ASTR 340: Origin of the Universe

Prof. Benedikt Diemer

Lecture 10 • General Relativity II

09/30/2021

Class survey results: Lectures

How do you find the pace of the lectures?

Way too slow for me		0 %
A bit too slow for me		0 %
About right for me	38 respondents	**69 %**
A bit too fast for me	16 respondents	29 %
Way too fast for me	1 respondent	2 %

Class survey results: Group discussions

How productive for your learning do you find the (group) discussions and in-class writing exercises?

Not productive at all	8 respondents	**15 %**
Somewhat productive	36 respondents	65 %
Quite productive	10 respondents	18 %
Most productive element of the course	1 respondent	2 %

Class survey results: Turning Point

How productive for your learning do you find the TurningPoint polls?

Not productive at all	1 respondent	**2 %**
Somewhat productive	7 respondents	13 %
Quite productive	42 respondents	76 %
Most productive element of the course	5 respondents	9 %

Class survey results: Quizzes

How productive for your leaning do you find the post-lecture quizzes?

Not productive at all		**0 %**
Somewhat productive	13 respondents	24 %
Quite productive	26 respondents	47 %
Most productive element of the course	16 respondents	29 %

Class survey results: Homework

How productive for your learning do you find the homework exercises?

Not productive at all	1 respondent	**2 %**
Somewhat productive	13 respondents	24 %
Quite productive	27 respondents	49 %
Most productive element of the course	14 respondents	25 %

Class survey results: Textbook

Are you using the textbook? If so, how productive for your learning do you find the textbook chapters that accompany the lectures?

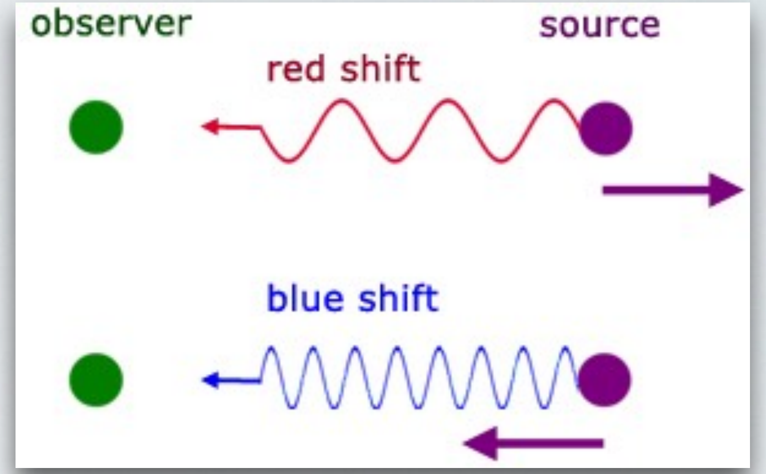
Not productive at all	2 respondents	**4 %**
Somewhat productive	2 respondents	4 %
Quite productive	1 respondent	2 %
Most productive element of the course	1 respondent	2 %
I don't use the text book	49 respondents	89 %

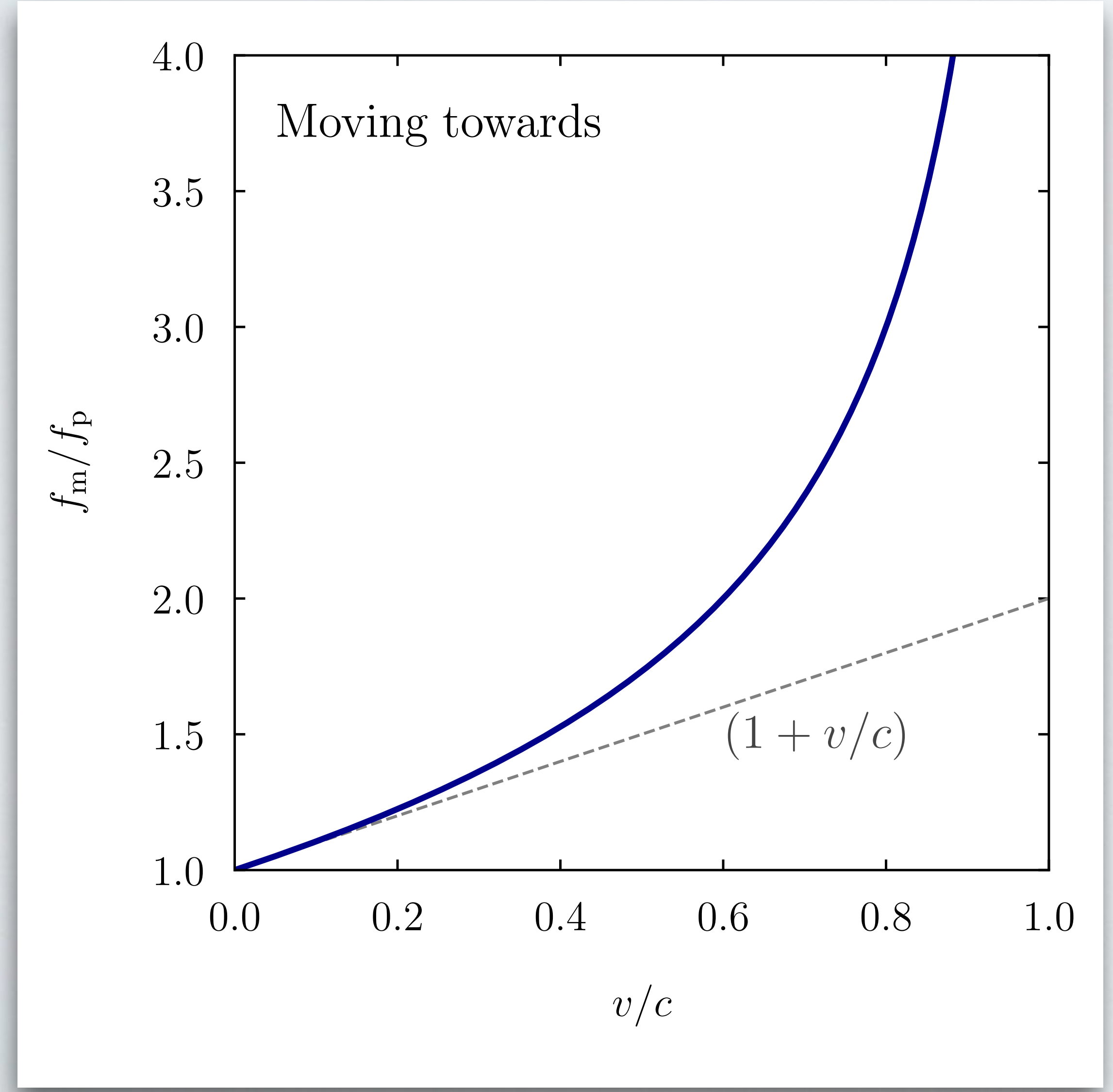
Other feedback / questions

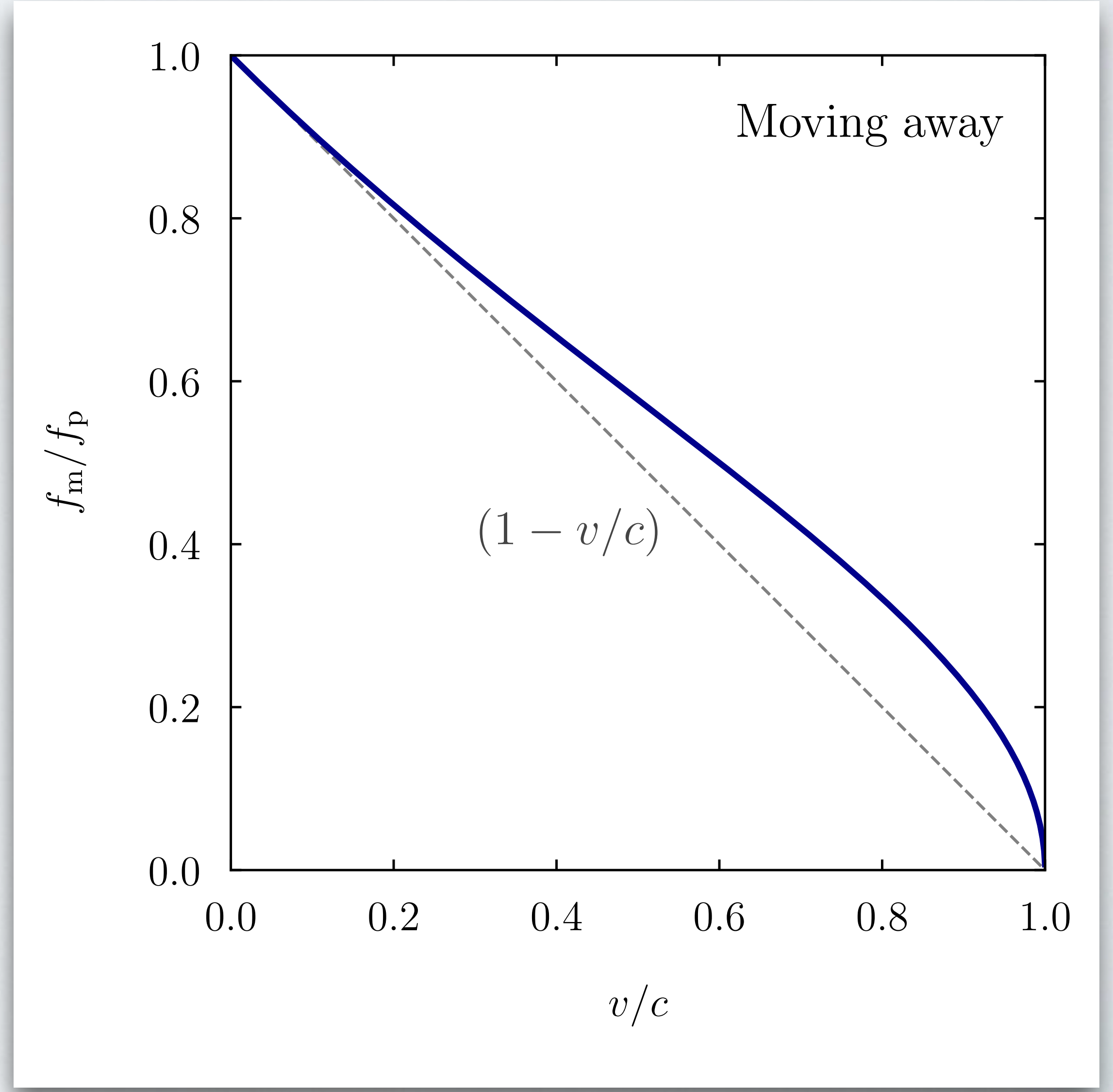
- Lecture recordings?
- Formula sheet?

Relativistic redshift

$$f_{\mathrm{m}} = f_{\mathrm{p}} \times \gamma\left(1 + \frac{v}{c}\right)$$







Part 0: Recap

Participation: Recap #1

TurningPoint:
What is the strong equivalence principle?

Session ID: diemer

30 seconds

Participation: Recap #2

TurningPoint:
How do free-falling objects move?

Session ID: diemer

30 seconds

Recap: General relativity

- Within free-falling frames of reference, Special Relativity applies
- Free-falling particles or observers move on geodesics (shortest paths) through curved space-time
- The distribution of matter and energy determines how space-time is curved

Space-time curvature tells matter/energy how to move, matter/energy tells space-time how to curve

Recap: General relativity vs. Newton

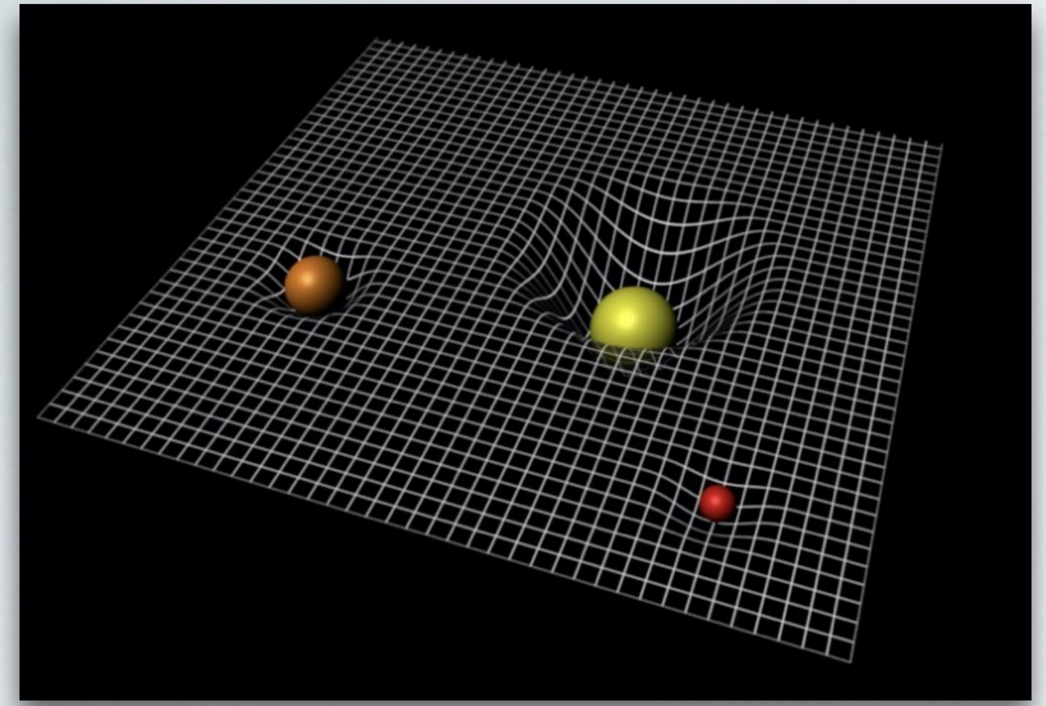
Newton

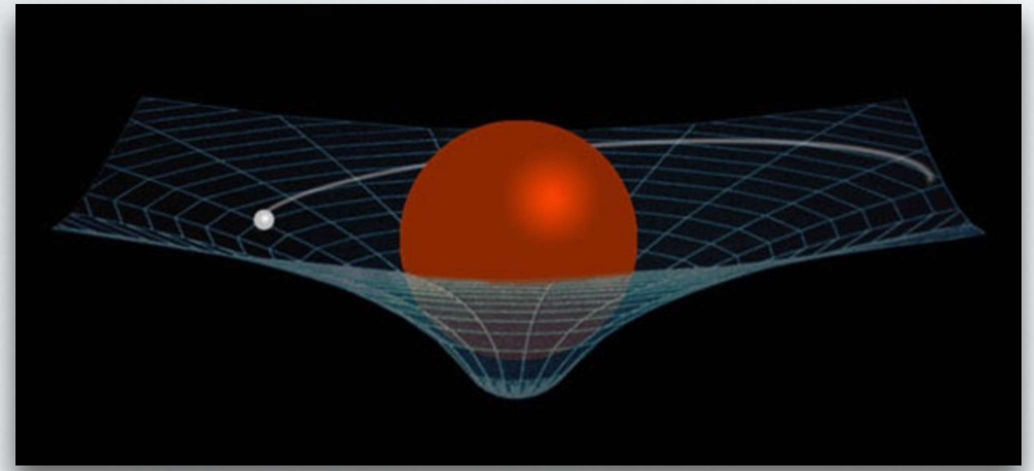
Mass tells gravity how to exert a force ($F = GMm/r^2$),
force tells mass how to accelerate ($a = F/m$)

