

## BLAST WAVES & SNRs

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SN explosion  $\rightarrow$  energy deposition in the ISM by compressive heating + radioactive decay. We'll discuss how the highly overpressured bubble expands, ~~and~~, and cools, sweeping the surrounding ISM into a shell.

The very first stage is nearly ballistic expansion, until the swept-up mass  $4\pi r_s^3 \rho_1$  is  $\sim$  the initial shell mass  $M_s$ .

$$\text{Typ. : } \rho_1 = 2 \times 10^{-24} \text{ g cm}^{-3} \\ M_s = 0.25 M_\odot \Rightarrow r_s \approx 1.3 \text{ pc}$$

$$\text{Initial energy of envelope } E = \frac{1}{2} M_s V_s^2 \approx 10^{51} \text{ erg}$$

$$(\text{e.g., } M_s = 0.25, V_s = 2 \times 10^4 \text{ km/s})$$

$$\text{To get to } r_s = 1.3 \text{ pc} \Rightarrow t = \frac{r_s}{V_s} = \frac{1.3 \text{ pc}}{2 \times 10^4 \text{ pc/Myr}} = 65 \text{ yr}$$

The second phase of the expansion is the "Sedov-Taylor" phase, where radiated energy  $\ll$  the initial energy (radiation is inefficient at these very high T)

$\Rightarrow E \approx \text{constant}$  ~~within~~ within the shocked region.

Momentum shock-jump condition is:  $\rho_1 V_1^2 + P_1 = \text{constant}$

$\Rightarrow$  If upstream medium is undisturbed  $V_1 = V_s$

If  $V_s^2 \rho_1 \gg P_1$  (i.e. ram pressure  $\gg$  thermal pressure)

$\Rightarrow$  Shock is considered a "blast-wave".

For  $\frac{P_1}{\rho_1} \sim (8 \text{ km s}^{-1})^2$  The solution will be a blast wave  
 as long as  $v_s \gtrsim 10 \text{ km/s}$  (recall  $v_s$  initially is  $10^4 \text{ km/s}$ )  
 (i.e.,  $v_s \gg$  typical thermal velocity of  $1 \text{ cm of s}^{-1}$  km/s)  
 (this assumes expansion occurs into warm, not hot, medium).

During this phase the flow approaches self-similar solution  $\Rightarrow$  The pattern of variation of  $\rho$ ,  $r$ , and  $P$  or  $t$  within the shocked region at any time "looks" the same as at any other time, within a simple rescaling of the axes.

### Natural scales

Total energy  $E$

ambient density  $\rho_1$

ambient  $P \approx 0$   $T_{\text{amb}} \approx 0$  neglected

$r_s$  must depend on these parameters and on time.

If we want to construct something with units of length from  $E$ ,  $\rho$ , and  $t$ , then

$$E = (\rho_1 L^3) L^2 / t^2 \Rightarrow L = \left( \frac{Et^2}{\rho_1} \right)^{1/5}$$

So we expect the outer edge of the shell,  $r_s$ , to vary as

$$r_s = \rho_0 \left( \frac{Et^2}{\rho_1} \right)^{1/5} \text{ for } \rho_0 \sim \text{constant.}$$

In interior positions in the shell then  $x = \frac{r}{r_s} = \frac{r}{\left( \frac{Et^2}{\rho_1} \right)^{1/5}}$

$$\propto r = x \cdot \xi_0 \left( \frac{Et^2}{\rho_1} \right)^{1/5} \text{ where } x \text{ runs from 0 to 1.}$$

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(A) What about the pressure in the interior?

$$[P] = \text{energy density} \quad \text{For perfect gas } \frac{\text{ener.}}{\text{Vol.}} = \frac{P}{\gamma - 1}$$

$$\gamma = 5/3 \text{ for atomic gas} \Rightarrow \frac{\text{ener.}}{\text{Vol.}} = \frac{3}{2} \frac{P}{\gamma - 1}$$

$$\Rightarrow P \propto \frac{E}{r_s^3} \propto \frac{E}{(Et^2/\rho_1)^{3/5}} \propto \frac{E^{2/5}}{t^{6/5}} \rho_1^{3/5}$$

$$\Rightarrow P(r=xr_s) = P(x) P_2, \quad P_2 = K \frac{E^{2/5}}{t^{6/5}} \rho_1^{3/5}$$

$P(x) = 1$  immediately inside the shell, where  $P = P_2$

Let's consider the shock jump conditions:

