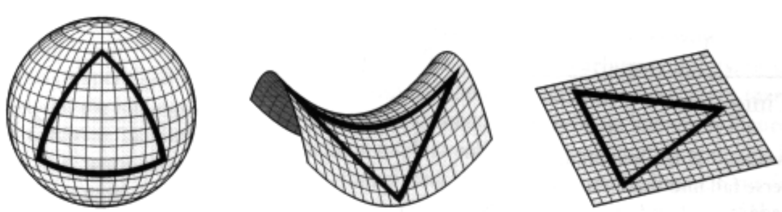
Class 6
The FRW Metric

- This class…
  - Recap the CP-compliant geometries
  - Finish discussion of metrics
    - Metrics of 3-flat space
    - Metrics of Minkowski space time
    - Metrics of 3-d spheres & 3-d hyperbolic surfaces
    - And then…
  - The FRW metric

O : Recap – possible geometries of the Universe

- The Cosmological Principle: there exists an observer who sees "simultaneous spatial slices" that are always isotropic and homogeneous… this powerful restriction limits the spatial geometry to one of three cases:
  - Spherical (positive curvature)
  - Flat
  - Hyperbolic (negative curvature)



I : Metrics (cont from Tuesday)

- Last class… discussed 2-d flat space
- Today, let's look at…
  - 3-d flat space
  - Spacetime of Special Relativity
  - 2-d spherical/hyperbolic spaces
  - 3-d spherical/hyperbolic spaces

II : The Friedmann-Robertson-Walker (FRW) Metric

- What is the metric for these spacetimes? It can be shown that the appropriate metric is…

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

- Notes:
  - This covers all three geometries
    - $k>0$: Spherical
    - $k=0$: Flat
    - $k<0$: Hyperbolic
  - r, θ, ϕ are co–moving coordinates
  - Role of $a(t)$ as scale factor is made clear
  - Substituting this metric into Einstein's Field Equations gives the Friedmann equation and the acceleration equation that we've already encountered (thus also encapsulates the fluid equation). The k above becomes the k of the Friedmann equation!