ASTR 340: Origin of the Universe

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Lecture 14 • Dark energy and the accelerated expansion

10/14/2021

Homework

“Ratio” can be expressed as a number (e.g., 0.4 / 0.8 = 0.5)

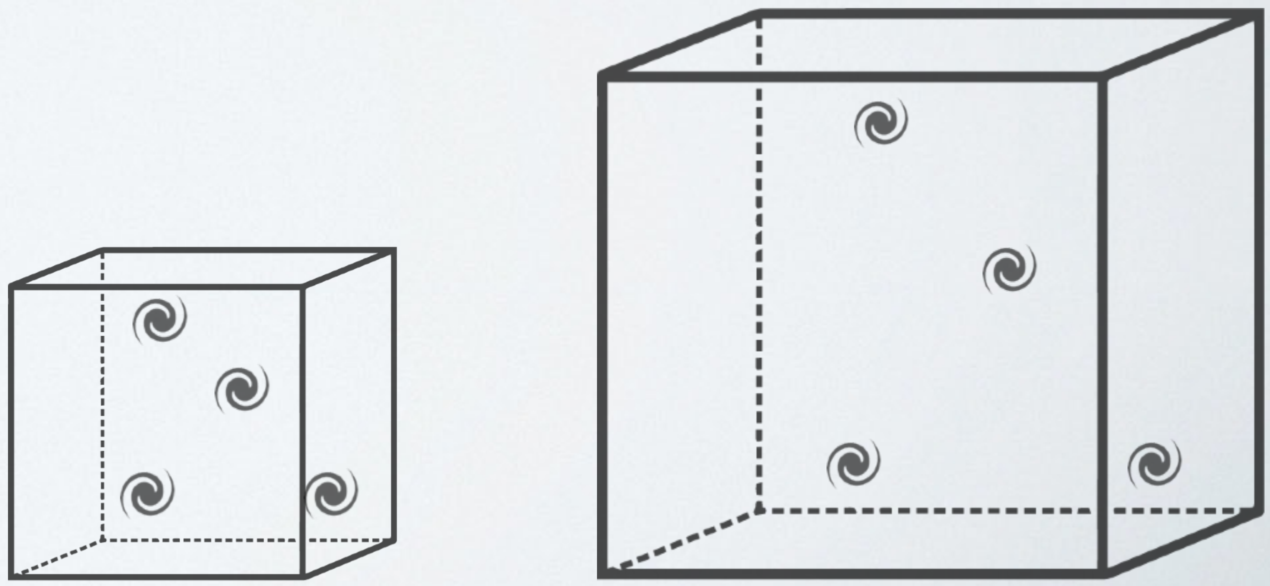
Part 0: Recap

Understanding the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2}$$

$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

- Assuming that no new matter is created or destroyed, the **density of matter decreases as space expands.**
- $\rho_0 = \rho(t_0)$ is the density today



Participation: Recap #1

TurningPoint:
What is the critical density?

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Critical density

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \quad \overset{k=0}{\Longrightarrow} \quad H^2 = \frac{8\pi G}{3}\rho$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

- For a given H_0, the Universe is **flat** if the density is the **critical density**

Critical density

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

1) Divide by H²:

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{kc^2}{a^2H^2}$$

$$= \frac{\rho}{\rho_c} - \frac{kc^2}{a^2H^2}$$

2) Define density parameter as fraction of critical density:

$$\Omega_m(t) \equiv \frac{\rho(t)}{\rho_c(t)}$$

3) Similarly define "density equivalent" for curvature:

$$\Omega_k(t) \equiv -\frac{kc^2}{a^2H^2}$$

4) Write total content of the Universe as:

$$\Omega_{tot} \equiv \Omega_m + \Omega_k = 1$$

In a Universe with only matter, density determines the geometry!

Participation: Recap #2

TurningPoint:

What happens to a Universe with Omega_m > 1?

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Participation: Recap #3

TurningPoint:

What is the curvature in a Universe with Omega_m > 1?

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Participation: Recap #4

TurningPoint:

What happens to the Hubble rate in a Universe with Omega_m > 1?

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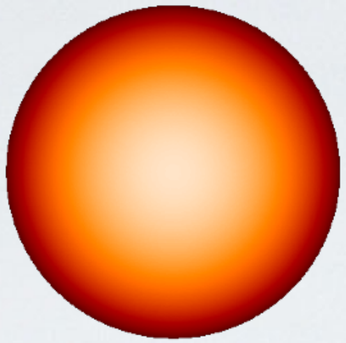
The fate of the Universe (with matter and curvature)

Case 1: Closed Universe

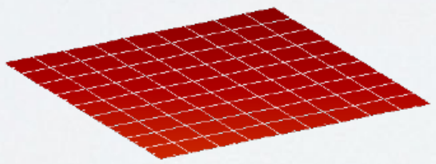
$$\Omega_m > 1 \implies \Omega_k < 0$$
$$\implies k > 0$$



Collapses eventually (big crunch)

Case 2: Flat Universe

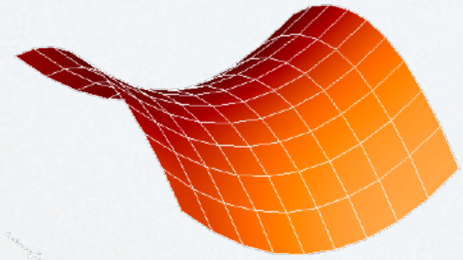
$$\Omega_m = 1 \implies \Omega_k = 0$$
$$\implies k = 0$$



