

# **ASTR 340: Origin of the Universe**

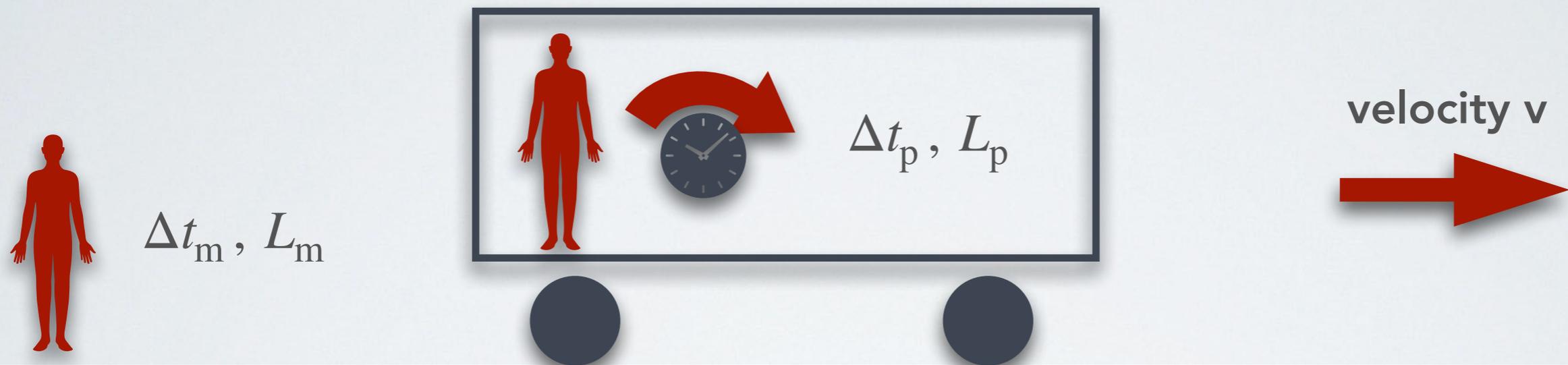
Prof. Benedikt Diemer

## **Lecture 8 • Special Relativity II**

09/23/2021

## Part 0: Recap

# Special Relativity Summary (so far)



# Participation: Recap #1 & 2



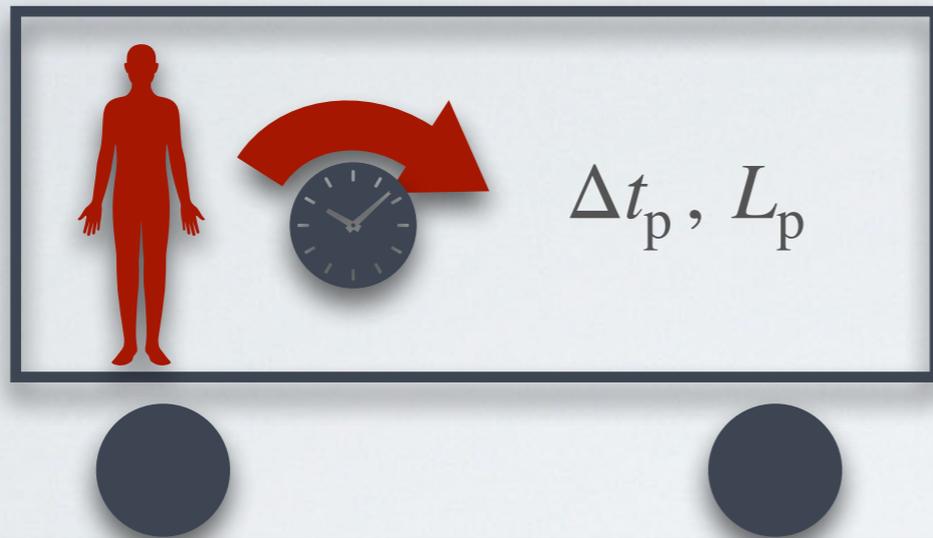
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# Special Relativity Summary (so far)



Time dilation:

$$\Delta t_m = \gamma \Delta t_p$$

Length contraction:

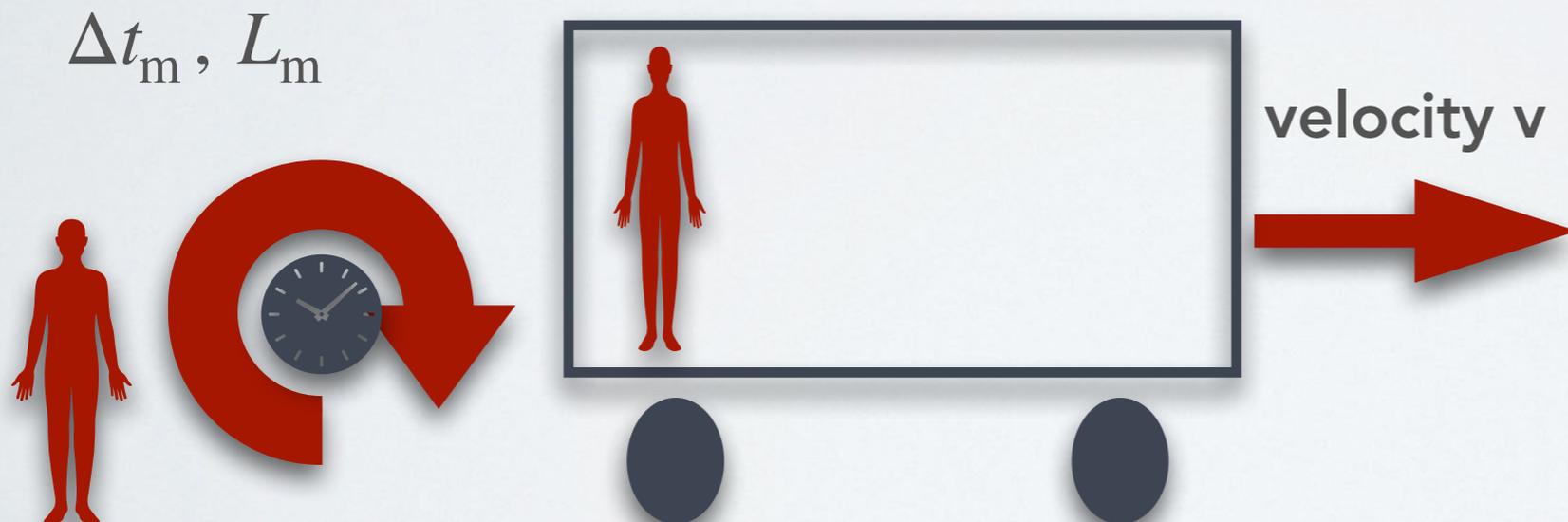
$$L_m = \frac{1}{\gamma} L_p$$

Lorentz factor:

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

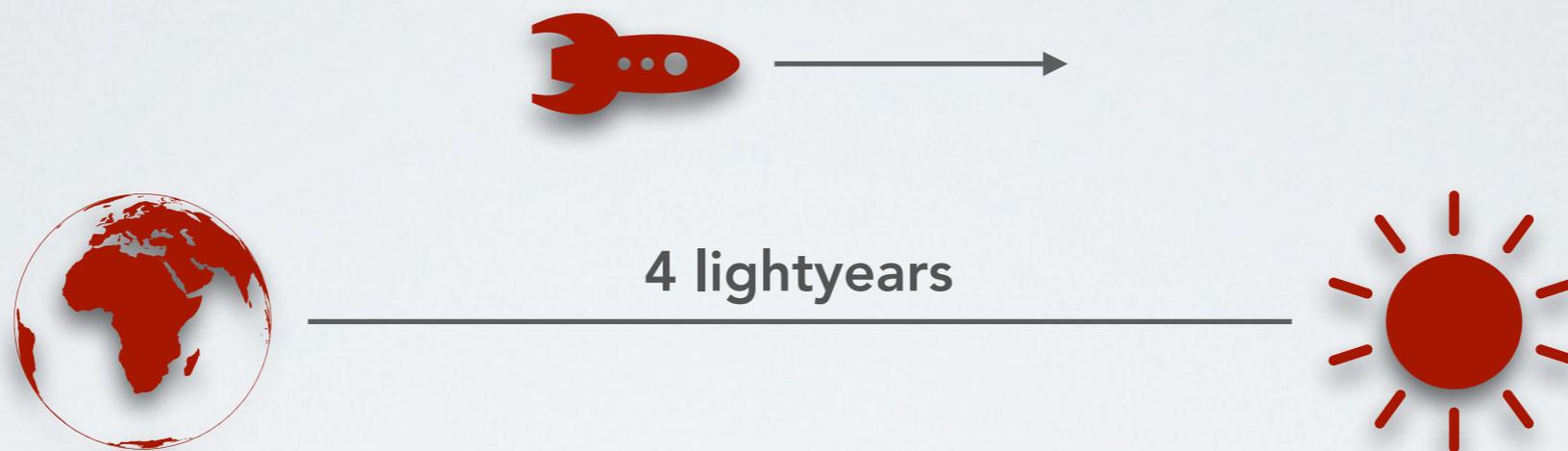
Velocity addition:

$$v_{\text{tot}} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$



# Proper time and length

We see a spaceship fly past earth at a speed of  $0.97c$  (or  $\gamma = 4$ ). They are going to Alpha Centauri, 4 lightyears away. **How long does the trip take according to the astronauts?**

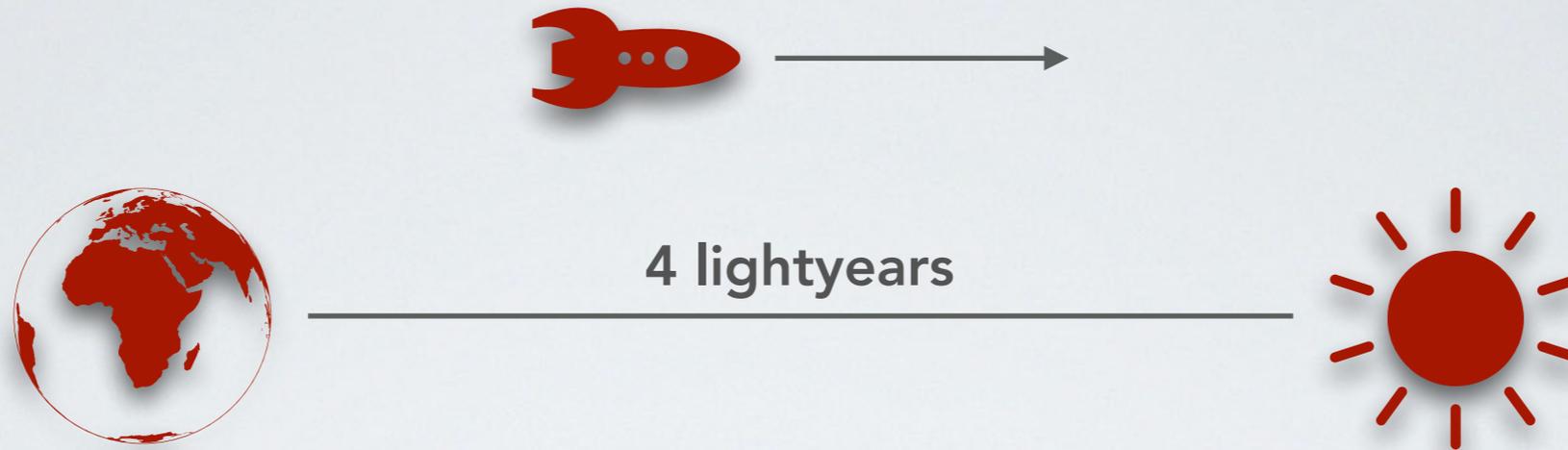


- Keyword: "according to the astronauts" -> asking about **proper time in frame of the astronauts**
- In Earth frame ("moving frame"), trip takes  $\sim 4 \text{ lightyears} / c = 4 \text{ years}$

$$\Delta t_m = \gamma \Delta t_p$$

$$\implies \Delta t_p = \frac{1}{\gamma} \Delta t_m = \frac{1}{4} 4 \text{ yr} = 1 \text{ yr}$$

# Proper time and length



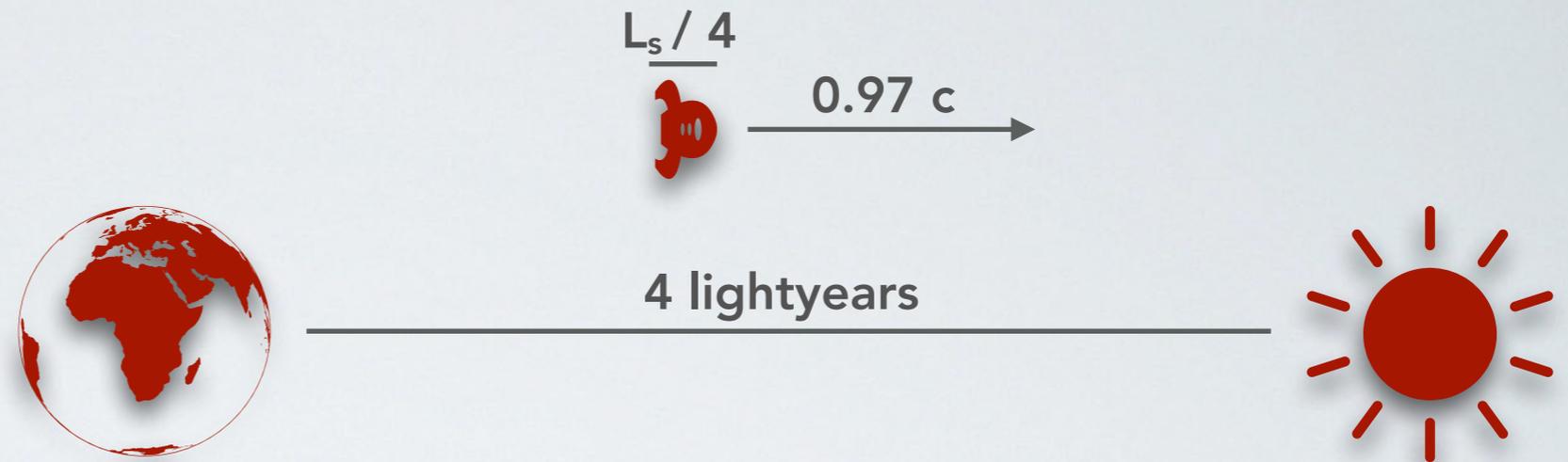
$$\Delta t_m = \gamma \Delta t_p$$

$$\implies \Delta t_p = \frac{1}{\gamma} \Delta t_m = \frac{1}{4} 4 \text{yr} = 1 \text{yr}$$

