Lecture #6: Plan

- Universal Gravitation
- Orbital Motion
- Surface Gravity
- Escape Velocity
First Law of Motion

• **Observations:** Orbital motion follows a *curved path*
  → A net force must be acting on them
  → Gravity!
First Law of Motion

If string is released when ball is here, ball goes straight toward A, not toward B, nor toward C.
Newton’s Universal Law of Gravitation

- Every mass exerts a force of attraction on every other mass.

\[ F_{\text{Gravity}} = \frac{GmM}{d^2} \]
Orbital Motion and Gravity
Orbital Motion

• Orbital velocity:

\[ V_{\text{orb}} = \sqrt{GM/d} \]

\[ V_{\text{orb}} \text{ (shuttle)} \sim 8 \text{ km/s} \]

\[ M = M_E = \text{Earth’s mass} \]

\[ d \sim R_E + 300 \text{ km} \]
Orbital Motion

- Orbital velocity:
  
  \[ V_{\text{orb}} = \sqrt{\frac{GM}{d}} \]

  \[ V_{\text{orb}} (\text{Earth}) \sim 30 \text{ km} / \text{s} \]

  \[ M = M_S = \text{solar mass} \]

  \[ d = 1 \text{ AU} \]
Masses from Orbital Speeds

• \( V_{\text{orb}} = \sqrt{\frac{GM}{r}} \)
• \( M = r V_{\text{orb}}^2 / G \)
• **Example:** Measuring the Earth’s mass from the orbiting space shuttle

\[
r = R_E + h \\
\sim 6400 + 300 \text{ km} \\
\sim 6700 \text{ km} \\
P \sim 90 \text{ minutes} = 5.4 \times 10^3 \text{ sec} \\
V_{\text{orb}} = 2 \pi \frac{r}{P} = 8 \text{ km / sec} \\
M = r V_{\text{orb}}^2 / G = M_{\text{Sun}} / 300,000
\]
Surface Gravity

- \( F = m \ a = \frac{G \ M \ m}{R^2} \)
- \( a = \frac{G \ M}{R^2} \equiv g \)
- Same for all objects! (Galileo @ Leaning Tower of Pisa!)
- \( W \equiv m \ g = \text{weight} \)
Galileo dropped balls of different masses off leaning tower in Pisa.

They hit the ground at the same time!
Weight vs Mass

- \( W = m \cdot g \)
- a force
- depends on environment (g)

\[ \text{“weightlessness”} \rightarrow W = 0 \]
\[ g = 0 \]
\[ m \neq 0 \]
Surface Gravity: Earth vs Moon

\[ g_E = \frac{GM_E}{R_E^2} \]
\[ g_M = \frac{GM_M}{R_M^2} \]
\[ \frac{g_E}{g_M} = \frac{(GM_E/R_E^2)}{(GM_M/R_M^2)} = \frac{(M_E/M_M) (R_M^2/R_E^2)}{(M_E/M_M) (R_M/R_E)^2} = (1/3.8)^2 \approx 81 \times (1/3.8)^2 \approx 6 \]
Surface Gravity

- On Earth, $g = 9.8 \text{ m/s}^2$
- $g$ on the Moon is around $1/6$ as much as on the Earth!
Escape Velocity

- The velocity needed to overcome the gravitational force of an object (e.g., planet, star) and not fall back on the object
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\[ V_{\text{esc}} = \sqrt{\frac{2GM}{R}} \]
Escape Velocity

- Important in determining whether a planet has an atmosphere:

\[ V_{esc} (\text{Earth}) = 11 \text{ km} / \text{s} \]
\[ V_{esc} (\text{Moon}) = 2.4 \text{ km} / \text{s} \]
Surface Gravity & Escape Velocity

- \( g = \frac{GM}{R^2} \)
- \( V_{\text{esc}} = \sqrt{\frac{2GM}{R}} \)

- Massive/compact objects have high surface gravity and escape velocity
- Low mass / extended objects have low surface gravity and escape velocity
Reminder: Homework #1

- Due: this coming Tuesday, February 16, 9:30 am
- Type your answers (do not scan handwritten notes)
- Single PDF file (not Word) with a distinct name (e.g., hw1_your_name_0101.pdf)
- Email to astro100-010N@astro.umd.edu where N = 1, 2, 3, 4, 5, or 6 according to your section number (e.g., astro100-0101@astro.umd.edu if you’re in Section 0101)
- VERY IMPORTANT: Use your own words!