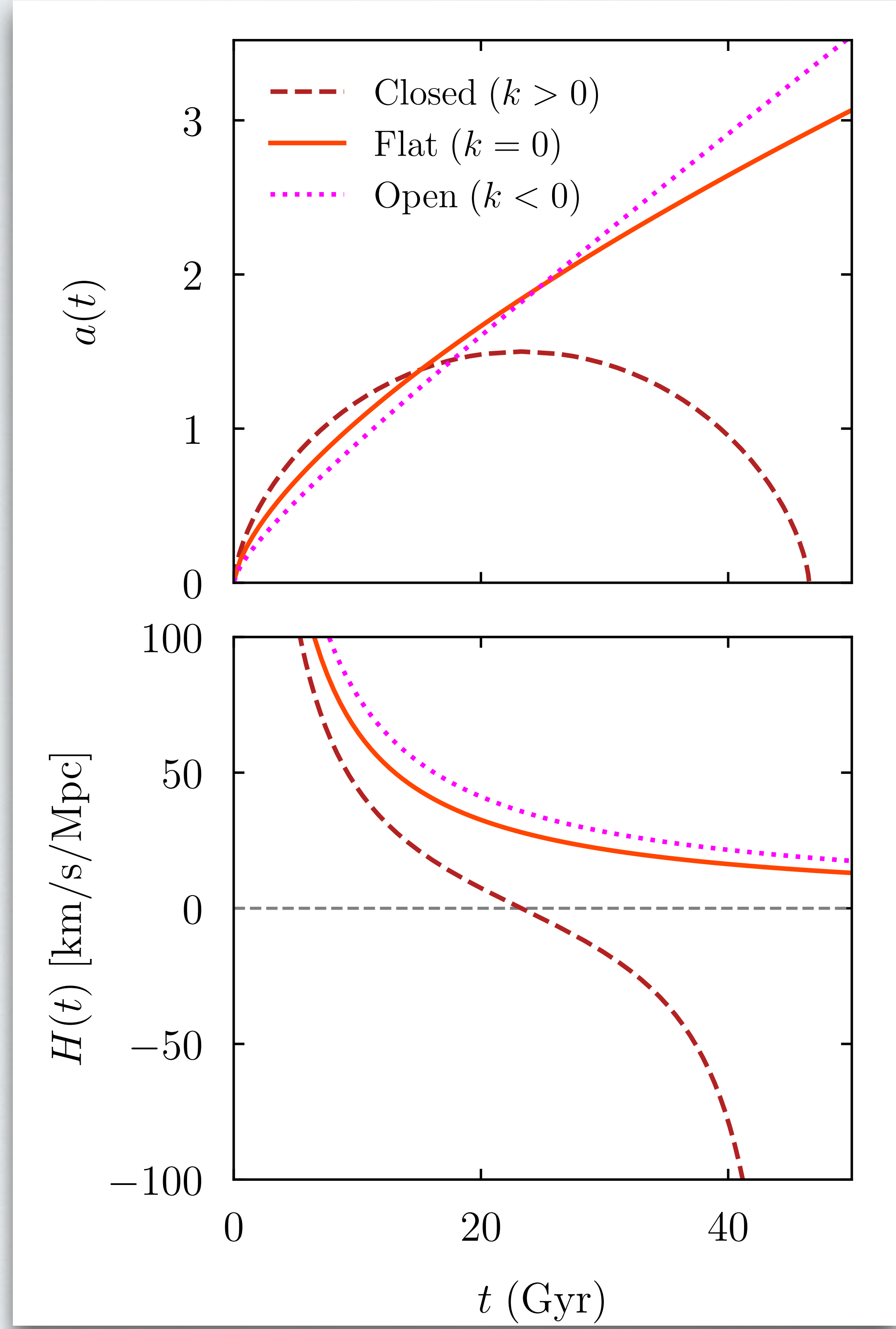
Slowly grinds to a halt

Case 3: Open Universe

$$\Omega_m < 1 \implies \Omega_k > 0$$
$$\implies k < 0$$



Expands forever



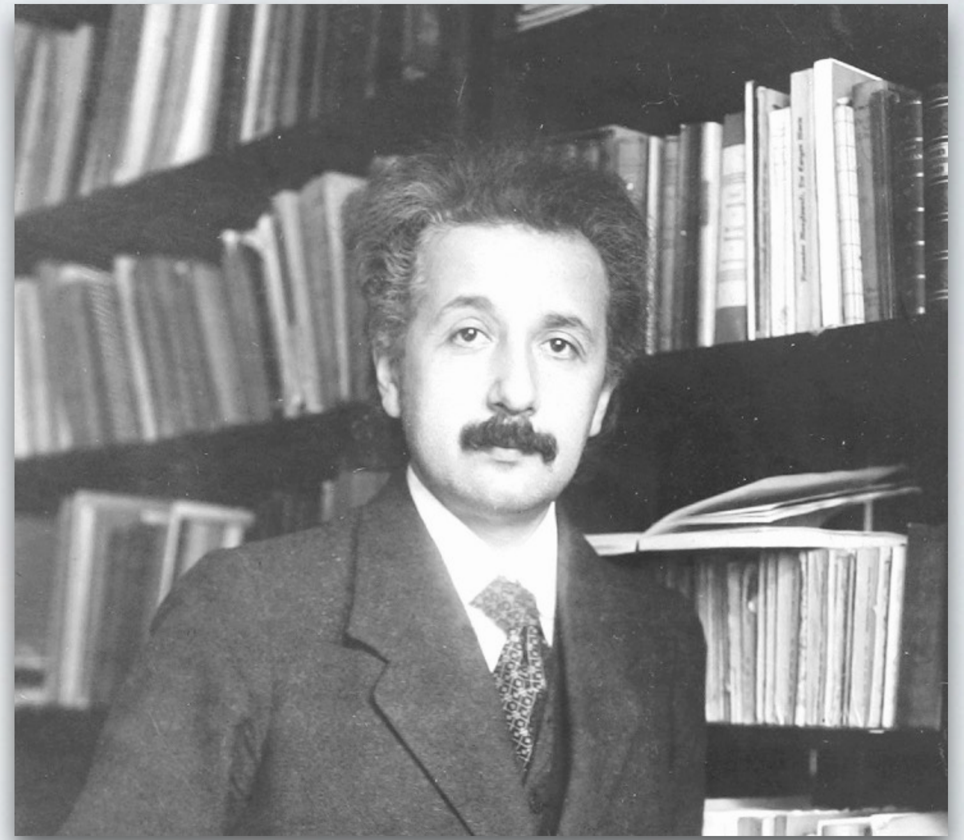
Today

- Cosmological constant
- Scenarios for the Universe
- The Universe we live in

Part 1: The cosmological constant

What happened to... Einstein?

- Einstein plugged the three homogeneous/isotropic cases of the FLRW metric into his equations of GR
- But this was before Hubble discovered the expansion; everybody thought the Universe was static
- For a static universe, $a(t) = \text{const}$, **only the spherical case ($k>0$)** worked as a solution to his equations
- If the Universe started off static, it would rapidly start collapsing because gravity attracts
- The only way to prevent collapse is to start off expanding; there would then be a phase of expansion followed by a phase of collapse
- Einstein could have used this logic to **predict** that the Universe must be expanding or contracting!
- Instead, Einstein **modified his GR equations**



Cosmological Constant

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

"Curvature tensor" that describes the curvature of 4D space-time

"Stress-energy tensor" that describes distribution of mass and energy

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

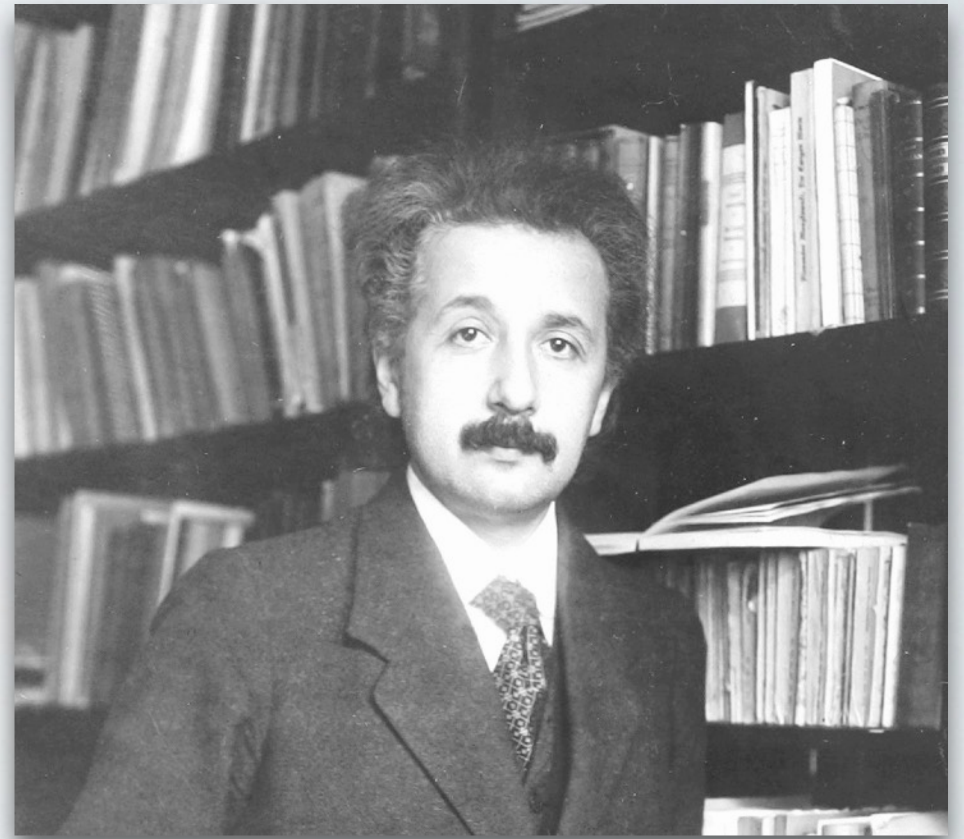
"Cosmological constant"

FLRW metric

- **Positive** cosmological constant **acts like repulsive force**
  - makes space expand faster
- **Negative** cosmological constant **acts like attractive force**
  - makes space expands more slowly
- **Constant** everywhere in space
- A (positive) cosmological constant is **one type of dark energy**
- **Dark energy is a property of space!**

The “greatest blunder”

- A positive cosmological constant can make the Universe static: it **balances gravity**
- But this situation is **unstable**:
  - if the Universe contracts slightly, gravity will increase and Λ decrease, leading to collapse
  - if the Universe expands slightly, gravity will decrease and Λ increase, leading to run-away expansion
- Soon after, Hubble discovered that the universe was expanding
- Einstein called the Cosmological Constant **“Greatest Blunder of My Life”**
- In 1998, however, the **accelerated expansion** was discovered — was he right after all?



