ASTR 340: Origin of the Universe

Prof. Benedikt Diemer

Lecture 23 • Black holes

11/23/2021

Logistics

- No office hours this week.
  Happy Thanksgiving!

Recap

Participation: Recap #1

TurningPoint:
Roughly what does the Einstein field equation say?

Session ID: diemer

30 seconds

General relativity

- Within free-falling frames of reference, Special Relativity applies
- Free-falling particles or observers move on geodesics (shortest paths) through curved space-time
- The distribution of matter and energy determines how space-time is curved

Space-time curvature tells matter/energy how to move, matter/energy tells space-time how to curve

Einstein field equation

Newton's gravitational constant

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

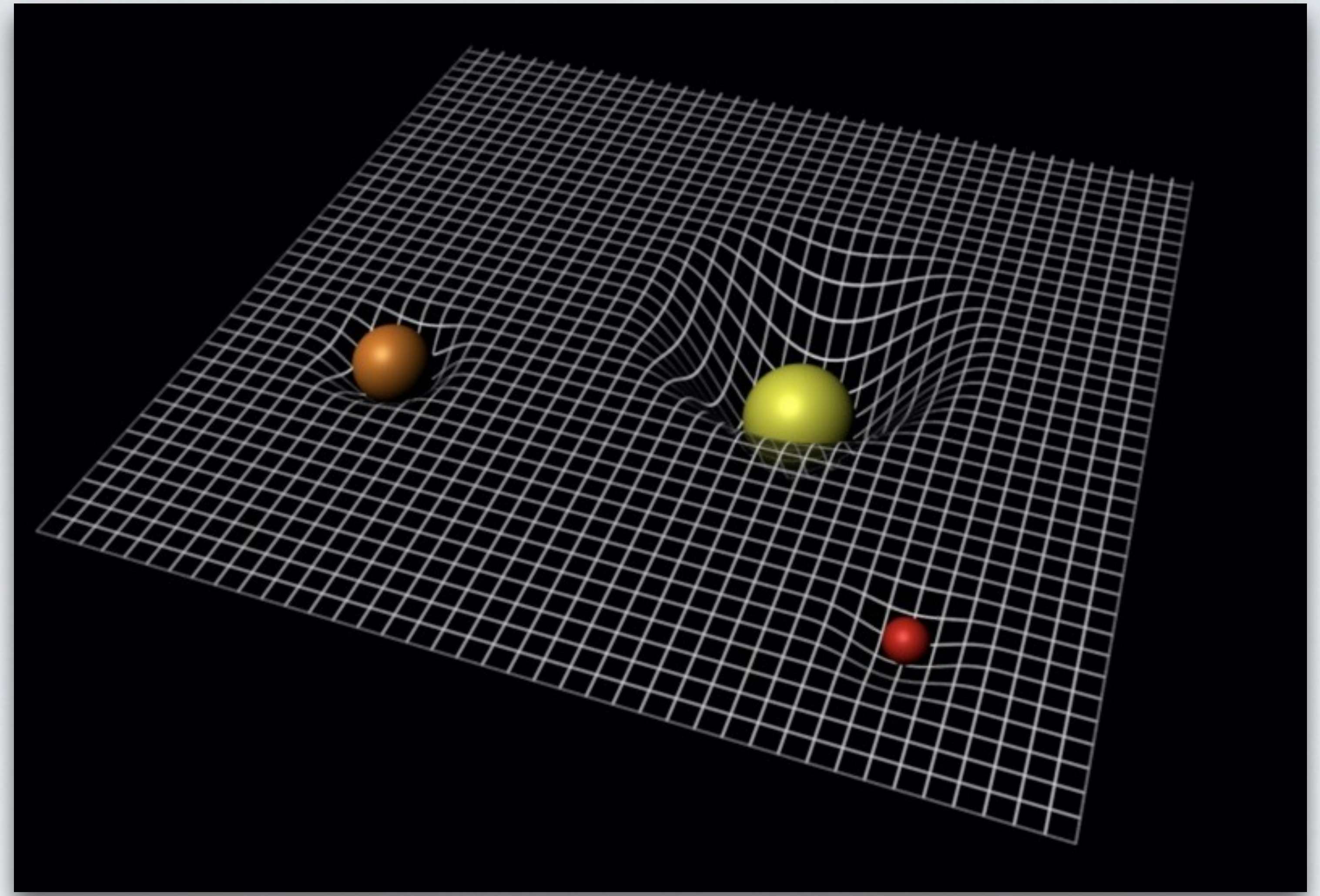
"Curvature tensor" that describes the curvature of 4D space-time

speed of light

"Stress-energy tensor" that describes distribution of mass and energy

- Geometry = constant * (matter + energy)
- G and T can be written in terms of **components**, similar to a matrix
- Horrendous set of **10 coupled equations!**
- For weak gravitational fields, this **reduces to Newton's law** of gravitation, to an excellent approximation

Warped spacetime



- Two-dimensional space as an analogy: rubber sheet with weights
- Amount that sheet sags depends on how heavy weight is
- Lines that would be **straight** become **curved (to external observer)** when sheet is "weighted"

Invariant spacetime interval

- Recall spacetime interval in flat space:
  - Invariant interval is equivalent to **c times proper time interval**
  - **Shorter** when traveling **faster!**
  - Space-time interval is **zero** for any two points on **light** world line

$$\Delta s_{\text{flat}} = \sqrt{(c\Delta t)^2 - \Delta x^2} = c\Delta t_{\text{p}}$$

- Generalize to **curved spacetime**
- Free-falling observers are like inertial obs. in SR, they have **maximal Δs**
- **Light** still moves on **"null" geodesics** with $\Delta s = 0$
- Spacetime distance is more complicated and described by **metric**

Participation: Recap #2

TurningPoint:

What are geodesics on the Earth's surface like?

