Virtual Petaflops to Simulate Solar System Formation *

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Abstract

The orbit of any one planet depends on the combined motion of all the planets, not to mention the actions of all these on each other. To consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds, unless I am mistaken, the forces of the entire human intellect.
—Isaac Newton 1687

We describe some problems in the formation and stability of planetary systems. Here, the challenge is to perform accurate integrations that retain Hamiltonian properties for 10^{13} timesteps.

1 COSMOLOGY MEETS COSMOGONY: PLANETARY SYSTEM FORMATION

Theories of Solar System formation are traditionally divided into four stages (Lissauer 1993): collapse of the local cloud into a protostellar core and a flattened rotating disk (Nebular Hypothesis); sedimentation of grains from the cooling nebular disk to form condensation sites for planetesimals; growth of planetesimals through binary collision and mutual gravitational interaction to form protoplanets (Planetesimal Hypothesis); and the final assembly to planets with the remaining disk cleansed by ejections from chaotic zones. The final configuration of the solar system is determined by the last two phases. We have attacking the third phase our new parallel adaptive code.

All that is required for a detailed simulation of the third stage is a model of the collisional physics and a code capable of dealing with a large number of particles. However, previous direct simulations of the planetesimal stage (summarized in Table 1) fall far short of capturing the full dynamic range of the problem. Our cosmology code has the potential to treat as many as 10^7 particles simulataneously for 10^7 dynamical times, a ten-million-fold improvement that makes us enthusiastic! Only statistical methods (Wetherill and Stewart 1989) employing prescriptions for the outcomes of gravitational encounters have been used to peek at this regime.

We reach an important threshold at $N \sim 10^7$ in our ability to follow planetesimal evolution. At early times, the relative velocities between planetesimals are small and inelastic physical collisions lead to "runaway" growth of planetary embryos (Beaugé and Aarseth 1990). Eventually gravitational scattering increases the planetesimal eccentricities to such an extent that collisions result in fragmentation, not growth. The embryos will continue to grow owing to their large mass, but at a slower rate as their "feeding zones" are depleted (Ida and Makino 1993). The total mass of our planetary system is $448 M_\oplus$ or $3.6 \times 10^4 M_{lunar}$, while the inner planetismal disk amenable to simulation had a mass $\sim 10^2 M_{lunar}$. To capture both growth and fragmentation (Wetherill and Stewart 1989) requires a minimum particle mass of $10^{-5} M_{lunar}$, leading to our target $N \sim 10^7$.

ref	N	t (yr)	Δa (AU)	Col?	G?
Lecar....	200	6×10^4	0.5–1.5	a	n
Beaugé...	200	6×10^5	0.6–1.6	abcf	n
Ida...	800	5000	0.3	–	n
Aarseth...	400	1.2×10^4	0.04	ab	n
Chambers...	100	10^8	0.5–2.0	a	y
Kokubo...	5000	2×10^4	0.4	a	n
Richardson...	10^5	10^3	1.2–3.6	a	y
goal	10^7	10^7	0.5–2.0	abcf	y

Table 1: Highlights of advances in direct simulations of the formation of the inner planets. N is the maximum number of planetesimals used in the simulation, t is the longest integration time, and Δa is either the width of the simulation region at 1 AU or the actual range in orbital distance. If collisions are included in the simulations, details are noted by: a=agglomeration; b=bouncing; c=cratering; f=fragmentation. The final column shows whether perturbations from one or more giant planets are included.

A detailed direct simulation of planet formation can address a variety of important questions, including: Was there runaway growth of a few embryos, or a continuously evolving homogeneous mass distribution? How does the primordial surface density alter the evolution? What fixes the spin orientation and period of the planets—uniform spin-up from planetesimal accretion (Lissauer and Safronov 1991), or a stochastic process

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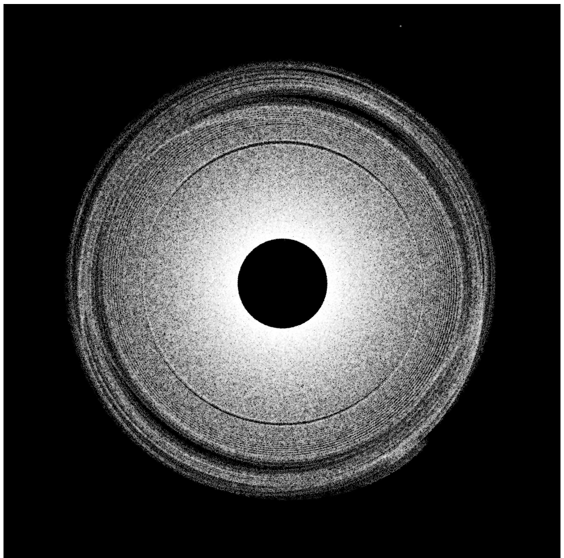