Solving the Universe in GR - now with Λ

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Homogeneous distribution of matter + energy

FLRW metric

$$\Delta s_{\mathrm{FLRW,curved}} = \sqrt{(c\Delta t)^2 - a^2(t)\left[\frac{\Delta r^2}{1-kr^2} + r^2(\Delta\theta^2 + \sin^2(\theta)\Delta\phi^2)\right]}$$

Get same Friedmann equation as before, but with an **extra term:**

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

Balance of contributions

Friedmann equation with Λ:

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

Critical density:

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

Density parameters:

$$\Omega_m(t) \equiv \frac{\rho(t)}{\rho_c(t)}$$

$$\Omega_k(t) \equiv -\frac{kc^2}{a^2 H(t)^2}$$

Similarly define the density parameter for Λ

$$\Omega_\Lambda(t) \equiv \frac{\Lambda}{3H(t)^2}$$

Sum total of all contributions is still unity at all times:

$$\Omega_{tot} \equiv \Omega_m + \Omega_k + \Omega_\Lambda = 1$$

Friedmann equation relative to today

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

$$\rho(t) = \frac{\rho_0}{a^3(t)}$$

Compare everything to values today:

$$\Omega_{m,0} \equiv \frac{\rho_0}{\rho_{c,0}}$$

$$\Omega_{k,0} \equiv -\frac{kc^2}{H_0^2}$$

$$\Omega_{\Lambda,0} \equiv \frac{\Lambda}{3H_0^2}$$

Take Friedmann equation (with time dependence):

$$H^2 = \frac{8\pi G}{3}\frac{\rho_0}{a^3} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

Divide by H_0 to make it relative to today:

$$\left(\frac{H}{H_0}\right)^2 = \frac{8\pi G}{3H_0^2}\frac{\rho_0}{a^3} - \frac{kc^2}{a^2 H_0^2} + \frac{\Lambda}{3H_0^2}$$

Obtain a more compact Friedmann equation:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \Omega_{\Lambda,0}$$

Participation: Cosmological Constant

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\mathrm{m},0}}{a^3} + \frac{\Omega_{\mathrm{k},0}}{a^2} + \Omega_{\Lambda,0}$$

TurningPoint:

What happens to the Hubble rate in a Universe dominated by a cosmological constant?

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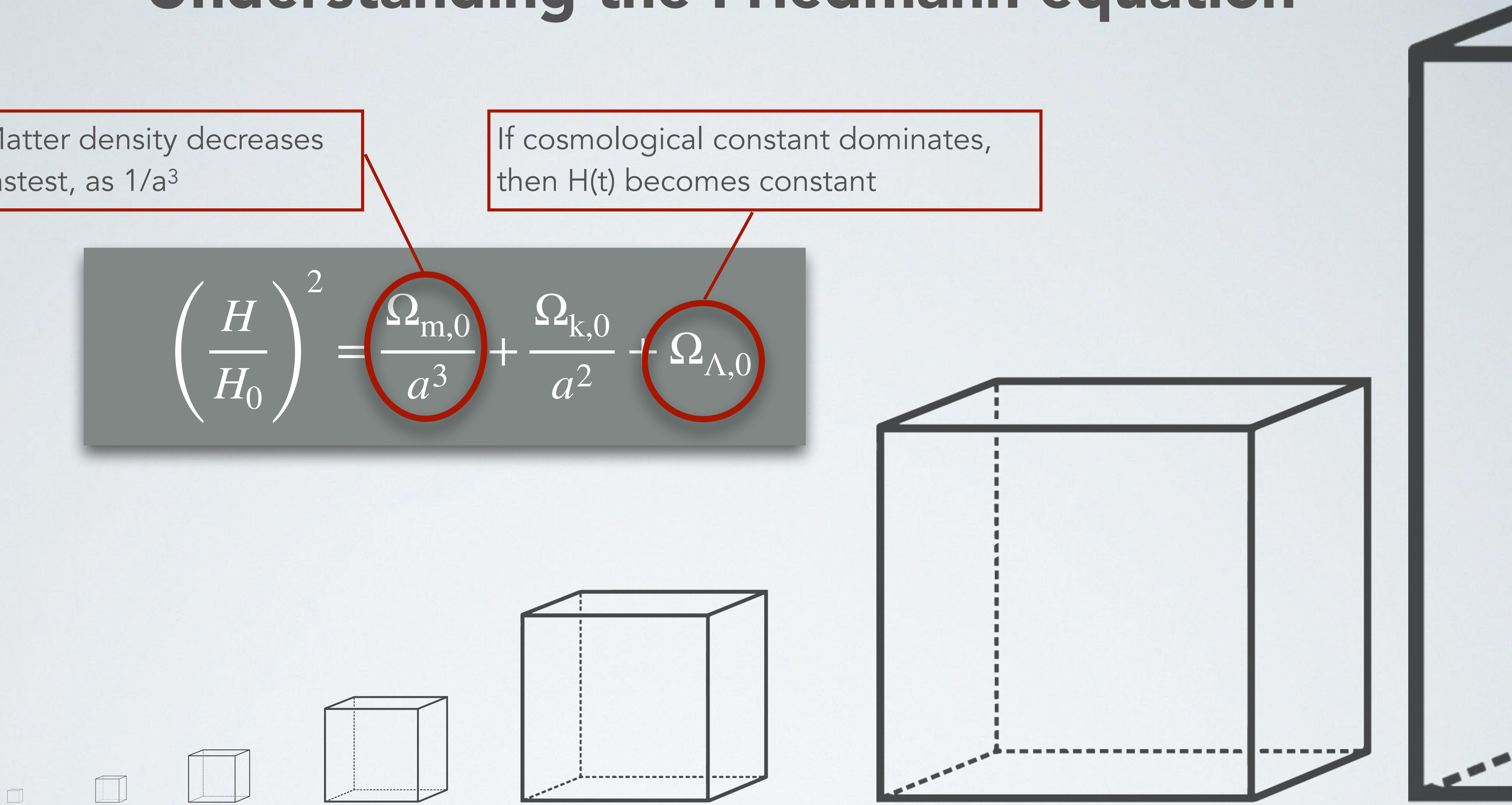
Understanding the Friedmann equation

