Class 4 :
Simple flat space cosmologies

- This class…
  - Recap of basic equations of cosmology
  - Flat space cosmology
  - Matter dominated Universes
  - Radiation dominated Universes
  - Mixed matter-radiation Universes

0 : Recap of basic equations

- Basic equations of cosmology…

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$ *Friedmann equation*

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$ *Fluid equation*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$ *Acceleration equation*

- Only two of these are independent… the acceleration equation can be derived from the other two

I : Flat space cosmology

- In fact, in a General Relativistic derivation of the Friedmann Equation, it becomes cleat that the "k" term tells us about the curvature of the space… we'll return to this next class
- The simplest set of cosmological models correspond to flat space
  - Mathematically, this corresponds to setting k=0 in Friedmann eqn
  - If space is flat, there is a unique relation between density and Hubble parameter

$$\rho = \frac{3H^2}{8\pi G} \quad \text{where} \quad H = \frac{1}{a}\frac{da}{dt}$$

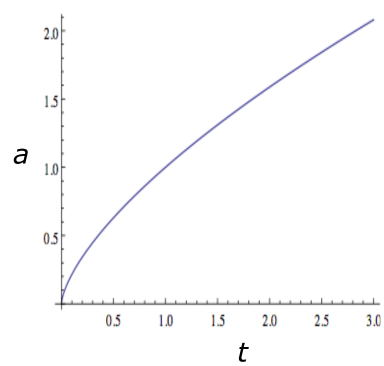
  - Even if Universe is not flat, the flat-space models are always a good approximation for the evolution of the early Universe… follows from an examination of Friedmann equation (see board)

II : Matter dominated flat Universe

- Matter dominated means that $\rho(a) \propto a^{-3}$
- Substitute this and k=0 into Friedmann eqn and solve… we get $a(t) \propto t^{2/3}$
- If a=0 at t=0, then the Hubble parameter is

$$H = \frac{1}{a}\frac{da}{dt} = \frac{2}{3t}$$

- Most importantly, this is a BIG BANG cosmology… the scale factor starts at a=0 from which the Universe "explodes"

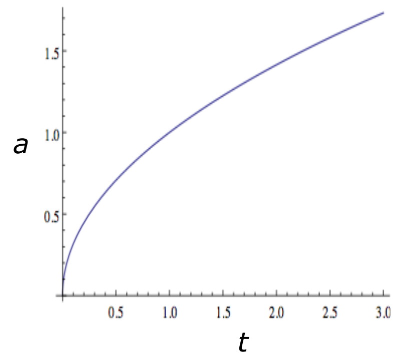


The Big Bang occurred everywhere at the same time. It was not an explosion into space, it was an explosion of space!

III: Radiation dominated flat Universe

- Radiation dominated means that $\rho \propto a^{-4}$
- Substitute this and k=0 into Friedmann eqn and solve… we get $a(t) \propto t^{1/2}$
- If a=0 at t=0, then the Hubble parameter is

$$H = \frac{1}{a}\frac{da}{dt} = \frac{1}{2t}$$



IV: Matter and radiation mixes

- What is there is both matter and radiation? Then,

$$\begin{aligned}\rho &= \rho_{mat} + \rho_{rad} \\ &= \rho_{mat,0}\left(\frac{a_0}{a}\right)^{-3} + \rho_{rad,0}\left(\frac{a_0}{a}\right)^{-4}\end{aligned}$$

- Exact solution to Friedmann equation for evolution of the Universe is quite complicated… but it works well to break the evolution into two phases:
  - At sufficiently early time, Universe will be **radiation dominated**… all of the results that we just derived for rad-domination will hold.
  - Radiation "dilutes" faster than matter as Universe expands… thus, at some point the Universe will be **matter dominated**. Again, all of the results that we derived for matter-domination will hold.
  - The time at which matter and radiation contribute equally is called the **epoch of equality**. In our Universe, this happens ~50,000 yrs after the big bang (this will be calculated later).