

## ASTR 220 Homework #7 Solutions

### Spring 2005

1. *Explain why more massive white dwarfs are smaller in size than less massive white dwarfs.*

If you add more mass to a white dwarf, the white dwarf's gravitational force upon itself increases. The increased gravitational force crushes the white dwarf to a smaller size.

2. *Jocelyn Bell discovered the first pulsar by observing that it emitted radio waves every 1.337301 s. The radio waves have a wavelength of 18 cm (0.18 m).*

- (a) *What is the speed of the radio waves?* All light travels at the same speed, the speed of light:  $3 \times 10^8 \text{ m/s}$ .
- (b) *What is the frequency of the radio waves she detected?* For light, we can use the equation:  $c = \lambda f$ . Rearrange this to solve for frequency and plug in our values:

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{0.18 \text{ m}} = 1.7 \times 10^9 \text{ s}^{-1} = 1.7 \times 10^9 \text{ Hz}$$

- (c) *From your previous answer, what is the period of the radio waves?* Period is the reciprocal of the frequency:

$$P = \frac{1}{f} = \frac{1}{1.7 \times 10^9 \text{ s}^{-1}} = 5.9 \times 10^{-10} \text{ s}$$

- (d) *Why isn't your answer in part c equal to 1.337301 s? (It shouldn't be.)* The period calculated in part c is the period of the light waves. The period of 1.337301 s is the period of rotation of the pulsar. This is how often the "beam" of the pulsar's magnetic field is pointed toward the Earth.
- (e) *If you had extremely sensitive hearing, would you be able to hear the pulses from the pulsar?* No. Radio is a form of light, which is inaudible.

3. *Ch. 13, Review Problems, #10.*

*What do we mean when we say a black hole really is a hole in spacetime?* The mass of any object bends spacetime: the effect of this is what we perceive as gravity. A black hole has so much mass packed into such a small volume that the curvature of spacetime near a black hole is extremely steep, and it's like the black hole is sitting in a bottomless well of spacetime.

*What is the event horizon of a black hole?* The event horizon is the imaginary line at which the escape velocity of the black hole is equal to the speed of light. Within that boundary, no object can escape from the black hole, not even light. However, it is not a physical boundary in any way.

*How is the event horizon related to the Schwarzschild radius?* The event horizon is in the shape of a sphere around the black hole, since an object could approach the black hole from any direction. The Schwarzschild radius is the radius of that sphere.

4. *Ch. 13, Sensible Statements, #19.*

This statement is sensible. The gravitational force between any two objects depends only on their masses and the distance between them. If the Sun was somehow crushed down into a black hole, its mass would not change. The distance between the center of the Sun and each planet would not change. Therefore, the gravitational force between the Sun and each planet would be the same as before the Sun was a black hole. Consequently, the orbits of the planets would not change.

5. *Calculate the Schwarzschild radius for each of the following.*

- (a) *A supermassive black hole with  $1 \times 10^8$  times the Sun's mass that is in the center of a quasar. (The Sun's mass is  $2 \times 10^{30} \text{ kg}$ .)*

The equation for the Schwarzschild radius is:

$$R_s = \frac{2GM}{c^2}$$

G is the gravitational constant,  $6.67 \times 10^{-11} \text{ m}^3/\text{s}^2/\text{kg}$ , M is the mass of the black hole, and c is the speed of light. In this case the mass is:  $M = (1 \times 10^8)(2 \times 10^{30} \text{ kg}) = 2 \times 10^{38} \text{ kg}$ . Plug this into the equation.

$$R_s = \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2/\text{kg})(2 \times 10^{38} \text{ kg})}{(3 \times 10^8 \text{ m/s})^2} = \frac{2.7 \times 10^{28} \text{ m}^3/\text{s}^2}{9 \times 10^{16} \text{ m}^2/\text{s}^2} = 3 \times 10^{11} \text{ m}$$

That's equal to 2 AUs!

- (b) *A black hole that has 5 times the Sun's mass and formed in the supernova explosion of a massive star.*

For this black hole,  $M = 5(2 \times 10^{30} \text{ kg}) = 1 \times 10^{31} \text{ kg}$ .

$$R_s = \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2/\text{kg})(1 \times 10^{31} \text{ kg})}{(3 \times 10^8 \text{ m/s})^2} = 1.5 \times 10^4 \text{ m}$$

That's only about 1 mile in radius!

- (c) *A mini-black hole that formed when an extremely advanced civilization of aliens decides to punish you (unfairly, of course) by squeezing you until you become so small you are a black hole. (You'll have to estimate your mass.  $1 \text{ kg} = 2.2 \text{ lbs}$ .)*

Let's assume your weight is approximately 150 lbs, which means it's 68 kg.

$$R_s = \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2/\text{kg})(68 \text{ kg})}{(3 \times 10^8 \text{ m/s})^2} = 1 \times 10^{-25} \text{ m}$$

Your Schwarzschild radius would be tiny!