Matter density decreases fastest, as $1/a^3$

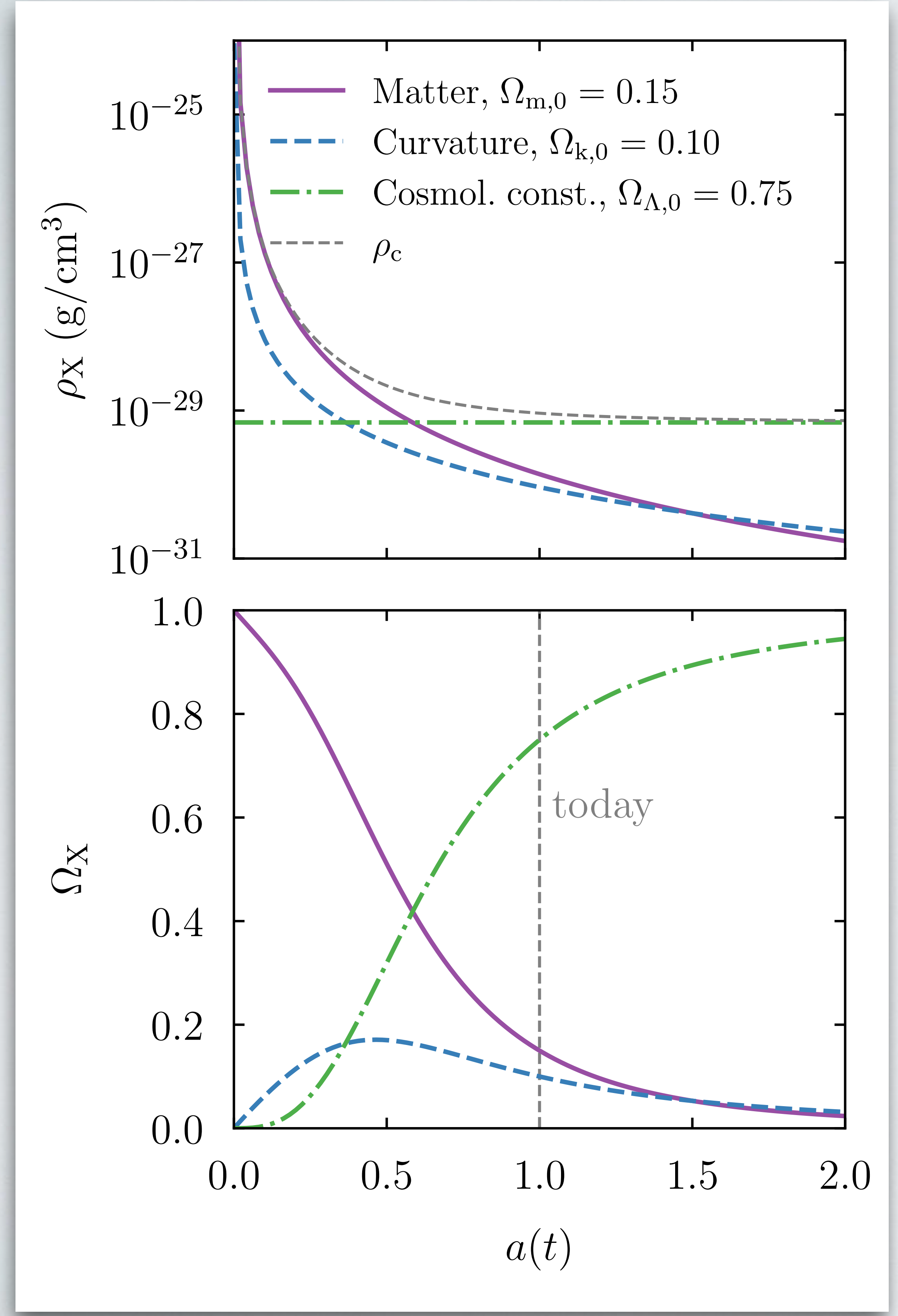
If cosmological constant dominates, then H(t) becomes constant

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{m,0}}{a^3} + \frac{\Omega_{k,0}}{a^2} + \Omega_{\Lambda,0}$$



- Positive Λ can create accelerating expansion
- Pure Λ means constant H(t), and thus exponential expansion

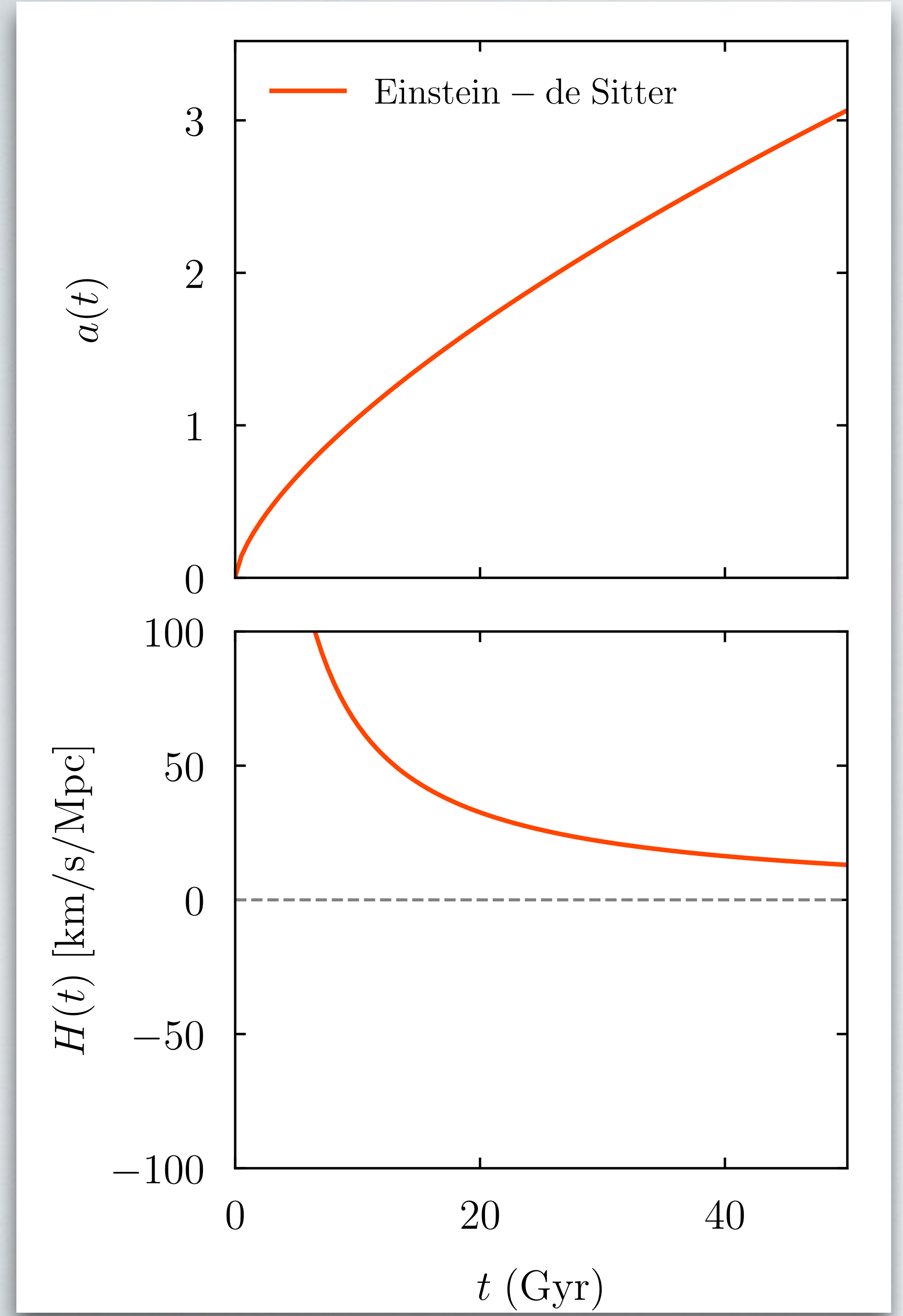
Understanding the Friedmann equation



- Density of matter decreases as $1/a^3$, whereas density of Λ is constant (by definition)
- As a result, the fractional density of the cosmological constant, Ω_Λ, increases whereas that of matter decreases
- The graph is for an unrealistic example Universe

