Lab 10 – Superluminal Motion of Quasar Jets

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ABSTRACT

In this lab we examine the phenomenon of superluminal motion, both in general and for the relativistic jet emitted from 3C 279. To understand the theory generally, we first consider apparent velocities of theoretical objects moving with various speeds at different angles to the observer's line of sight, finding that the angle at which apparent velocity peaks at is a function of its actual velocity. Next, we measured the superluminal motion of the jet emitted from 3C 279 using given data and found that the bulk motion of the jet has a velocity $v_{app} = (7.32 \pm 2)\ c$. Using this apparent velocity, we calculated a lower bound for the true velocity, $v = (0.991 \pm 0.007)\ c$. These results are similar to what we expected, with an apparent velocity greater than the speed of light and a true velocity less than, but close to the speed of light.

1. Introduction

A well-known consequence of Einstein's theory of special relativity is that maximum speed is finite: nothing can travel faster than the speed of light in a vacuum. Theoretically, this limit applies to everything in the universe. However, some astronomical objects, such as quasars and radio galaxies, appear to emit matter at speeds higher than the speed of light. This phenomenon, called superluminal motion, does not, in fact, contradict relativity; the faster-than-light motion is an illusion caused by matter moving at relativistic speeds at a small angle to an observer.

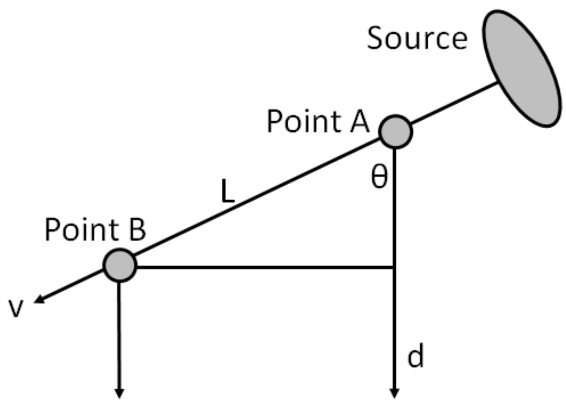


Figure 1: Diagram of Superluminal Motion

Figure 1 illustrates this phenomenon: the source galaxy emits a jet traveling with velocity v on a line with angle θ to an observer's line of sight. It passes through Point A, then Point B, a distance L apart. Because the light emitted from the jet at Point A must travel a farther distance d to reach the observer than light emitted from Point B, they two emissions will appear closer in time than they are, giving the impression that the object is moving faster than it is.

Where we would usually calculate the velocity by dividing the distance L by the time between Points A and B, the apparent velocity due to super luminal motion is calculated by instead using a 'distance' normal to the line of sight, L_{app}. From this, an expression for the apparent velocity of the jet can be derived with some simply geometry:

$$v_{app} = \frac{v \sin\theta}{1 - \frac{v}{c}\cos\theta}$$

Because the ratio $\frac{v}{c}$ is often expressed as β, the previous equation can also be written:

$$\beta_{app} = \frac{\beta \sin\theta}{1 - \beta\cos\theta}$$

In this lab, we will look for a lower limit on the true velocity of a jet. In order to find this limit, one can take the derivative of the right-hand-side of the previous equation and set it equal to zero, finding that

$$\beta_{app,max} = \gamma\beta$$

By finding β, we can calculate the minimum true velocity.

2. Methodology

To understand the relationship between angle to line of sight and apparent velocity of an object due to superluminal motion, we first calculated apparent beta for a range of angles for six different values of betas. This data was plotted in Figure 2, and is effectively a representation of apparent velocity for different actual velocities (because of how we defined beta).

Next, we calculated the apparent velocity of 3C 297. In order to do this, we were given images showing the movement of this galaxy's jet over the course of several years (included in Appendix 3), as well as the redshift of the galaxy, $z = (0.5362 \pm 0.0004)$. Using the redshift, we calculated the distance to the galaxy by using the relativistic redshift equation and Hubble's Law, assuming $H_o = (74.2 \pm 3.6)$ km s^{-1} Mpc^{-1}:

$$D = \frac{zc}{H_o} = 1635.3 \text{ Mps}$$

Because distances on the given image are measure in milliarcseconds, we then needed to determine the scale between distance and angular distance for an object at this distance. We did this by calculating the diameter d of an object at distance D with an angular size θ of 1", which gave us the conversion from angular size to physical size:

$$d = D \sin\left(\frac{\theta}{2}\right) = 0.007928 \text{ Mpc/as}$$

Next, we measured the distance l the jet moved between two points in time. For this, we used the first and fifth image because they show the jet over the longest period of time before it becomes unclear. To determine the distance, we measured the separation of the core of the galaxy and the center of the furthest blob at the two points in time and subtracted the two, and converted this value from an angle to a distance using the above scale:

$$l = x_2 - x_1 = 2.5569 - 1.8013 = 0.7575 \text{ mas} = 6.01 \times 10^{-6} \text{ Mpc} = 1.85 \times 10^{-6} \text{ m}$$