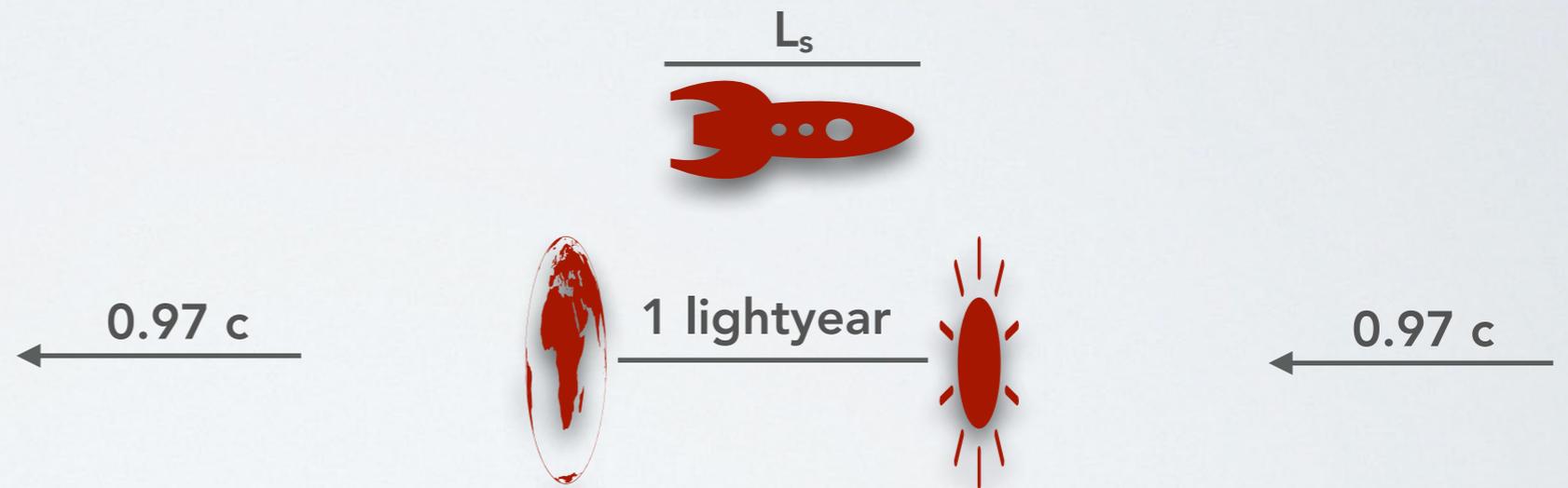
- This is what we mean by “we see the astronauts aging more slowly” (i.e., they have aged one year while “we” aged four years)
- People also say “moving clocks tick more slowly than in rest frame” (i.e., the time interval between ticks is longer)
- **Solution: always identify the proper frame for the given question!**

# Proper time and length

Earth frame



Spaceship frame



- Test: **physics** must come out the **same in all frames**
- Astronauts: travel at  $\sim c$ , take 1yr to travel distance of 1 lightyear
- Earth: travel at  $\sim c$ , take 4yr to travel distance of 4 lightyears
- $\rightarrow$  We agree on what happens: they arrive!

# Participation: Recap #3



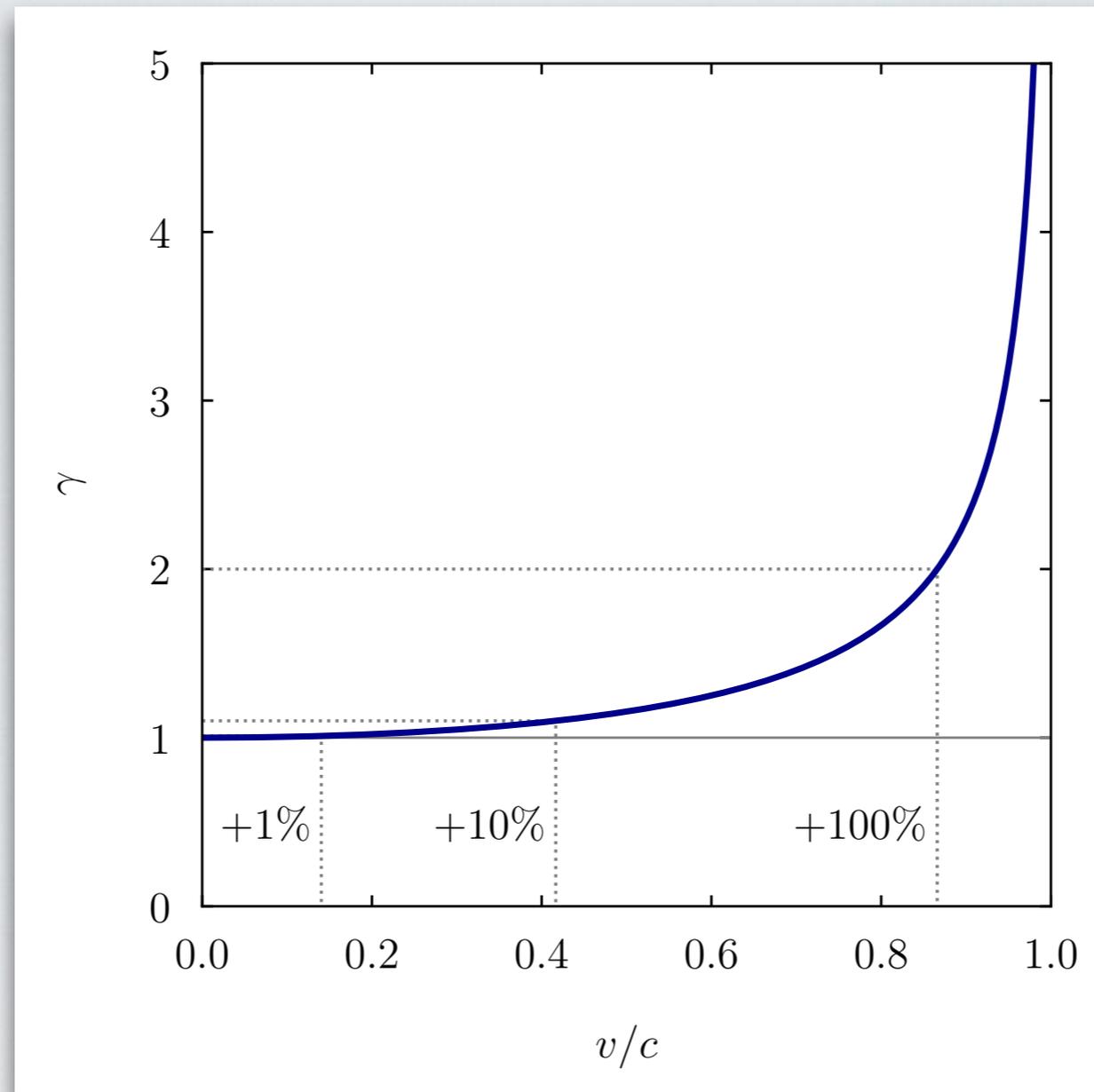
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# Special Relativity Summary (so far)



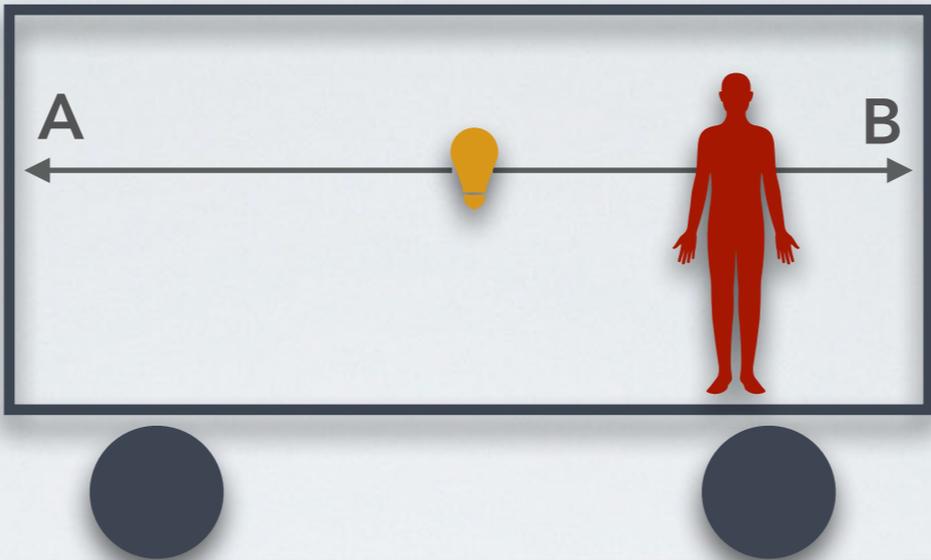
$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Today

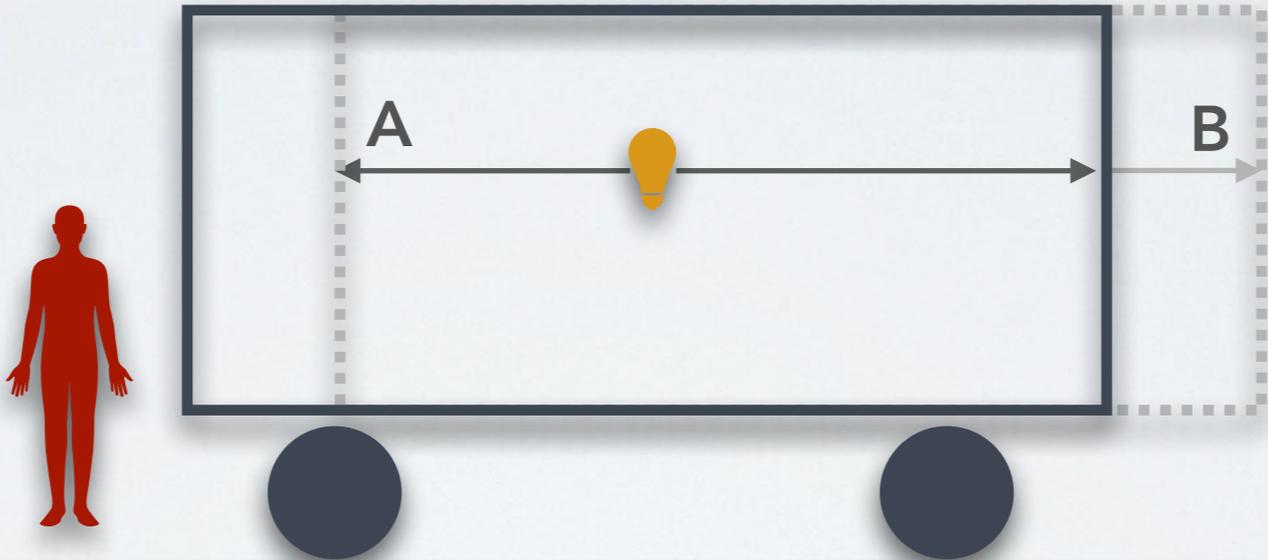
- Spacetime diagrams & causality
- The twin paradox
- Mass & Energy

# Part 1: Spacetime diagrams & Causality

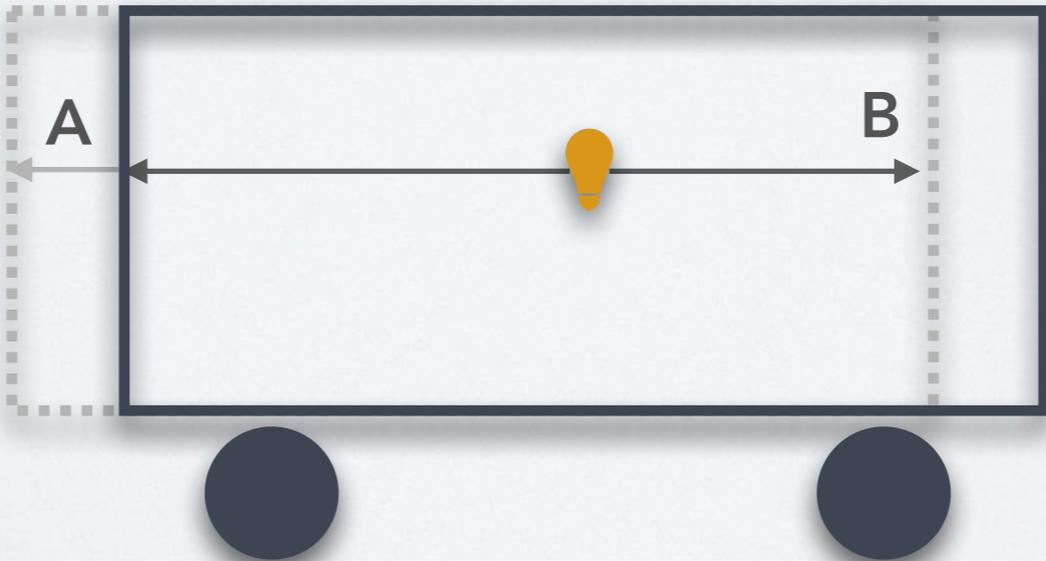
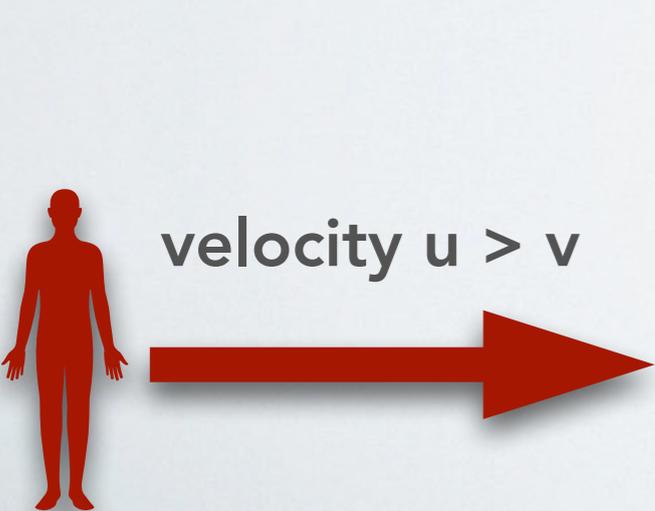
# Simultaneity