Einstein

Space-time curvature tells matter/energy how to move,
matter/energy tells space-time how to curve

Warped spacetime





- Two-dimensional space as an analogy: rubber sheet with weights
- Amount that sheet sags depends on how heavy weight is
- Lines that would be **straight** become **curved (to external observer)**

Today

- Curved spacetime
- Light, gravity & lensing
- Gravitational time dilation

Part 1: Curved spacetime

Invariant spacetime interval

- Recall spacetime interval in flat space:
  - Invariant interval is equivalent to **c times proper time interval**
  - **Shorter** when traveling **faster!**
  - Space-time interval is **zero** for any two points on **light** world line

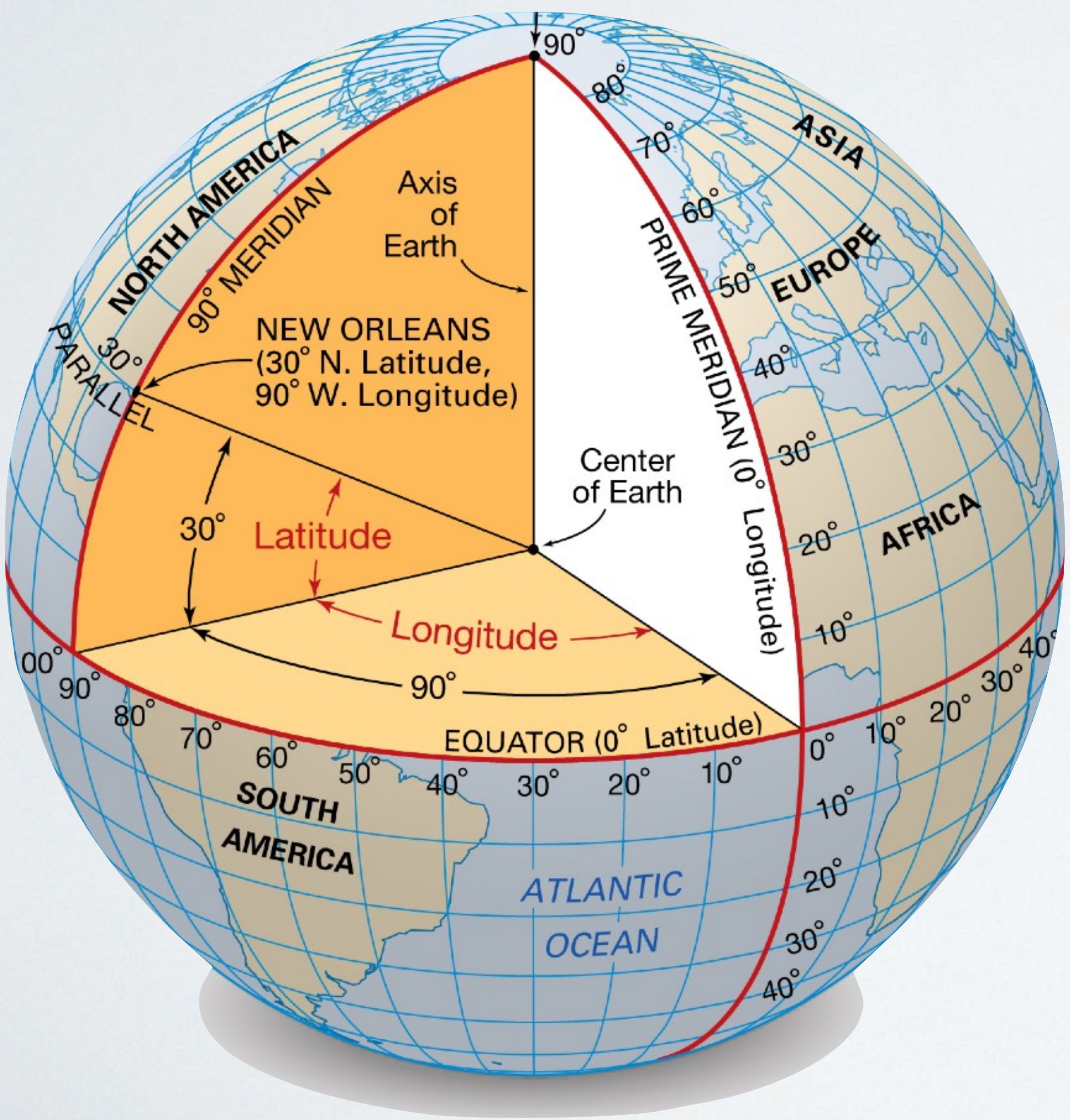
$$\Delta s_{\text{flat}} = \sqrt{(c\Delta t)^2 - \Delta x^2} = c\Delta t_{\text{p}}$$

- Generalize to **curved spacetime**
- Free-falling observers are like inertial obs. in SR, they have **maximal Δs**
- **Light** still moves on **"null" geodesics** with Δs = 0
- Spacetime distance is more complicated and described by **metric**
  - E.g., for a particular class of geometries (Riemann):

$$\Delta s_{\text{curved}} = \sqrt{\alpha c^2\Delta t^2 - \beta c\Delta t\Delta x - \gamma\Delta x^2}$$

Geodesics on Earth

- Coordinate system: two angles



Images: Encyclopaedia Brittanica

Participation: Geodesics #1

TurningPoint:

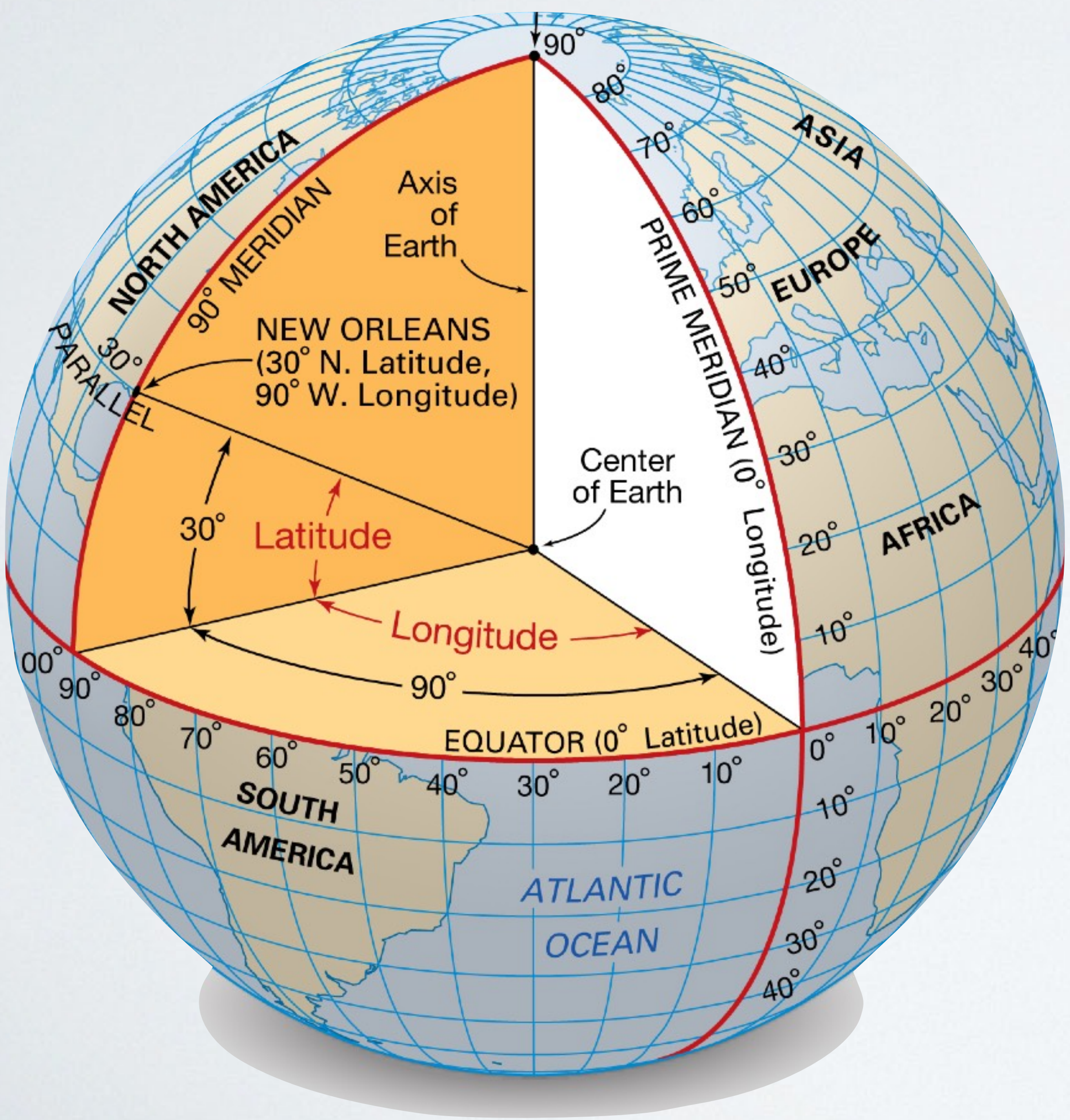
Are meridians on a sphere geodesics?

Session ID: diemer

30 seconds

Geodesics on Earth

- Coordinate system: two angles
- Constant-longitude lines (**meridians**) are **geodesics**



Images: Encyclopaedia Brittanica

Participation: Geodesics #2

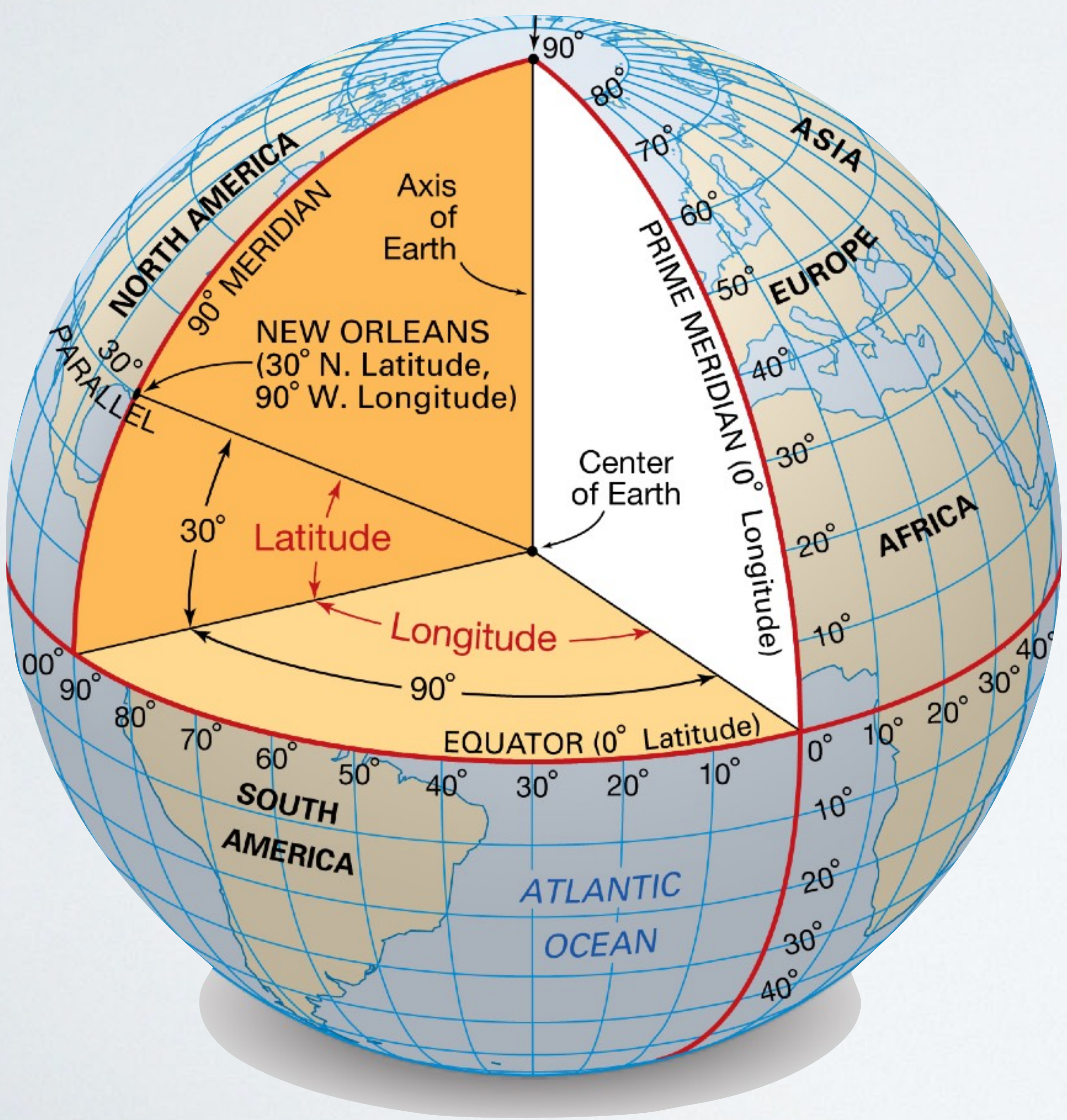
TurningPoint:
Are parallels on a sphere geodesics?

