Class 10 : Photons, distance, and the observable Universe

- This class
  - General motivation : observational probes of the dynamics of the Universe
  - Photon propagation in the FRW metric
  - Scale factor and redshift
  - Finite nature of the observable Universe
  - Luminosity distance
  - Angular size distance

0 : Taking Stock…

- Cosmological Principles ✔
- Cosmological models ✔
  - Basic equations ✔
  - Models with various types of "fluid" ✔
  - Models with various curvatures ✔
  - Density-Curvature-Dynamics connection ✔
- Which model describes our Universe? ✔
  - Measurements of matter density ✔
  - The nature of Dark Matter ✔

I : Our View of the Universe

- From Earth, we look out into Universe that is…
  - Expanding
  - Has a finite beginning in time
  - May be curved

- How does this affect our view of the Universe? We will see that...
  - Concept of "distance to an object" now more complex
  - We can only see a finite volume of space, even if the Universe is infinite

II : Photon propagation in FRW metric

- In General (and Special) Relativity, photons follow paths such that the space-time interval is zero, $ds^2=0$. These are called **<u>null geodesics.</u>**

- Recall the FRW metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

  - Consider a photon propagating in the r-direction. Put $ds^2=0$ and re-arrange to get

$$\frac{c\,dt}{a} = -\frac{dr}{\sqrt{1-kr^2}}$$

$$\int_{t_e}^{t_r} \frac{c\,dt}{a} = \int_0^{r_0} \frac{dr}{\sqrt{1-kr^2}}$$

  - Here, t_e is time photon is emitted, t_r is time photon is received

- From this, we can provide a more rigorous proof of the redshift/scale-factor relation that we motivated from considerations of the evolution of radiation energy density…

$$1 + z = \frac{a(t_r)}{a(t_e)}$$

- See prove on board…
- Notes:
  - This directly relates changes in scale factor to measured redshift.
  - Same statement as saying that λ stretches in proportion to scale factor… we've already seen this result before (when discussing fluid equation)

III : Size of the Observable Universe

- Suppose a photon is emitted at the time of the big bang
- How far will it have propagated in time t?
- For simplicity, let's think about a flat, matter-dominated Universe. So we can put k=0 and $a=(t/t_0)^{2/3}$ to get

$$r_0 = 3ct_0$$

- Notes:
  - Despite fact that $da/dt \rightarrow \infty$ as one approaches big bang, photons emitted at big bang travel finite distance. True for all geometries.
  - r_0 is larger that ct_0… photons cover more ground (in terms of coordinate r) when scale factor is smaller
  - r_0 defines the **size of the observable Universe**… it is impossible to obtain information from any further than this distance since no signal could have traveled further within the age of the Universe
  - So, our "view" of the big bang is an infinite redshift sphere surrounding us at a proper distance of r=3ct.

IV : Different definitions of distance

- **Suspect you look at a galaxy with a redshift of z… how far away is it?**
- For z<<1, we get the answer from a simple application of Hubble's law…

$$z \approx \frac{v}{c} \quad \Rightarrow \quad d \approx \frac{zc}{H_0}$$

- For higher redshift, we need to integrate the FRW metric. We find that we have to be careful about what we mean by distance.
- Illustrate with k=0 matter dominated universe with age t_0. Find that the **coordinate distance** is

$$r_z = 3ct_0 \left[1 - \frac{1}{\sqrt{1+z}} \right]$$

- Practically, a very common way of measuring distance is to measure the flux of an object with a known luminosity and apply the inverse square law.
- But, in a curved evolving universe, the flux F does not go down as $1/r_0^2$. We define a **luminosity distance** d_{lum} such that F **is** proportional to $1/d_{lum}^2$. We find that

$$d_{\text{lum}} = r_z(1+z)$$

- Another possible way to assess distance is to look at the angular size of something on the sky that has a known size. This gives the **angular diameter distance**:

$$d_{\text{diam}} = \frac{r_z}{1+z} = \frac{d_{\text{lum}}}{(1+z)^2}$$