velocity  $v$



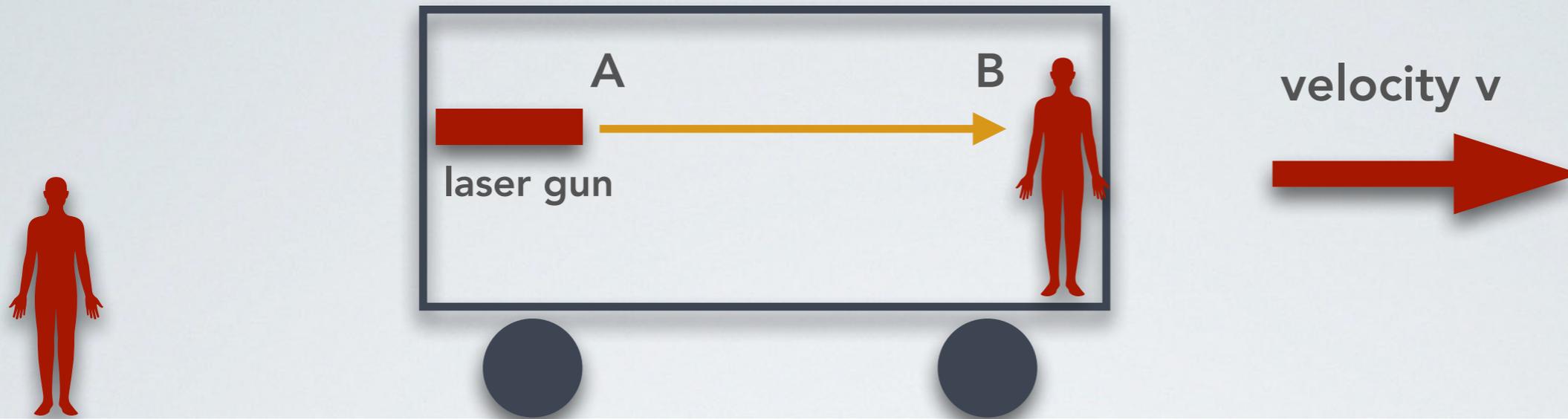
velocity  $v$



velocity  $v$



# Causality



- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end
- Laser gun fires (event A) and then victim gets hit (event B)
- Can we **change the order of these events** by changing the frame of reference? Can the victim get hit before the gun fires?

# Participation: Order of events



**Respond to the poll on TurningPoint**

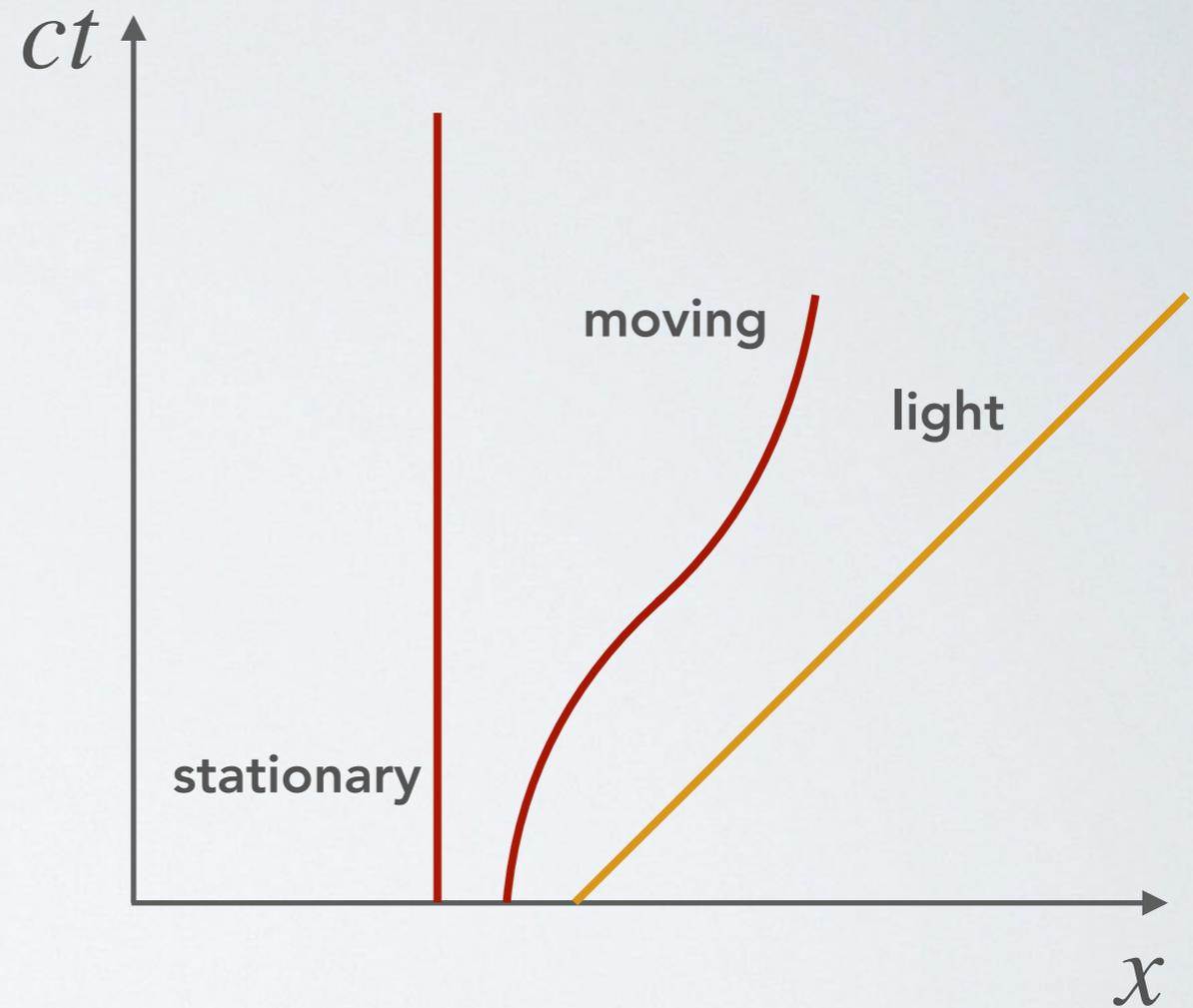
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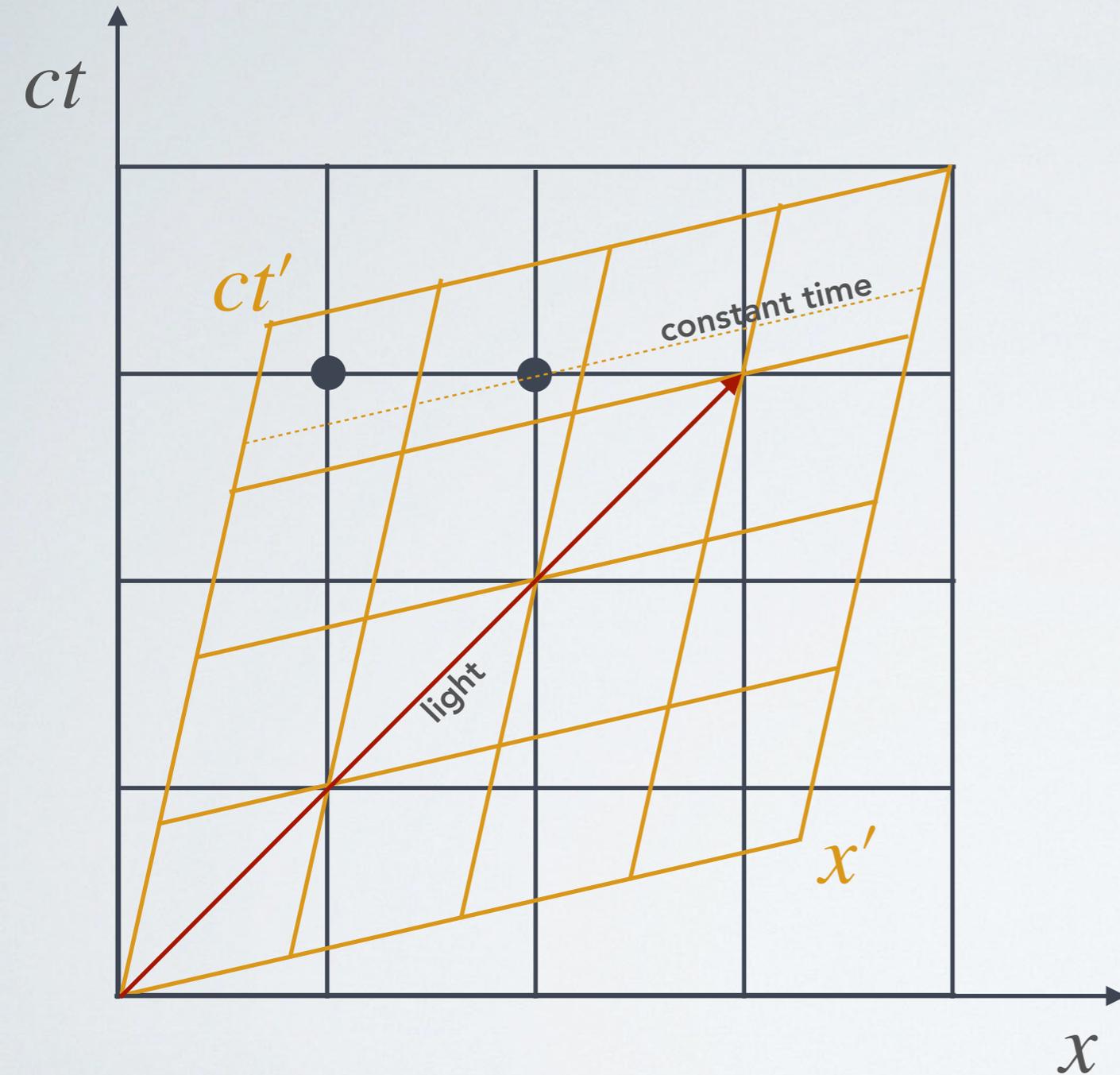
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# Space-time diagrams

- Because space and time are “mixed up” in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates
- Plot as a **2D graph of  $x$  and  $ct$**
- Can be generalized to events taking place in 3D space using a 4D graph, but difficult to visualize
- Lines in this diagram are called **world lines**

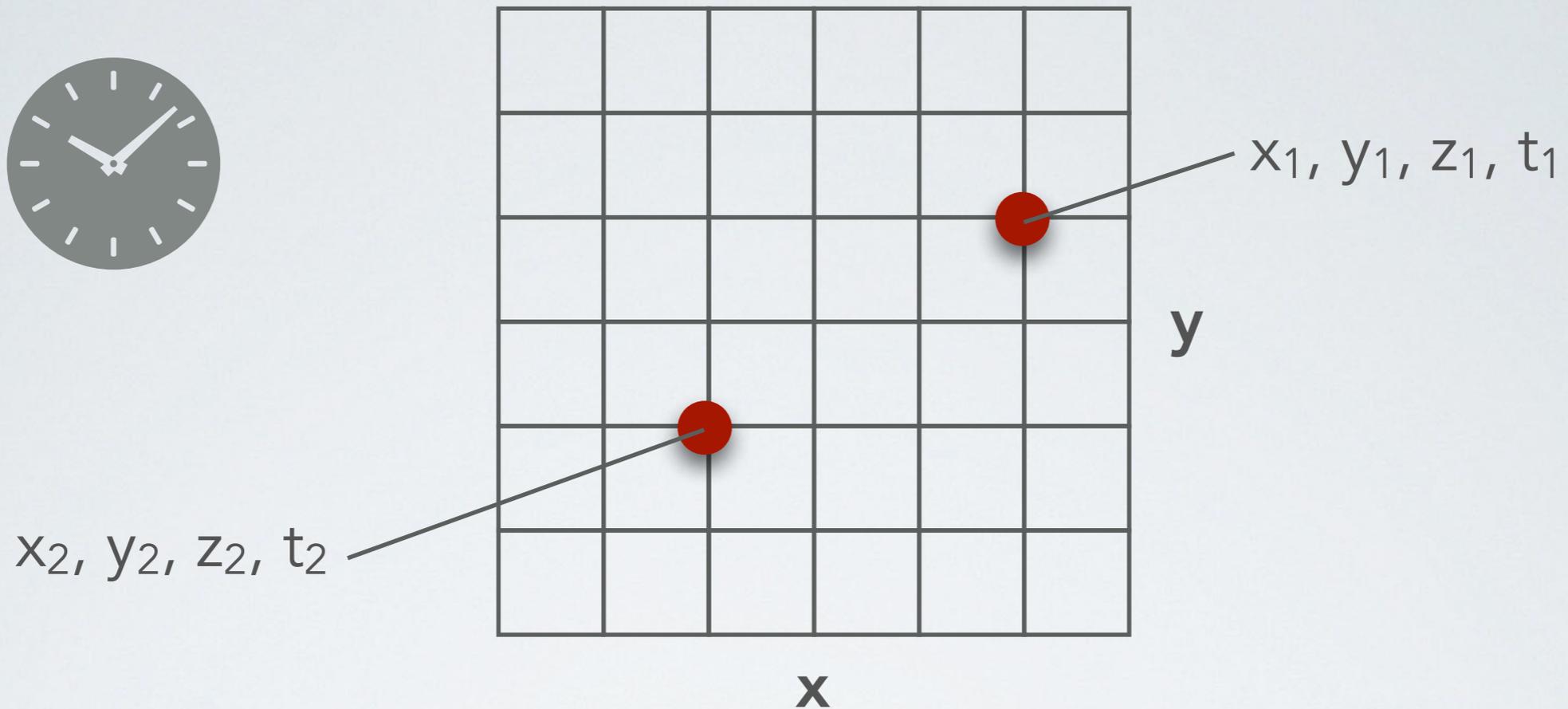


# Lorentz transformation in space-time diagrams



- Changing from one reference frame to another...
  - Affects **time** coordinate (time dilation)
  - Affects **space** coordinate (length contraction)
  - Leads to a **distortion** of the space-time diagram
- Events that are **simultaneous** in one frame are not simultaneous in another frame
- Everyone agrees on **light**

