1. **8 pts** Write a program to do simplified Comptonization. Specifically, assume that the initial energy of photons is always $\hbar \omega = 0.001 m_e c^2$, and that electrons have a speed $v = \beta c$ where $\beta = 0.1$, and an isotropic distribution of directions. To simplify things yet further, assume that the probability of scattering off an electron is independent of the energy of the photon and the direction of the electron relative to the photon. Assume that in the rest frame of the electron the scattering probability is independent of the direction. However, you need to compute the change in energy of the photon by electron recoil, and you need to do the Doppler shifts into and out of the electron rest frame. Plot the number of photons versus photon energy after 10, 100, and 1000 scatterings (you’ll need to follow at least 100,000 photons for good statistics), on a log-log scale. As before, please send me (by e-mail) a copy of your code, in a form that I can compile and run on the astro machines.

2. **4 pts** Polarization of the cosmic microwave background has been observed with WMAP, Planck, and other experiments. Let’s focus on observations at 1 GHz. If the universe had a strong enough magnetic field with a direction that varied on angular scales smaller than the beam of the detector (note that the “beam” of the detector is its angular resolution limit), Faraday depolarization would have destroyed the polarization signal. Your task will be to determine what this means for the possible strength of the universe’s magnetic field.

(a) First, let’s think about the current universe. The polarization was observed at approximately the expected strength. Assuming that the universe is fully ionized hydrogen at a number density of $2 \times 10^{-8} \text{ cm}^{-3}$, place an upper limit (to within two orders of magnitude) on the net parallel magnetic field strength on scales smaller than the beam. The size of the universe at low redshift is around $10^{28} \text{ cm}$.

(b) Now what about at earlier times? Consider the universe in the redshift range $z_0/2$ to $z_0$. Assume that the number density scales as $(1 + z)^3$, and that the path length from $z$ to $z + dz$ is roughly $10^{28} \text{ cm}(1 + z)^{-2} dz$. Also remember that if we observe a frequency $\omega$, the frequency in the rest frame was $\omega(1 + z)$. Within two orders of magnitude, what is the limit on the magnetic field that existed at $z_0 = 1000$? We’re still assuming fully ionized hydrogen.

(c) A primordial magnetic field that expands passively with the universe has a strength that scales as $n^{2/3}$, where $n$ is the number density. Let us suppose that reionization happened at $z = 6$; that is, when we do our physics properly we realize that the universe was actually mainly neutral from the CMB epoch $z \approx 1100$ until stars and the first quasars reionized the universe at $z = 6$. Thus take your magnetic field limit from part (b), evaluated at $z = 6$, and passively evolve that primordial field to $z = 0$. What does that imply about the upper limit to the *current* strength of the primordial component of the magnetic field?
3. **4 pts** Dr. Sane has become fascinated by the problem of missing mass. He thinks he has the perfect candidate: “Sane nebulae”. Sane nebulae are clouds of fully ionized pure hydrogen, that typically have temperatures of $10^5$ K and number densities of $0.1 \, \text{cm}^{-3}$. They have no internal energy sources. Dr. Sane thinks that Sane nebulae have existed with these properties more or less unchanged for the last 10 billion years. Evaluate his claim, assuming that Sane nebulae have negligible interaction with anything external to them. **Hint:** Dr. Sane’s nebulae are small enough that any radiation generated in them escapes without interacting further.