In order to determine velocity, we need distance as well as time; to find the time, we used the same method we used to find the distance, but vertically along the image. By dividing the distance between the two times by the distance between consecutive markings of the year, we can find the amount of time that has passed between the two images:

$$\Delta t = \frac{\Delta t_{pix}}{1\ year_{pix}} = 2.6759 \text{ y} = 8.4458 \times 10^7 \text{ s}$$

Again, we measured from the cores of the galaxy, assuming the brightest point was the center of the galaxy.

3. Analysis

Before calculating the apparent velocity of the jet, we determined the uncertainties of all the values that will be used in the calculation. First, the uncertainty in the distance to 3C 297 was found by using error propagation and the given values:

$$\sigma_D = dc\sqrt{\left(\frac{\sigma_z}{z}\right)^2 + \left(\frac{\sigma_{H_o}}{H_o}\right)^2} = 79.35 \text{ Mps}$$

Using this uncertainty in D, we also used error propagation to find the uncertainty in the conversion factor between distance and angular size on the image:

$$\sigma_d = \frac{d\sigma_D}{D} = 0.0029 \text{ Mpc/as}$$

Though we could use error propagation on the uncertainties of the distances measured from the image based on the intrinsic precision of the ruler we used, the uncertainty associated with the measurement was dominated by the difficulty in defining the position of the points we were measuring. Therefore, we estimated a more conservative error based on the range in which we would define the centers, assigning the value l an uncertainty of 0.25 mas, or $5\text{x}10^{16}$ m. Similarly, the uncertainty of the time measurement was dominated by ambiguity of measurement points; realistically, the uncertainty is about $5\text{x}10^{6}$ s.

Finally, we can calculate the apparent velocity and its uncertainty:

$$v_{app} = \frac{l}{\Delta t} = 2.19 \times 10^9 \text{ m s}^{-1}, \qquad \sigma_{v_{app}} = v_{app}\sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \left(\frac{\sigma_{\Delta t}}{\Delta t}\right)^2} = 6.06 \times 10^8 \text{ m s}^{-1}$$

Which corresponds to an apparent beta:

$$\beta_{app} = \frac{v_{app}}{c} = 7.32, \qquad \sigma_{\beta_{app}} = \left|\frac{\sigma_{vapp}}{c}\right| = 2.02$$

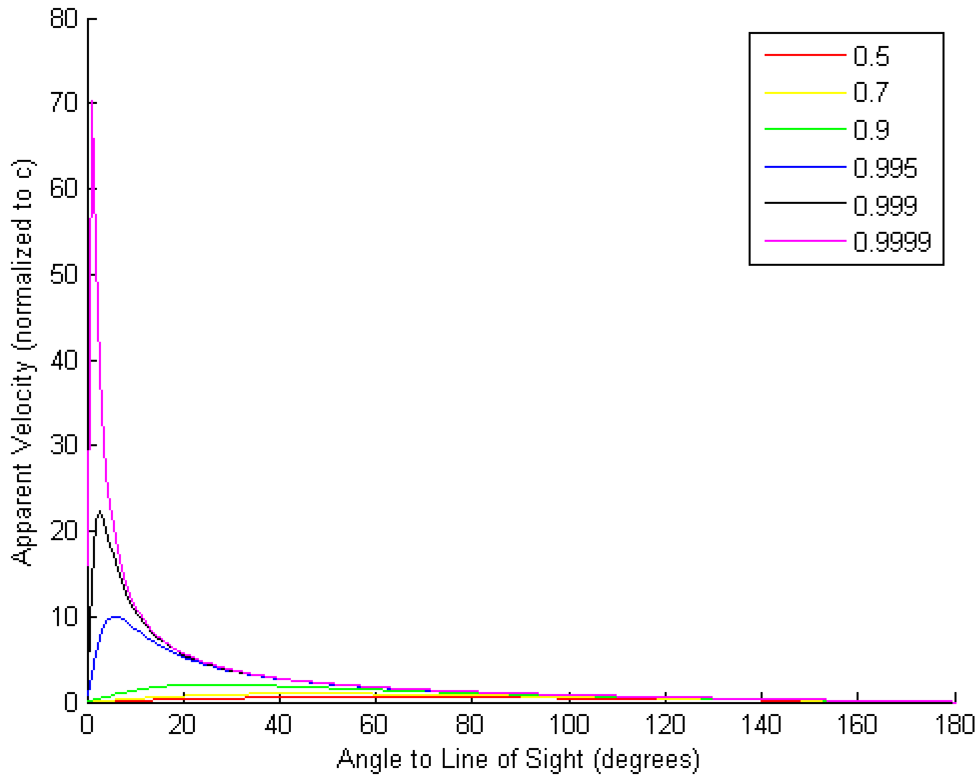
Finally, using the final equation in the introduction, we used β_{app} to calculate true β, and thereby the true velocity:

$$\beta = \frac{\beta_{app}}{\sqrt{1 + {\beta_{app}}^2}} = 0.991, \qquad \sigma_\beta = \frac{{\sigma_{\beta_{app}}}^2}{(\beta_{app} + 1)^3} = 0.0071$$

This value v for a lower bound of the true velocity of the jet is given by this β:

$$v = \beta c = 0.991\ c, \qquad \sigma_v = 0.007\ c$$

Figure 2: Apparent Velocity as a Function of Angle



4. Discussion

The first indication that we made reasonable measurements for the apparent velocity of the jet $v_{app} = (7.32 \pm 2)\ c$ and the true velocity $v = (0.991 \pm 0.007)\ c$ is the fact that the former is greater than the speed of light, and the latter is lower, but close. By definition, superluminal motion is motion that appears to be greater than c, which our $v_{app}$is, and it only occurs for objects moving very close to c, which our v is.

We can also use our plot in Figure 2 to confirm that our results are reasonable: the bright blue line shows apparent velocity for an object with a true β of 0.995, a little higher than our value, $\beta = (0.991 \pm 0.007)$. The plot peaks at a β_{app} of about 10, just a little higher than our value, $\beta_{app} = (7.32 \pm 2)$.

The uncertainties in most of our measurements and calculations is dominated by the difficulty in defining the position of the jet in the given image. Though we should have easily been able to measure hundredths of a milliarcsecond, we did not have an objective method for determining the center of the galaxy's core or the extended jet, so we estimated uncertainties based on how large our guess of the center was. To improve our precision, we could potentially fit the intensities near the core to a Gaussian and determine the peak, giving us a less subjective definition of the center.

5. Appendices

Appendix 1: Function for calculating β_{app}

```
function f=bapp(b,t) %Function that will take inputs b for beta and t for
angle and give apparent beta
trad=pi().*t./180 %convert t from degrees to radians
f=sin(trad)*b./(1-b.*cos(trad)); %Calculation of beta
```

Appendix 2: Script for plotting Figure 2

```
theta = linspace(0, 180, 200); %Creates a set of angles theta for input
beta = [0.5, 0.7, 0.9, 0.995, 0.999, 0.9999]; %Different beta values we'll
look at
clrs = ['r', 'y', 'g', 'b', 'k', 'm'];
figure(1); clf; %Figure showing apparent beta as a function of angle for a
set of betas
hold on;
for i = 1:6
    bA = bapp(beta(i),theta);
    plot(theta, bA, clrs(i))
end
title('Apparent Velocity')
xlabel('Angle to Line of Sight (degrees)')
ylabel('Apparent Velocity (normalized to c)')
legend('0.5', '0.7', '0.9', '0.995', '0.999', '0.9999', 'Location',
'NorthEast');
hold off;
figure(2); clf; %Same plot, but zoomed in on the y axis to show detail
hold on;
for i = 1:6
    bA = bapp(beta(i),theta);
    plot(theta, bA, clrs(i))
end
title('Apparent Velocity - Details')
xlabel('Angle to Line of Sight (degrees)')
ylabel('Apparent Velocity (normalized to c)')
legend('0.5', '0.7', '0.9', '0.995', '0.999', '0.9999', 'Location',
'NorthEast');
ylim([0 10])
hold off
```

Appendix 3: Images of 3C 279