Part 2: Scenarios for the Universe

Flat matter-only Universe ("Einstein-de Sitter")



$$\Omega_{\rm m} = 1 \qquad \Omega_{\rm k} = 0 \qquad \Omega_\Lambda = 0$$

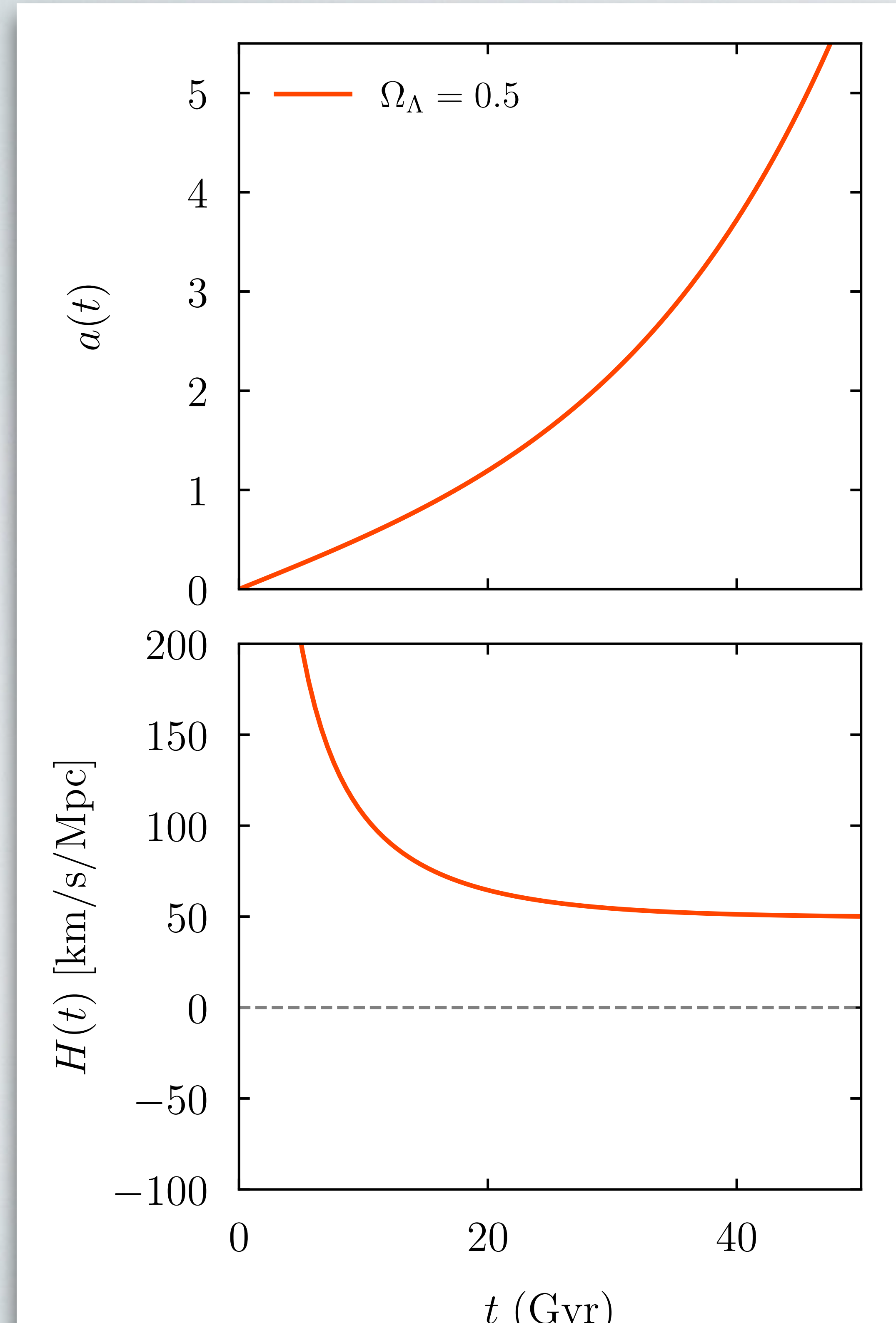
$$\implies a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\left(\frac{H}{H_0}\right)^2 = \frac{1}{a^3} \implies H(t) = \frac{2}{3t}$$

$$\implies t = \frac{2}{3H} = \frac{2}{3} t_{\rm H}$$

- a(t) keeps growing, but more and more slowly
- True age of Universe is 2/3 of Hubble time

Dark energy only ("de Sitter")



$$\Omega_m = 0 \quad \Omega_\Lambda > 0 \quad \Omega_k = 1 - \Omega_\Lambda$$

$$a(t) \rightarrow e^{H_0(t-t_0)}$$

- Initially, curvature matters, then Hubble rate becomes constant
- Exponential growth

Early and late times

If there is any matter, there is an early time (a << 1) where matter dominates

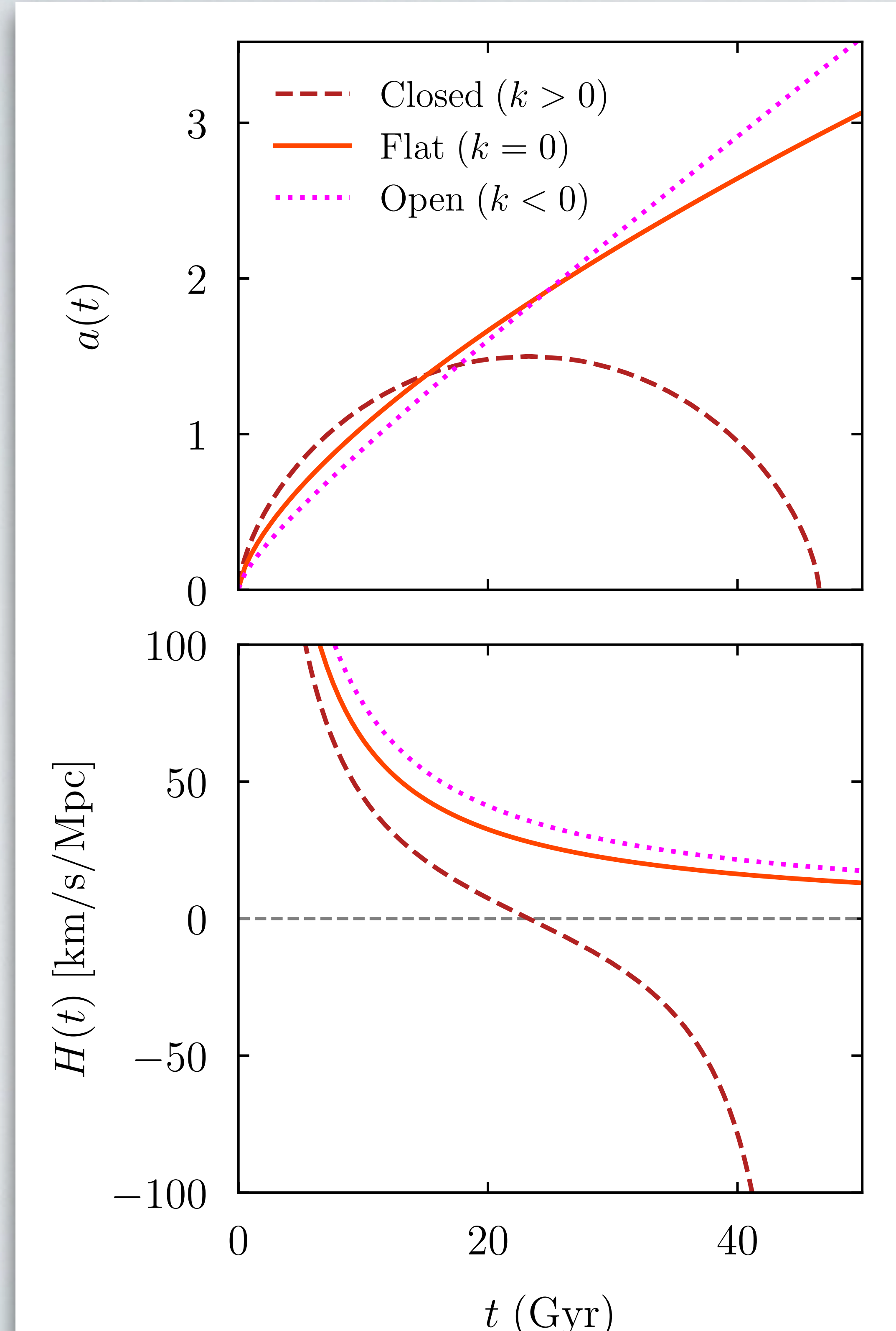
If there is any Λ and the Universe does not collapse, there is a late time (a >> 0) where Λ dominates