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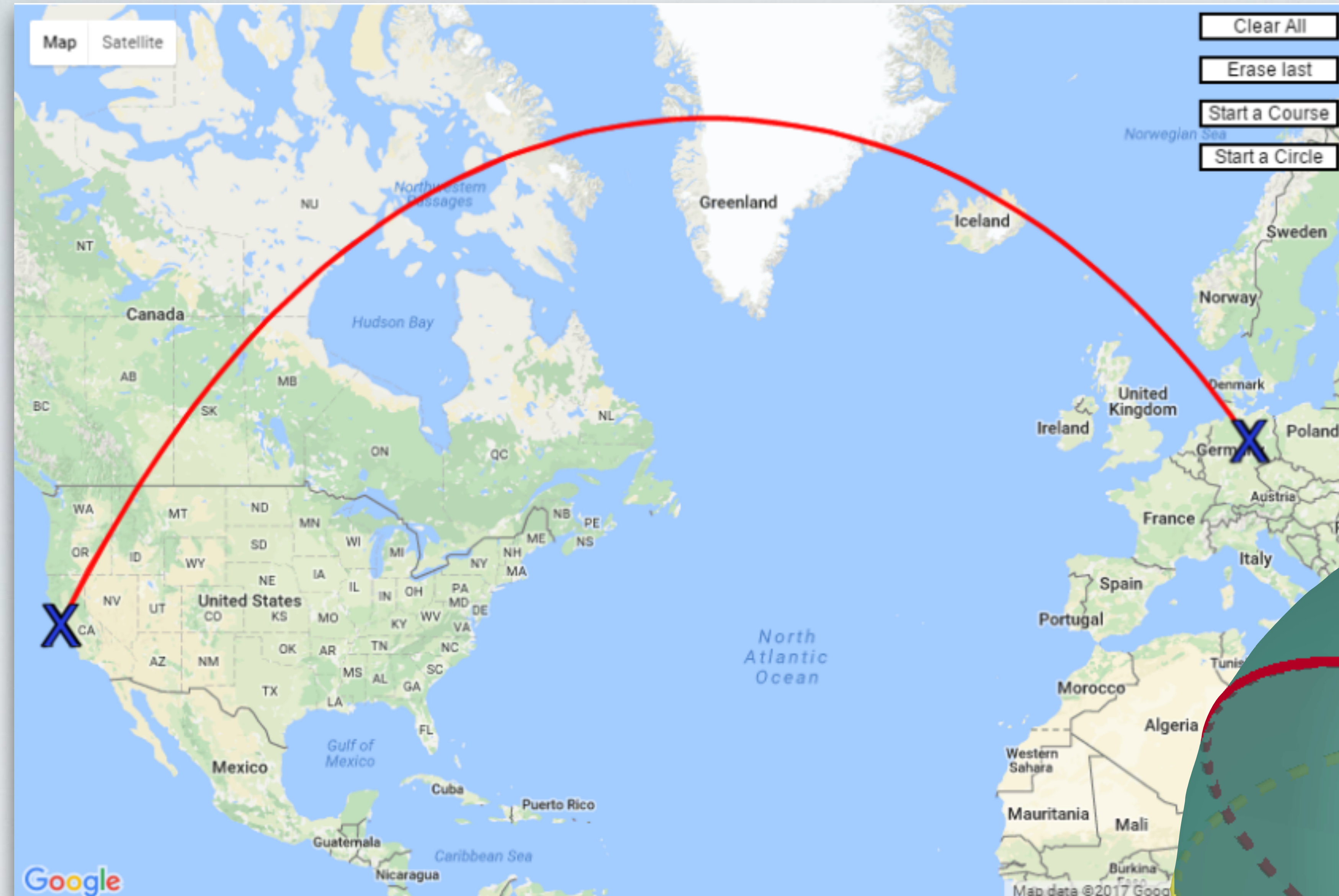
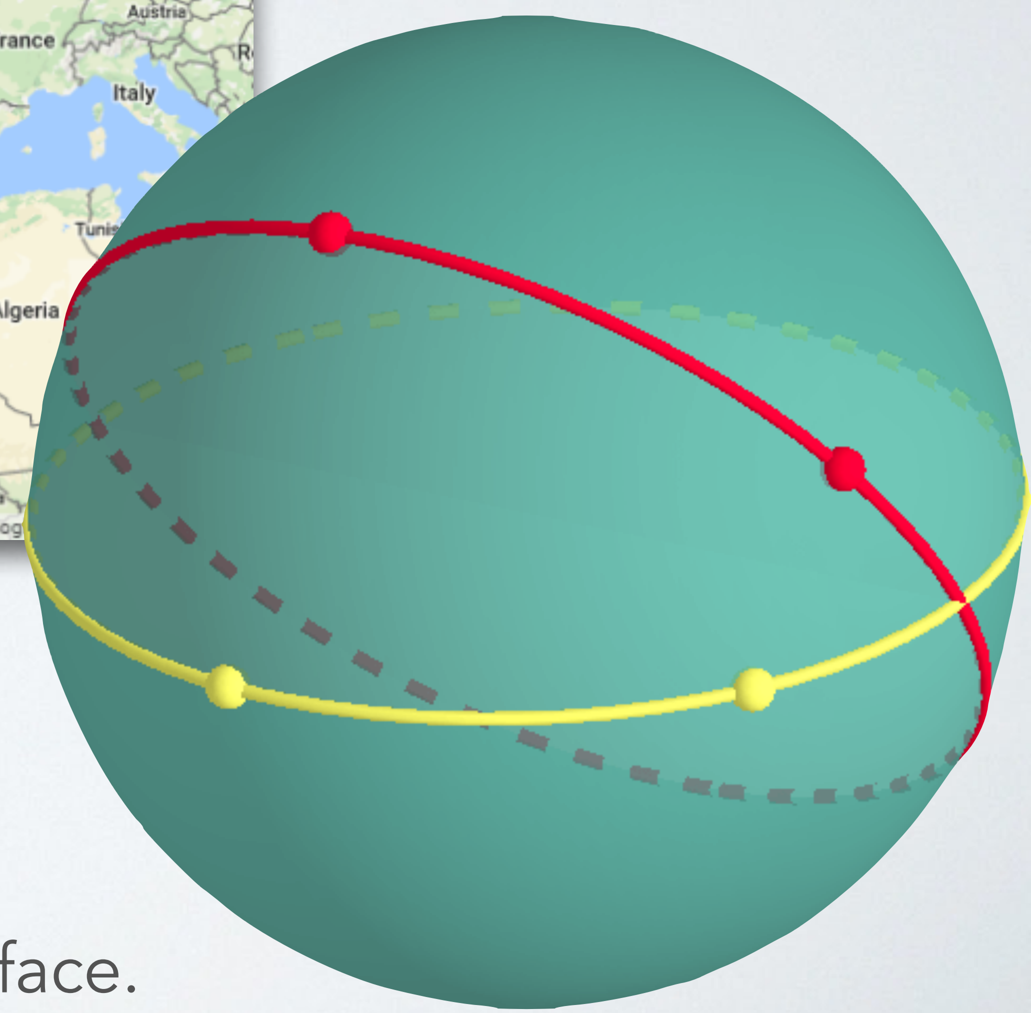
30 seconds