# Invariant interval



$$\Delta x = x_2 - x_1$$
$$\Delta t = t_2 - t_1$$
$$\Delta S_{\text{space}} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

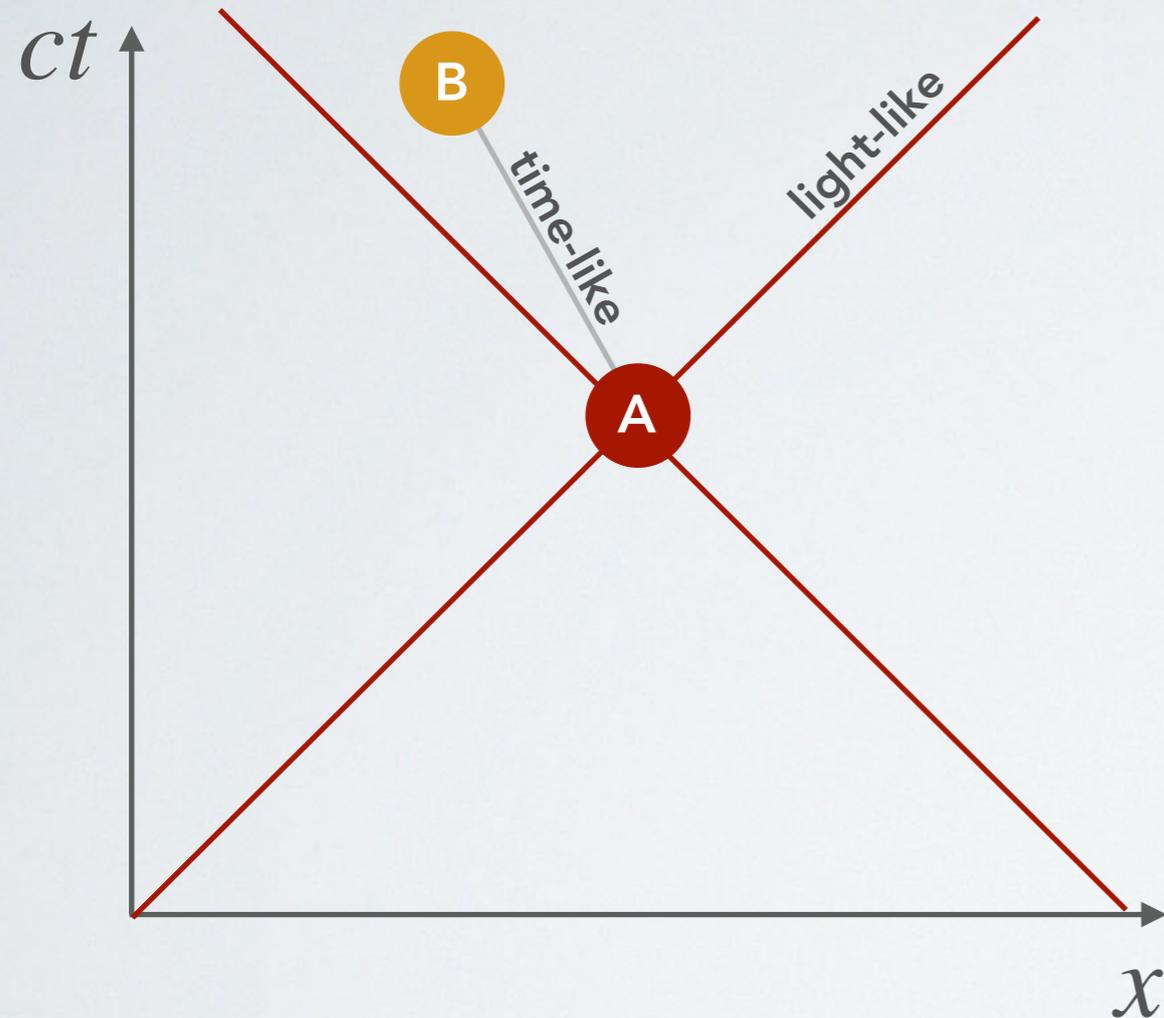
$$\Delta S_{\text{space-time}} = \sqrt{(c\Delta t)^2 - \Delta x^2}$$

# Invariant interval

- Two events A and B in space-time are separated by an **invariant interval**
  - Analogous to Pythagorean equation, but **modified** to account for the **difference between space (x) and time (ct)**
  - **Independent of the frame** in which it is measured; all observers agree on it!
- Invariant interval is equivalent to **c times proper time interval**
  - **Shorter** when traveling **faster!**
  - Space-time interval is **zero** for any two points on **light** world line

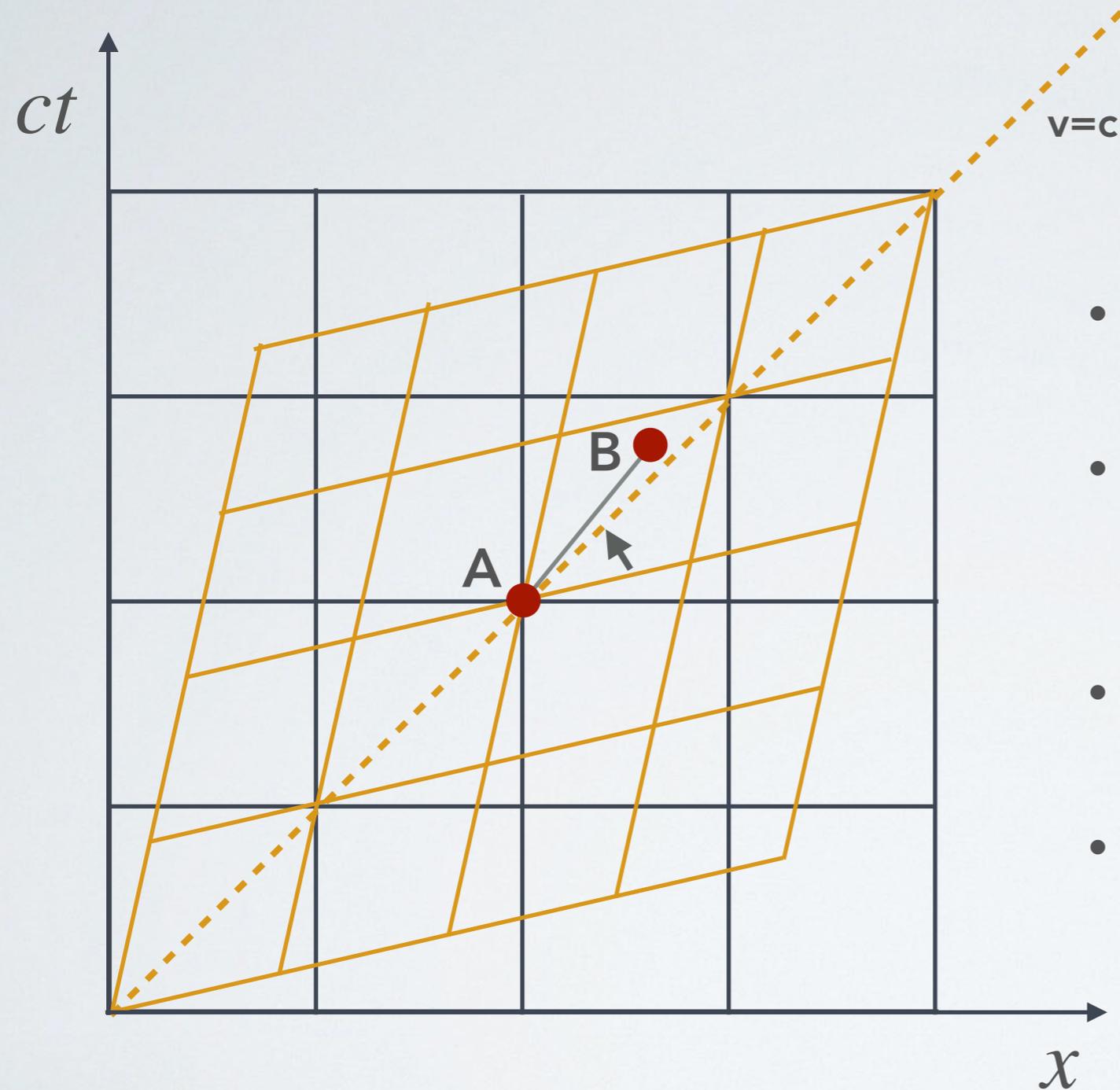
$$\Delta s_{\text{space-time}} = \sqrt{(c\Delta t)^2 - \Delta x^2} = c\Delta t_p$$

# Causality in space-time diagrams



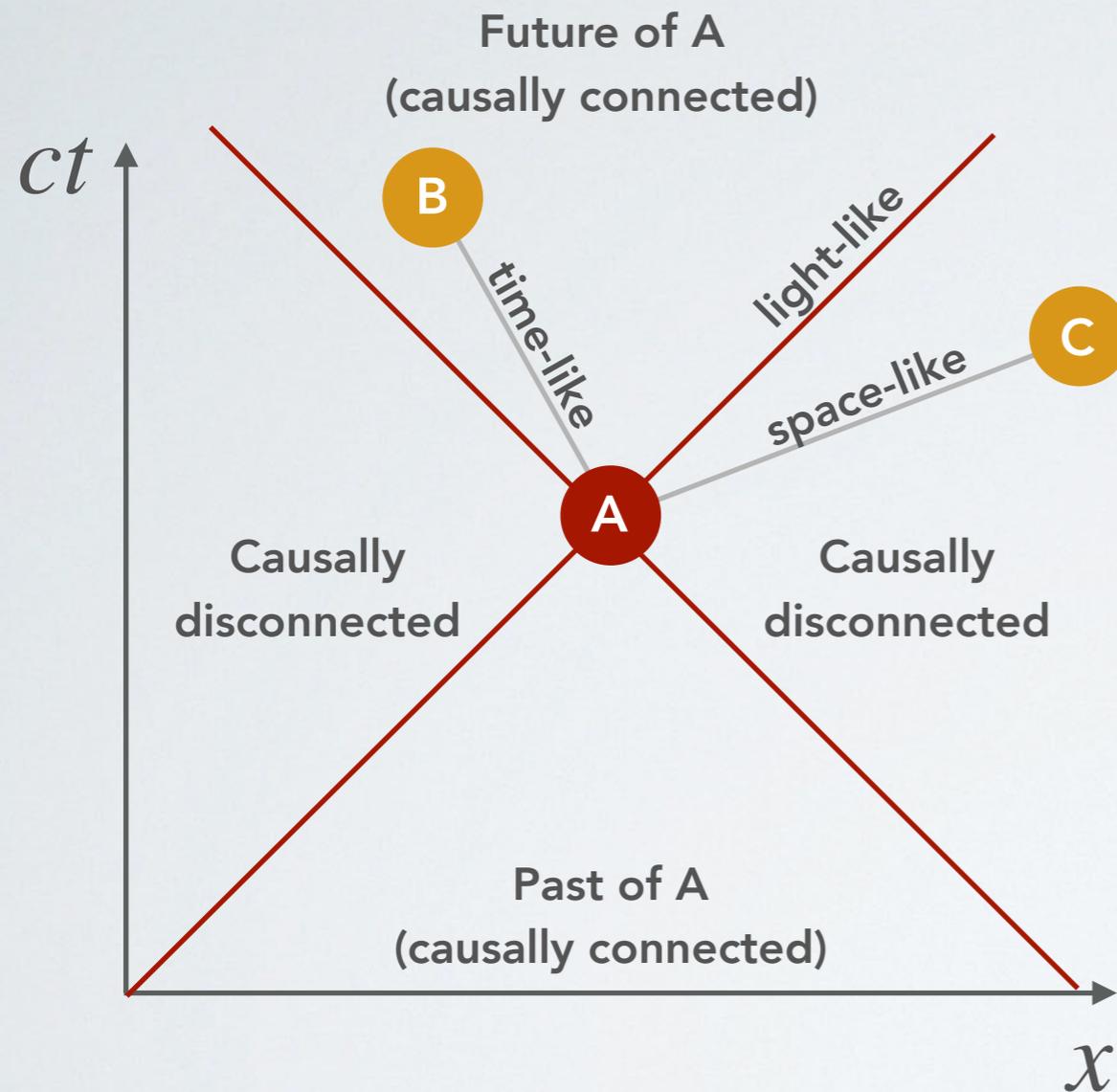
- Types of trajectories:
  - Time-like:  $\Delta s^2 > 0$
  - Light-like:  $\Delta s^2 = 0$
- Events A and B...
  - **Cannot change order of A and B** by changing frames of reference

