ASTR422 Cosmology (Fall 2016)

Homework 5

Due date; Tuesday 29th November 2016

1. (based upon Liddle Q12.2) Consider the epoch of primordial nucleosynthesis which begins at $t = 340\,\mathrm{s}$ after the Big Bang. Suppose that each neutron present at that time is instantly locked up into a ^4He nucleus, and that all of the left over protons form hydrogen.

   (a) Explain why the resulting mass fraction of helium (i.e. the mass in helium divided by the total baryonic mass) is given by $Y = 2f_{\rm pns}/(1 + f_{\rm pns})$, where $f_{\rm pns} = n_n/n_p$ is the neutron-to-proton ratio immediately before primordial nucleosynthesis.

   (b) In class, we showed that the neutron-to-proton ratio coming out of the lepton era (a few second after the Big Bang) is $n_n/n_p \approx 0.2$. Given that the neutron has a half life of $614\,\mathrm{s}$, calculate $f_{\rm pns}$ and hence an estimate for Y.

   (c) Suppose that the neutron had a half life of $100\,\mathrm{s}$. What would be the expected helium mass fraction resulting from primordial nucleosynthesis, Y?

2. **The structure of dark matter halos :** In a simple model for a spherically symmetric dark matter halo, the dark matter density is given by

$$\rho_{DM} = \frac{M_v}{4\pi r_v r^2} \tag{1}$$

   where M_v is enclosed mass at the virial radius r_v.

   (a) Let $\sigma_c(r)$ be the velocity of a dark matter particle that is executing a circular orbit in the halo with radius r centered on $r = 0$. Show that σ_c is independent of radius, and obtain its value in terms of M_v and r_v.

   (b) Assuming that σ_c typifies the velocity of a dark matter particle, what is the average gravitational potential energy of a dark matter particle. Explain how you got your answer. [Hint — there is a fast route to this!]

   (c) Suppose that this dark matter halo has formed recently, and that the average density of the halo (measured out to the virial radius) is $\Delta \approx 200$ times greater than the average matter density today. Derive an expression for the mass of the halo (in units of solar masses) as a function of σ_c, scaled to a fiducial σ_c of $100\,\mathrm{km/s}$.

3. **Reionization and the smallest dark matter halos :** At a redshift of $z \approx 10$, the intergalactic baryons underwent a process called *reionization*: the radiation from the first stars and black holes was sufficient to ionized hydrogen and heat the baryons from very low temperatures up to temperatures of $10^4\,\mathrm{K}$. This question explores some implications of this for galaxy formation.

   (a) Assume that the reionized gas consists of protons and electrons such that the average particle mass is μm_p with $\mu \approx 0.5$. Using the expression given in class, calculate the sound speed of the reionized gas.

   (b) Noting that the sound speed is also approximately the average speed of a particle in the gas, explain why the process of reionization will drive baryons out of the smallest dark matter halos thereby preventing any star formation in those halos.

   (c) Consider the first population of dark matter halos which where formed at high redshift ($z \sim 20$ or greater). For these halos, it can be shown that $M_v \approx 3 \times 10^6 (\sigma/10\,\mathrm{km\,s^{-1}})^3\,\mathrm{M_\odot}$. Estimate the smallest halo that can still retain its baryons following reionization.