Figure 1: Mass density of a 10^6-particle simulation after 250 yr. Bright shades represent regions of high density. The dot at the top right is Jupiter. The disk extends from 0.8 AU, just inside Earth's present-day orbit, to 3.8 AU, near the outer edge of the asteroid belt. The gaps and spiral structures in the disk are associated with Jupiter mean-motion resonances.

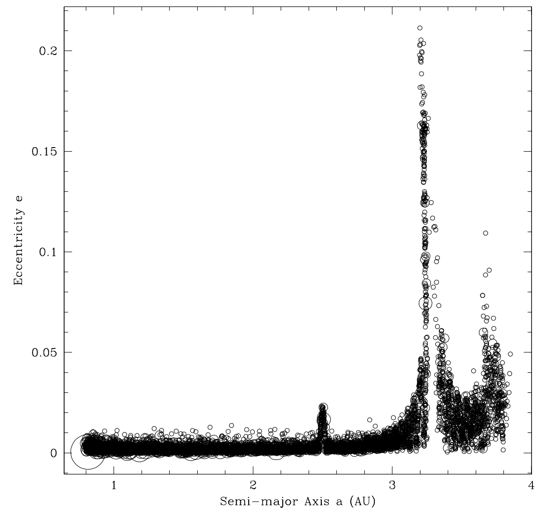

Figure 2: Plot of semi-major axis a *vs.* eccentricity e for the 10^6-particle simulation, showing only every 100^{th} particle to prevent overcrowding. The peaks in e correspond to mean-motion resonances with Jupiter; there are similar features in plots of a *vs.* inclination i (not shown). The circles are scaled by mass for emphasis.

dominated by the very last giant collisions (Dones and Tremaine 1993)? Is it feasible that the Earth suffered a giant impact late in its growth that led to the formation of the Moon (Benz *et al.* 1986)? How much radial mixing was there and can it explain observed compositional gradients in the asteroid belt (Gradie, Chapman and Tedesco 1989)? Finally, what is the dominant physical mechanism that drives the late stages of growth—are intrinsic gravitational instabilities between embryos sufficient, or are perturbations by the giant gas planets required? This last point is of key importance to future searches for terrestrial planets. We strongly suspect that the end result of our research may be the assertion that one should concentrate searches for terrestrial planets in those systems that have giant planets.

We have begun to address these issues with a modified version of a code developed for simulations of large scale structure. Collisions are detected (rather than "softened away") and the outcomes are determined by the impact energy, the lowest energies generally leading to mergers and the highest energies leading to fragmentation (presently merging and bouncing are implemented). Integrations are carried out in the heliocentric frame and may include the giant planets as perturbers. Auxilliary programs are used to generate appropriate initial conditions and to analyze the results of the simulation, but the main work is performed by the modified cosmology code.

Figures 1 and 2 show the mass density and a *vs.* e, respectively, at the end of a 250-yr run that began with 10^6 identical cold planetesimals in a disk from 0.8 to 3.8 AU with surface density proportional to $r^{-3/2}$. The present-day outer planets were included in the calculation. The simulation took 60 hours to finish on a Cray T3E with 128 dedicated processors using a fixed timestep of 0.01 yr. The effect of Jupiter on the disk, which extends well into the present-day asteroid belt, can be seen clearly in the density plot: there is a large density gap at the 2:1 resonance at 3.2 AU and a narrow groove at the 3:1 at 2.5 AU along with spiral wave patterns and other telltale features. There are corresponding features in Fig. 2 which show how Jupiter stirs up planetesimals at the mean-motion resonances. Note that conservation of the Jacobi integral accounts for the slight bending of the e peaks toward smaller a. Meanwhile, planetesimal growth has proceeded unmolested in the inner region of the disk (under the assumption of perfect accretion). The largest planetesimal at the end of the run is 8 times its starting size. As far as we are aware, this is the largest simulation of a self-gravitating planetesimal disk that has ever been attempted.

