Class 3 : Cosmological Fluids

- This class…
  - Recap of Friedmann Equation
  - The Cosmological Fluid Equation
  - Behavior of "radiation fluids"
  - The Acceleration Equation

0 : Friedmann Equation (recap)

- Friedmann equation is the basic (differential) equation that describes the time evolution of the scale factor

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

  - Here, ρ is total equivalent mass density of the Universe
  - k is a constant (−, 0, or +)
  - c is speed of light
  - From now on, we will use "dot" for the time derivative
- Full derivation comes from applying assumptions of homogeneity and isotropy to the Field Equations of General Relativity. Can cheat a little and more-or-less derive it with Newtonian arguments.

I : The Fluid Equation

- To solve Friedmann equation and determine a(t), we need to know how density depends on scale factor ρ(a)
- Starting with the first law of thermodynamics, we can show that

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

Derivation on Board

  - The two terms in the bracket correspond to the (1) dilution of the energy by the expansion and (2) the loss of energy due to the work done on the surroundings.
  - **Important case**: Pressure-less matter ("dust")

$$p = 0 \Rightarrow \dot{\rho} + 3\frac{\dot{a}}{a}\rho = 0 \Rightarrow \rho \propto a^{-3}$$

"Matter-dominated" Universe

  - This simply describes how a constant mass gets diluted as the volume increases (mass=ρV=const)

II : Behavior of radiation fluids

- Suppose that the Universe were dominated by radiation and not matter!
- In fact, as we'll see later, this is precisely the situation for the first 50,000 yrs of the Universe's history.
- For radiation:

$$p = \rho c^2/3$$

- Substituting into Fluid Equation, we get

$$\dot{\rho} + 4\frac{\dot{a}}{a}\rho = 0 \Rightarrow \rho \propto a^{-4}$$

Radiation-dominated Universe

- This has an interesting interpretation…

III : The acceleration equation

- Combining the Friedmann and Fluid equations, we can derive the **acceleration equation**:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

Derivation on Board

- Notes:
  - This equation does not depend upon "k"
  - Pressure forces increase the gravitational deceleration of the Universe! Pressure acts to SLOW DOWN the expansion, not drive it. WHY???