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} M_i^2 \Rightarrow P_2 = \frac{2\gamma}{\gamma+1} \frac{P_1 v_i^2}{\gamma P_1} \rho_1$$

$$P_2 = \frac{2}{\gamma+1} \rho_1 v_i^2$$

Since the pre-shock material is stationary  $\Rightarrow$

$$v_i = v_s \Rightarrow \boxed{P_2 = \frac{2}{\gamma+1} \rho_1 v_s^2}$$

$$\text{The shock velocity is } v_s = \frac{dr_s}{dt} = \frac{2}{5} \xi_0 \left( \frac{E}{t^3 \rho_1} \right)^{1/5}$$

$$\text{Hence } P_2 = \frac{2}{\gamma+1} \rho_1 \left( \frac{2}{5} \xi_0 \right)^2 \left( \frac{E}{t^3 \rho_1} \right)^{2/5} \propto \frac{E^{2/5} \rho_1^{3/5}}{t^{6/5}}$$

$\Rightarrow$  we can write  $P = P(\omega) P_2(t)$  as above

(B) The density just inside the shock, for a strong shock, is

$$\xi_2 = \frac{\gamma+1}{\gamma-1} \xi_0 \Rightarrow \text{This sets the density scale for interior, } \rho = \xi_2 \cdot \alpha(\omega)$$

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(c) Velocity just inside the shock in the shock frame

$$|v_2| = |v_1| \frac{\gamma - 1}{\gamma + 1} = v_s \frac{\gamma - 1}{\gamma + 1}. \text{ Back in the observer frame } \Rightarrow$$

$$v_{2, \text{obs. frame}} = v_s - |v_2| = v_s \left( 1 - \frac{\gamma - 1}{\gamma + 1} \right) = v_s \frac{2}{\gamma + 1} = \frac{2}{\gamma + 1} \frac{2}{5} \zeta_0 \left( \frac{E}{t^2 p_i} \right)^{1/5}$$

In fact this, we have  $v_{0, \text{obs}} = \frac{2}{\gamma + 1} v_s(t) \text{ where}$

The ~~second~~ forms above for  $P_2$ ,  $\rho_2$ , and  $r_{2, \text{obs}}$  give the boundary conditions. We have scaling & solutions for everything in terms of  $x = r/r_s$  such that

$$\frac{\partial}{\partial t}( ) = \frac{\partial}{\partial x}( ) \frac{\partial x}{\partial t} = \frac{\partial}{\partial x}( ) x \left( -\frac{2/5}{t} \right) \text{ and } \frac{\partial}{\partial r} = \frac{\partial}{\partial x} \frac{\partial x}{\partial r} = \frac{\partial}{\partial x}( ) \cdot \frac{x}{r}$$

$\Rightarrow$  In the hydro-Euler eqs all of the factors depending on  $t$  and  $r$  then factor out due to the assumed scalings for  $\rho$ ,  $v$ ,  $P$  and we are left with coupled ODEs in  $x$ .

The exact solution of this interior problem, determining  $p(x)$ ,  $v(x)$ ,  $\alpha(x)$ , and  $\zeta_0$ , was first obtained separately by Sedov (1946) and Taylor (1950), with direct comparison to the first atomic bomb blasts.

The numerical value of  $\zeta_0$ , for the case of atomic gas, is  $\zeta_0 = 1.17$ .

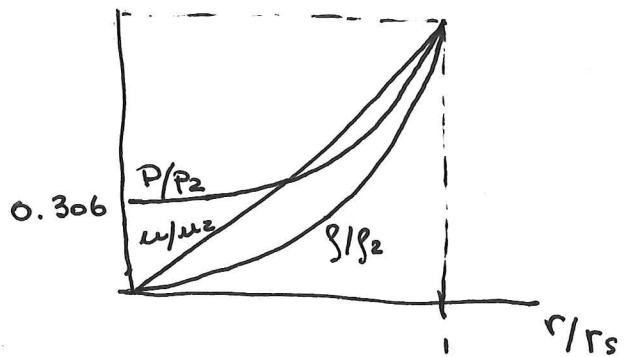
$$\Rightarrow r_s(t) = 1.17 \left( \frac{Et^2}{P_i} \right)^{1/5} = 0.26 \left( \frac{t/yr}{\text{yr}} \right)^{2/5} \cdot \left( \frac{E}{4 \times 10^{50} \text{ erg}} \right)^{1/5} \text{ pc}$$

$$\left( \frac{m_H/\text{cm}^{-3}}{\text{cm}^{-3}} \right)^{1/5}$$

Expansion speed is

$$v_s(t) = \dot{r}_s(t) = \frac{2}{5} \frac{r_s}{t} = 10^5 \text{ km s}^{-1} \cdot \frac{(t/\text{yr})^{-3/5}}{\left(\frac{m^4}{\text{cm}^3}\right)^{1/5}} \cdot \left(\frac{E}{4 \times 10^{50} \text{ erg}}\right)^{1/5}$$

The interior solution looks like



half of mass is in outer 6.1% of radius.  $\frac{3}{4}$  of mass is in ~~the~~ outer 12.6%.

