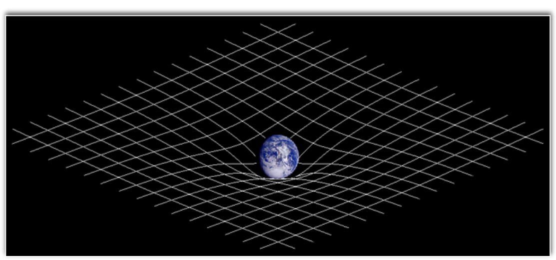
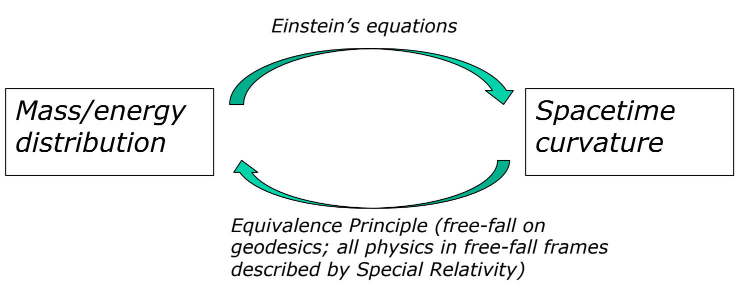
Class 5 : General Relativistic Cosmology

- This class…
  - Recap : Flat space cosmology
  - Basic structure of GR and curvature
  - Isotropic & Homogeneous spaces : 3 cases
  - Metrics
  - The FRW metric

I : Reminder about GR

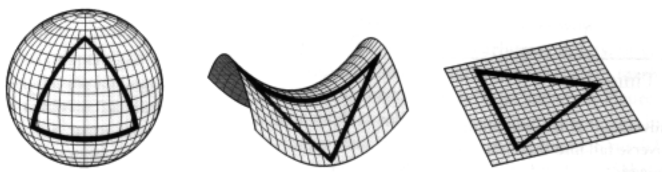




- What is the meaning of curvature?

- Space-time curvature is an intrinsic property
  - You do NOT need to step into a higher dimension to be able to tell that you're living in a curved space
  - How can you tell?

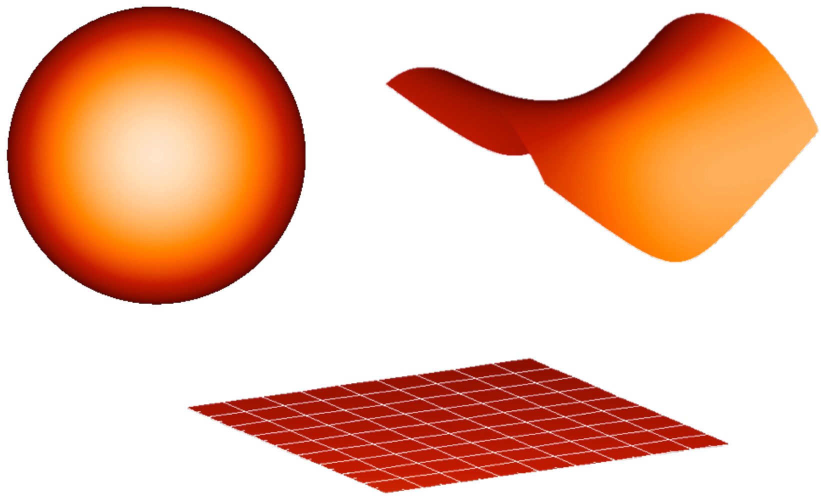
- Characteristic properties of curved space
  - Parallel lines can cross (or diverge)
  - Interior angles of triangles do not sum to 180°



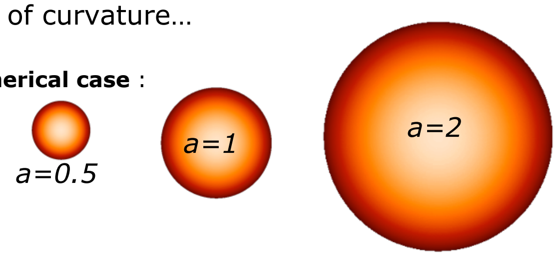
II : The geometry of the Universe

- Spacetime is 4-dimensional (3 space + 1 time)
- We can imagine breaking this spacetime up into a 1-d "time stack" of 3-d "space slices"
  - This is called a foliation of the space-time
  - A central aspect of GR is that the foliation is not unique since there is no unique definition of the time coordinate.
  - (see discussion on board)

- The Cosmological Principle: there exists an observer who sees "simultaneous spatial slices" that are always isotropic and homogeneous… this powerful restriction limits the spatial geometry to one of three cases:
  - Spherical (positive curvature)
  - Flat
  - Hyperbolic (negative curvature)



- What is the meaning of the "scale factor" a for these geometries? For curved cases, it corresponds to the radius of curvature…
  - **Spherical case** :



  - **Hyperbolic case** :

III : Metrics

- How do you mathematically describe curvature? We define a metric tensor.
  - Definition : Suppose some region of space is spanned by coordinates $x^1, x^2, \dots x^N$. The distance ds between two nearby points separated by coordinates $dx^1, dx^2, \dots dx^N$ is

$$ds^2 = \sum_{ij} g_{ij} dx^i dx^j$$

where g_{ij} is the metric. We often just present the metric by writing down the expression for ds^2 ("line element")

[See discussion on board]

IV : The Friedmann-Robertson-Walker (FRW) Metric

- What is the metric for these spacetimes? It can be shown that the appropriate metric is…

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- Notes:
  - This covers all three geometries
    - k>0 : Spherical
    - k=0 : Flat
    - k<0 : Hyperbolic
  - Role of a(t) as scale factor is made clear
  - Substituting this metric into Einstein's Field Equations gives the Friedmann equation and the acceleration equation that we've already encountered (thus also encapsulates the fluid equation)