The figures show however that to get to the regime of runaway growth ($\sim 10^4$–10^5 yr), a new timestepping approach is needed. We are currently developing a technique to exploit the near-Keplerian motion of the planetesimals. For weakly interacting particles, we divide the Hamilonian into a Kepler component, implemented using Gauss' f and g functions, and a perturbation component owing to the force contributions of all the other particles. In this regime, timesteps can be of order the dynamical (*i.e.* orbital) time, resulting in compuational speedups of 10–100. For strongly interacting particles (defined as particles with overlapping Hill spheres), the Hamiltonian is factored into the standard kinetic and potential energy components, with the central force of the Sun as an external potential. In this regime, particles are advanced in small steps, which allows for the careful determination of collision circumstances. It also allows the detection of collisions in the correct sequence even if a single particle suffers more than one collision during the interval.

The challenge is to predict when particles will change between the regimes of weak and strong interaction. One method we are considering is to construct a new binary tree ordered by perihelion and aphelion. Those particles with orbits separated by less than a Hill sphere are flagged for further testing. This screening has a cost of $N \log N$ and is only performed once per long Kepler step. Flagged pairs of particles with phases that are certain to stay separated over the integration step are reset. The remaining particles are tested by solving Kepler's equation in an elliptical cylindical coordinate system to determine the time of actual Hill sphere overlap. Switching between Hamiltonians is not strictly symplectic, but it occurs infrequently enough for any given particle that it is not a concern. Dissipating collisions are inherently non-symplectic anyway. Once particles separate beyond their Hill spheres (or merge), they are returned to the Kepler drift scheme.

Although much work remains to be done, the reward will be the first self-consistent direct simulation of planetesimals evolving into planets in a realistic disk. The results can be used to study related problems, such as the formation of planetary satellites, orbital migration of giant planets in a sea of planetesimals, and ultimately the ubiquity and diversity of extra-solar planetary systems.

2 Summary: Virtual Petaflops

Past planetesimal simulations used codes with an algorithmic complexity that would be similar to the point labeled "full tune" in Figure 3 and computers with speeds of $\sim$ 10 Mflops. (Special purpose "GRAPE" hardware of $\sim 10^6$ Mflops has been used, but such implementations involve sums over all interactions so they are closer to the direct sum case in floating point cost. Our algorithms result in a collective speed-up $\gtrsim 10^8$ for simulations with $N \sim 10^7$ (with rough accounting for the

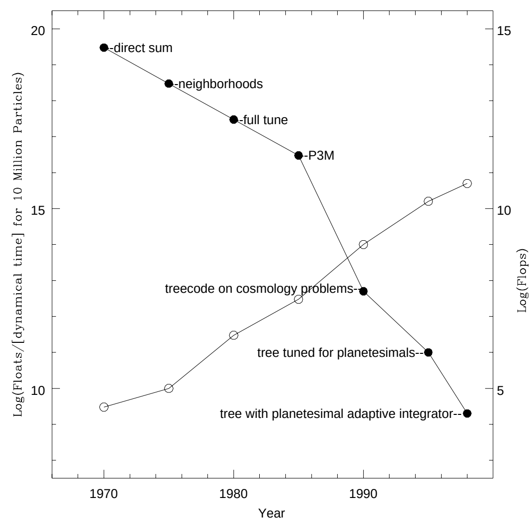

Figure 3: Gains in hardware and algorithms are compared for the N-body simulations. Algorithms are shown as filled points with the scale to the left, while hardware is open points with the sclale on the right. The final algorithmic point should be considered a hopeful projection for 1999.

reduction in N over the course of the simulation). It must be emphasized that to attain the desired performance, both hardware *and* algorithm improvements are required. Figure 3 shows that the speed-up factor from algorithms vastly exceeds that of the hardware. It is not sufficient to simply wait for computers to get better, nor does it seem to pay to build special hardware. McMillan *et al.* (1997) asserted that if Grape-6 is built, it could use its Petaflop speed to follow 10^6 planetesimals by the year 2000. They claimed that this would be a seven-year advantage over General Purpose Computers that would only be able to follow 10^4 particles by the year 2000. Our test simulations *without the new integrator* are already 10 years ahead of their projections. We argue that our approach will beat efforts that rely on special purpose hardware with encoded algorithms for at least the next decade.

Sir Isaac would love to see the enhancement of *"the entire human intellect"* by high performance computing.

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