Session ID: diemer

30 seconds

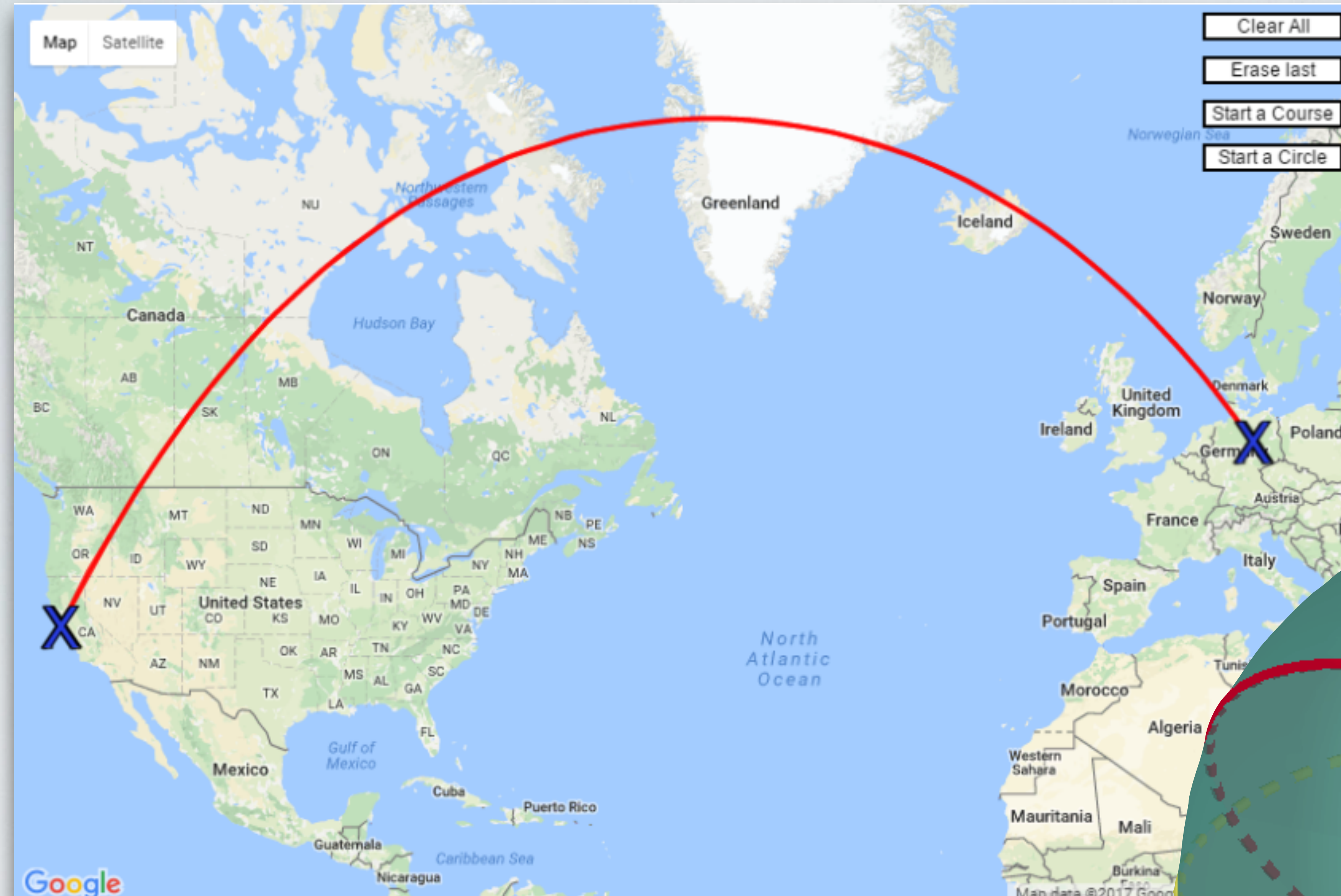
Geodesics on Earth

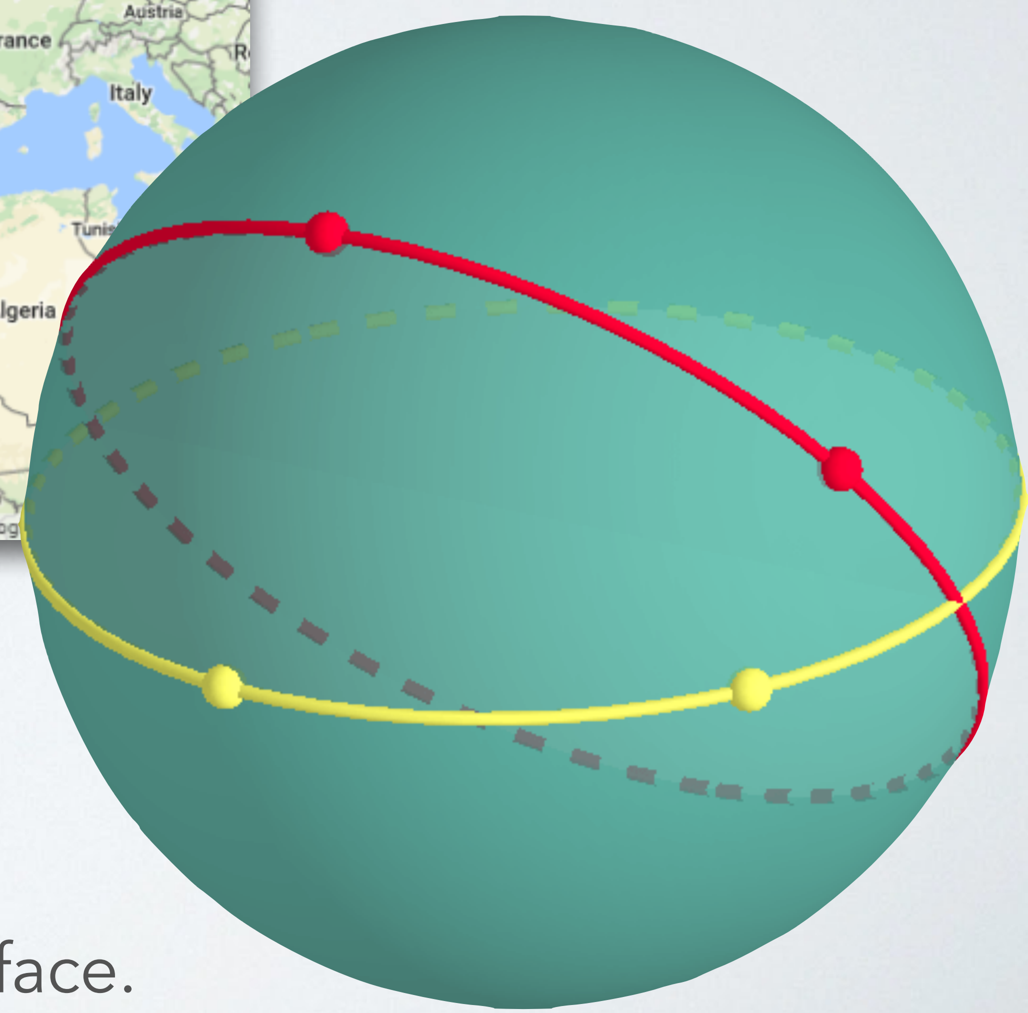
- Coordinate system: two angles
- **Meridians** (north-south lines) are **geodesics**
- **Parallels** (east-west lines) are **not geodesics**



Images: Encyclopaedia Brittanica

Shortest flight paths

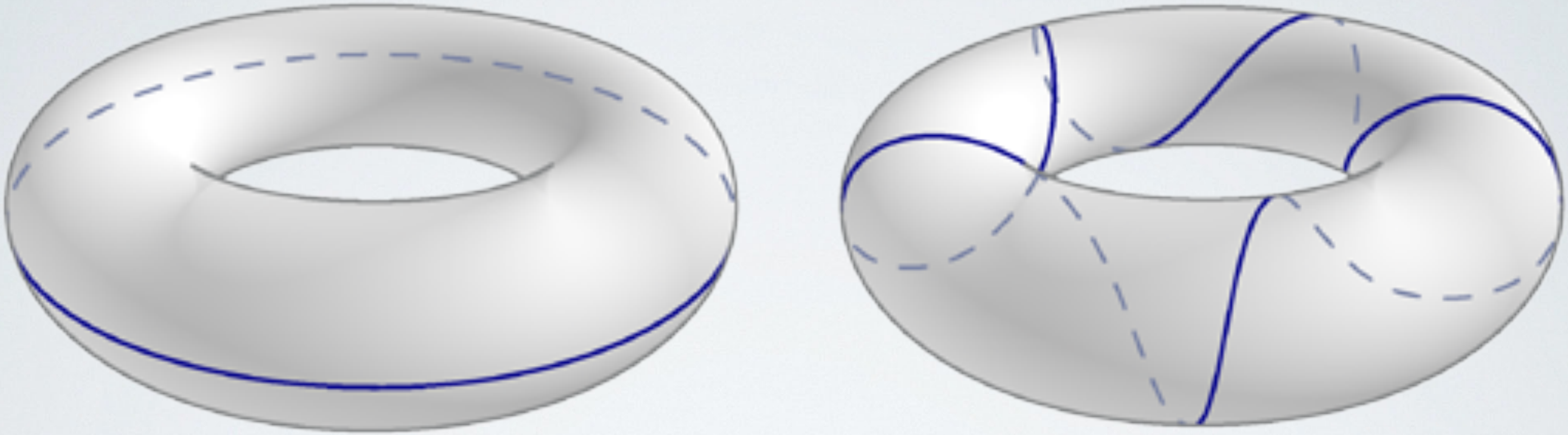




On a sphere, geodesics are **Great Circles**, the shortest distance between two points on the surface.

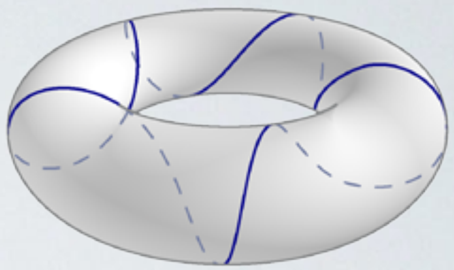
Images: malinc.se

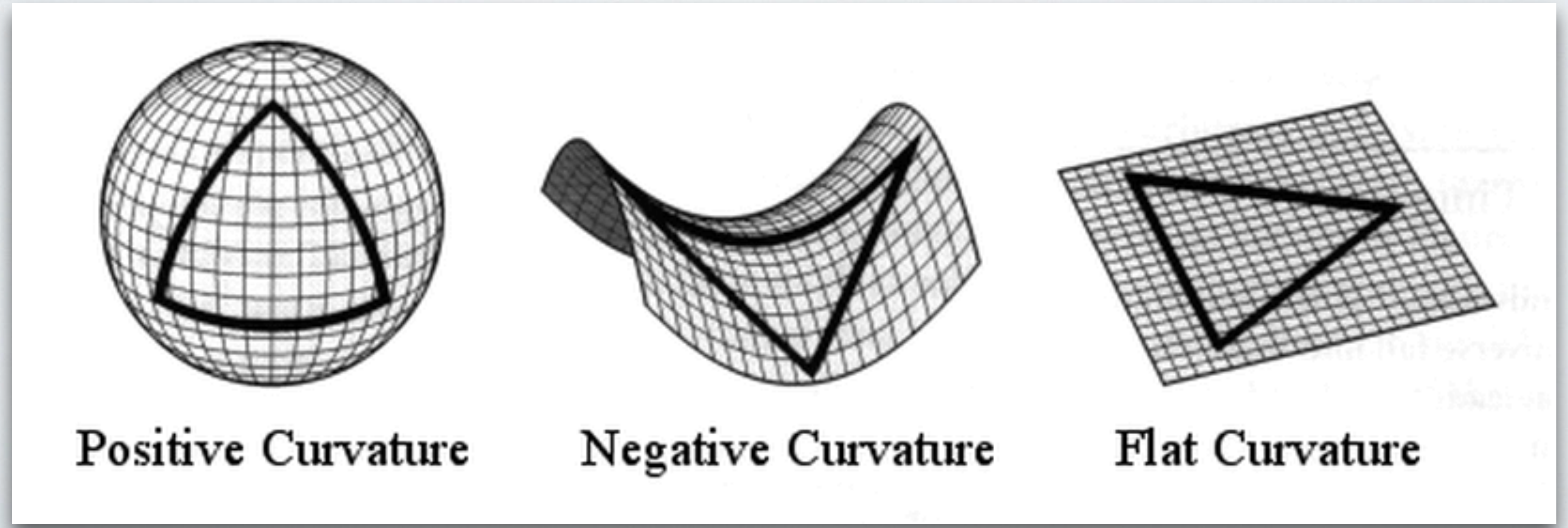
More complicated geodesics



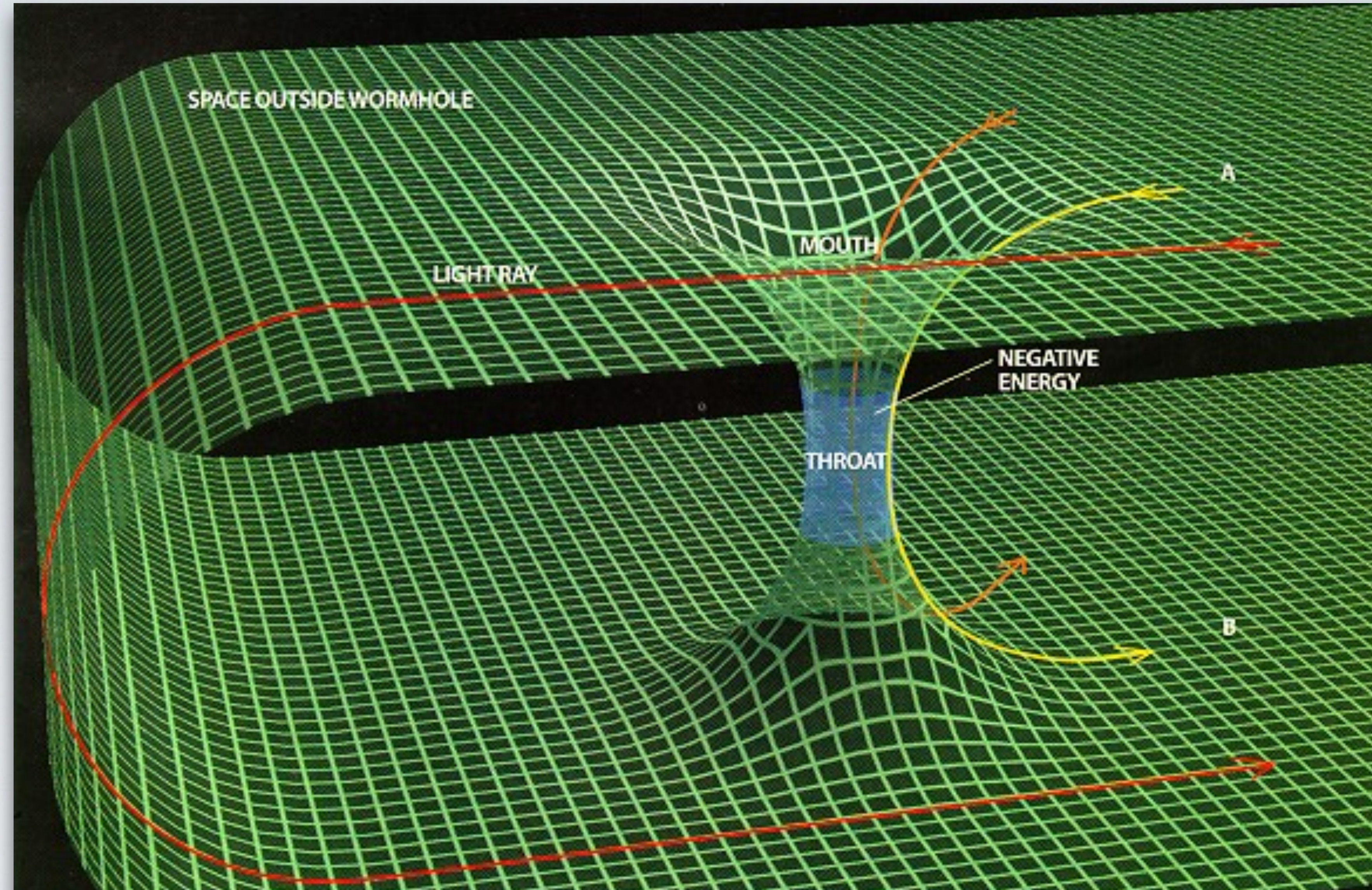
Riemann spacetimes

- Spacetime must be **locally flat** for strong equivalence principle
  - This is true for Riemannian spacetimes (no powers greater than 2 in metric)
  - Basically "smooth" surfaces
- For the Universe as a whole, must be **homogeneous & isotropic**
  - **Flat** (Euclidean space, Minkowski spacetime)
  - **Positively curved** (like a sphere)
  - **Negatively curved** or **hyperbolic** (like a saddle point everywhere; but no equivalent in 2D/3D)





