1. Repeat Problem 2 from Homework #3 for $D$, the range of a projectile. Use the solution set to guide your work. a) Make the “flat Earth” and constant gravity approximations and solve for the maximum horizontal distance (range) that a projectile will travel in terms of gravity $g$, angle of launch $\theta$, and launch velocity $v$.  

b) Use dimensional analysis to get an expression for $D$ using the true $1/r^2$ gravity of the Earth. Neglect the Earth’s spin and treat the Earth as a perfect sphere. Start by identifying four (or five) variables that the solution will depend upon. See if you can form a dimensionless constant from the four variables and remember that the most general solution is the one that is multiplied by the most general dimensionless function.

c) Finally, solve for the maximum range of a projectile (measured along the Earth’s surface) exactly and check your answer against part b). Start by arguing that trajectories will be segments of conic sections. What conservation laws can you use? After you get an answer, consider all of the limiting cases that you can think of as tests for your answer.

2. Mass of the Oort cloud. In our Solar System, Jupiter formed at about 6.0AU and was dragged inward to its current 5.2AU. This happened as Jupiter interacted with icy planetesimals and threw them out of the Solar System. Imagine that the planetesimals all were in eccentric orbits at Jupiter’s distance to begin with, and that they were thrown out of the Solar System on parabolic trajectories. Use a conservation law to estimate how much mass must have been ejected from the Solar System to account for the changes in Jupiter’s orbit.

3. Tidal forces between the Earth and the Moon are pushing the Moon’s orbit away from the Earth and slowing down the Earth’s spin. What happens in the end?  
a) Calculate the Earth’s spin angular momentum, the Moon’s spin angular momentum, and the Moon’s orbital angular momentum. Make the approximations that the lunar orbit is circular and that the angular momenta are aligned.

b) Assume that only tidal forces between the Earth and the Moon are operating and argue that the total angular momentum of the Earth-Moon system is conserved. Is the orbital energy conserved? Explain your reasoning.

c) In the end state of orbital evolution, the Moon will be further away from the Earth, than it is now, and the Earth will always keep one face toward the Moon. This is the synchronous state (i.e. the “month” will equal the “day”). Use conservation of angular momentum to determine how far the Moon will be from the Earth then, and how long the “day” will be. You may neglect the spin angular momentum of the Moon - use your answer from a) to justify this. The final equation cannot be solved analytically; try to get a solution with a numerical or graphical method.

d) That would be the end of it, but we’ve neglected the effects of the Sun. Tides raised on Earth by the Sun slowly slow Earth’s rotation. How will the Moon’s orbit react? (assume that the Moon has already reached the synchronous state that you found in part c) - because of the Sun’s influence, the synchronous state actually will occur with the Moon somewhat closer to the Earth). Describe what happens to the Moon next using Angular Momentum and Energy arguments.