Shortest flight paths



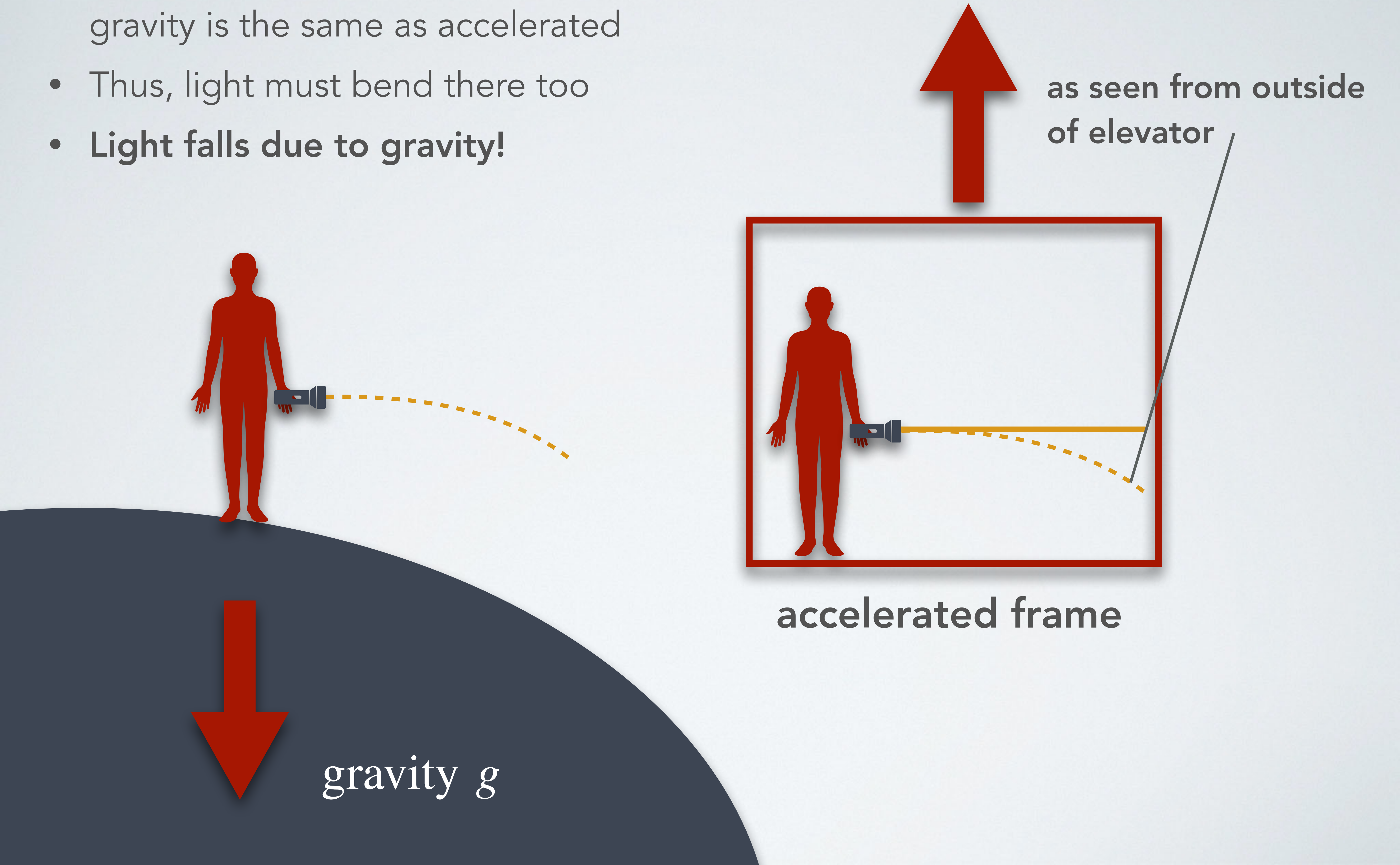


On a sphere, geodesics are **Great Circles**, the shortest distance between two points on the surface.