# Causality in space-time diagrams



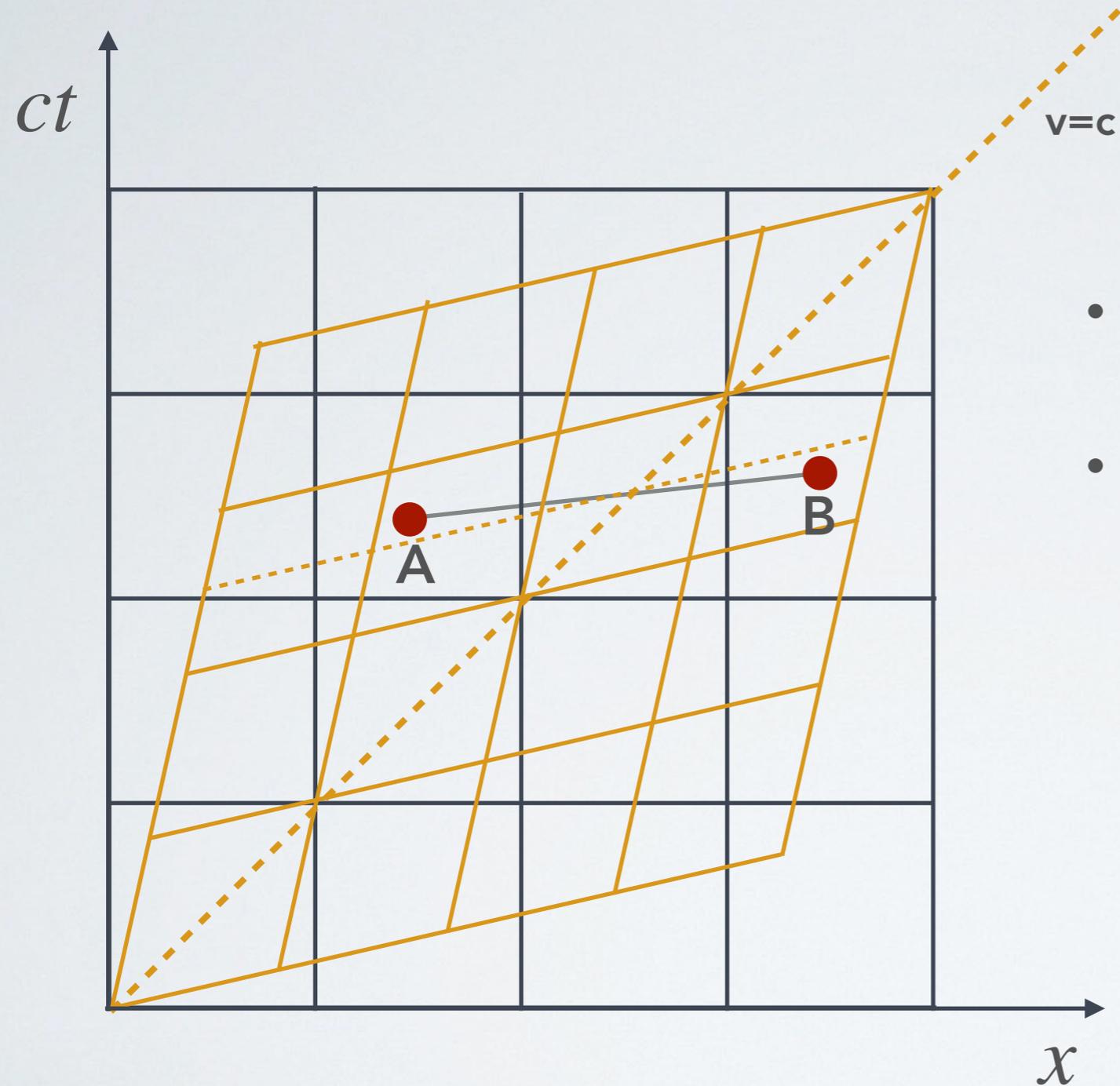
- Why can order of two time-like events not be changed?
- We cannot squeeze the yellow grid more than making the time-lines almost 45 degrees
- Time-like means angle steeper than 45 degrees
- The order is still preserved

# Causality in space-time diagrams



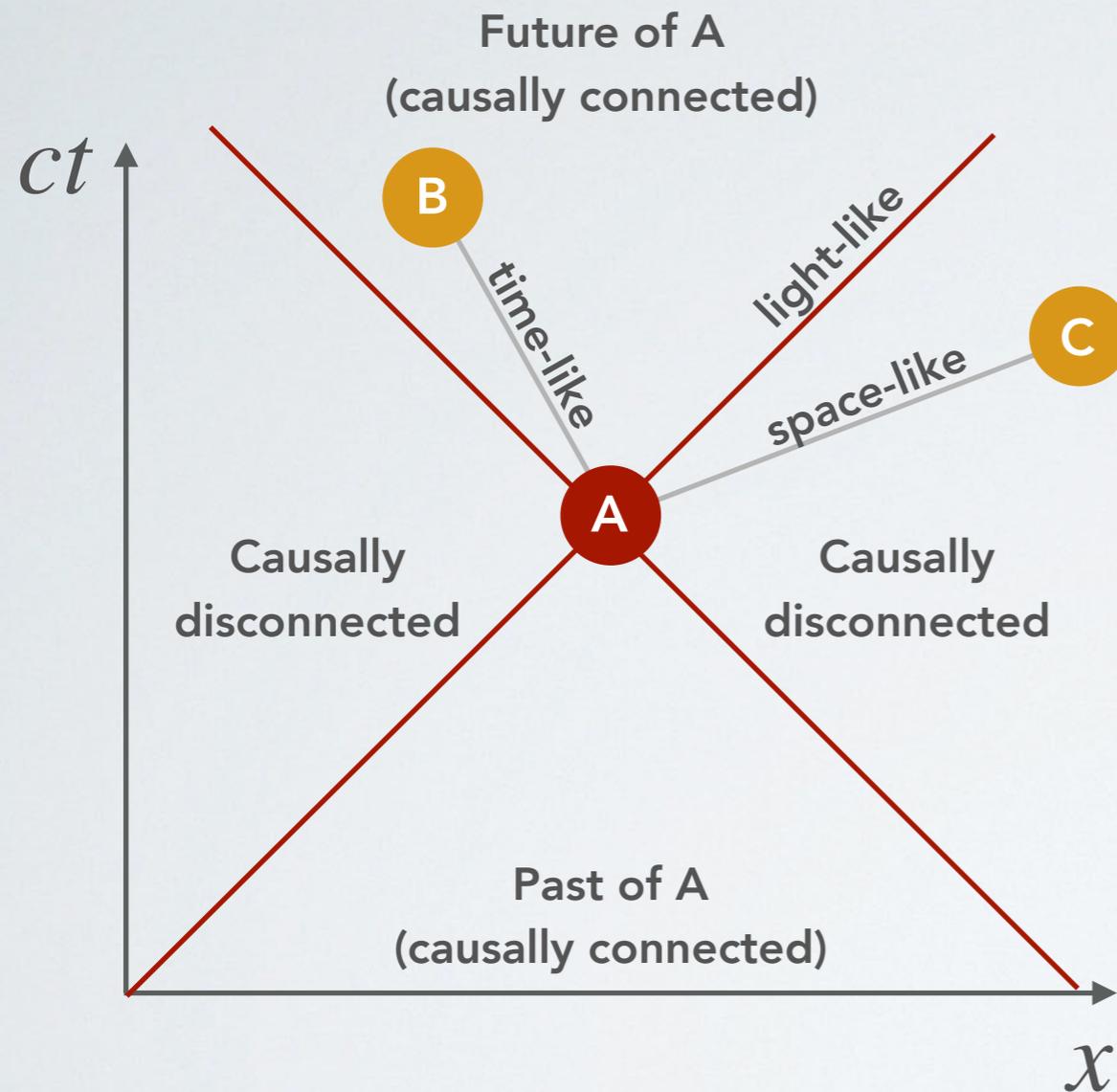
- Types of trajectories:
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  - Space-like:  $\Delta s^2 < 0$
- Events A and B...
  - **Cannot change order of A and B** by changing frames of reference
  - A can send **information** to B at, or less than, the speed of light
  - Thus, A and B are **causally connected**

# Causality in space-time diagrams



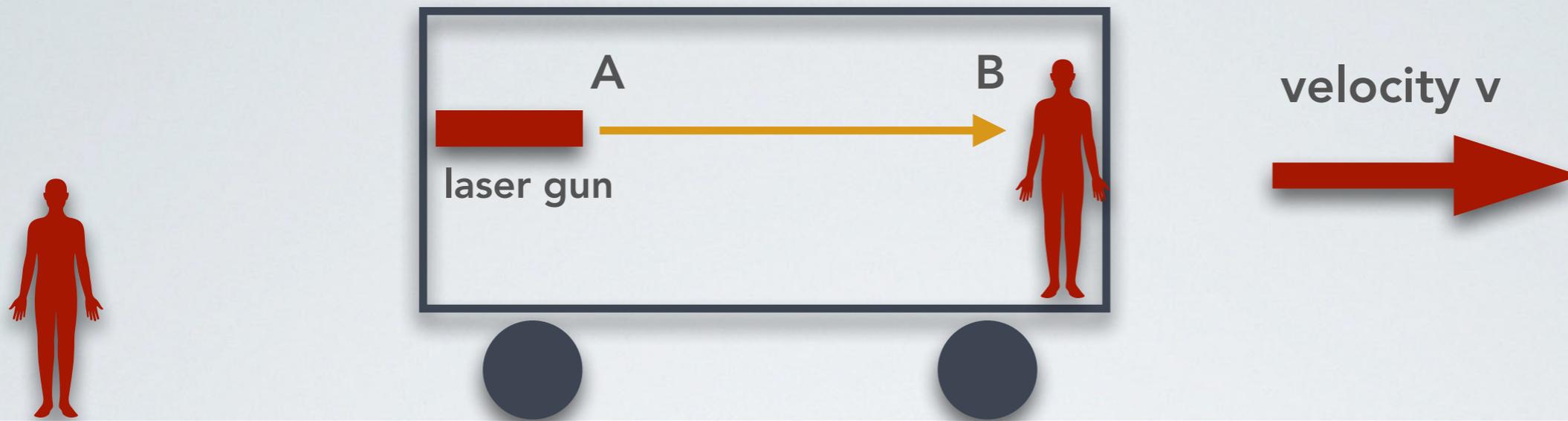
- Why can order of two space-like events be changed?
- If angle is less than 45 degrees, constant-time lines in squeezed grid can cross the line between the two events, changing their order in time

# Causality in space-time diagrams



- Types of trajectories:
  - Time-like:  $\Delta s^2 > 0$
  - Light-like:  $\Delta s^2 = 0$
  - Space-like:  $\Delta s^2 < 0$
- Events A and B...
  - **Cannot change order of A and B** by changing frames of reference
  - A can send **information** to B at, or less than, the speed of light
  - Thus, A and B are **causally connected**
- Events A and C...
  - **Can change the order of A and C** by changing frame of reference
  - **No communication** between A and C at speed of light or slower
- If idea of cause and effect is to have any meaning, we must conclude that **no communication can occur at a speed faster than the speed of light**

# Causality

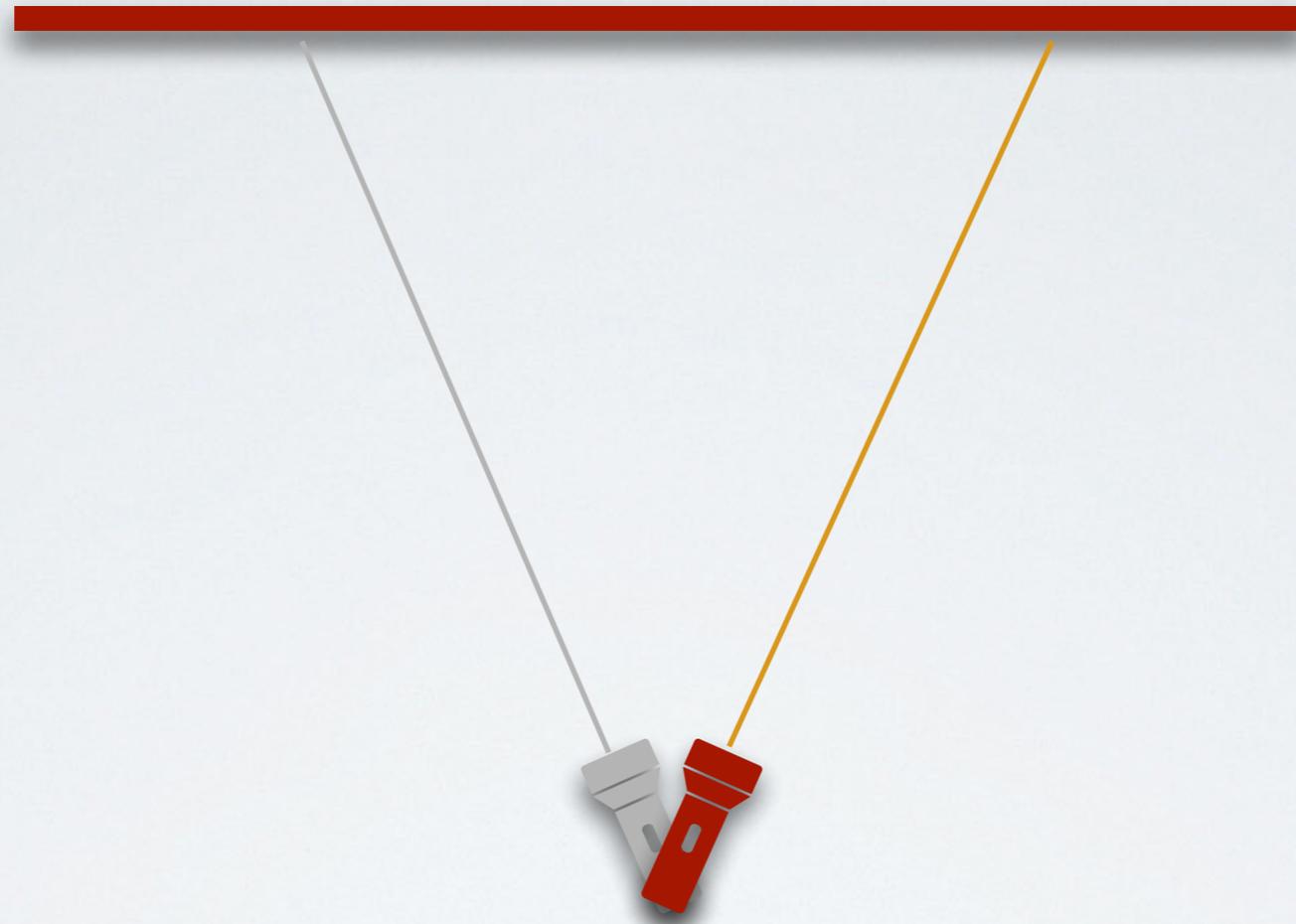


- This is a question of **causality**
  - The events described are **causally connected** (i.e. one event affects the other event).
  - **Not possible to change the order of causally connected events** by changing frames, according to Special Relativity
- This is true provided that the laser blast does **not travel faster than the speed of light**

