1. Consider the Earth and Moon as a binary system. Find the following:

(a) The distance of the barycenter (center of mass) of the Earth-Moon system from the center of the Earth. Give your answer in km and in Earth radii.
(b) The velocity of the Moon about the barycenter.
(c) The velocity of the Earth about the barycenter.
(d) The velocity of an observer on the Earth’s equator due to the Earth’s rotation.
(e) Would an astronomer trying to detect extra-solar planetary systems by observation of the Doppler shift in stellar spectra need to consider the above velocities? I.e., how do these velocities compare to those the astronomer wants to measure?

(The mass of the Earth is $5.98 \times 10^{27}$ g. The mass of the Moon is $1/81.33$ Earth masses. The period of the orbit is 27.32 days. The Earth’s radius is $6.378 \times 10^3$ cm.)

2. In class, we showed by direct integration that the gravitational force exerted by a shell of mass $M$ and radius $R$ on a unit mass located at a distance $r$ from the center of the shell, where $r > R$, is $F_r = -GM/r^2$. Repeat this derivation for the case $r < R$, and show explicitly that the force inside the shell is zero.

3. Consider a sphere of radius $R$ in which the density varies from $\rho_c$ at the center to $\rho_s$ at the surface in a linear fashion, so that we can write $\rho(r) = \rho_c[1 - \beta(r/R)]$, where $\beta = (\rho_c - \rho_s)/\rho_c$.

(a) Integrate over the sphere and express $\rho_c$ in terms of $R$ and the mass $M$.
(b) Obtain an expression for the gravitational force as a function of $r$ for $r < R$.
(c) Show that if $\beta > 2/3$, the force is a maximum at some point interior to $R$. If $\rho_s = 0.1 \rho_c$, find the value of $(r/R)$ for which the force is a maximum.
(d) Find the expression for the gravitational potential $\Phi(r)$ within this sphere. Make sure you normalize $\Phi(r)$ so that it is continuous with the value for $r > R$.

4. Two stars of equal mass $M$ are in circular orbits about their center of mass. The separation of the stars is $a$. For a point mass rotating with the binary, and on the line between the two stars, write down the expression for the total force on this third body. Eliminate the angular velocity $\omega$ with Kepler’s third law, and thus express the condition that the force be zero as a 5th degree polynomial for the fractional distance, $r/a$, where $r$ is the distance of the particle from the CM.

Find the real root of this equation if you can. At least find an approximate solution by graphing the function between 1 and 1.4 to see where it has a zero.

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