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{\mathrm{m},0}}{a^3} + \frac{\Omega_{\mathrm{k},0}}{a^2} + \Omega_{\Lambda,0}$$

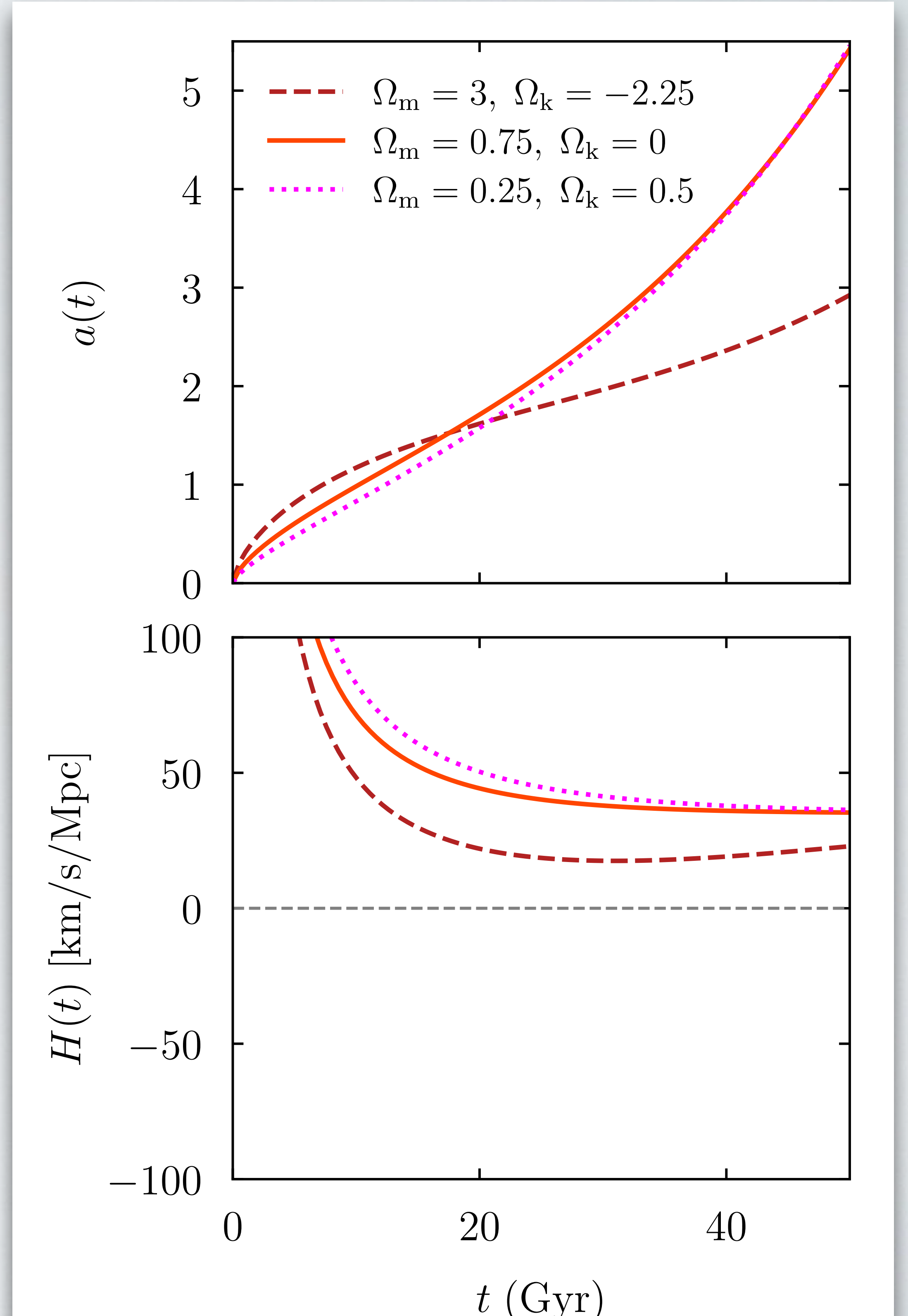
- In the **beginning**, the Universe behaves like a **flat matter-only** Universe (except for photons, which we have not included yet)
- At **late** times, the Universe **expands exponentially** if Λ>0

