1\text{H} &\rightarrow 16^\text{O} + \gamma \\
16^\text{O} &\rightarrow 12^\text{C} + ^4\text{He} + \gamma
\end{align*}$$

**III : The main sequence lifetime of a star**
- A star leaves the main sequence once it exhausts its supply of hydrogen in the core
- This lifetime depends upon
  - The luminosity $L$
  - The efficiency of the fusion $\eta$
  - The mass of the star $M$
  - The fraction of the stellar mass that can participate in the fusion reactions $f$
- Total energy available to the star is
  $$E = \eta f M c^2$$
- So (assuming constant luminosity), lifetime is
  $$\tau = \frac{E}{L} = \frac{\eta f M c^2}{L}$$
What determines f?

- This is controlled by how effectively gas within the star is mixed
- Convective zones are good at mixing, radiative zones are very poorly mixed

For Sun, \( f = 0.1 \), so \( \tau \approx 1.0 \times 10^{10} \) yr

What about other stars?

- We find observationally that, for all but the most massive stars, \( L \propto M^{3.5} \)
- So, using formula for lifetime...

\[
\tau \propto \frac{M}{L} \propto M^{-2.5}
\]

- Scaling from the Sun we get,

\[
\tau \approx 1.0 \times 10^{10} \left( \frac{M}{M_\odot} \right)^{-2.5} \text{ yr}
\]

- Very important result!
  - Low mass stars live for a long time (but are very dull)
  - High mass stars have short lives (but burn so brightly!)
For a main-sequence star, high mass means high luminosity... 

...while low mass means low luminosity.