ASTR422 Cosmology (Fall 2016)

Homework 1

Due date; Thursday 15th September 2016

1. Liddle Q2.1
2. Liddle Q2.2
3. **The onset of acceleration :** Consider a Universe (such as ours!) containing only non-relativistic matter and dark energy with a cosmological-constant like equation of state ($p_\Lambda = -\rho_\Lambda c^2$, where p_Λ and ρ_Λ are the pressure and density of the dark energy components), i.e., radiation and relativistic matter plays a negligible roll in such a Universe (unless we consider very early times).
   (a) Explain why dark energy becomes relatively more dominant as the Universe expands.
   (b) With reference to the relevant cosmological field equation, show that the Universe starts accelerating ($\ddot{a} > 0$) when $\rho_\Lambda > \frac{1}{2}\rho_\mathrm{m}$, where ρ_m is the density of the non-relativistic matter.
   (c) Today, the ratio of the density dark energy to matter is $\rho_\Lambda/\rho_\mathrm{m} \approx 2.25$. Using this information, calculate the redshift at which the Universe switched from a decelerating to an accelerating expansion.
   (d) There is, of course, a non-zero amount of radiation and relativistic matter in our Universe. Today, the ratio of the density of the radiation/relativistic matter to that of the dark energy is $\rho_\mathrm{rad}/\rho_\Lambda \approx 1.31 \times 10^{-4}$. Calculate the redshift at which the radiation energy density becomes equal to the dark energy density.