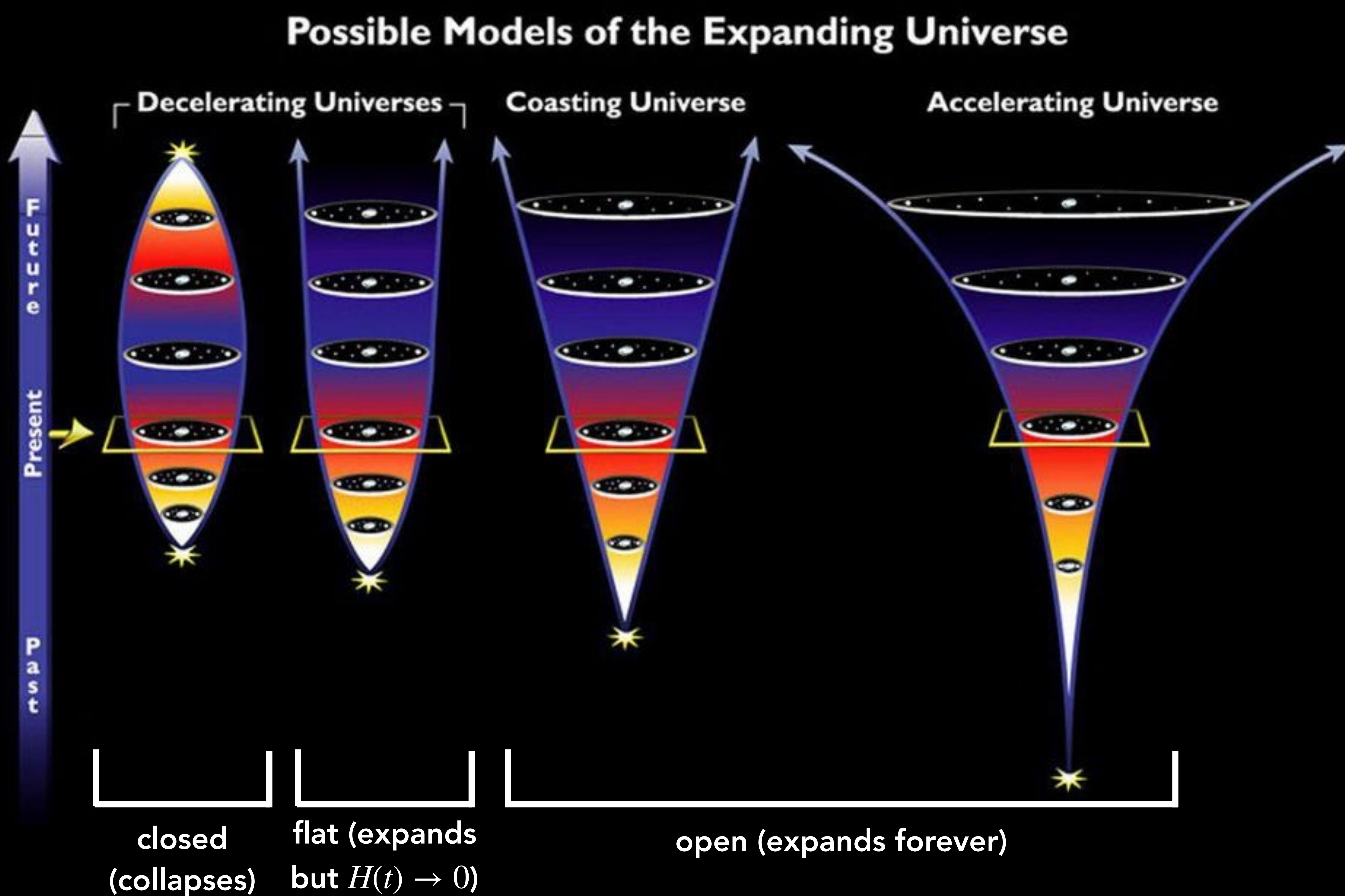
Matter + Dark Energy

No dark energy ($\Omega_m + \Omega_k$)



With 25% dark energy ($\Omega_m + \Omega_k + \Omega_\Lambda$)





With dark energy, even positively curved Universes can be open!

Image: Ethan Siegel

Part 3: The Universe we live in

Participation: Our Universe #1

TurningPoint:

Which component dominates in our Universe today?

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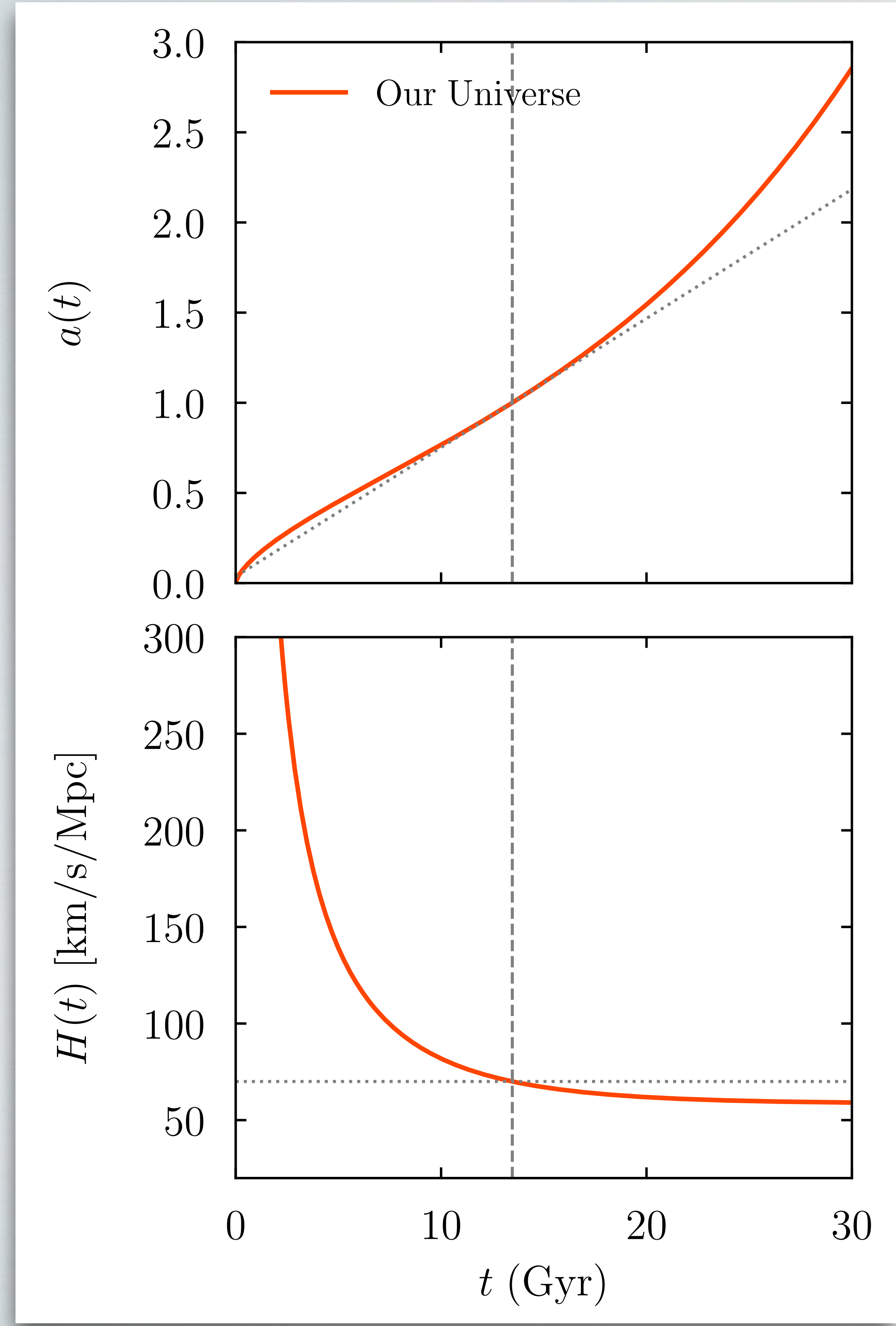
30 seconds