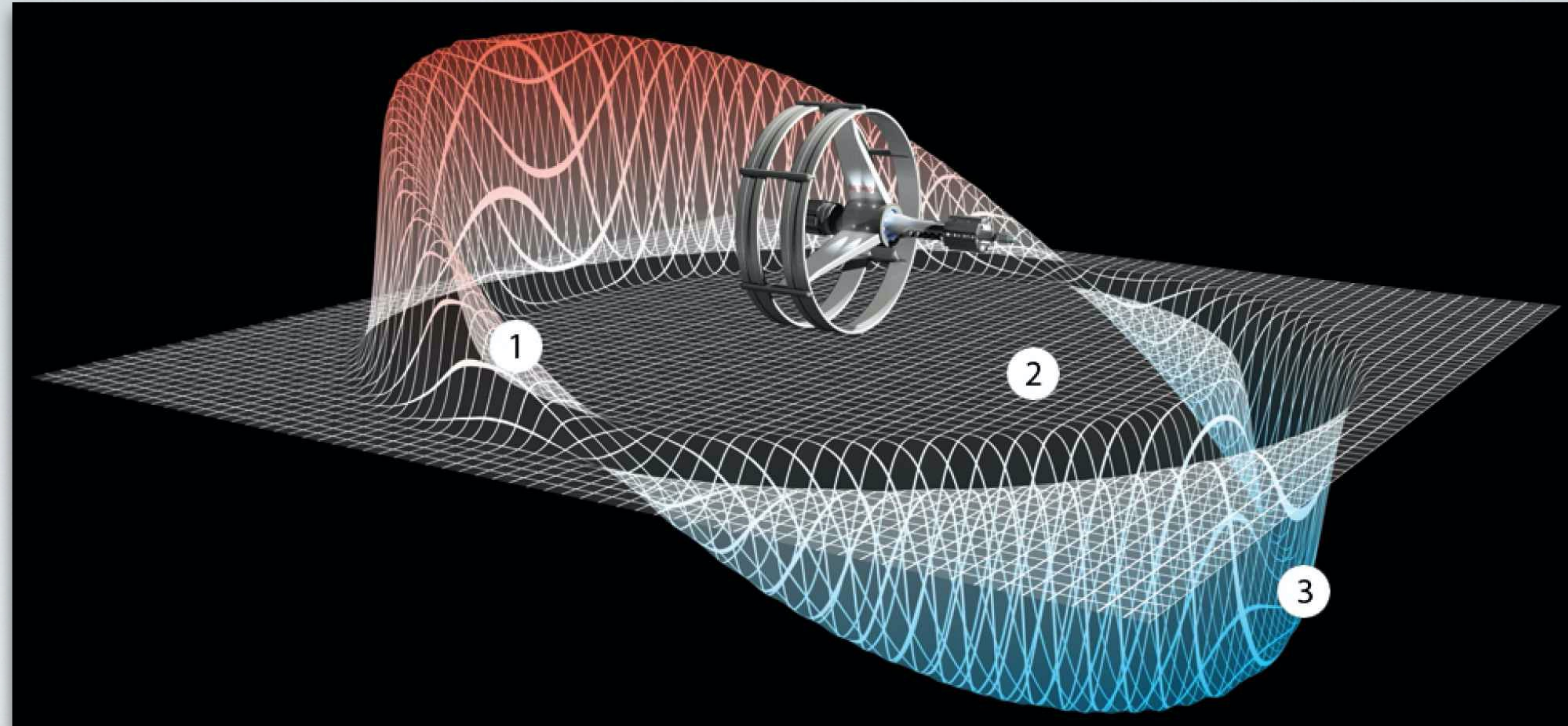
Wormholes



- **Particular metric** that allows travel between disparate points in spacetime
- Often relies on symmetric solution for a black hole known as a white hole
- Rosen & Einstein (1935) developed a metric for a standard BH, but not stable
- Other solutions exist, e.g. Ellis wormhole:

$$ds^2 = -c^2\,dt^2 + d\ell^2 + (k^2 + \ell^2)(d\theta^2 + \sin^2\theta\,d\varphi^2)$$

Alcubierre Drives

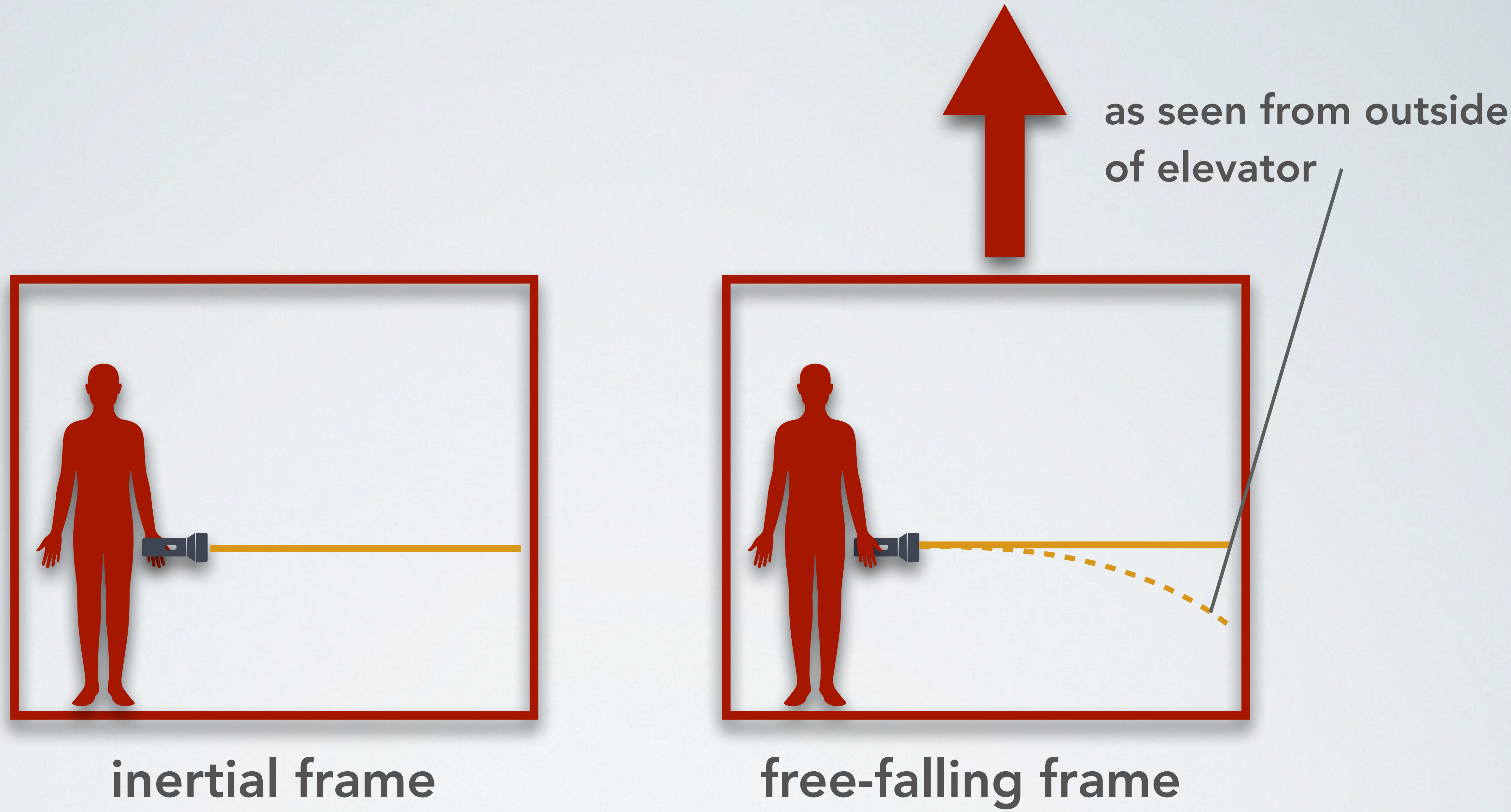


- Contracts space in front of spaceship, expands it behind it
- Needs exotic matter (or **negative mass/energy**)
- Metric mathematically developed by Miguel Alcubierre:

$$ds^2 = -\left(\alpha^2 - \beta_i \beta^i\right)\, dt^2 + 2\beta_i\, dx^i\, dt + \gamma_{ij}\, dx^i\, dx^j,$$

Part 2: Light, gravity & lensing

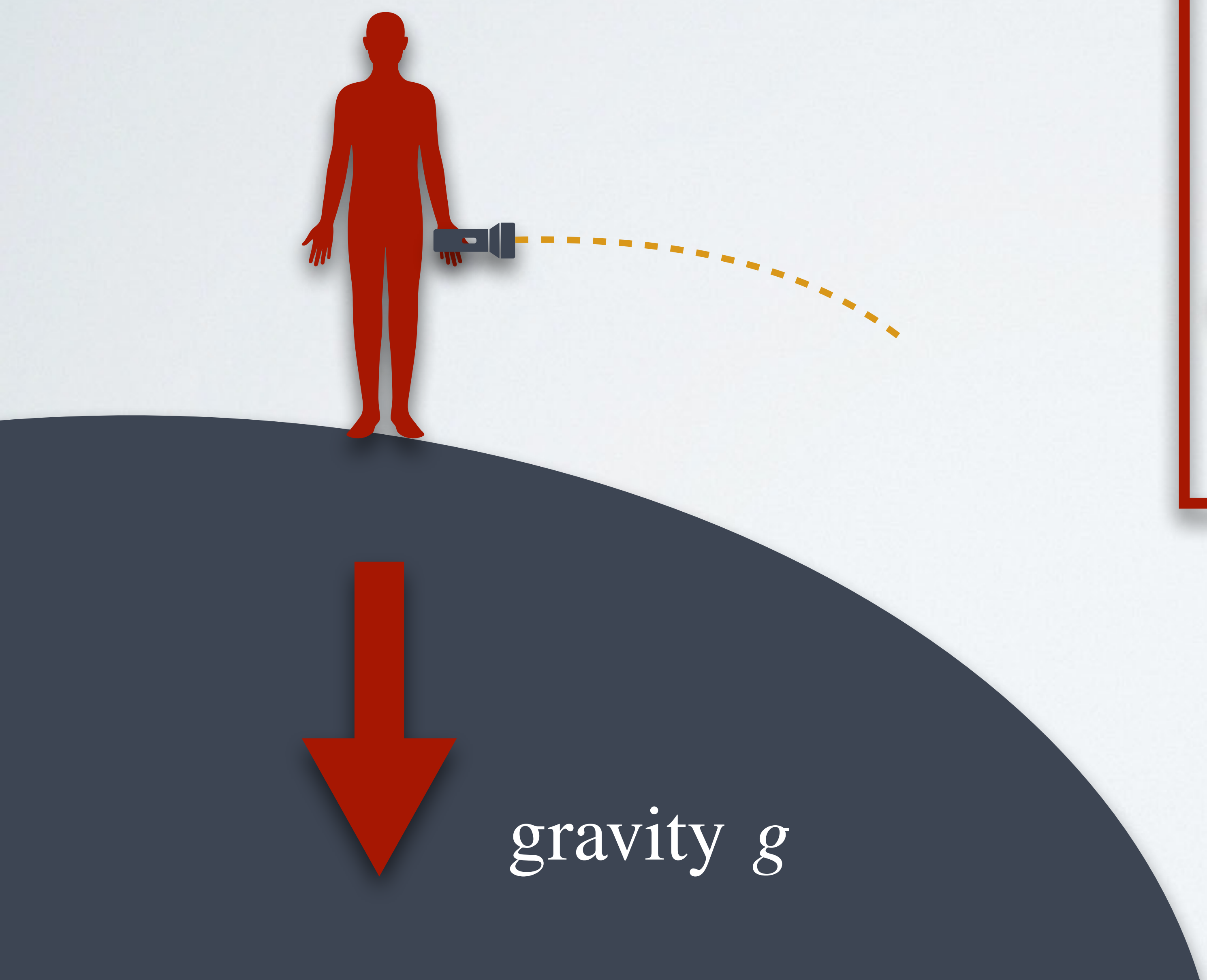
Light in gravitational fields

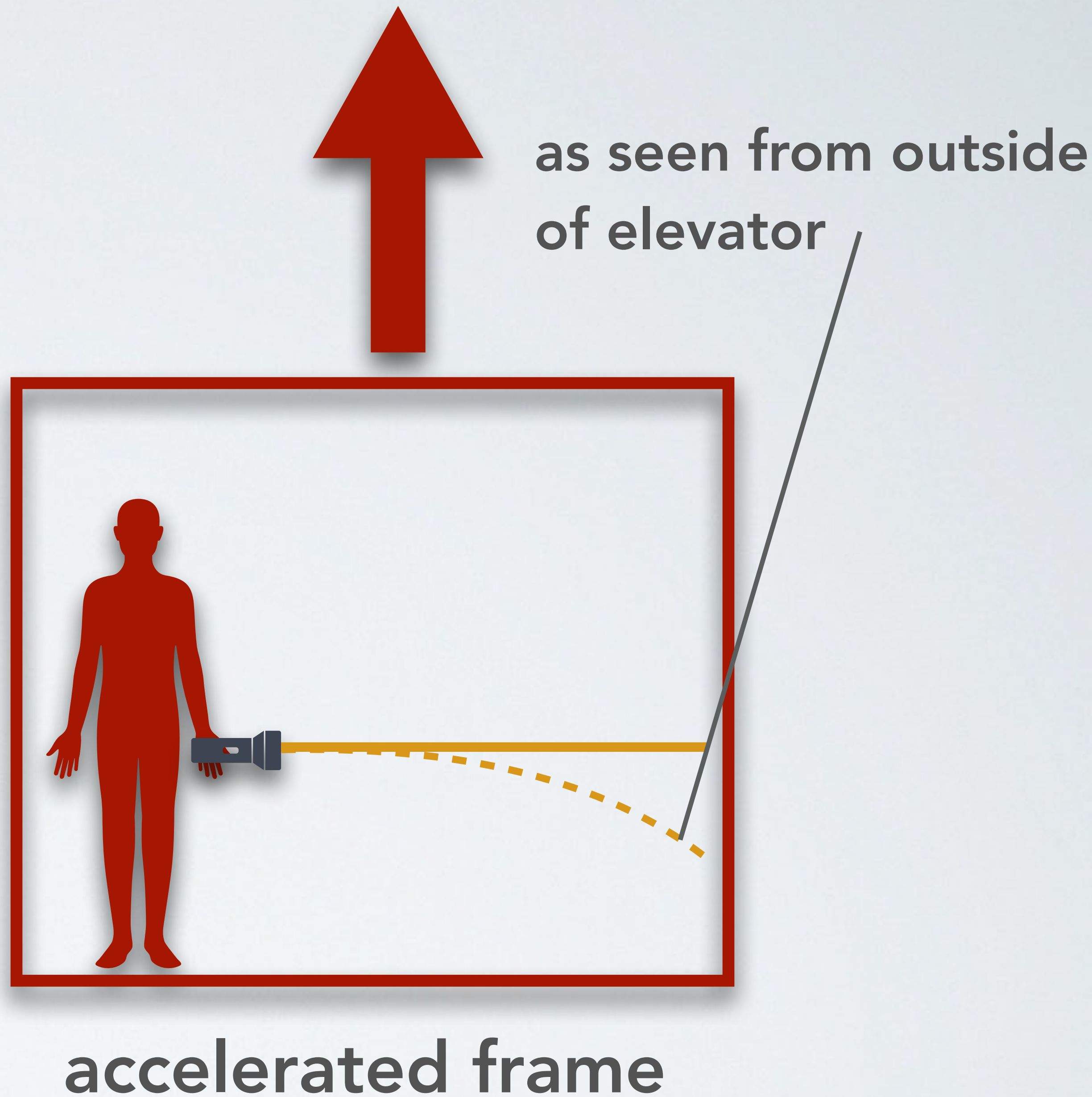


- Inertial and free-falling frames are equivalent: light goes in straight beam
- Thus, light bends as seen from another frame

Light in gravitational fields

- Weak equivalence principle: frame with gravity is the same as accelerated
- Thus, light must bend there too
- **Light falls due to gravity!**





Participation: Light bending in Newtonian picture

TurningPoint:

Would light bend in gravity in Newtonian physics?