Note that central  $u$  and  $P$  are zero, but central  $\gamma$  is  $\approx 30\%$ .

of the pressure just inside the edge of the shock  $\Rightarrow$  Temperature is greatest at the center of the remnant. That is because the shock was stronger when it shocked the central region.

Recall that  $T_2 = \frac{P_2 u}{\gamma_2 k}$ ,  $P_2 = \frac{2}{\gamma+1} \rho_1 v_i^2$ ,  $\gamma_2 = \frac{\gamma+1}{\gamma-1} \gamma_1$

$$\Rightarrow T_2 = \frac{\frac{2}{\gamma+1} \rho_1 v_i^2 u / k}{\frac{\gamma+1}{\gamma-1} \rho_1} = \frac{2(\gamma-1)}{(\gamma+1)^2} \cdot \frac{v_i^2 u}{k} = \frac{2(\gamma-1)}{(\gamma+1)^2} r_s^2 \frac{u}{k}$$

with  $v_i = r_s \propto t^{-3/5} \Rightarrow T_2 \propto t^{-6/5}$  immediate post-shock

$$T_2 = \frac{1.5 \times 10^{10} \text{ K}}{(t/\text{yr})^{6/5}} \frac{1}{m^{2/5}} \left( \frac{E}{4 \times 10^{50} \text{ erg}} \right)^{2/5}$$

Inside SNR,  $T/T_2 \sim \left(\frac{r}{r_s}\right)^{-4.3}$

The overall temperature drops as  $\frac{P}{\rho} \propto t^{-6/5}$  within the shocked region

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⇒ As time advances the rise of the cooling curve at lower  $T$  means the radiative losses become more important, surpassing the sharing of energy with the shocked region

The strongest X-ray radiation is for the outer part, because it has the highest  $m$  and the lowest  $T \Rightarrow$  strongest  $\Lambda m^2$

When radiative losses become important, the SNR enters the "dense shell formation" stage,  $t \approx 2 \times 10^4 \text{ yr}$ ,  $r \approx 14 \text{ pc}$

⇒ Shock becomes a "cooling shock" rather than the energy conserving shock.

⇒ post-shock density approaches the isothermal limit,

$\rho_2 = M_1^2 \rho_1$ , which can be quite large as long as  $v_1 = v_s$  remains  $\gg 10 \text{ km/s}$  ( $M_1$ )  
 $C_1 \approx 6 \text{ km/s}$  ( $m=1$ ,  $P/k \approx 3000$ )

⇒ For  $t = 2 \times 10^4 \text{ yr}$ ,  $v_s = \frac{2}{5} \frac{r_1}{t} = 260 \text{ km/s}$  for the fiducial parameters  $\Rightarrow \rho_2/\rho_1 \approx 2 \times 10^3$ !

If the shock becomes isothermal  $\Rightarrow r_2 = \frac{v_s}{M_1^2}$

In the obs. frame:  $v_s - \frac{v_s}{M_1^2} = v_s \left(1 - \frac{1}{M_1^2}\right) \approx v_s$

⇒ The material is not left behind the shock front, but it is carried along with it forming a dense shell, of thickness  $\sim$  the cooling length  $\sim \frac{3}{2} \frac{k t}{m_2 \Lambda} v_2$  behind shock front

$$v_2 = \frac{\gamma-1}{\gamma+1} v_s, \quad kT_2 = \frac{2(\gamma-1)}{(\gamma+1)^2} v_s^2 \mu$$

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$$\Rightarrow L_{\text{cool}} \sim \frac{\frac{3}{2}^2 \frac{(2/3)}{(8/3)^2} v_s^2 \mu \frac{2/3}{8/3} v_s}{\frac{8/3}{2/3} m_i \Lambda} = \cancel{\frac{2(2/3)^2}{(8/3)^4} \frac{v_s^3 \mu}{m_i \Lambda}}$$

$$L_{\text{cool}} \sim \frac{9}{8^3} \cdot \frac{(v_s/100 \text{ km/s})^3 10^{21} \text{ cm}^{-1} \cdot 1.67 \times 10^{-24} \text{ erg s}^{-1}}{(m_i/\text{cm}^{-3}) \frac{1}{10^{-23} \text{ erg cm}^{-3} \text{s}^{-1}}} \sim \text{pc} \frac{(v_s/100 \text{ km/s})^3}{(m_i/10^{-23} \text{ erg s}^{-1})}$$

After cooling becomes strong (when  $T_2 \sim 1.5 \times 10^5 \text{ K}$  or so, which happens at  $t \sim 10^5 \text{ yr}$  in the Sedov-Taylor solution)  
 $r_s \sim 26 \text{ pc}, v_s \sim 100 \text{ km/s}$   
 The SNR enters the "snow-plow" or momentum-conserving phase.