6. *How many orbits around the Milky Way galaxy will the Sun make in its lifetime?*

From pg. 353 of the book, we see that the Sun takes about 230 million years to orbit the Milky Way once. That's  $2.3 \times 10^8 \text{ yr}$ . The Sun's adult lifetime is 10 billion years, or  $1 \times 10^{10} \text{ yr}$ . We can determine the number of orbits by dividing the two numbers:

$$N = \frac{1 \times 10^{10} \text{ yr}}{2.3 \times 10^8 \text{ yr}} = 43.5$$

The Sun will make about 43.5 orbits around the Milky Way in its lifetime.

7. *In class, we discussed how the spiral arms in the Milky Way cannot be created by a fixed pattern of the same stars. You are going to show this by drawing diagrams of the motions of three stars in the disk of the galaxy. You will draw five separate diagrams as described below. (Instructions abbreviated here.)*

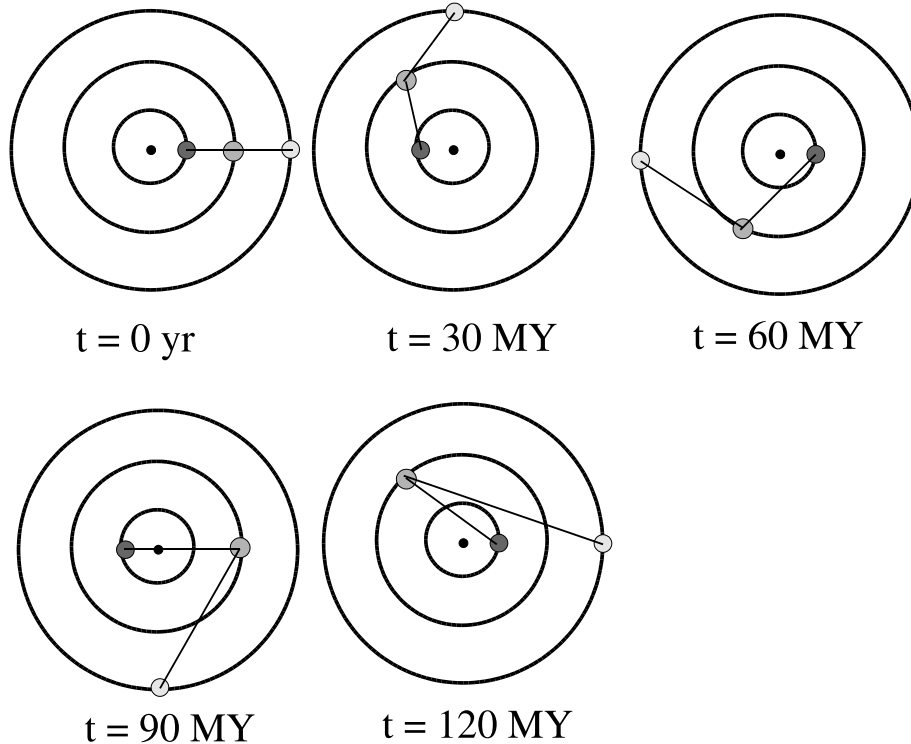
*Now draw a diagram for each of the following times (on the next page). On each diagram, mark where each star is, and draw a line connecting the three stars. (This line represents a possible spiral arm.)*

- $t=0$ . (The starting time. You can line up all three stars right in a row.)
- $t = 30$  million years.

- $t = 60$  million years.
- $t = 90$  million years.
- $t = 120$  million years.

After you are finished with the diagrams, use them to help you explain why spiral arms cannot be made of fixed patterns of the same stars.

Example diagrams are below.



From the diagrams, we can see that any pattern we begin with in the galaxy will be destroyed over time by the rotation of the galaxy. Since stars at different distances from the center of the galaxy have different orbital periods, no pattern can stay fixed. The stars in the outer part of the galaxy lag behind and smear out any pattern.

8. *Ch. 14, Sensible Statements, #17.* This statement is not sensible. The Milky Way galaxy is 100,000 light-years across. Even if we could send out a probe that traveled at the speed of light, it would take thousands of years for it to get far enough away to see the whole galaxy at once. We have no pictures of the Milky Way from the “outside”.
9. *Ch. 14, Sensible Statements, #24.* This statement is sensible. Spiral arms are created by density waves. Gas and stars bunch up, creating the appearance of spiral arms. Gas clouds that bunch up in the spiral arms get squeezed a bit, which can trigger star formation. The highest-mass stars that form in a molecular cloud live for such a short period of time that they die in supernovae before the new bunch of stars has the chance to move out of the spiral arm.