# Causality

- To preserve causality, the **speed of light must set the upper limit** to the speed of anything in the Universe
- Can causality be proved? No, it is an **axiom** of physics
- What if causality doesn't hold?
  - Then the Universe returns to being **random**, unconnected events that can't be understood or predicted
  - This would be a true "end of science"

# Moving faster than the speed of light?



- Spot on screen can move faster than light
- But it does not transmit information

## Part 2: The twin paradox

# Twin Paradox

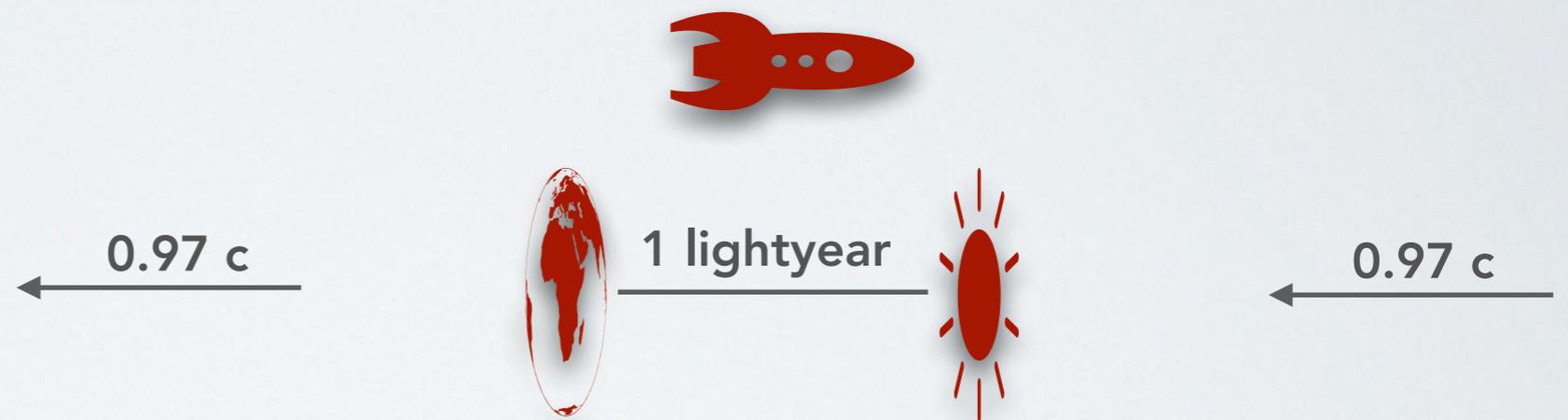
- Andy (A) and Betty (B) are twins
- **Andy stays on Earth, while Betty travels** (at a large fraction of the speed of light) to Alpha Centauri and returns
- When Betty gets home, she finds **Andy is greatly aged** compared with herself

# Twin Paradox

Andy's frame



Betty's frame



- But what about **reciprocity**?
- Doesn't **Betty observe Andy's clock as dilated** from her point of view? Wouldn't that mean she would find him younger when she returns?
- **Who's really older?**

# Participation: Twin paradox



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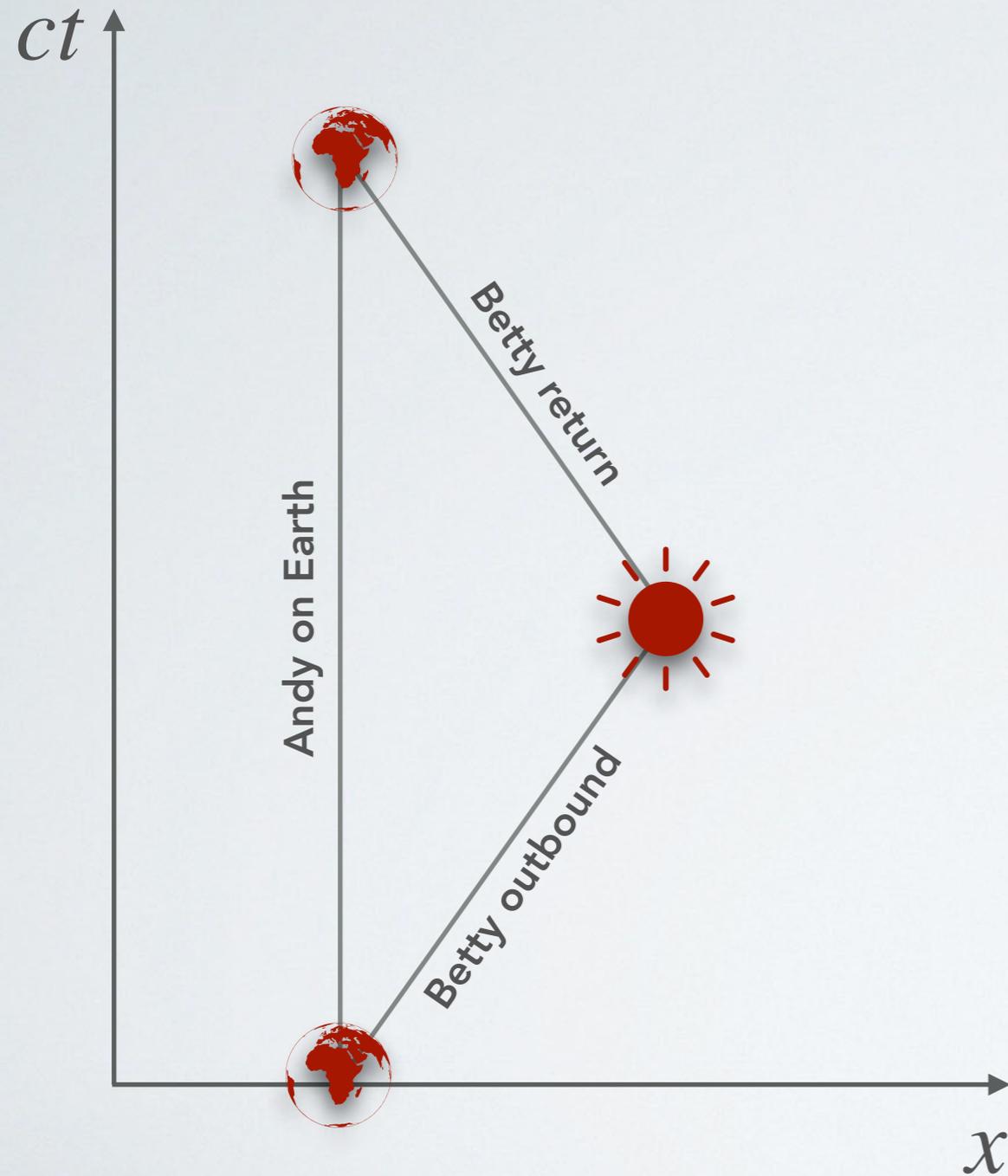
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# Twin paradox and space-time diagrams

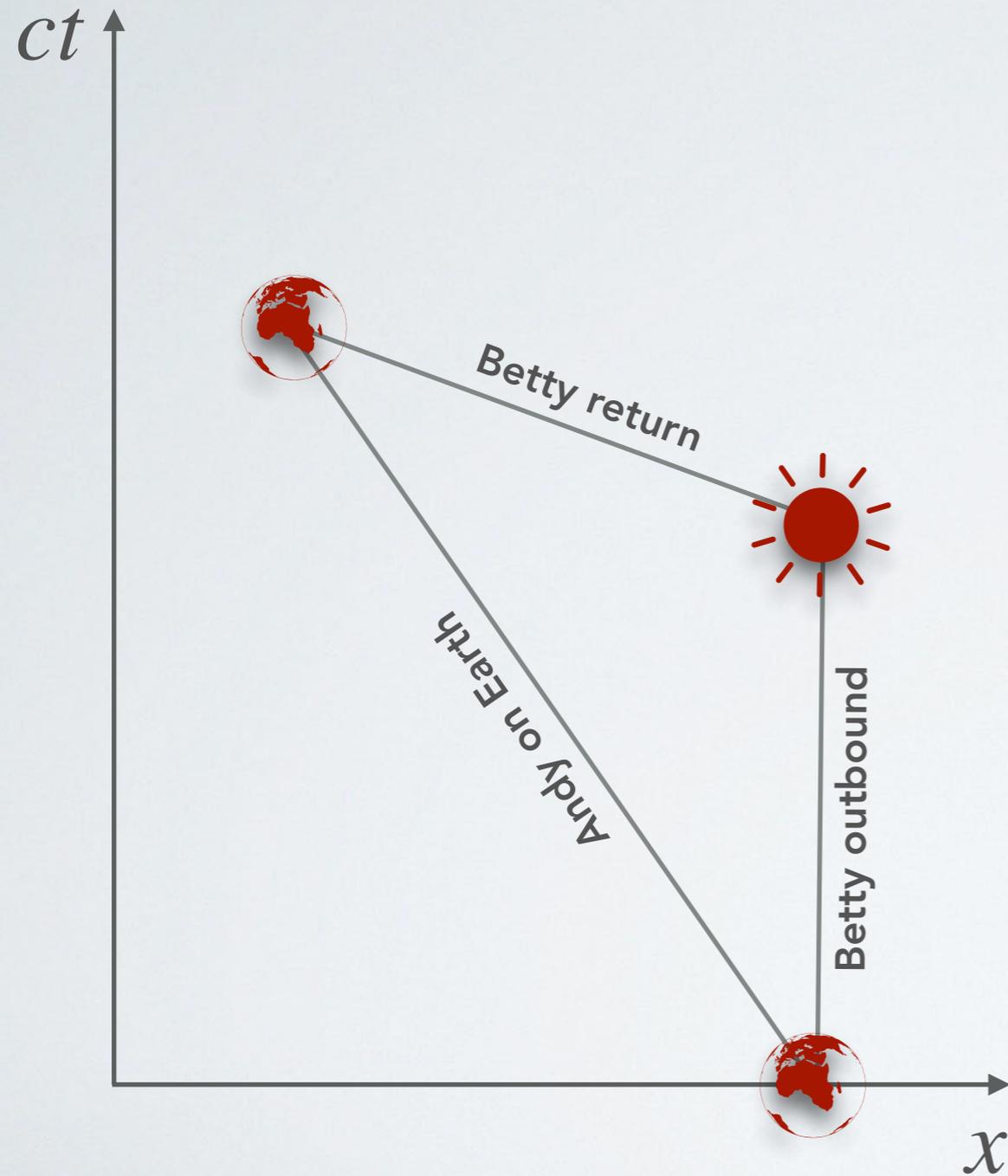
## Andy's frame



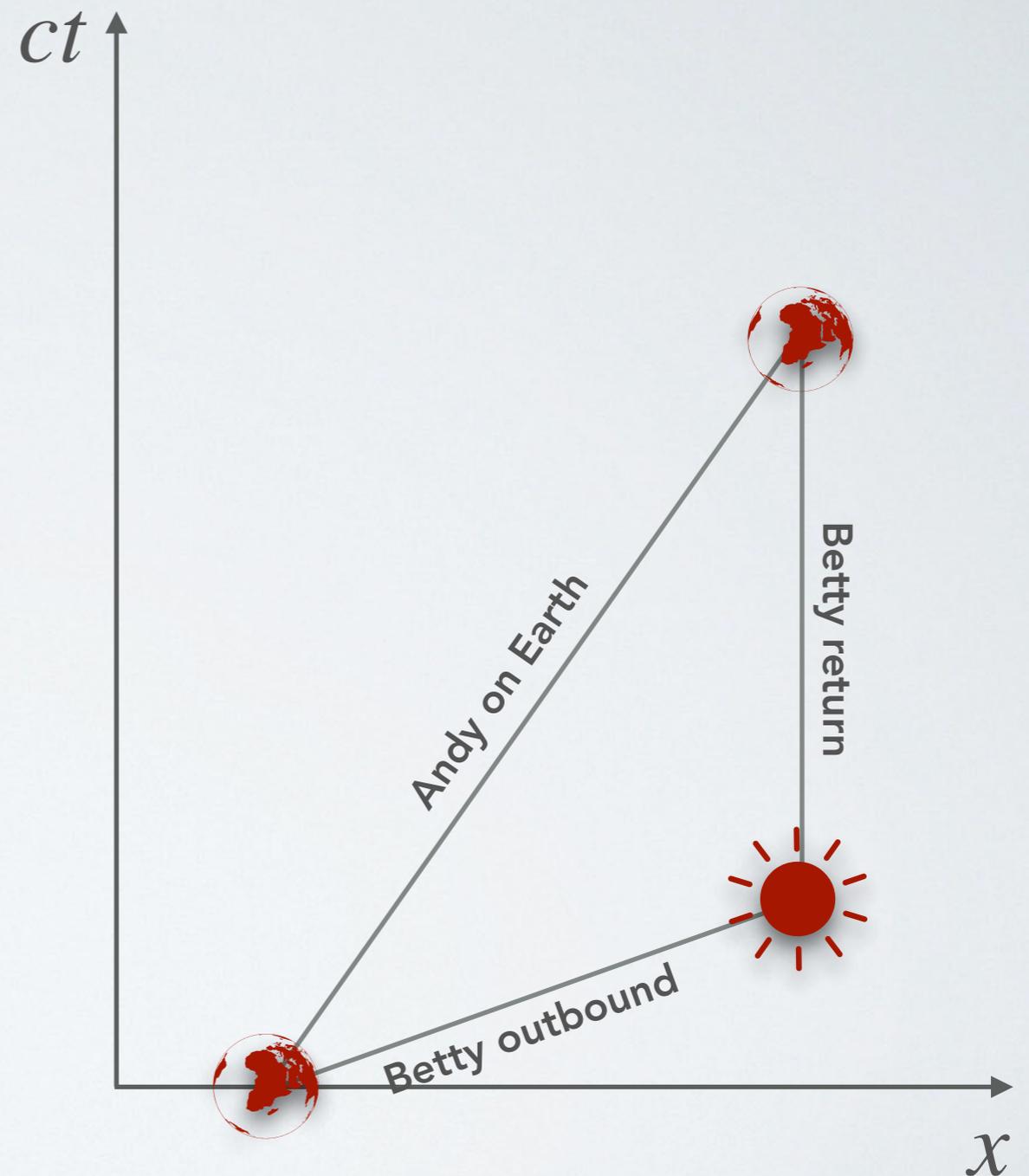
- Andy's world line, in his own frame, is a straight line
- Betty's world line has two segments for outbound and return (angle  $<45^\circ$ )
- But what if we look at it from Betty's point of view?