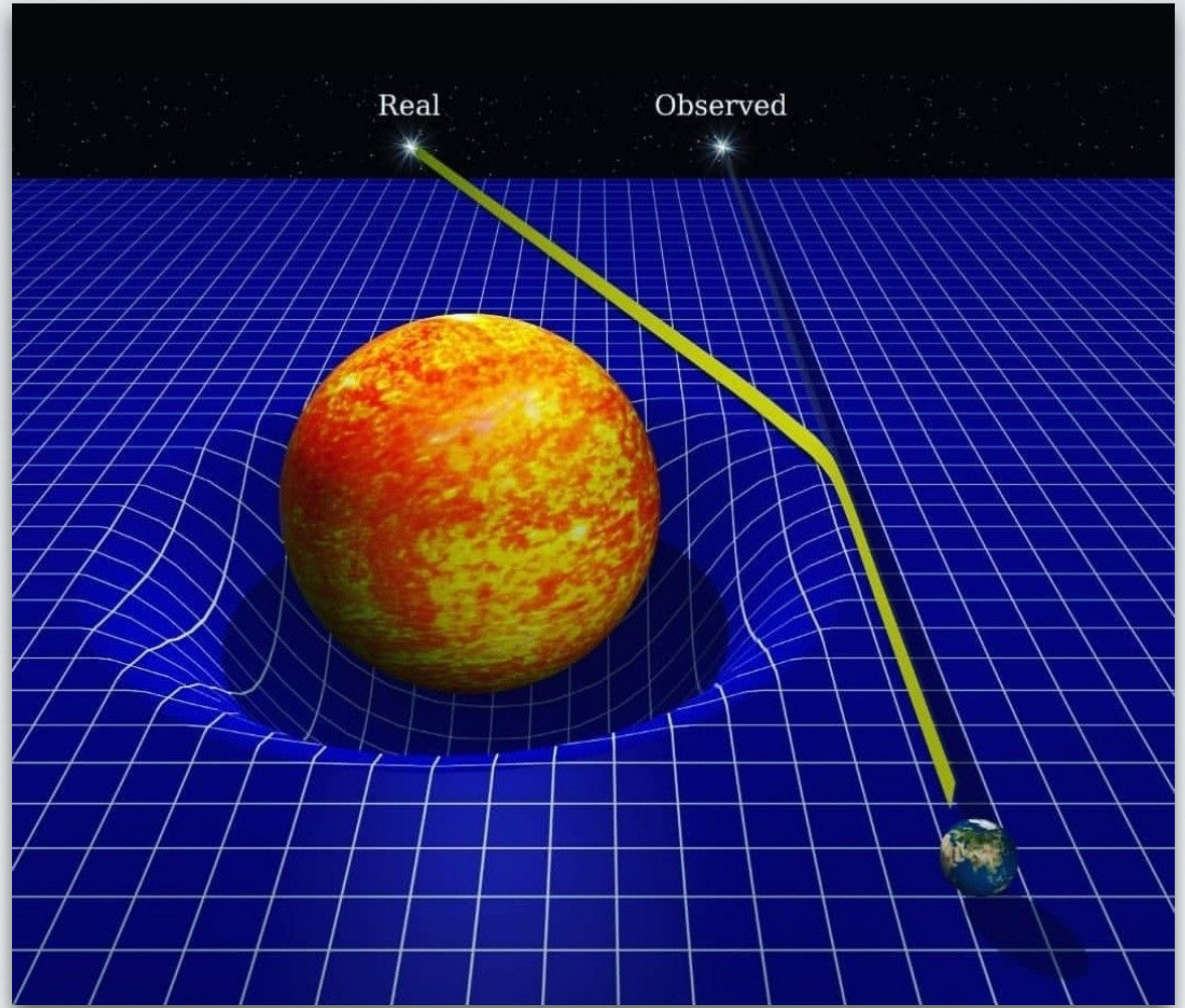
Images: malinc.se

Light in gravitational fields

- Weak equivalence principle: frame with gravity is the same as accelerated
- Thus, light must bend there too
- **Light falls due to gravity!**



Light in the Sun's warped spacetime



Light rays follow ("null") geodesics

Participation: Recap #3

TurningPoint:
What happens to time near masses?

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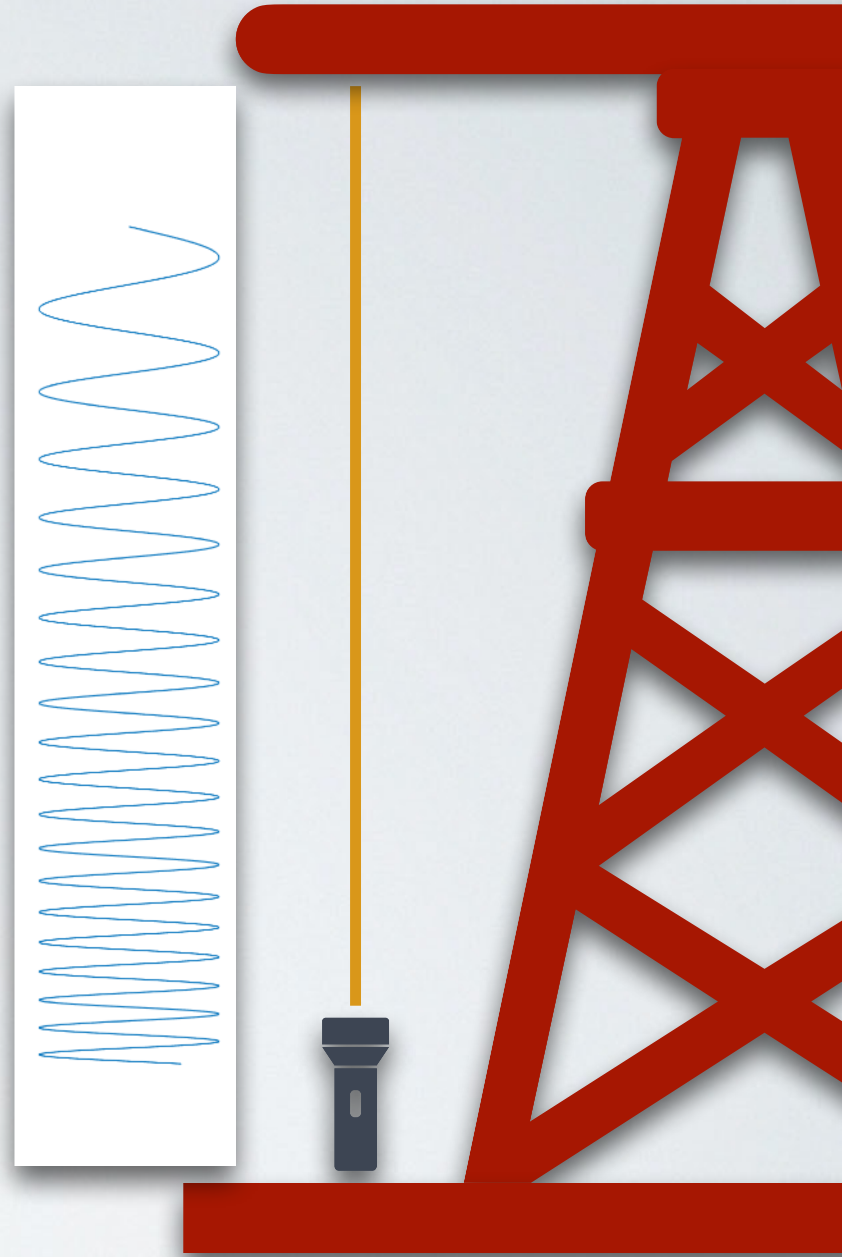
30 seconds

Gravitational time dilation

- Light beam loses energy as it climbs up (gravitational redshifting)
- **Frequency decreases**
- Imagine a **clock based on frequency** of laser light: 1 tick = time taken for fixed number of crests to pass
- Gravitational redshifting **slows down the clock**
- **Clocks in gravitational fields run slower**



$$\Delta t_{\text{grav}} = \sqrt{1 - \frac{2GM}{c^2 r}}\,\Delta t_{\text{space}}$$