Our Universe



$\Omega_{m,0} \approx 0.3$

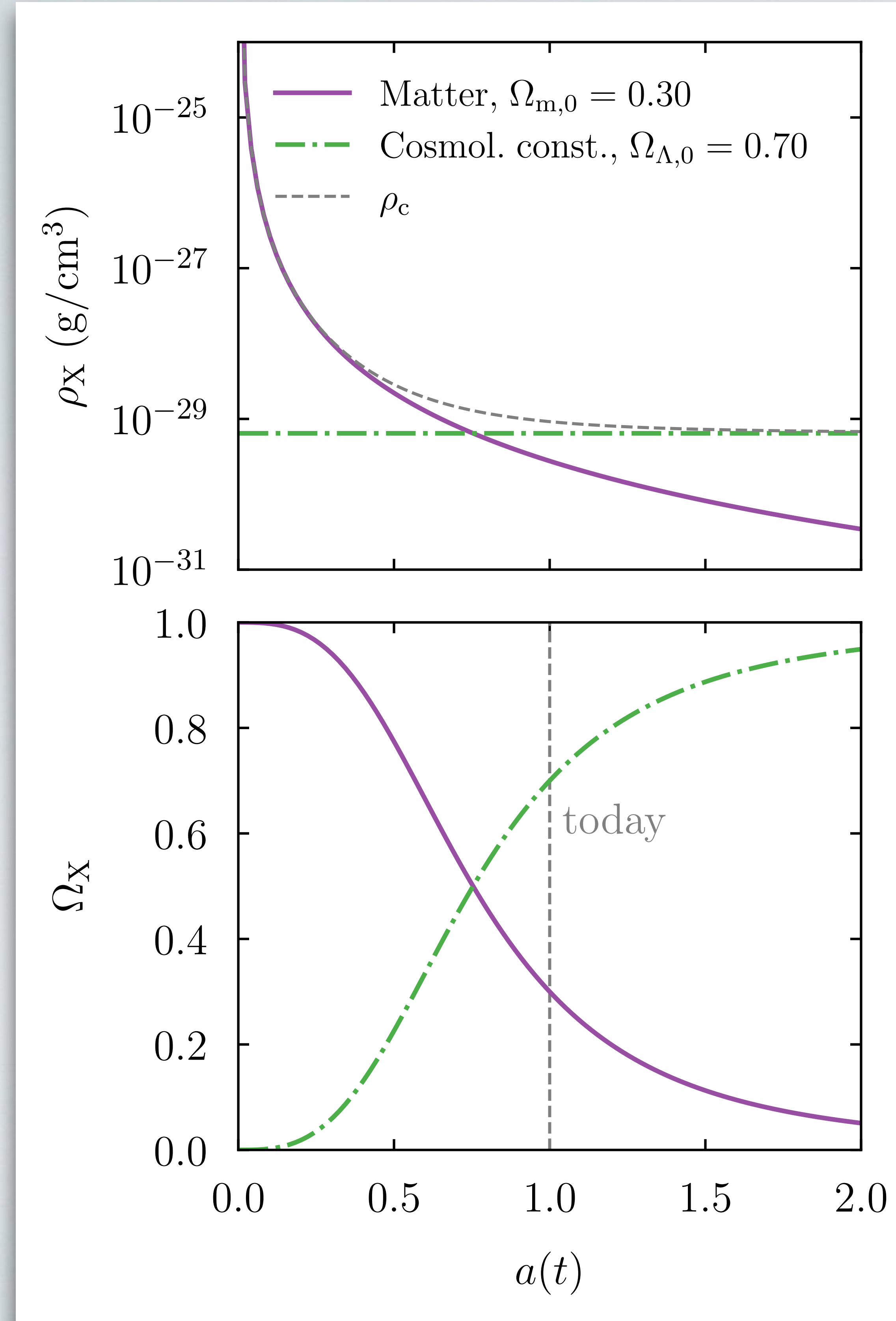
$\Omega_{\Lambda,0} \approx 0.7$

$\Omega_{k,0} \approx 0$

$H_0 \approx 70$ km/s/Mpc

- **Flat** (as far as we can tell)
- Dominated by **dark energy** (since $t \approx 10$ Gyr)
- DE looks like **cosmological constant**
- Will undergo **accelerated expansion** forever (unless we're missing something)
- Hubble time is (coincidentally) quite close to true age of Universe

Understanding the Friedmann equation



$$\Omega_{m,0} \approx 0.3$$
$$\Omega_{\Lambda,0} \approx 0.7$$
$$\Omega_{k,0} \approx 0$$
$$H_0 \approx 70 \text{ km/s/Mpc}$$

- Matter dominates in the beginning
- About 10 Gyr after the Big Bang ($a \approx 0.75$, $z \approx 0.3$) DE (the cosmological constant) becomes the dominant component
- DE will continue to become more dominant in the future

Participation: Our Universe #2

TurningPoint:
Given what we have learned, do you think our Universe is finite or infinite?

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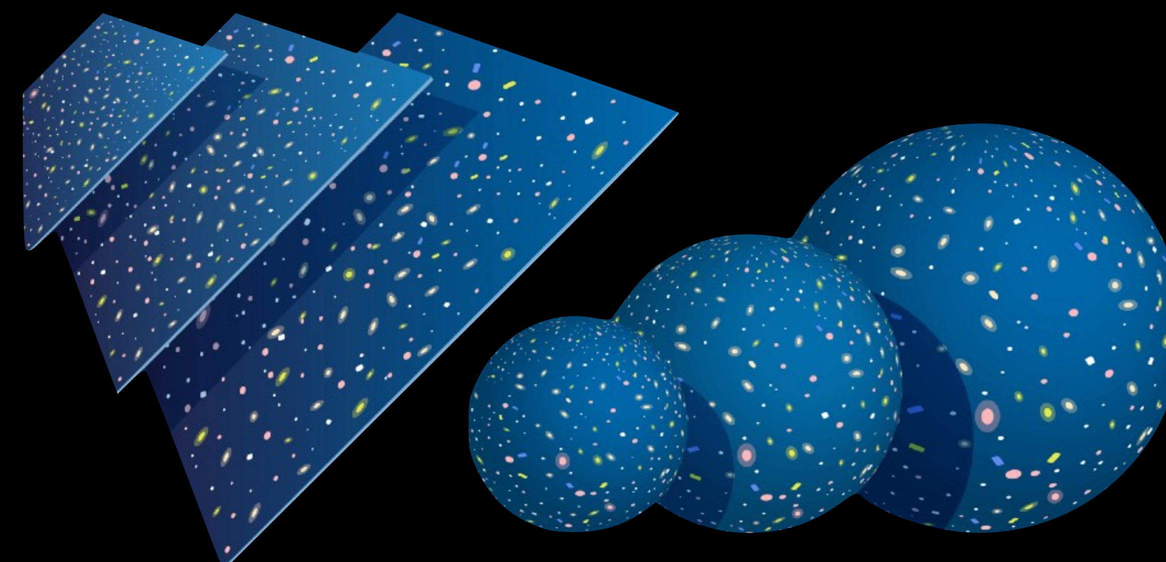
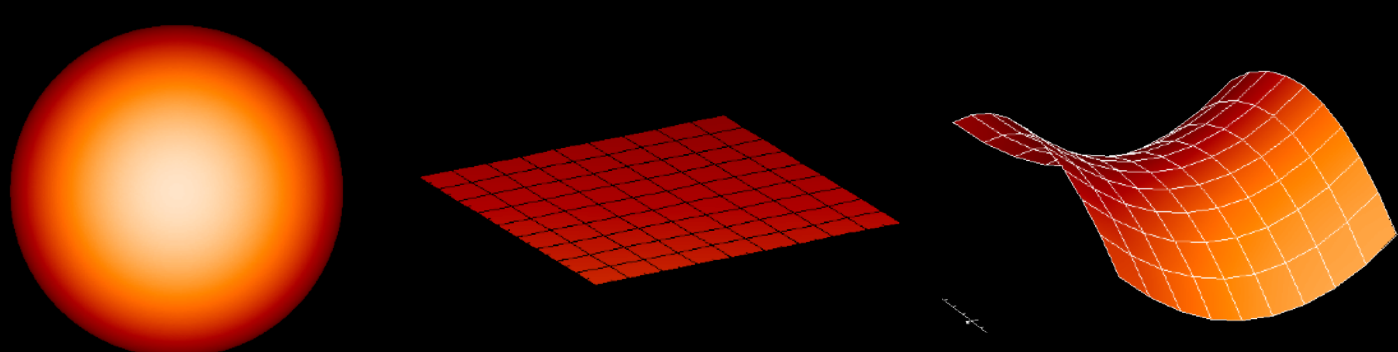
Is the Universe finite or infinite?

- If the Universe is **positively curved**, it is **finite**
- If the Universe is **flat or negatively curved**, it is probably **infinite**
- However, it could theoretically have a non-simple ("multiply connected") geometry, which could be finite
- We can only test flatness, homogeneity, and isotropy within the part of the Universe that we can see





Images: Quanta Magazine / Forbes

Take-aways

- A **cosmological constant** is one possible type of dark energy; it is a property of space itself
- Once the cosmological constant dominates the energy density, it leads to **exponential expansion** forever
- As far as we know, we live in a **flat matter-Λ Universe** which has recently become dominated by dark energy

Next time...

We'll talk about:

- The very early Universe (after the midterm)

Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #3 (by Wednesday 10/20)

Reading:

- H&H Chapters 1-3, 6-8, 10-11 (for midterm)