# Twin paradox and space-time diagrams

Betty's frame (outbound)



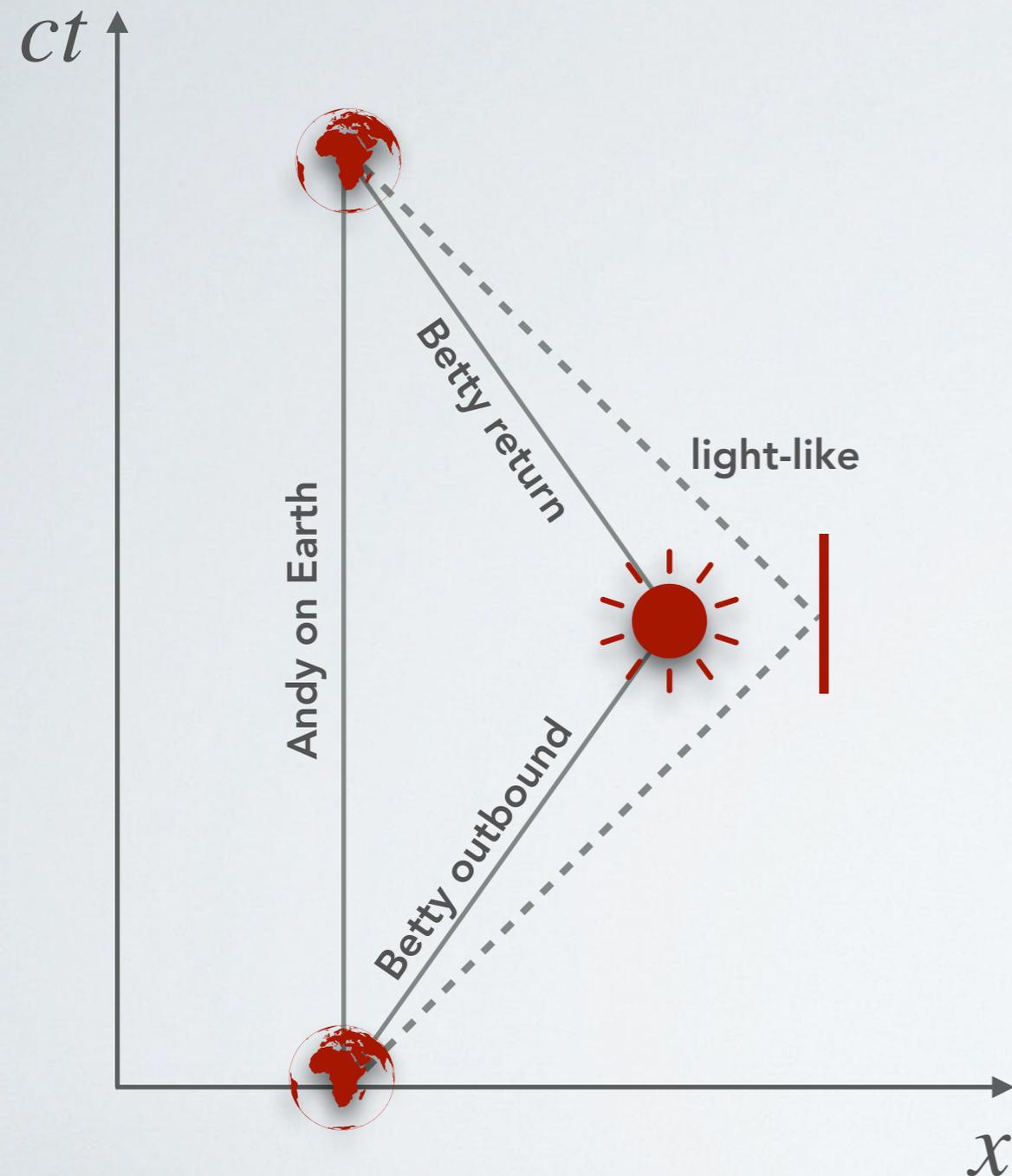
Betty's frame (return)



- Andy is moving now, but there isn't one inertial frame for Betty's journey

# Twin paradox and space-time diagrams

## Andy's frame



- From any perspective, Andy's world line has a single segment whereas Betty's has two
  - No single inertial frame for Betty's trip, so **no reciprocity**
  - Betty's trip involves **accelerations**
- Proper time ( $=\Delta s/c$ ) between two events is computed by **adding segments**
- Even if two world lines start and end at the same place, they may result in **different proper time intervals**
- The world line with the **longest proper time (or  $\Delta s$ )** is always the **straight world line** that connects two points
- Light-like world lines have the shortest proper time: zero!
- Betty's world line is less straight, so her proper time is shorter: **Betty is younger!**

## Part 3: Mass & Energy

# Mass and energy in relativity

- Einstein reworked Newton's laws of mechanics using his new **relativistic formulae**
- Energy of a moving object with **mass  $m_0$  and speed  $v$** :

$$E = mc^2 = \gamma m_0 c^2$$

total energy

$$\approx m_0 c^2 + \frac{1}{2} m_0 v^2 + \dots$$

rest energy      Newtonian kinetic energy

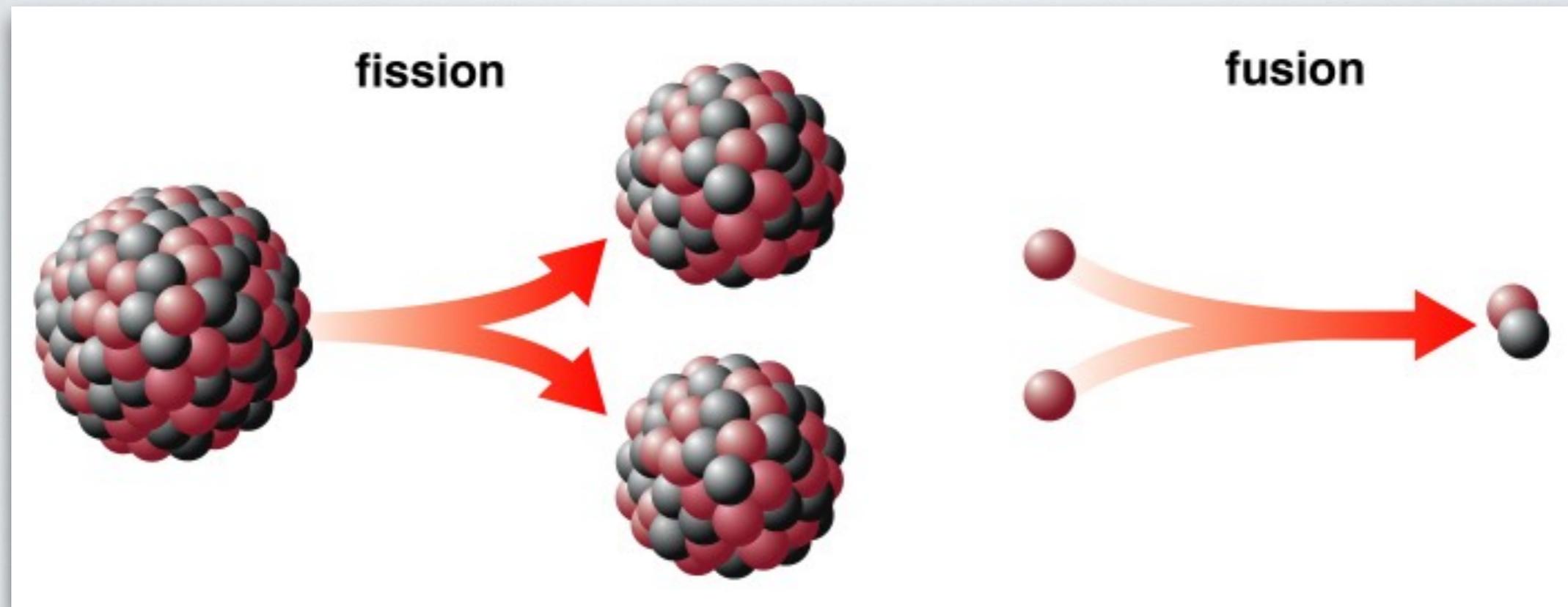
- Energy increases as the speed increases
- Energy **would become infinite** if  $v$  approaches  $c$

# How wrong was Newton?

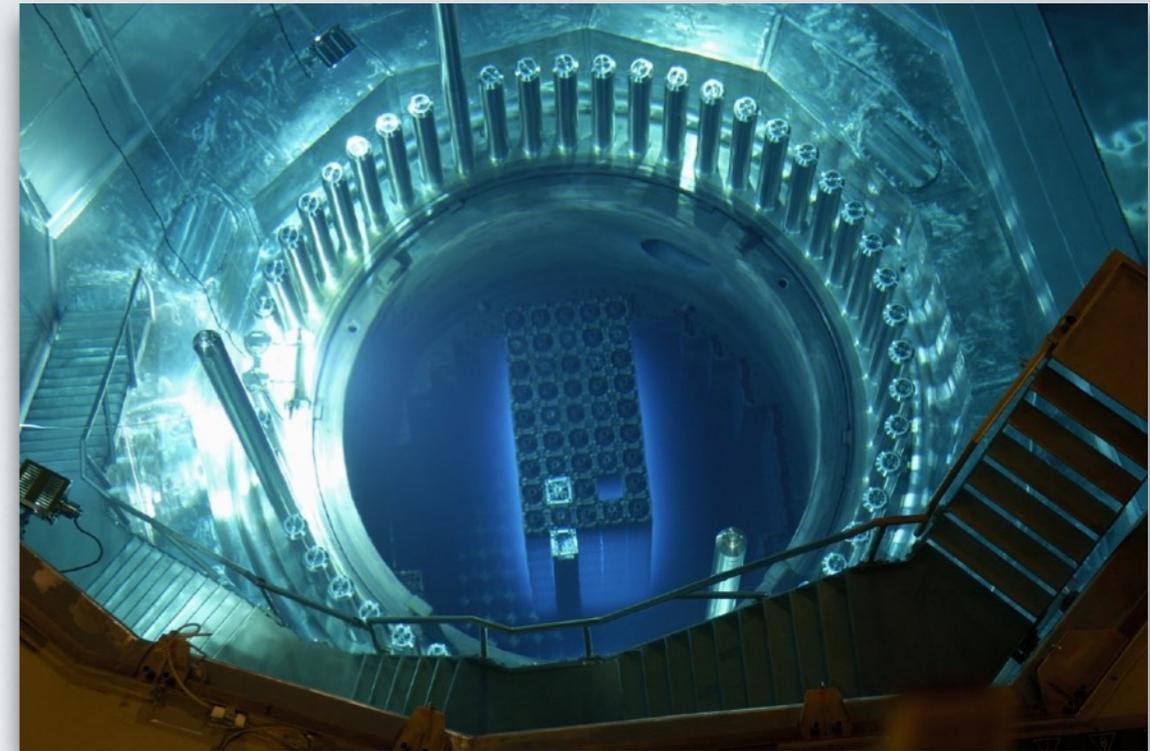
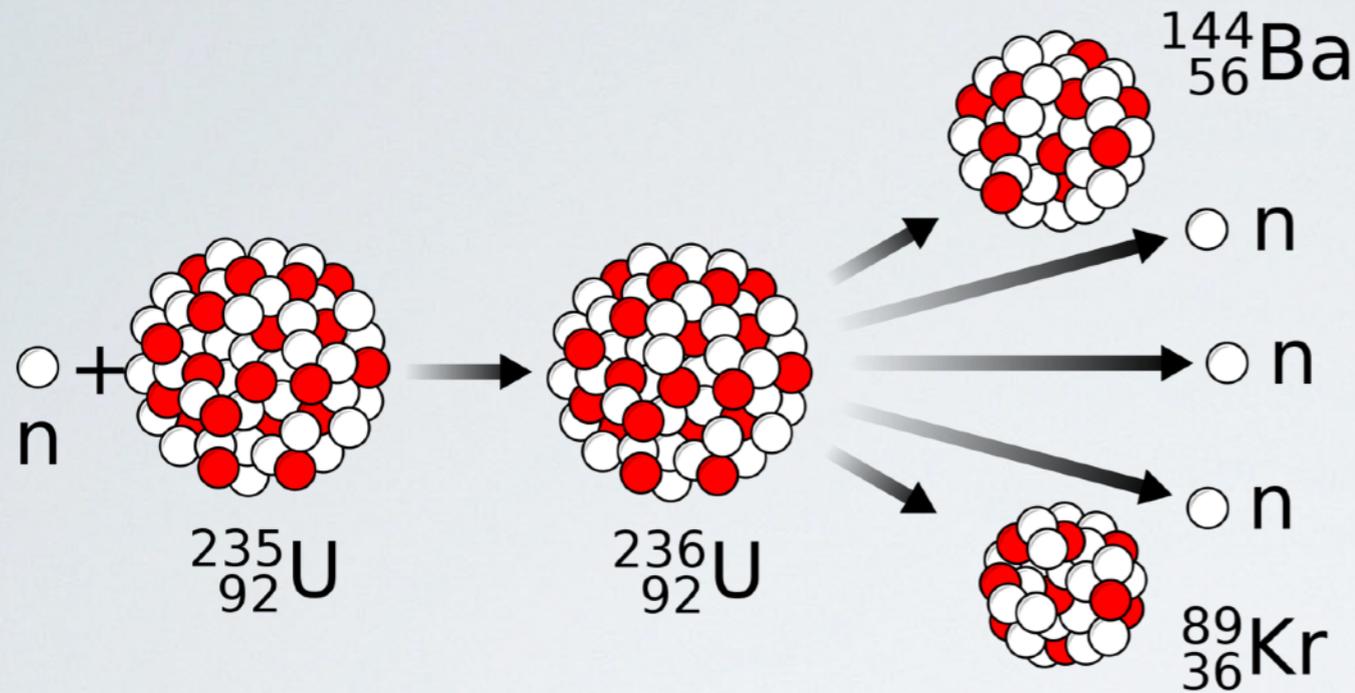
$$E \approx m_0c^2 + \frac{1}{2}m_0v^2 + \dots$$

