Class 2 : Hubble's Law Renewed & the Friedmann Equation

- This class…
  - Recap of Cosmological Principles
  - A new view of Hubble's law
  - The Friedmann Equation

O : Recap The Cosmological Principle

- **The Cosmological Principle** states that the Universe looks the same where-ever you are and which-ever direction you look

- Two immediate consequences of the Cosmological Principle…
  - Universe is **homogeneous** : every place in the universe has the same conditions as every other place, on average.
  - Universe is **isotropic** : there is no preferred direction in the universe, on average.

I : Fresh look at Hubble's Law

- Hubble's Law (original form) : A galaxy a distance *R* from us will be moving away from us with velocity *V* given by

$$V = H_0 R$$

- Notes:
  - The constant H_0 is the **Hubble constant**. Subscript "0" refers to the current cosmic time.
  - Current estimates are H_0=67.8±0.9 km/s/Mpc (Planck Collaboration)
  - As written, this is only true in the non-relativistic limit (V<<c). Once you start to look far enough out, the value of Hubble's constant changes! For this reason, it is best called the **Hubble parameter, H.**

- We can think of the Hubble flow as an overall homogeneous & isotropic expansion of space
- Imagine a coordinates that expand with the space…
  - These are called **co-moving coordinates**
  - Can then write

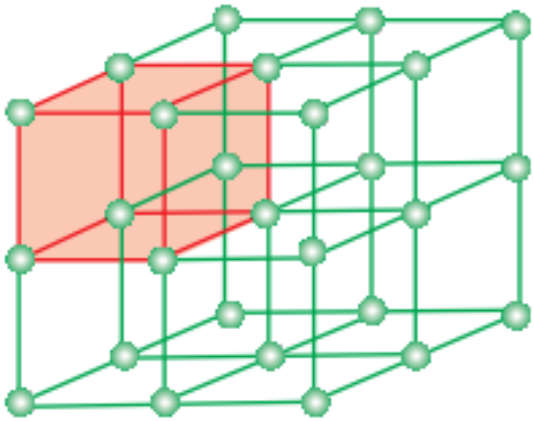
$$R = a\,x$$

proper distance; Scale factor; comoving distance

  - Cosmological Principle ➔ scale factor is purely function of t
  - So, a galaxy's velocity can be decomposed into two pieces:

$$V \equiv \frac{dR}{dt} = \frac{da}{dt}x + a\frac{dx}{dt} = \left(\frac{1}{a}\frac{da}{dt}\right)R + a\frac{d}{dt}\left(\frac{R}{a}\right)$$

Hubble Flow; Peculiar velocity



- We define the **Hubble parameter** by

$$H \equiv \frac{1}{a}\frac{da}{dt}$$

- So, if a galaxy has zero peculiar velocity, then it is carried along by the Hubble Flow and we have

$$V = \left(\frac{1}{a}\frac{da}{dt}\right) R = HR$$

- Notes
  - We have recovered Hubble's Law
  - Hubble parameter can be a function of time (depending upon the mathematical form of the scale factor)

II : The Friedmann Equation

- Friedmann equation is the basic (differential) equation that describes the time evolution of the scale factor

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

  - Here, ρ is total equivalent mass density of the Universe
  - k is a constant (−, 0, or +)
  - c is speed of light
  - From now on, we will use "dot" for the time derivative
- Full derivation comes from applying assumptions of homogeneity and isotropy to the Field Equations of General Relativity
- With a little cheating, we can also derive the Friedmann equation from Newtonian arguments… [see discussion on board]