Session ID: diemer

45 seconds

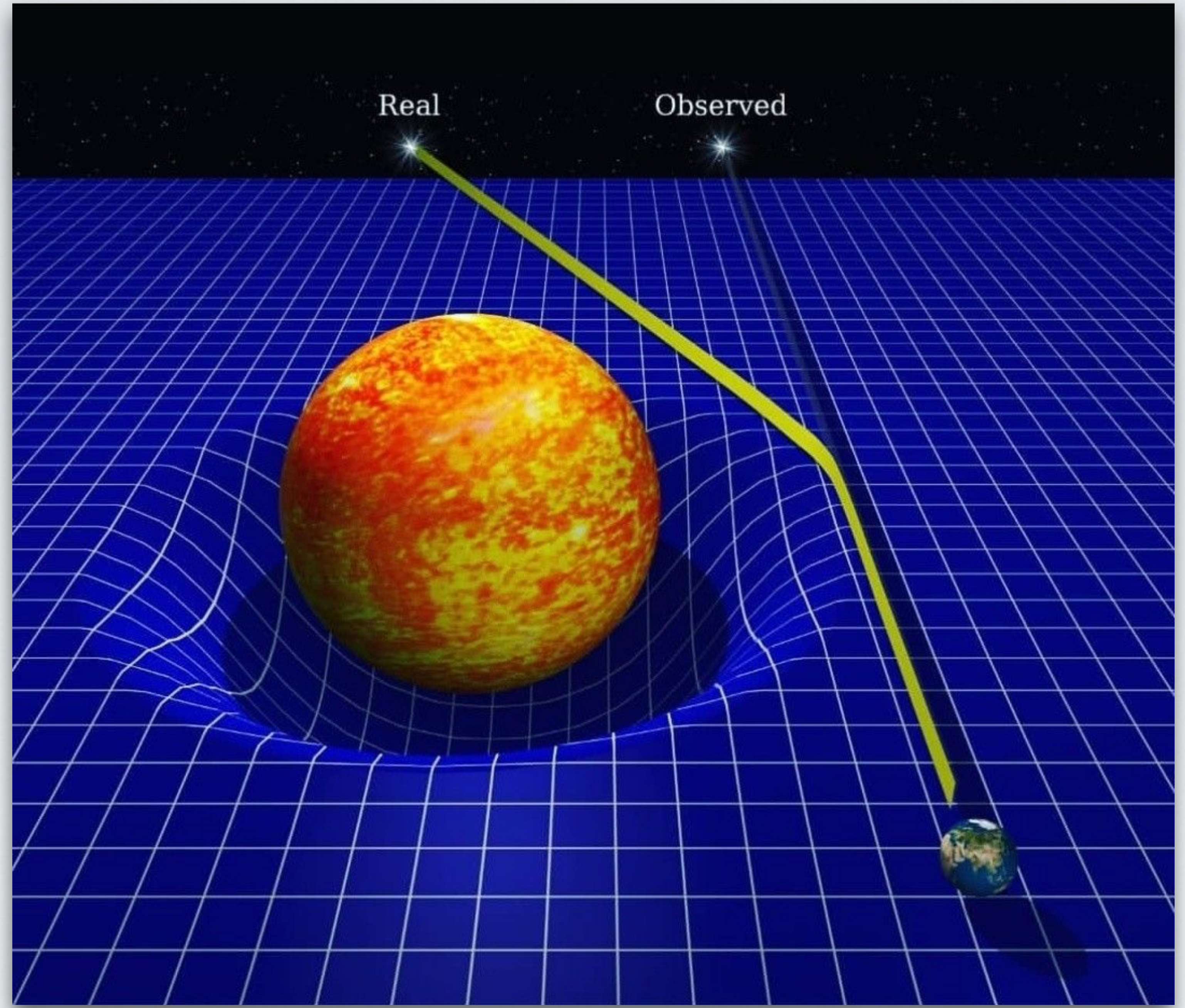
Light bending in Newtonian physics

Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action (caeteris paribus) strongest at the least distance?

Newton, 1704

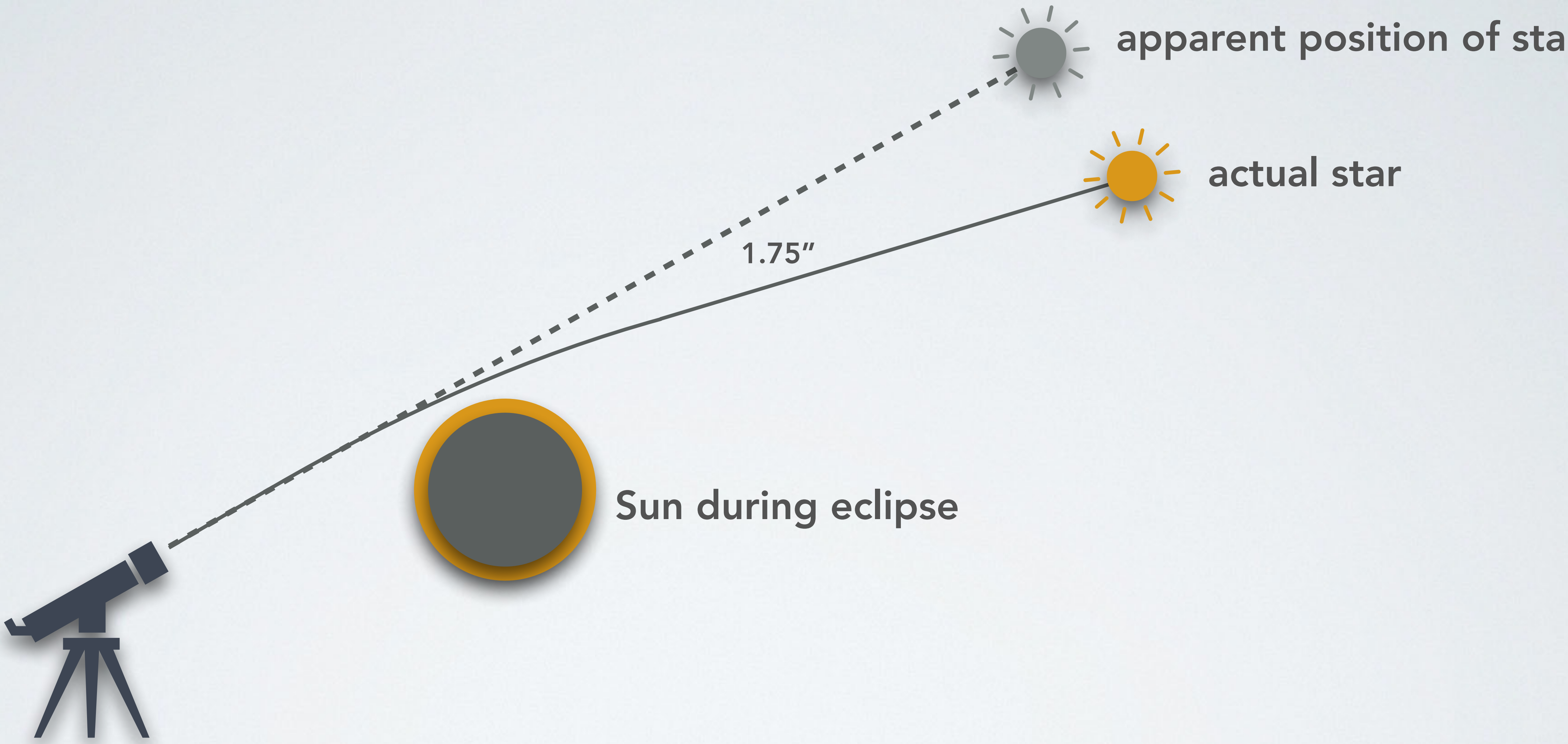
- Could argue that gravitation should only work on massive particles, but photons have no mass
- On the other hand, the mass does not matter for the acceleration (weak equivalence principle). So is light accelerated?

Light in the Sun's warped spacetime



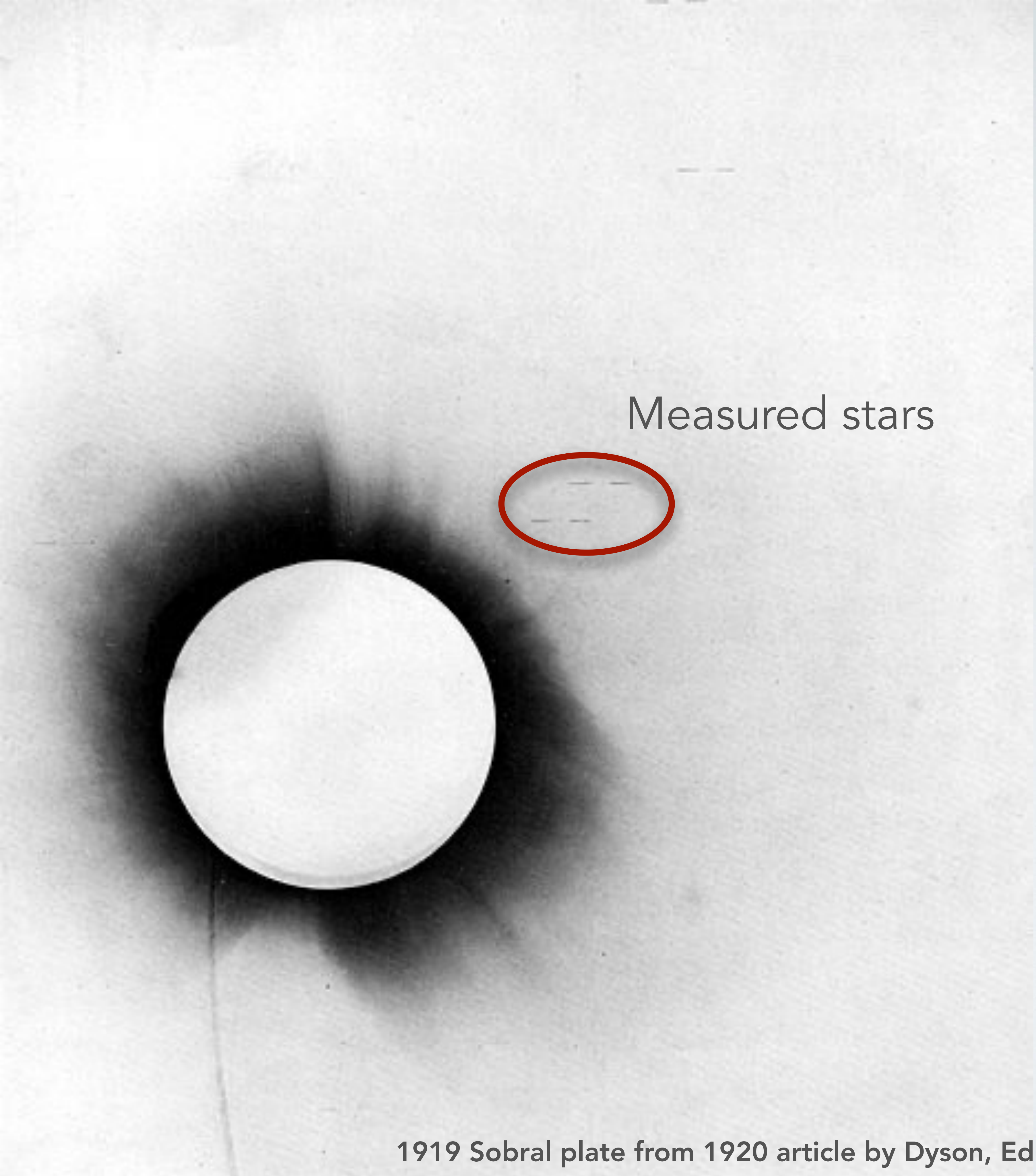
Light rays follow ("null") geodesics

Gravitational bending of light



Eddington experiment

- 1919: first "accessible" **total Solar eclipse** since Einstein proposed GR
- British astronomer Arthur Eddington organized **expeditions** to Principe (West Africa) and Sobral (Brazil) to observe the eclipse
- Looking for effects of gravitational light bending in positions of stars just next to the Sun
- He found them, exactly **as predicted!**



