Lecture 9: Special Theory of Relativity III

- A new velocity addition law
- Momentum, Energy and Mass

I: A NEW VELOCITY ADDITION LAW

- Once we’ve taken into account the way that time and distances change, what’s the new law for adding velocities? Result is...

\[ V_{\text{add}} = \frac{V_1 + V_2}{1 + \frac{V_1 V_2}{c^2}} \]

- We will discuss implications of this new formulae in class...
II: MASS AND ENERGY

- Einstein reworked Newton's laws of mechanics using his new relativistic formulae.
- He found that the relationships between mass \( M \), velocity \( v \), momentum \( p \) and energy \( E \) are different than those found in Newtonian mechanics.

\[
p = \gamma M v \\
E = \gamma M c^2
\]

\[
\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}
\]
Was Newton just plain wrong?

* What about objects moving at “small velocity”?
  * It can be shown that:
    \[ E \approx mc^2 + \frac{1}{2} mV^2 \]
  * The \( \frac{1}{2} mV^2 \) is the Newtonian expression for the kinetic energy of a moving object.
  * What counts as “small velocity”?
    * For car going at 30mph, approximate formula is wrong by 1 part in \( 10^{30} \)
    * For rocket going at 30,000mph, this approximate formula is wrong by 1 part in \( 10^{18} \)
    * So, approximation is fine for all velocities experienced in everyday life.
Rest mass energy

If we put $V=0$ in Einstein’s energy formula, we get...

$$E = mc^2$$

What does this mean?
- Maybe it is some fundamental “irreducible” (i.e., inaccessible) energy that every object possesses?
- Or, perhaps this energy can be accessed? In other words, maybe mass can be turned into “usable” energy? It turns out that this is correct!
- Also, this can go the other way - energy can be turned into mass.

III : EXAMPLES OF CONVERTING MASS TO ENERGY

- Nuclear fission
- Nuclear fusion
Fission

**Nuclear fission (e.g., of Uranium)**

- Nuclear Fission - the splitting up of atomic nuclei
- E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is

\[ ^{235}U + n \rightarrow 3n + ^{89}Kr + ^{144}Ba \]

From web site of Georgia State University
**Fission**

- **Nuclear fission (e.g., of Uranium)**
  - Nuclear Fission - the splitting up of atomic nuclei
  - E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is
    
    \[ {}^{235}\text{U} + n \rightarrow 3n + {}^{89}\text{Kr} + {}^{144}\text{Ba} \]
  - Mass of products of reaction (neutrons, Krypton, Barium) is slightly less than mass of initial Uranium nucleus + neutron
  - That mass “lost” has been converted into energy (gamma-rays and kinetic energy of fragments):
    \[ E = mc^2 \]

**Fusion**

- **Nuclear fusion (e.g. hydrogen)**
  - Fusion - the sticking together of atomic nuclei
  - Much more important for Astronomy (and life on Earth!) than fission
    - Power source for stars, including the Sun
    - Path to making heavy elements (C, N, O, Si, Fe...)
  - Important example – hydrogen fusion.
    - Ram together 4 hydrogen nuclei to form helium nucleus
    - Spits out couple of “positrons” and “neutrinos” in process
    
    \[ 4^1\text{H} \rightarrow ^4\text{He} + 2e^+ + 2\nu \]
The proton-proton fusion cycle

Fusion

- Mass of final helium nucleus plus positrons and neutrinos is less (by about 0.7%) than original 4 hydrogen nuclei
- \( E = mc^2 \)
- Mass has been converted into energy (gamma-rays and kinetic energy of final particles)
- This nuclear reaction (and similar ones) is the energy source for...
  - Hydrogen Bombs (about 1 kg of mass converted into energy gives equivalent of 20 Megatons of TNT)
  - The Sun (about \( 4 \times 10^9 \) kg of matter per second is converted into energy, ultimately yielding sunlight)
Antimatter

- Anti-matter
- For every kind of particle, there is an antiparticle...
  - Electron ↔ anti-electron (also called positron)
  - Proton ↔ anti-proton
  - Neutron↔ anti-neutron
- Anti-particles have opposite properties from the corresponding particles (e.g., opposite charge)... but exactly the same mass.
- When a particle and its antiparticle meet, they can completely annihilate each other... all of their mass is turned into energy (gamma-rays)!

III: EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy (e.g., gamma-rays) can produce particle/anti-particle pairs
- Very fundamental process in Nature... we’ll see that this process, operating in early universe, is responsible for all of the mass that we see today!
Particle production in a particle accelerator
- Can reproduce conditions similar to early universe in modern particle accelerators...