ASTR422 Cosmology (Fall 2016)

Homework 2

Due date; Thursday 29th September 2016

1. Liddle Q4.1
2. Liddle Q4.2
3. Liddle Q5.3
4. Liddle Q9.2
5. **Fun with 3-spheres :** The metric of a 3-dimensional spherical surface with radius of curvature a is
$$ds^2 = \frac{dr^2}{1 - r^2/a^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right). \quad (1)$$
   (a) Derive an expression for the physical volume contained within the region defined by $r \leq r_0$, where r_0 is some constant. *(Hint : In class, we derived an expression for the surface area of the 2-d sphere with $r =$ constant. Use this and the metric above as the starting point to calculate the physical volume of a shell $r \rightarrow r + dr$).*
   (b) Show that this volume reduces to the usual flat space result when $r_0/a \ll 1$.
   (c) There is a maximum allowable value for r_0. What is this value?
   (d) What is the *total* volume of the 3-sphere?
6. **Spherical radiation-dominated Universes :** Solve the Friedmann equation for a radiation-pressure dominated Universe with $k > 0$ (i.e. the spherical/closed case) in order to derive an algebraic expression relating scale factor a and time since the big bang t, putting your answer in the simplest form possible. Show that, at early times, the answer approximates the flat-space result, $a \propto t^{1/2}$. [Hint : Recall that the density of radiation is $\rho_{\rm r} = \rho_{r,0}(a_0/a)^{-4}$, where subscript 0 refers to the current time. When you evaluate the relevant integral, use the limits of integration to encode the condition that $a = 0$ at $t = 0$, i.e., to define the origin of time].