Gravitational time dilation

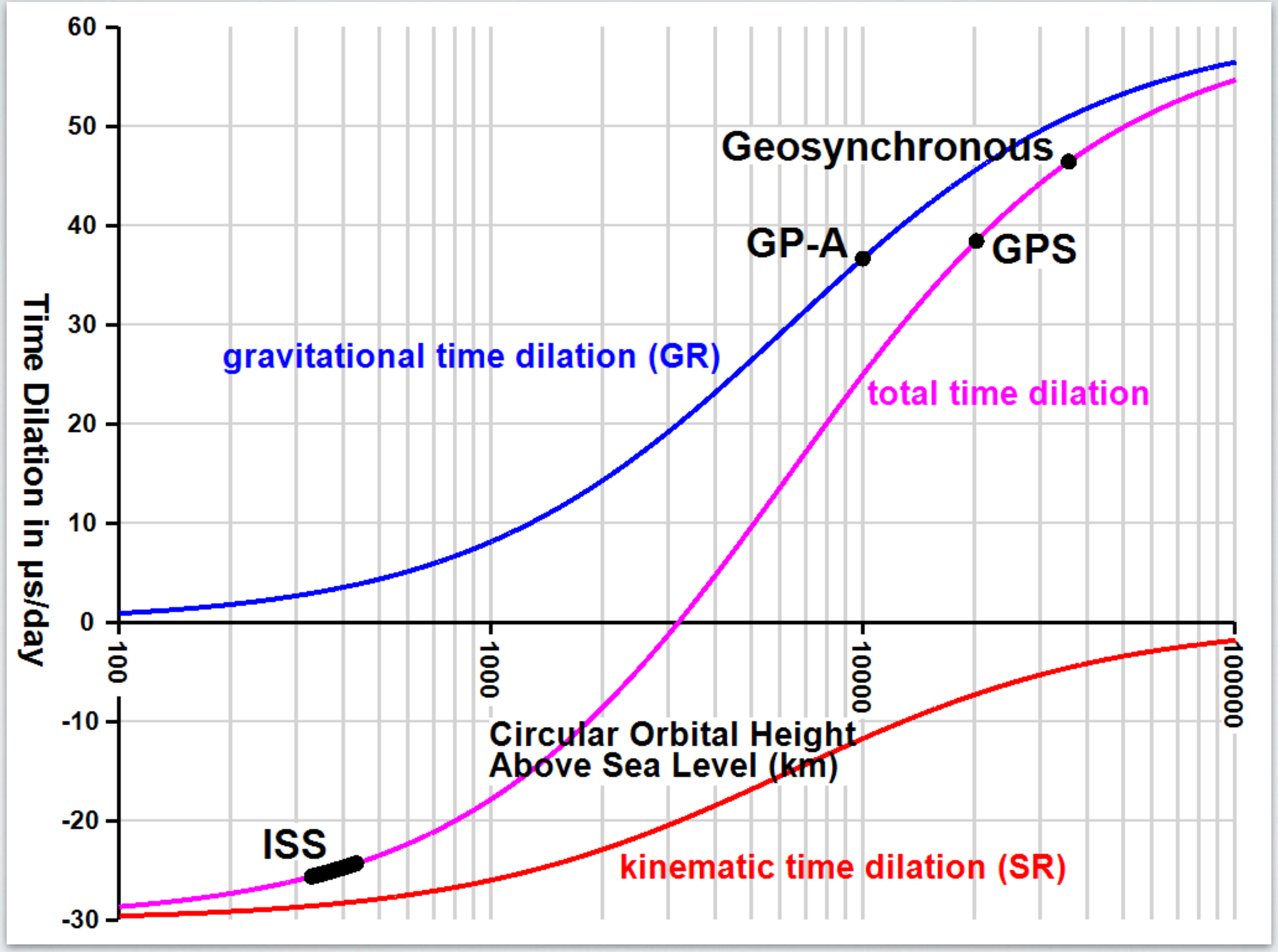


Image: Wikipedia

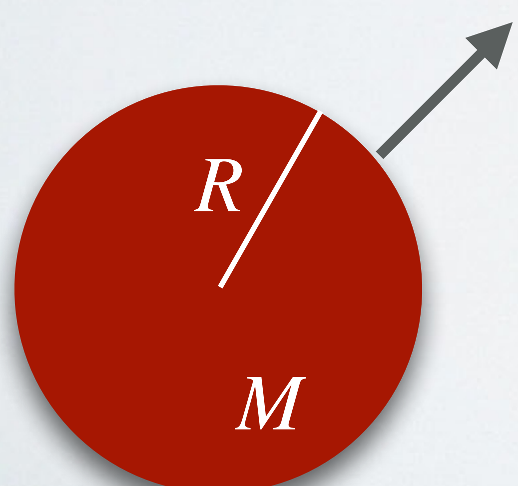
Today

- Black hole theory
- Black holes in the Universe

Part 1: Black hole theory

Escape velocity

- Escape velocity is reached when kinetic energy is greater than gravitational potential
    - $v < v_{\rm esc}$: object falls back to Earth
    - $v > v_{\rm esc}$: object never falls back to Earth
    - For Earth, v_{esc} = 11 km/s
    - For Sun v_{esc} = 616 km/s
- Larger mass or smaller radius mean larger escape velocity
- Setting v_{esc} = c gives R-M relation where light cannot escape



$$\frac{mv^2}{2} > \frac{GMm}{R}$$

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}}$$

$$v_{\rm esc} = c \implies R = \frac{2GM}{c^2}$$?