1919 Sobral plate from 1920 article by Dyson, Eddington & Davidson

Smithsonian Castle

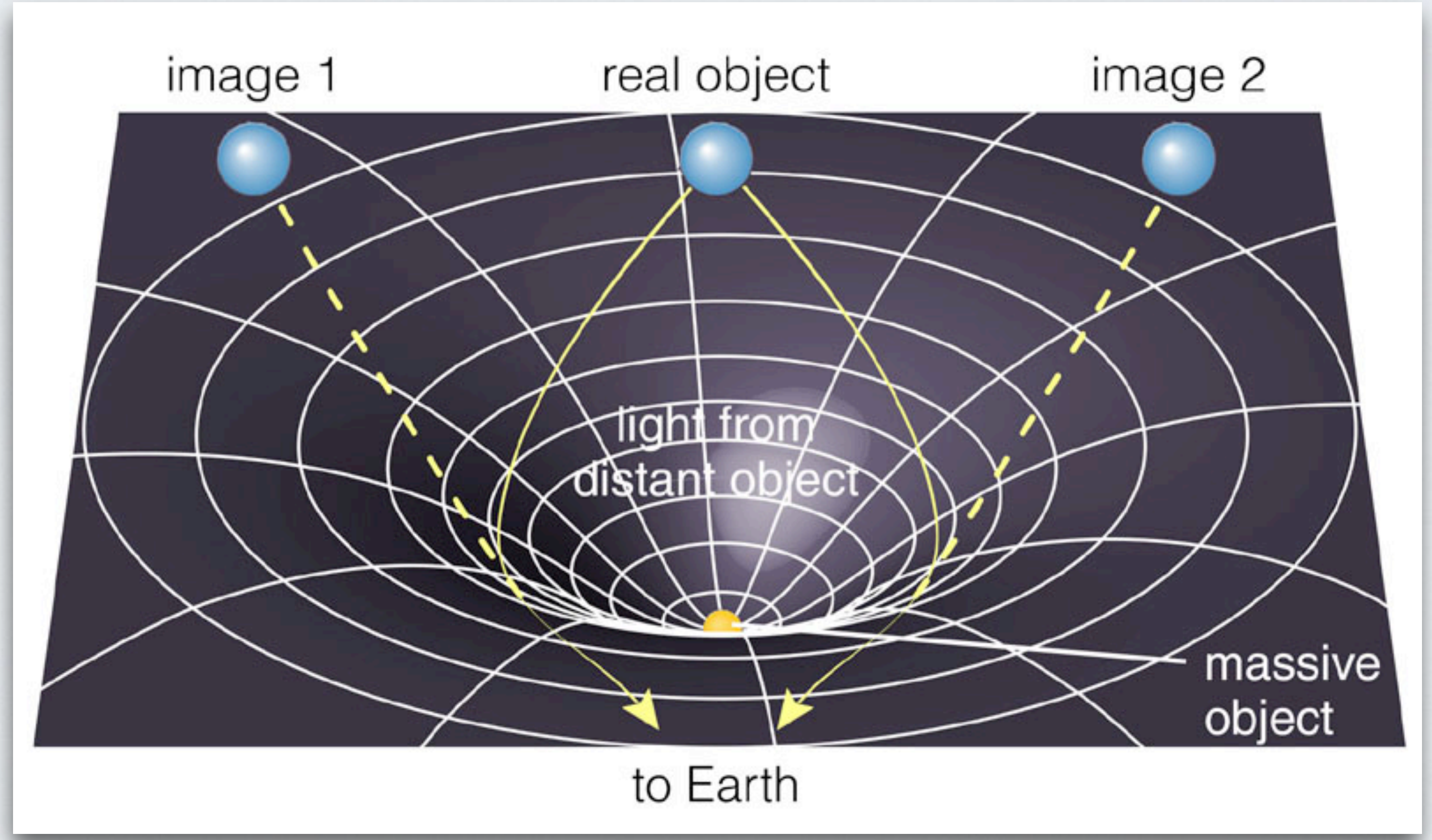
Emilio Falco

Galaxy lensing

Lensing Galaxy

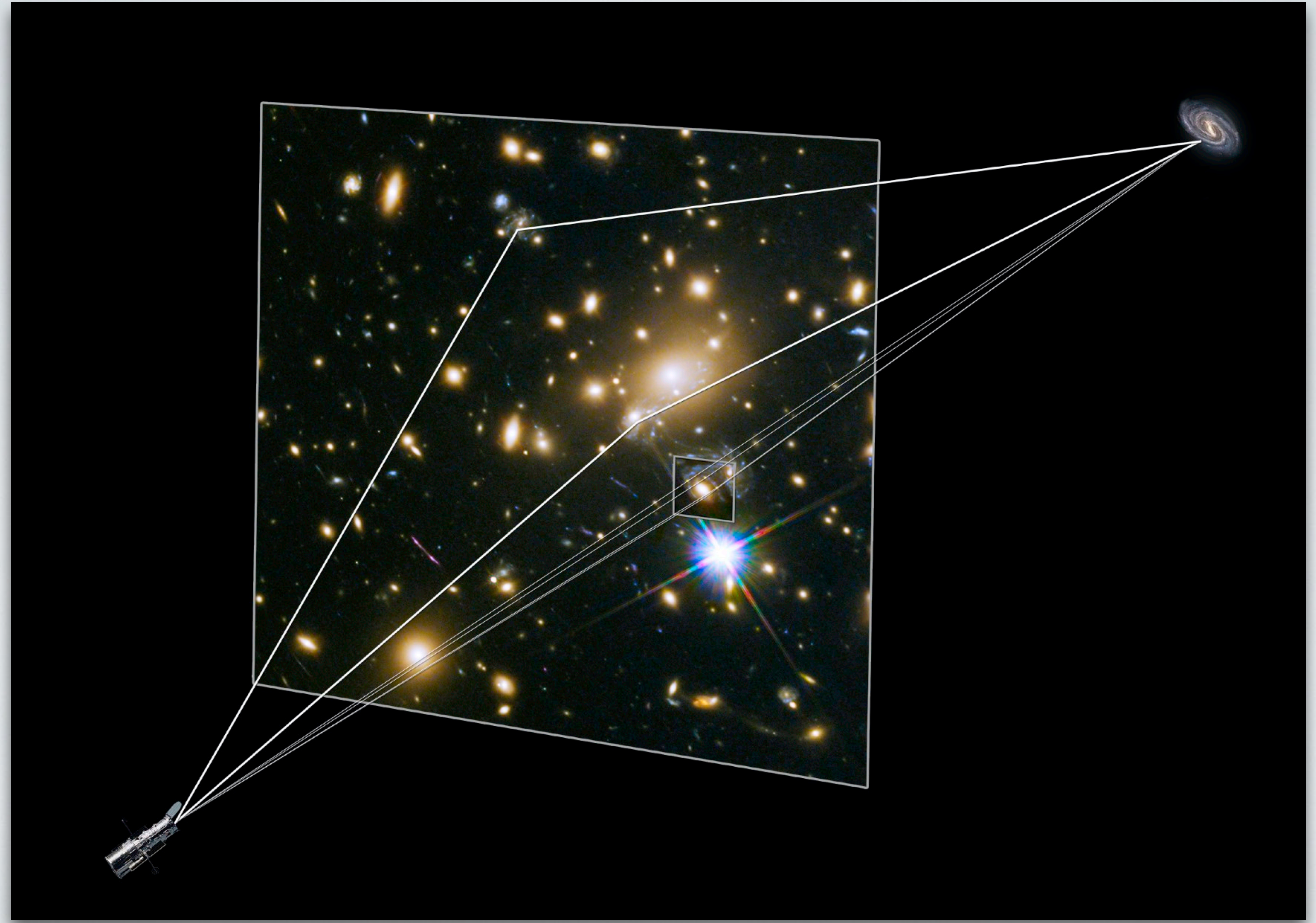
Lensing from galaxy clusters

- Light rays from distant quasar or galaxy provide **background source**
- Massive galaxy or cluster is **foreground lens**
- Two or more **images** can appear



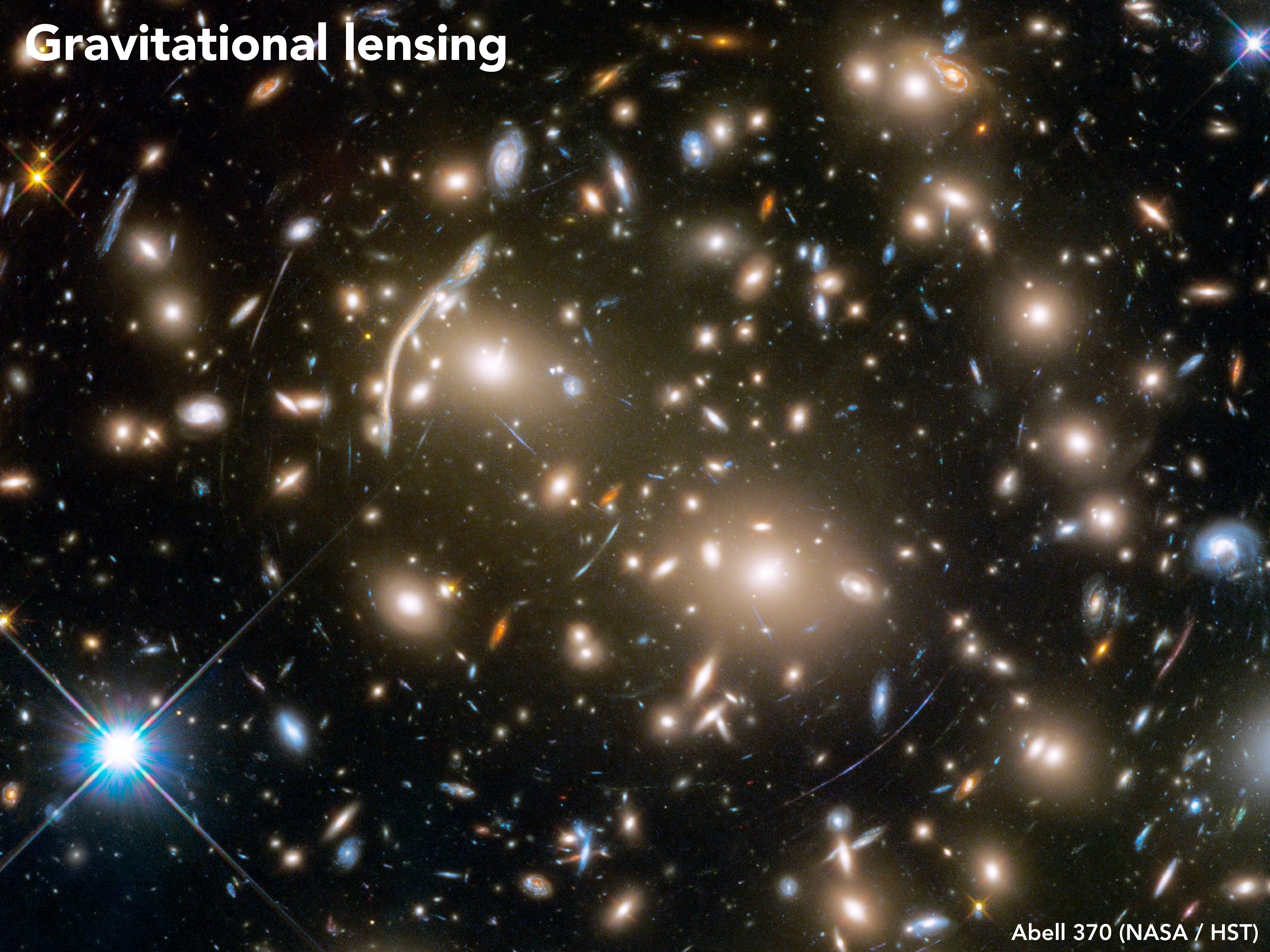
Images: NASA / University of Georgia

Lensing from galaxy clusters



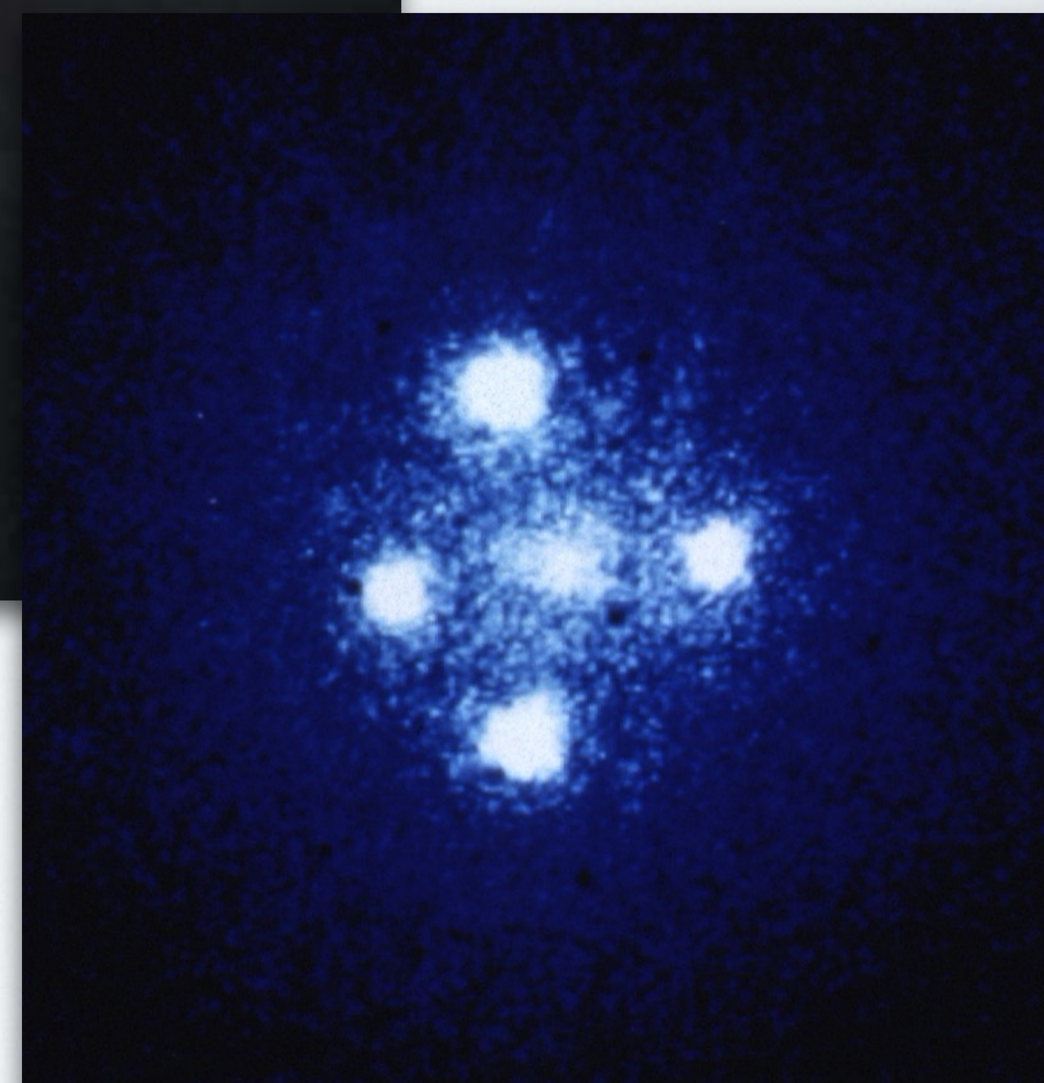
Images: NASA

Gravitational lensing



Abell 370 (NASA / HST)