Suppose initial shell velocity is  $v_0$ , radius is  $r_0$ , and mass is  $M_0 = \frac{4}{3} \pi r_0^3 \rho_0$ , e.g.  $r_0 = 100 \text{ km}^{-1}$  and  $M_0 = 2.6 \times 10^3 M_\odot$

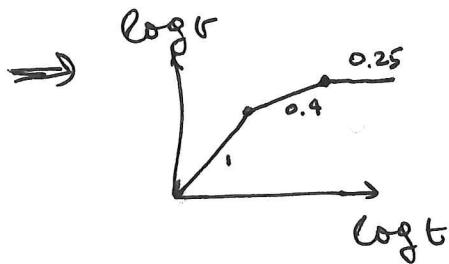
$$\Rightarrow M(t-t_0) = \frac{4}{3} \pi r^3 \rho_0$$

$$\text{To conserve momentum, } v(t-t_0) = \frac{v_0 M_0}{M(t-t_0)} = \frac{v_0 r_0^3}{r^3}$$

$$v(t-t_0) = \frac{dr}{dt} \Rightarrow r^3 dr = v_0 r_0^3 dt \Rightarrow \frac{r^4}{4} \Big|_{r_0}^r = v_0 r_0^3 (t-t_0)$$

$$\Rightarrow \frac{r^4 - r_0^4}{4} = v_0 r_0^3 (t-t_0)$$

For  $r \gg r_0 \Rightarrow r \approx (4 v_0 r_0^3 (t-t_0))^{1/4} \Rightarrow r \propto t^{1/4}$  in the snowplow phase instead of  $r \propto t^{2/5}$  in the adiabatic phase and  $r \propto t$  for the ballistic phase



Expansion is slower because momentum, shell not internal pressure, carries ~~it~~ forward.

$$r^4 \sim 4v_0 r_0^3 t, \quad t \sim \frac{r^4}{4v_0 r_0^3} = \left(\frac{r}{r_0}\right)^3 \frac{r}{4v_0} = \frac{M}{M_0} \frac{r}{4v_0}$$

$$r \propto (4r_0^3 v_0 t)^{1/4} \Rightarrow r(t) \sim \frac{1}{4} \frac{r}{t} \sim \frac{1}{4} \left(\frac{4r_0^3 v_0}{t^3}\right)^{1/4}$$

$$\Rightarrow \frac{v(t)}{v_0} \sim \left(\frac{r_0}{4v_0 t}\right)^{3/4}$$

⇒ E.g. for  $v_0 = 100 \text{ km s}^{-1}$  and  $r_0 = 26 \text{ pc}$ , velocity slows to

$$10 \text{ km/s when } \frac{1}{10} = \frac{1}{4} \left(\frac{26 \text{ pc}}{\text{pc} \cdot 100 \text{ yr}}\right)^{3/4} \Rightarrow t \sim 8 \times 10^5 \text{ yr}$$

~~and reaches a steady state at  $r = 26 \text{ pc} \times 10^{13/3} = 56 \text{ pc}$~~

⇒ At this point the expansion is comparable to thermal velocity in the warm ISM, and external pressure becomes comparable to ram pressure, inhibiting further expansion.

The total mass is:  $v_0 t M_0 / v M$

$$M \sim \frac{v_0 M_0}{v} \sim 10 M_0 \sim 2.6 \times 10^4 M_\odot$$

Comparing final to initial energy, the efficiency for kinetic energy injection is  $\frac{\frac{1}{2} M_{\text{final}} v_{\text{final}}^2}{E} \sim \frac{1}{2} \frac{2.6 \times 10^4 M_\odot (10 \text{ km/s})^2}{4 \times 10^{50} \text{ erg}} = 0.07$

$$= 7\%$$

Precise calculations show  $\eta \sim 3\%$  for  $n_H = 1 \text{ cm}^{-3}$

6% for  $n_H = 0.01 \text{ cm}^{-3}$  ← radiation is inefficient

Is this enough to equilibrate energy dissipation  
in the ISM? Yes if SNe were uncorrelated (Spitzer § 11.1b)  
but not if they are correlated because most of this  
energy is channeled through chimneys and is not  
injected in the plane.

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