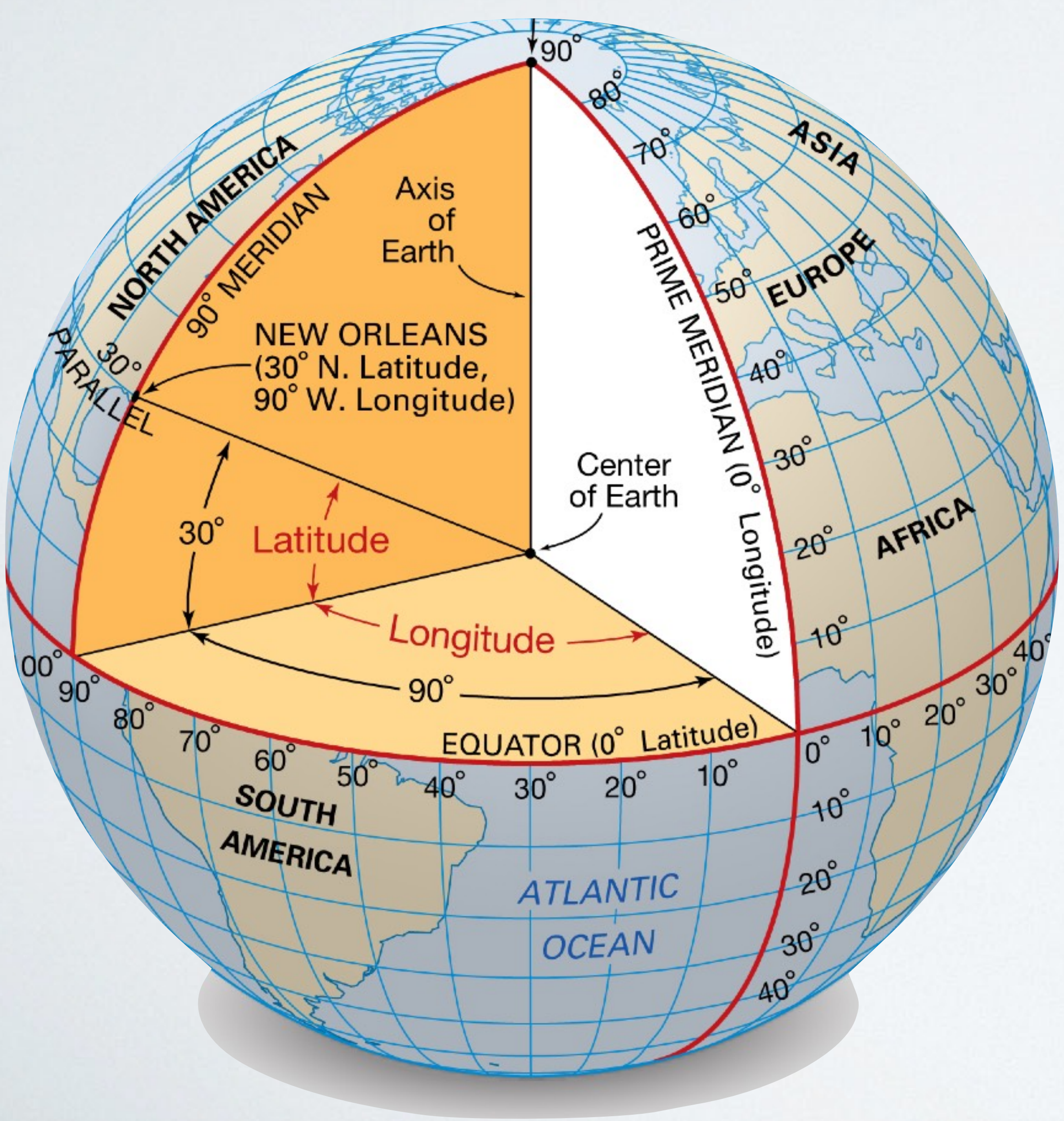
18th century

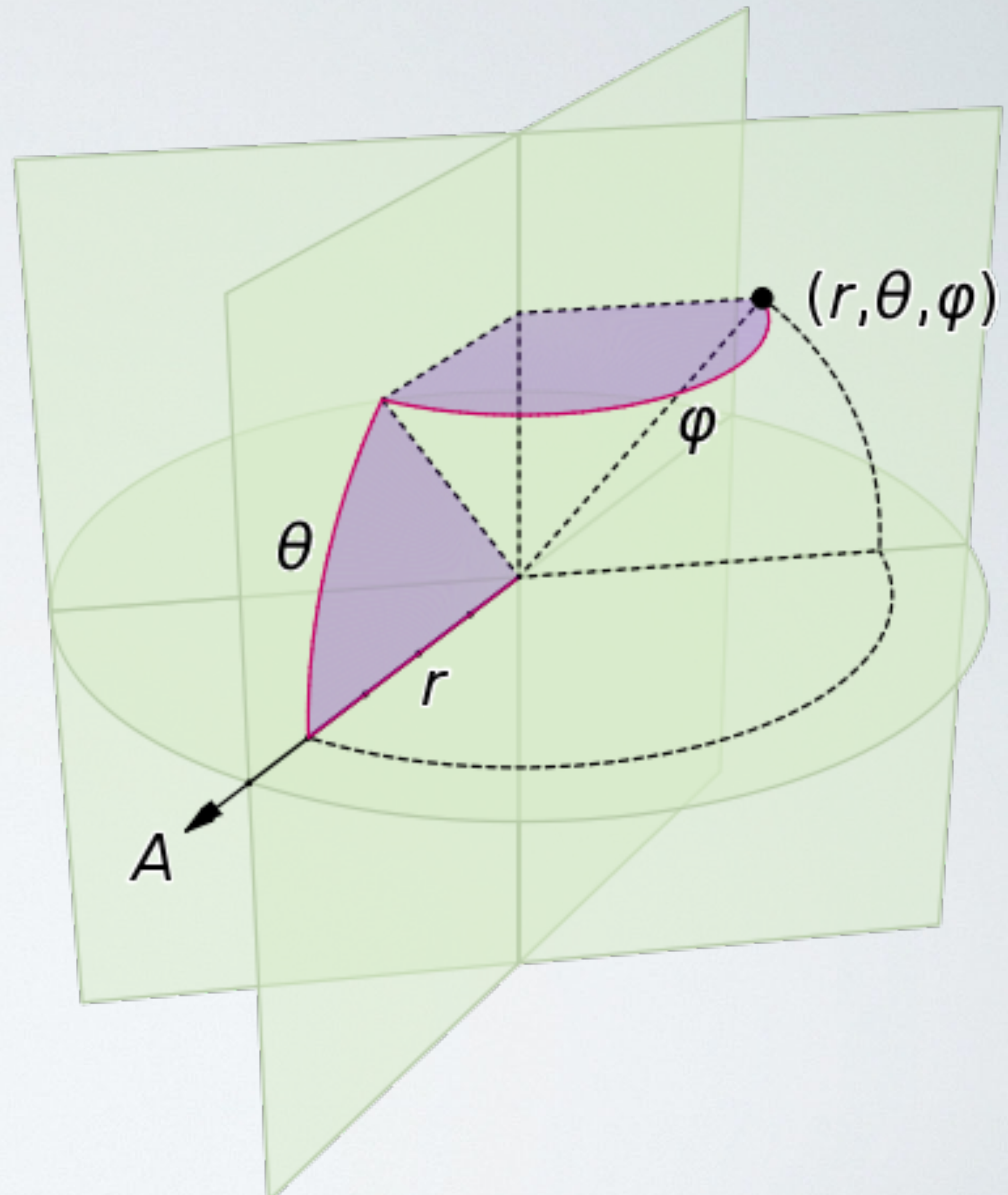
- First suggested as “dark stars” by the **Reverend John Mitchell** in 1784:
  - “If there should really exist in nature any bodies, whose **density is not less than that of the sun**, and whose **diameters are more than 500 times** the diameter of the sun, **since their light could not arrive at us** … of the existence of bodies under either of these circumstances, we could have **no information from sight** … the consideration of them [is] somewhat beside my present purpose, I shall not prosecute them any further.”
- **Pierre-Simon Laplace** in 1798:
  - “A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, **would not, in consequence of its attraction, allow any of its rays to arrive at us**; it is therefore possible that the **largest luminous bodies in the universe** may, through this cause, be **invisible**.”
- When wave nature of light was discovered, the idea fell out of fashion (not clear how gravity would affect a light wave rather than a light particle)