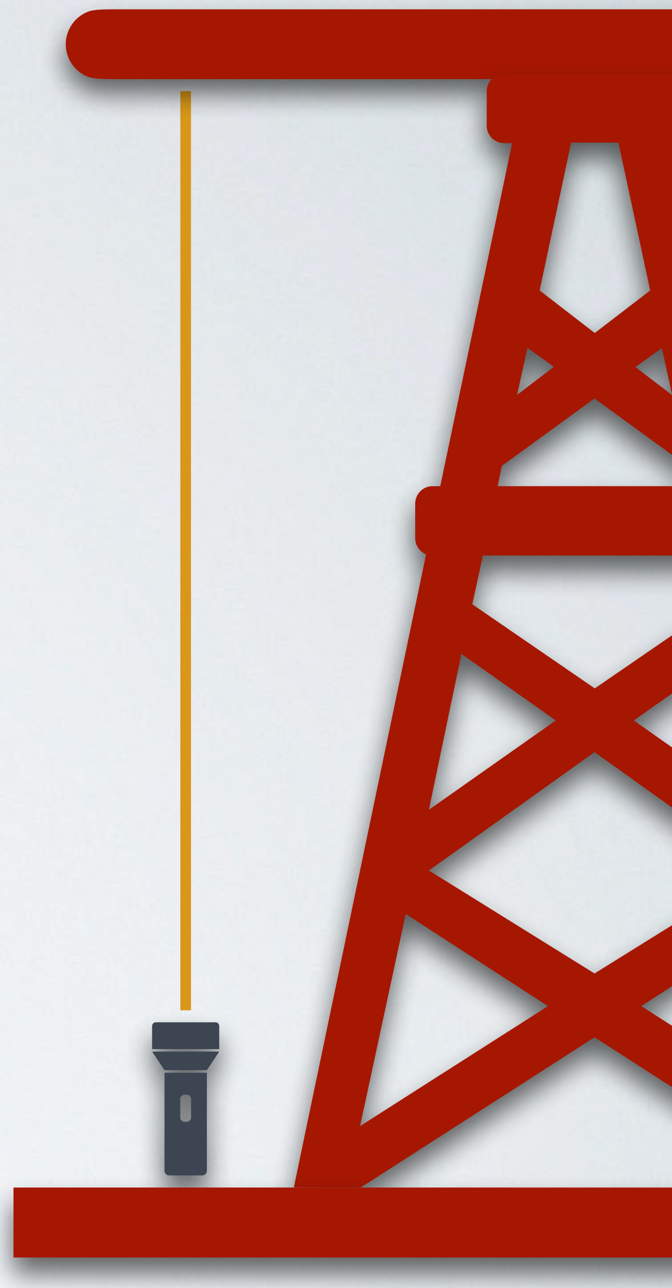
Einstein Cross



Images: NASA / HST

Part 3: Gravitational time dilation

Gravitational time dilation



Participation: Recap

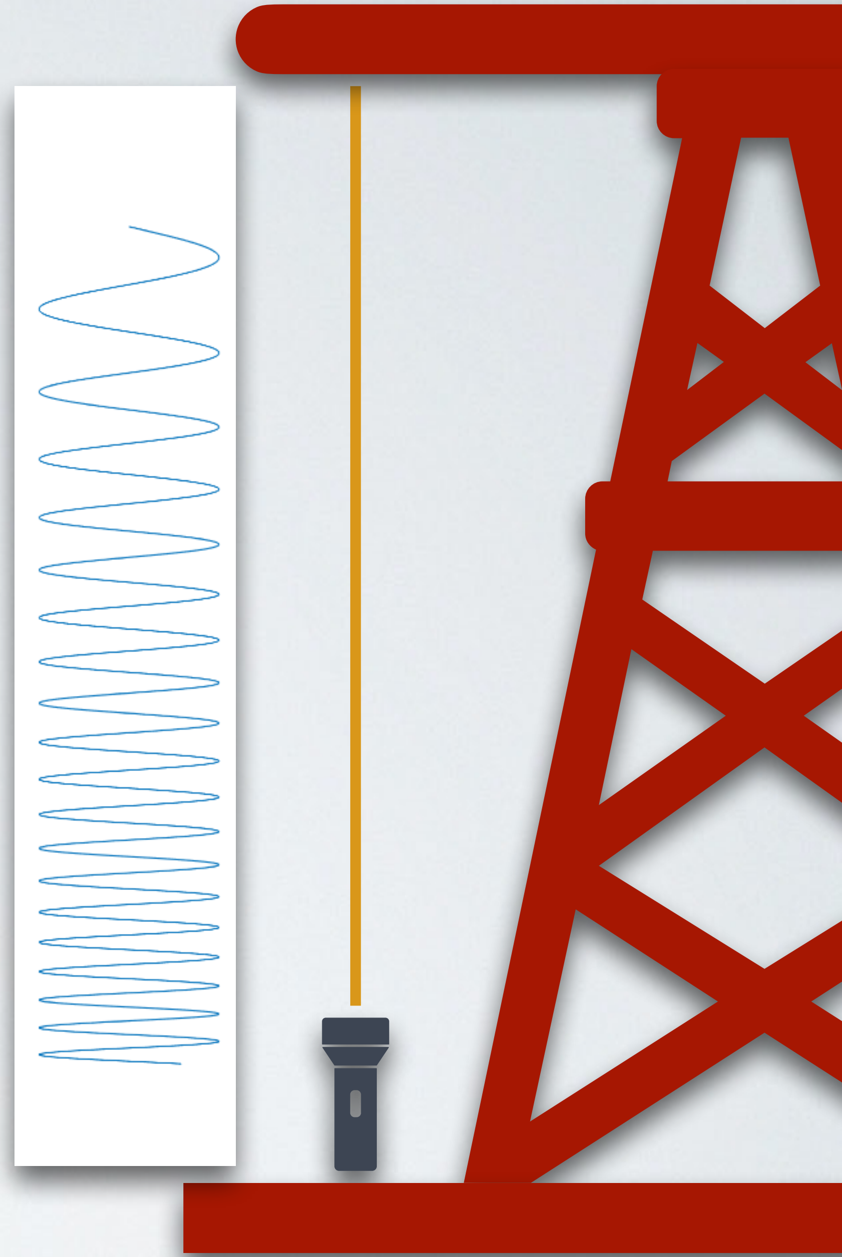
TurningPoint:
What happens to the beam sent upwards as seen by an observer far away?

Session ID: diemer

30 seconds

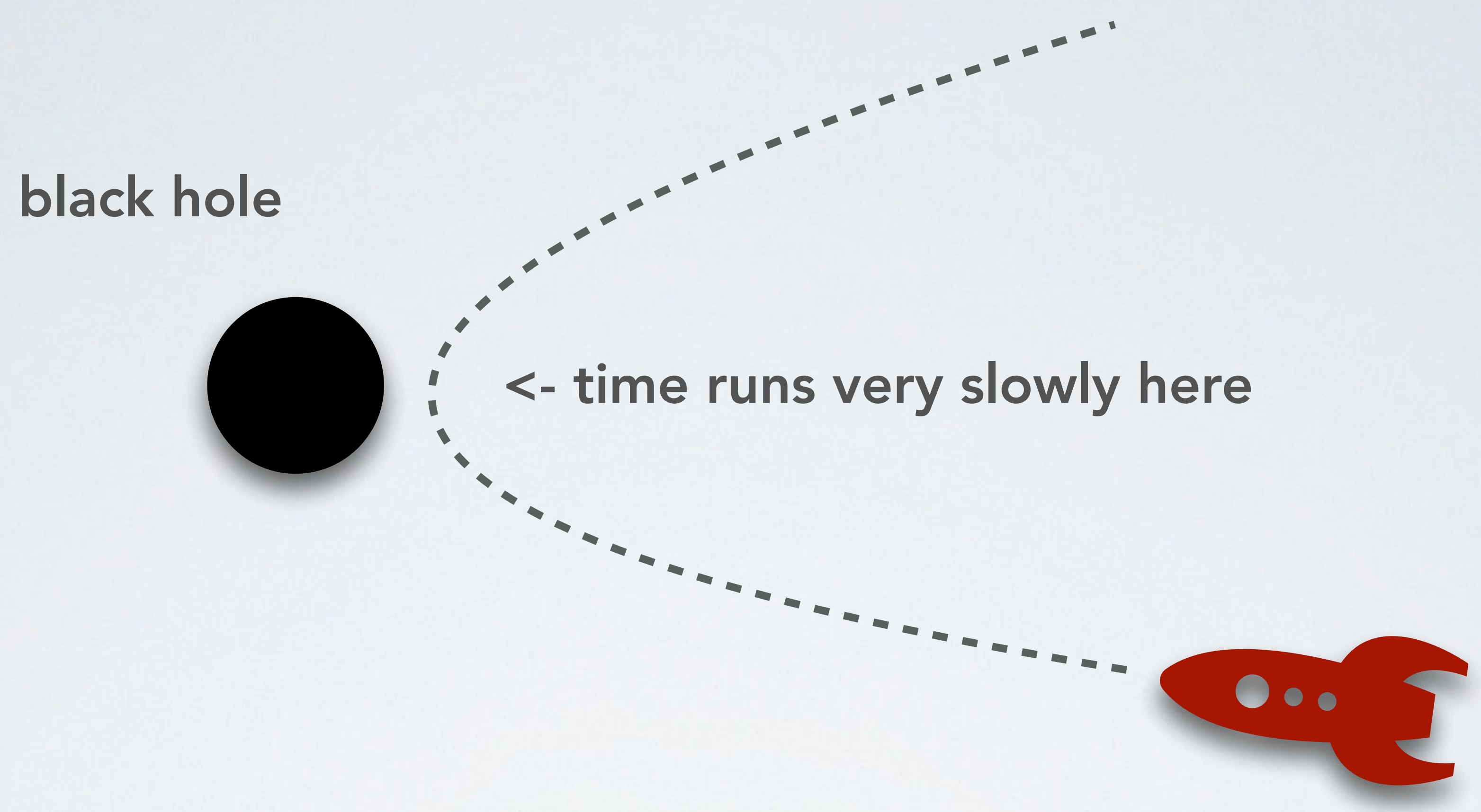
Gravitational time dilation

- Light beam loses energy as it climbs up (gravitational redshifting)
- **Frequency decreases**
- Imagine a **clock based on frequency** of laser light: 1 tick = time taken for fixed number of crests to pass
- Gravitational redshifting **slows down the clock**
- **Clocks in gravitational fields run slower**



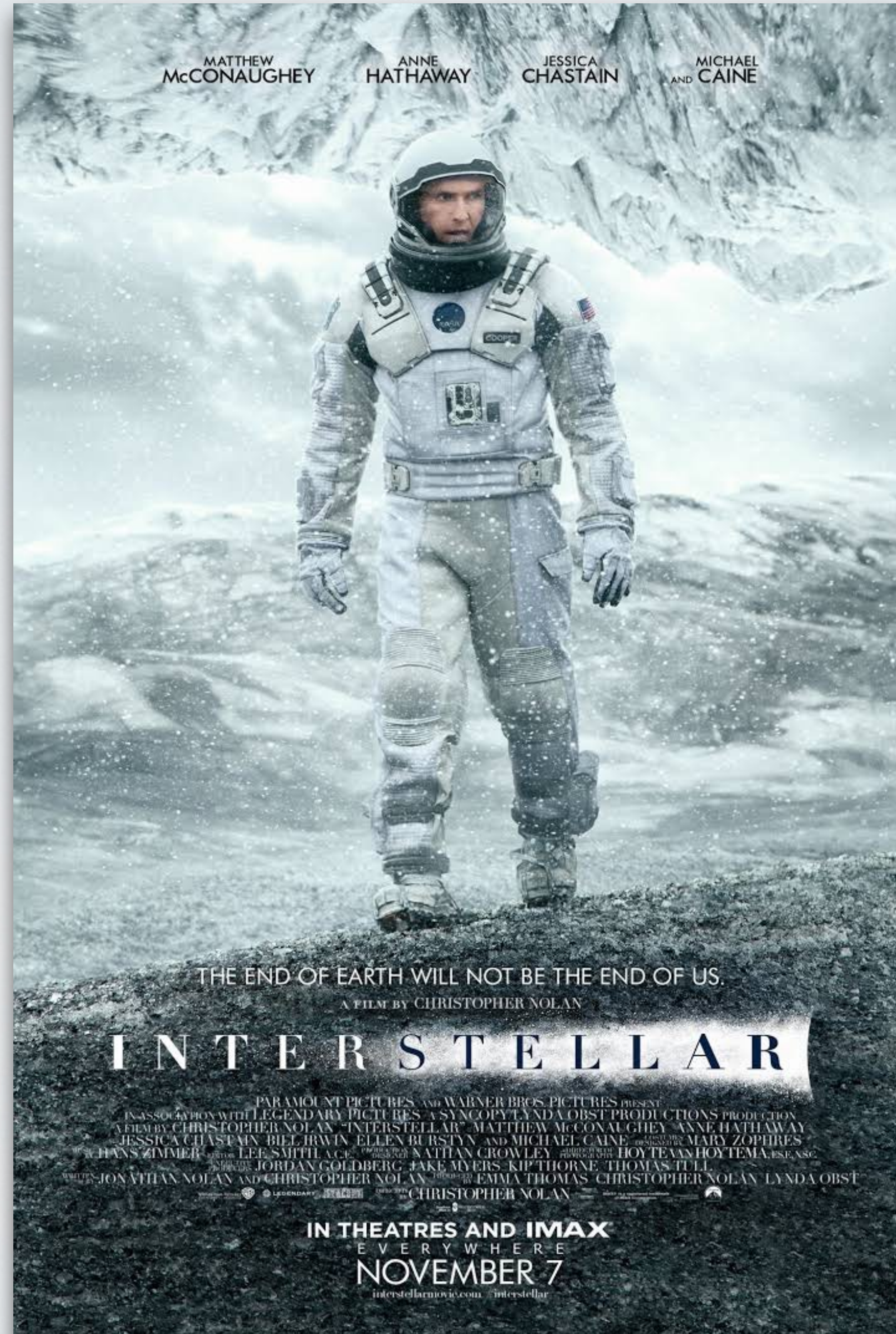
$$\Delta t_{\text{grav}} = \sqrt{1 - \frac{2GM}{c^2 r}}\Delta t_{\text{space}}$$