- Error in Newtonian formula
  - For car going at 30 mph, approximate formula is wrong by 1 part in  $10^{30}$
  - For rocket going at 30,000 mph, this approximate formula is wrong by 1 part in  $10^{18}$
- At rest ( $v = 0$ ), we get **rest energy**
  - Fundamental, “irreducible” energy that every object possesses
- This energy can be accessed!
- And vice versa: energy can be turned into mass
- **Mass and energy are equivalent** in special relativity

# Mass to energy



# Fission (Uranium)



- Uranium-235 nuclei split into fragments when capturing moving neutron
- Mass of products (neutrons, Krypton, Barium) is slightly less than mass of initial Uranium nucleus + neutron
- Lost mass is **converted into energy** as  $E = mc^2$
- $c^2$  is a large number...

# Participation: Mass to energy #1



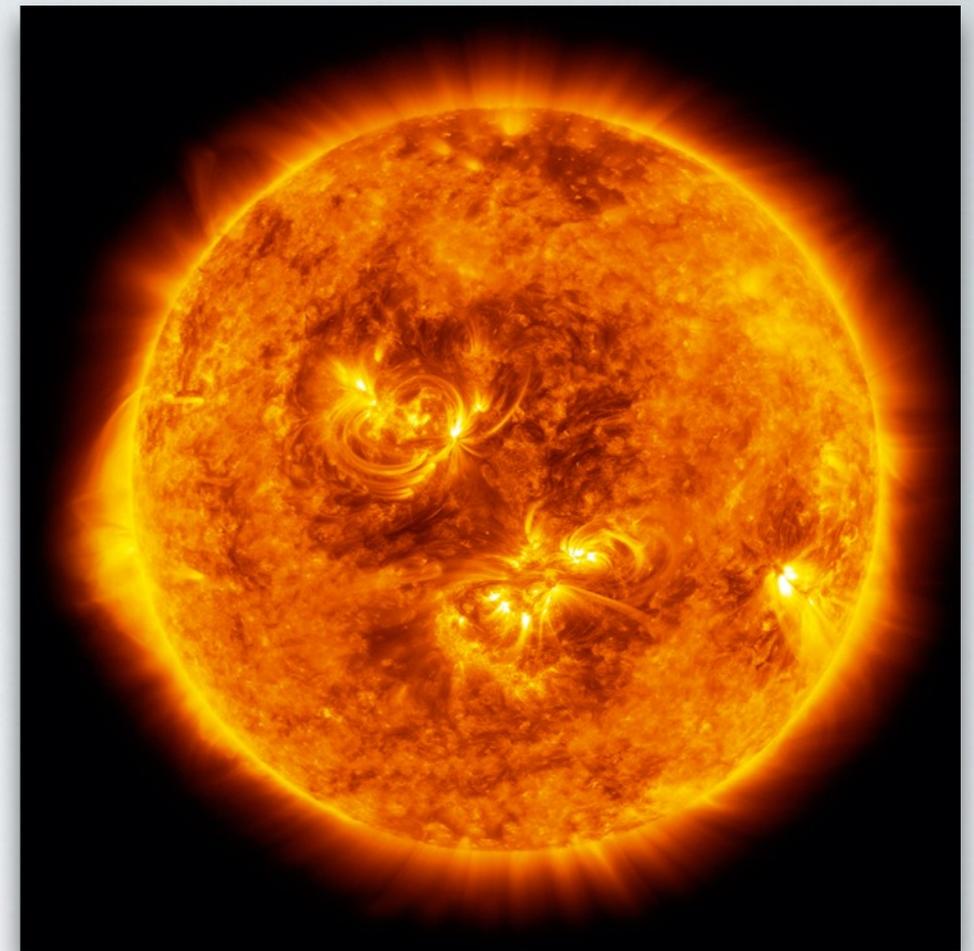
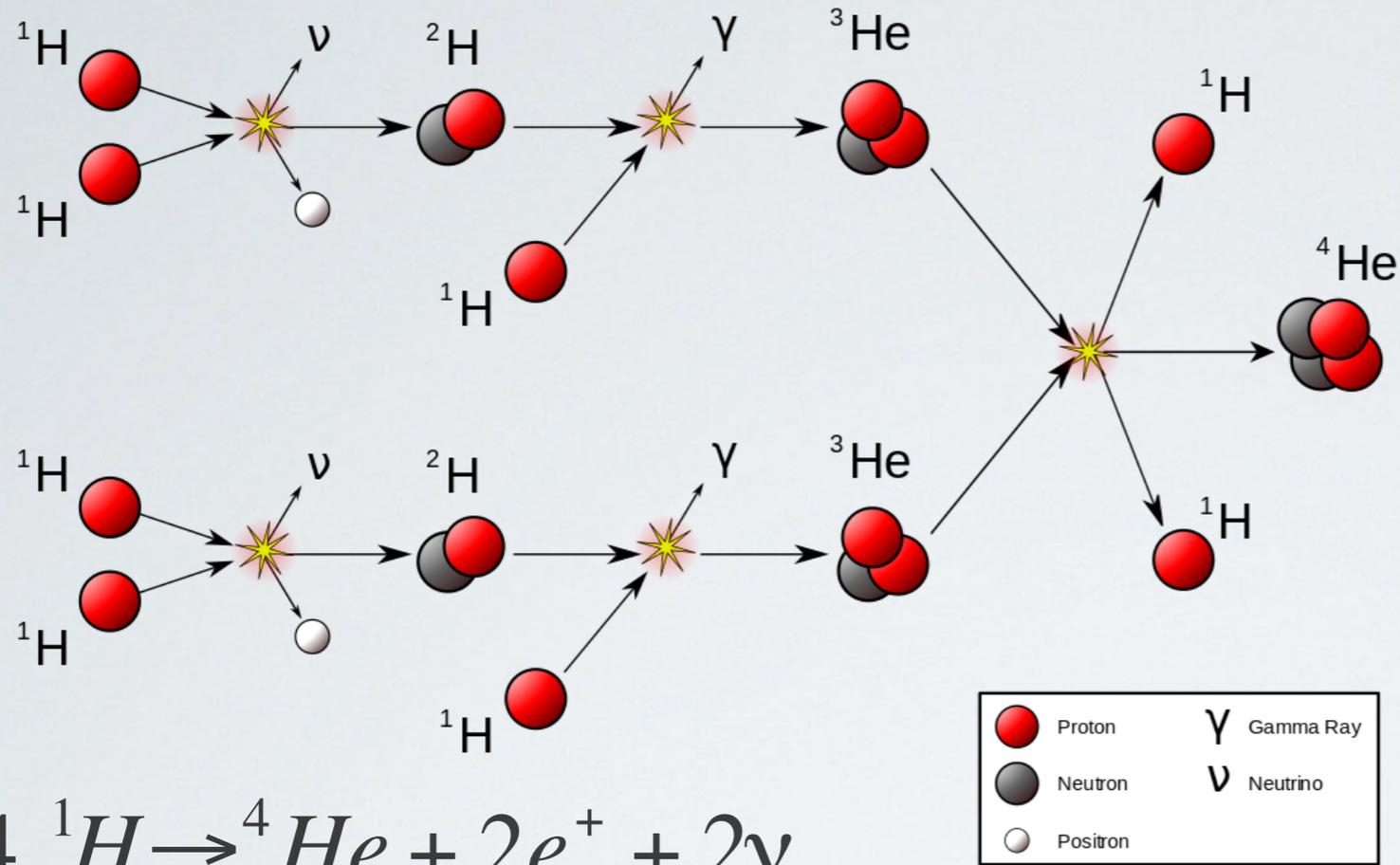
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# Fusion (hydrogen to helium)



- Much more important for astronomy (and life on Earth!) than fission
  - **Power source for stars**, including Sun (about  $4 \times 10^{12}$  g/s burned)
  - Path to making heavy elements (C, N, O, Si, Fe...)
- Products about **1% lighter** than 4 hydrogen nuclei

# Participation: Mass to energy #2



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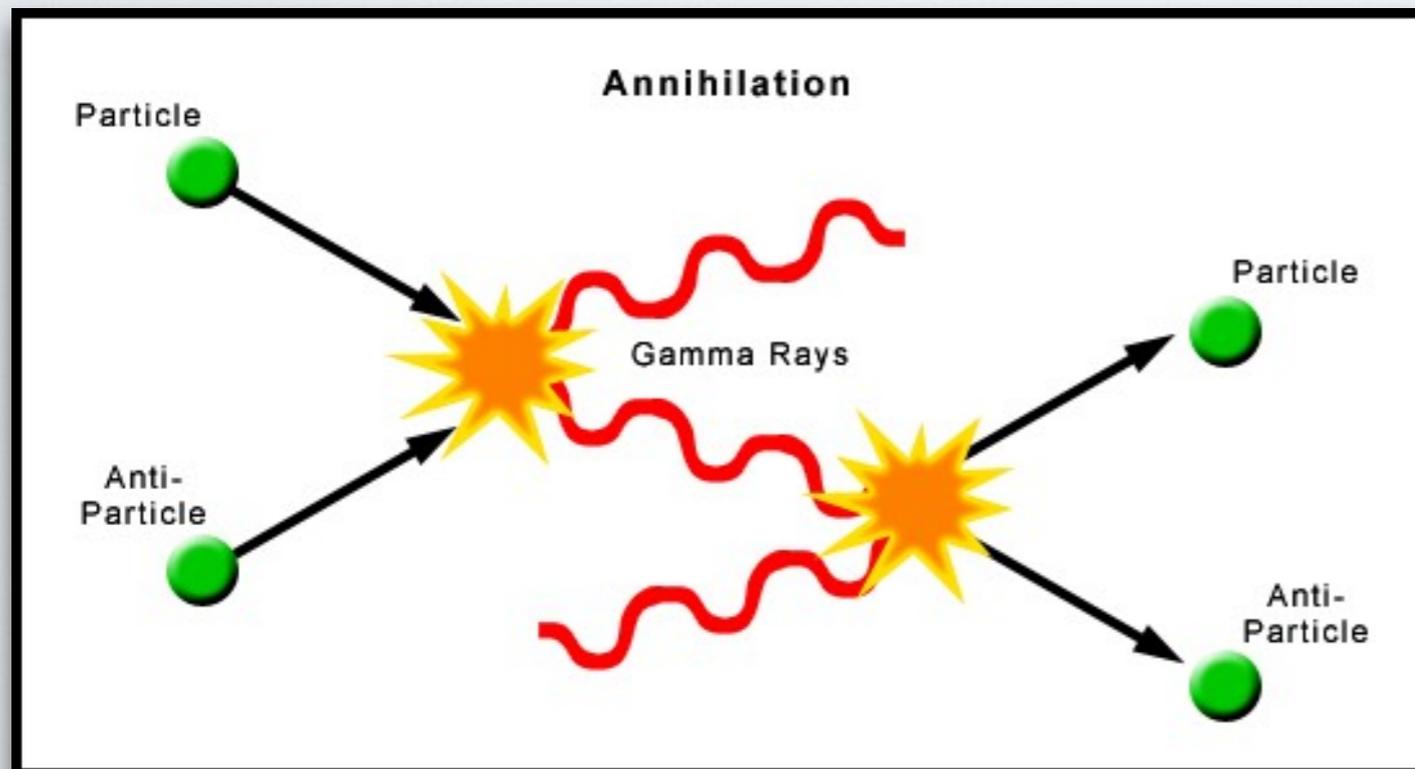
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# Conversion from energy to mass: antimatter

- For every particle, there is an **antiparticle** (e.g., electron / positron)
- Antiparticles have opposite properties, same mass
- Particles and anti-particles can **annihilate** to radiation (energy) and be **created** from it



# Take-aways

- Events are **causally connected** if they can communicate with the speed of light or less; the order of such events cannot be changed by switching frame of reference
- In space-time diagrams, the straightest line between events corresponds to the **longest proper time** interval; this resolves the twin paradox
- **Mass and energy are interchangeable** in relativity; objects possess both a rest mass and kinetic energy

# Next time...

## We'll talk about:

- General Relativity

## Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #1 (by tonight)
- Homework #2 (by Thu 10/07)

## Reading:

- H&H Chapter 8