Gravitational time dilation



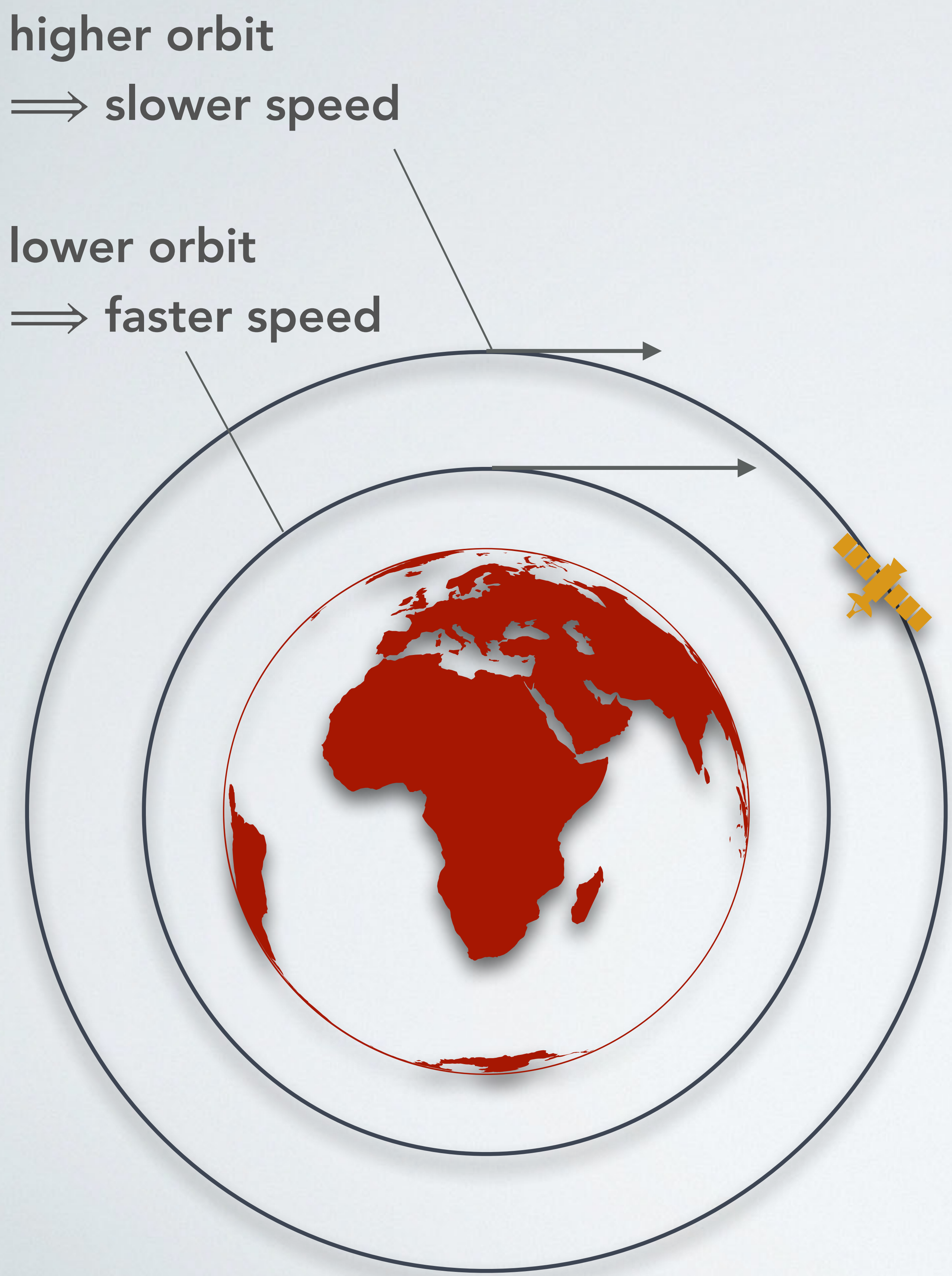
- Wanna live longer? Go where gravity is very strong!
- Observer on Earth would see astronaut's clock running very slowly when close to black hole; astronaut would age very slowly
- **No reciprocity!**

Gravitational time dilation




$M_{\mathrm{BH}} \approx 10^8 M_\odot$
Interstellar

GR & SR time dilation



As seen from Earth:

SR:

$$\Delta t_{\text{earth}} = \gamma \Delta t_{\text{sat}}$$

GR:

$$\Delta t_{\text{earth}} = \sqrt{1 - \frac{2GM}{c^2 r}} \Delta t_{\text{free-space}}$$

We're seeing satellite time go by slower, or our time faster, due to SR. But our time passes more slowly due to gravity (GR)!

Gravitational time dilation

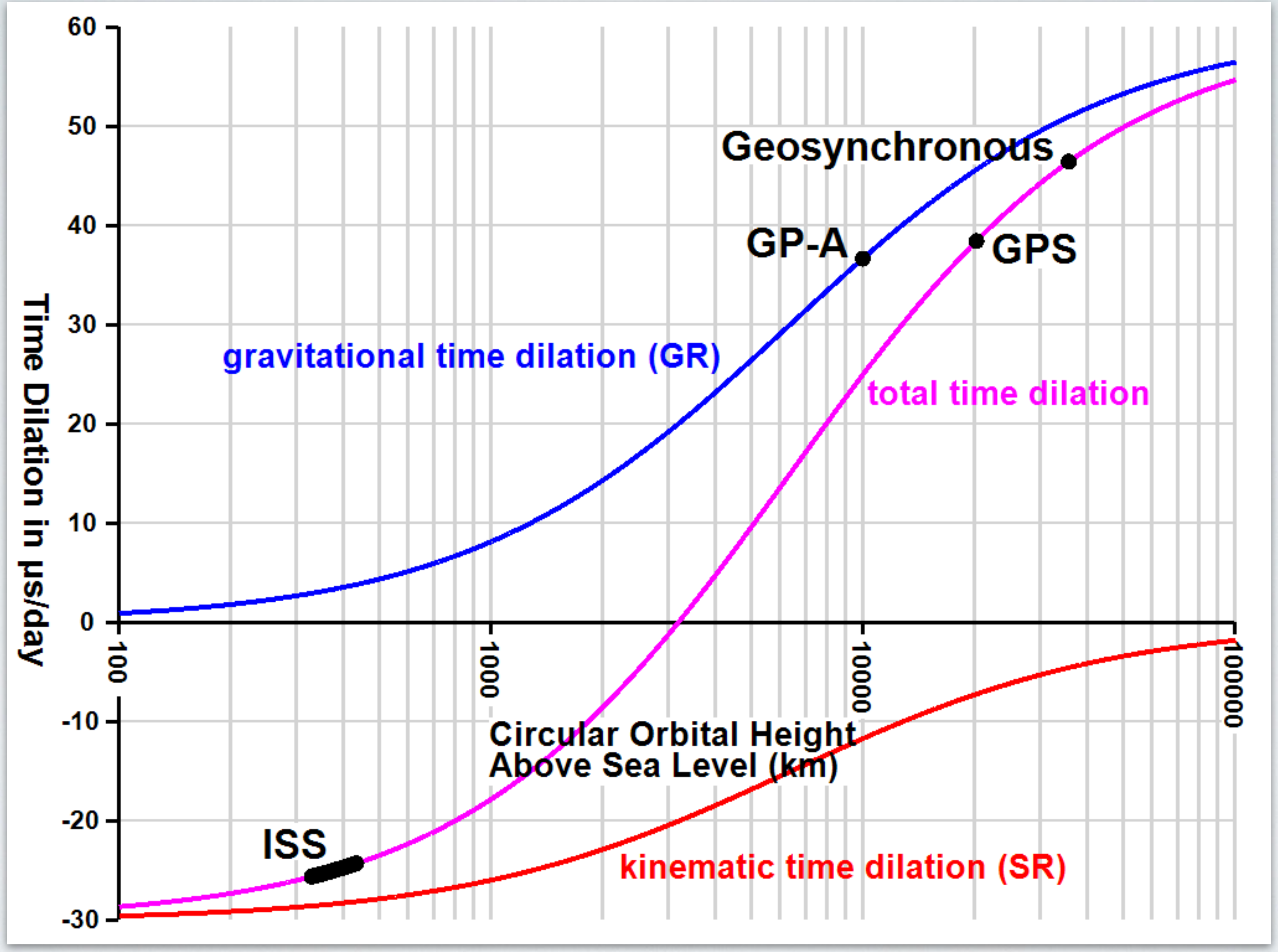
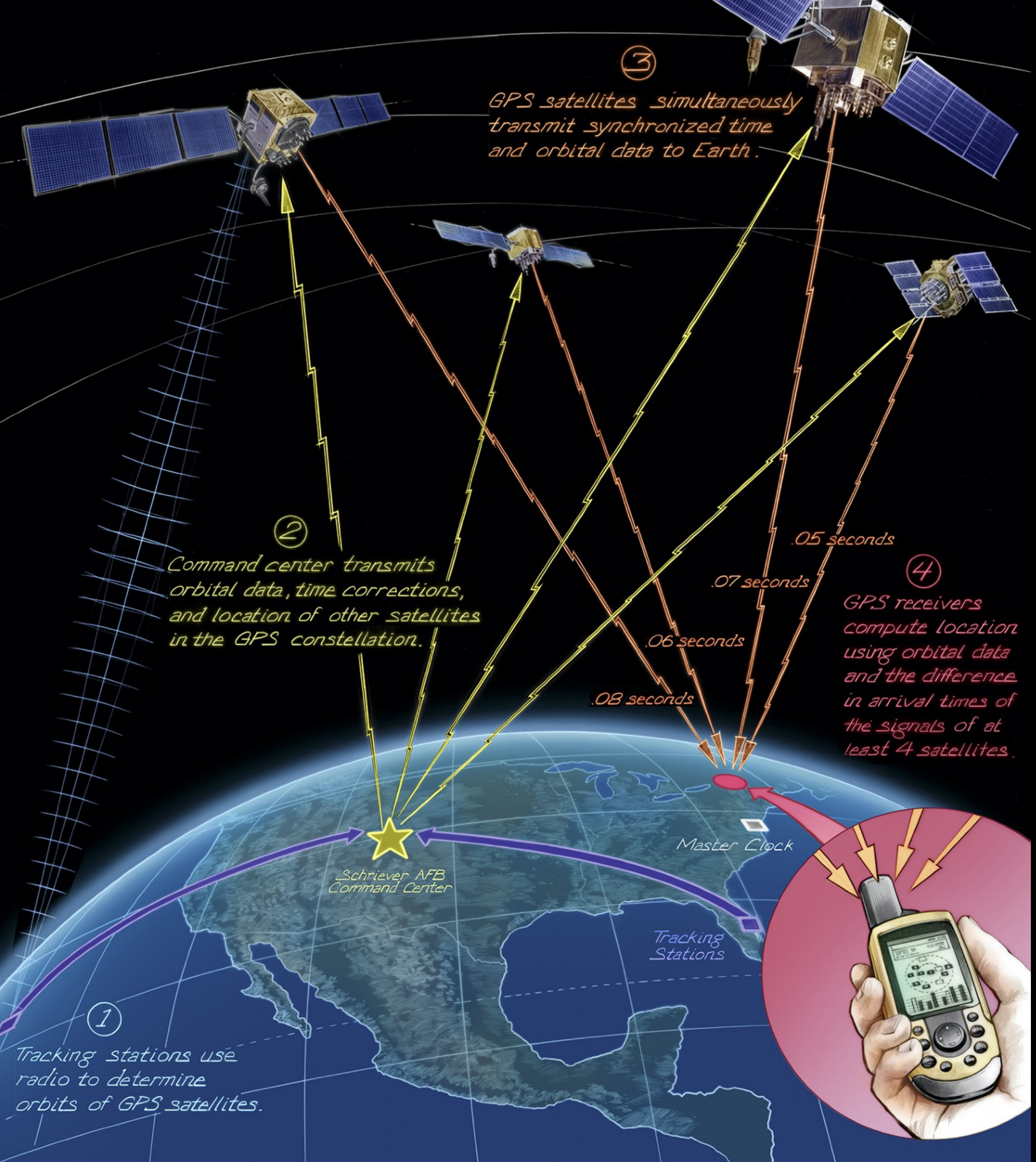


Image: Wikipedia

GPS

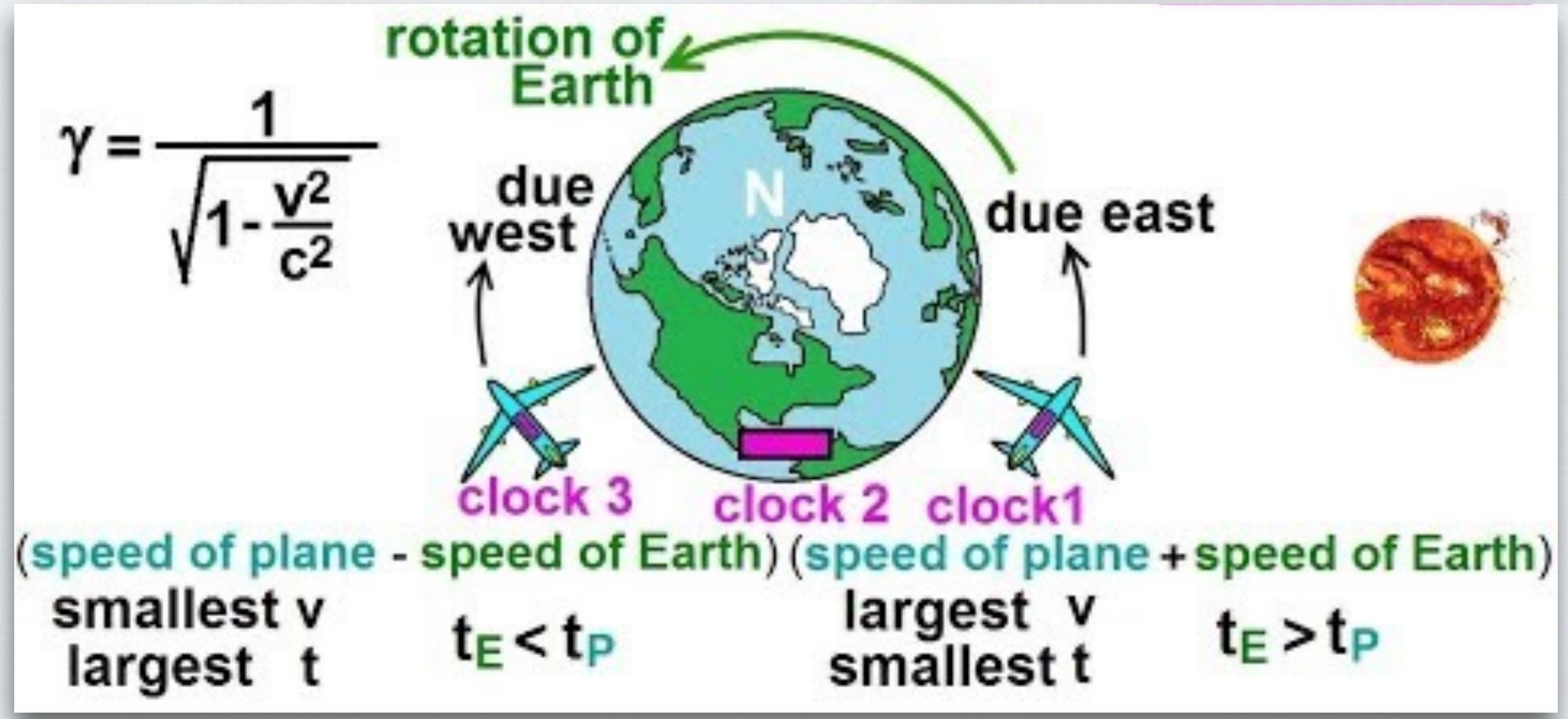


- 10m positioning requires **~30ns time accuracy**
- Satellites are at varying heights; **clocks run at varying rates**
- Satellite clocks drift by **~38μs per day** wrt Earth
- If GR effects were not included, GPS positions would drift from true position by km's per day!

Smithsonian Institution

Hafele-Keating experiment (1971)

- Fly atomic clocks around the world
- Compare to stationary clock
- Get both special and general relativistic effects
- Found **-59 / +273 nanoseconds** difference to ground
- Compatible with theory



Take-aways

- Free-falling observers and light move on **geodesics**, which are determined by the **metric** (geometry) of a spacetime
- Light also feels gravity, leading to gravitational **lensing**
- Clocks in gravitational fields run slower than clocks in free space, leading to **gravitational time dilation**

Next time…

We'll talk about:

- Modern Cosmology, finally!

Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #2 (by 10/07)

Reading:

- H&H Chapter 10