Quotes from Wikipedia

Spherical coordinates

- Spherical coordinates:
  - Radius (r)
  - Angle "up-down" (θ)
  - Angle "around" (ϕ)

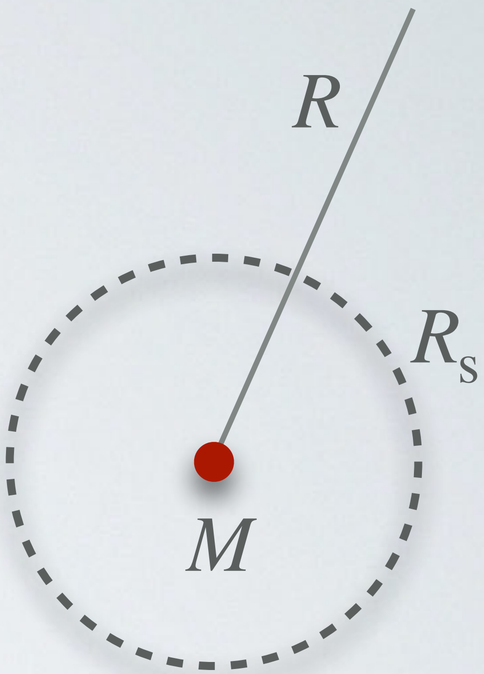




Images: Encyclopaedia Brittanica

Schwarzschild Metric

- First exact solution of the equations of GR (by Karl Schwarzschild in 1916)
- Describes gravitational field in (empty) space around a point mass
- Described by spacetime interval
- Features of Schwarzschild's solution:
    - Radius of the sphere representing the **event horizon** is called the **Schwarzschild radius**
    - Reduces to Newton's law of gravity / flat space at large R >> R_s
    - Space-time curvature becomes infinite at center (**R = 0, called a space-time singularity**)
- R_s is same as Newtonian solution, but that is a little coincidental



$$\Delta s^2_{\text{flat}} = (c\Delta t)^2 - \Delta r^2 - \text{angle terms}$$

Schwarzschild metric:

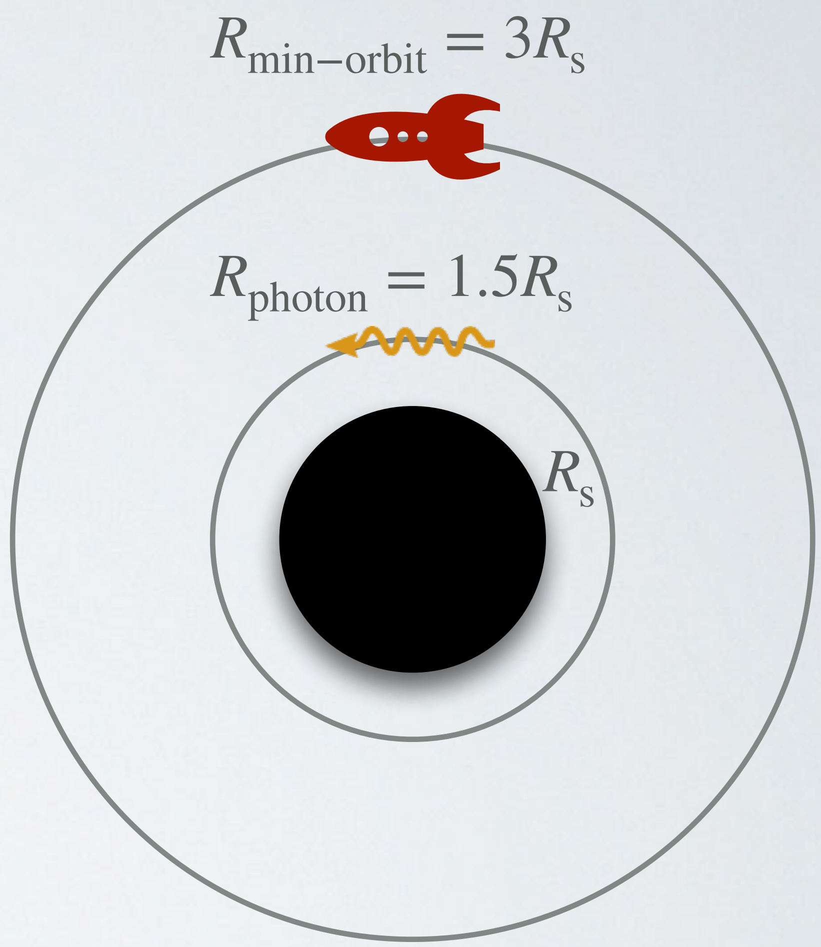
$$\Delta s^2 = \left(1 - \frac{R_s}{R}\right)(c\Delta t)^2 - \frac{\Delta r^2}{\left(1 - \frac{R_s}{R}\right)} - \text{angle terms}$$

Schwarzschild radius:

$$R_s = \frac{2GM}{c^2}$$

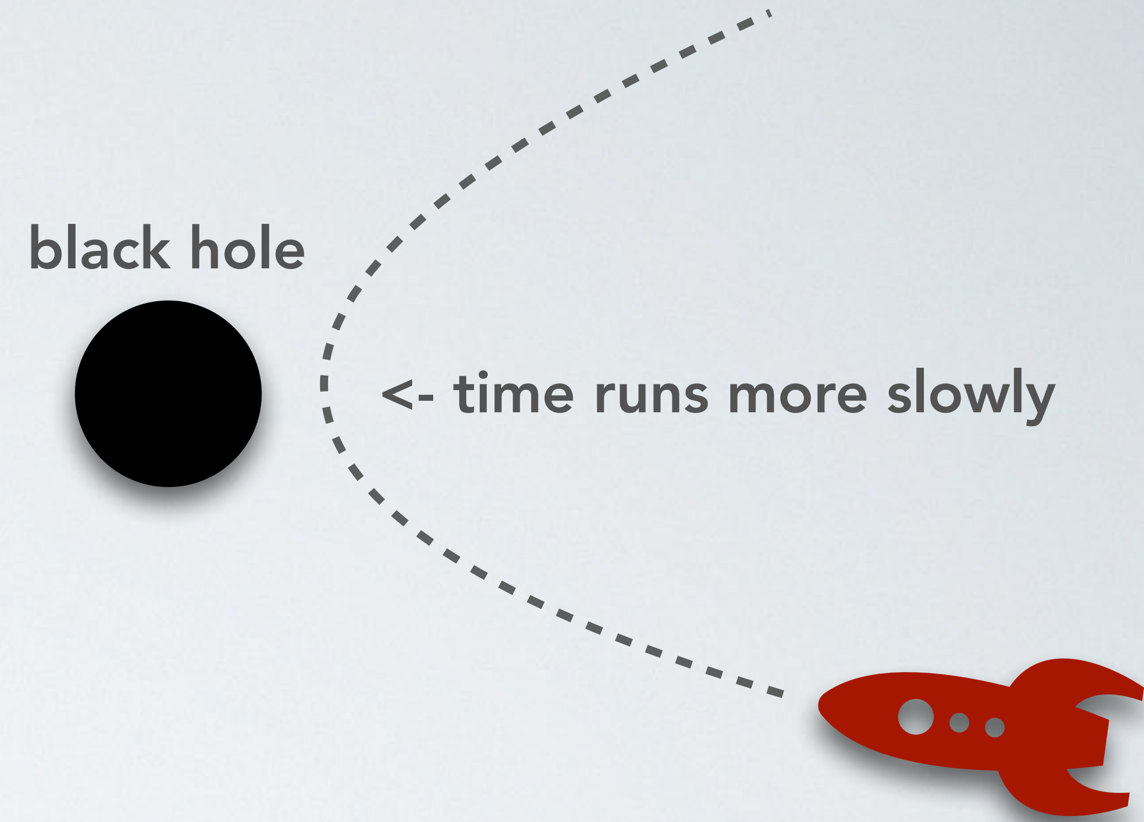
Event horizon, photon orbit, matter orbit

- **Schwarzschild radii are small**
  - $R_s = 2GM/c^2 = 3\ \text{km}\ (M/M_\odot)$
  - For BH at center of Milky Way, $R_s = 0.08\ \text{AU}$
- Events inside the event horizon are **causally disconnected** from events outside of the event horizon (no information can be sent from inside to outside the horizon)
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Any light emitted at R_s is **infinitely redshifted** (and cannot be observerd from outside)
- **Light rays** can orbit at 1.5 R_s (forming a sphere of photons)
- Stable, **circular orbits for matter** are not possible inside 3 R_s



Time dilation

- At the event horizon, **time dilation seems infinite** to observer far away
- This singularity occurs only in the chosen coordinate system; time will continue normally for an observer falling into the event horizon



$$\Delta s^2 = \left(1 - \frac{R_s}{R}\right)(c\Delta t)^2 - \frac{\Delta r^2}{\left(1 - \frac{R_s}{R}\right)} - \text{angle terms}$$

$$\Delta t_{\text{grav}} = \sqrt{1 - \frac{R_s}{R}}\,\Delta t_{\text{space}}$$

$$R_s = \frac{2GM}{c^2}$$

Participation: Are BHs simple?

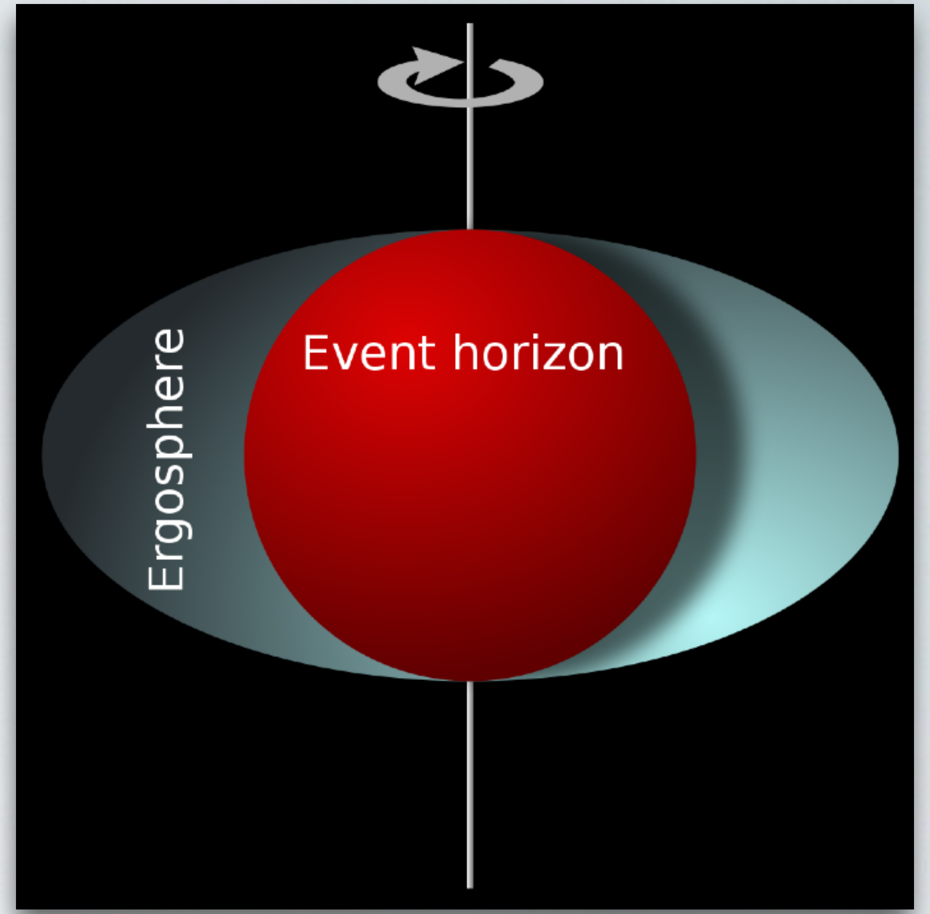
TurningPoint:
What does the physics of a black hole depend on?

Session ID: diemer

30 seconds

Rotating black holes

- No-hair theorem: black holes are characterized **by only mass, spin**, and electric charge
- In practice, electric charge would get evened out immediately
- Roy Kerr (1963) discovered solution to Einstein's equations corresponding to a rotating black hole
- Similar to Schwarzschild solution; Kerr solution describes all black holes found in nature
- Space-time near rotating black hole is dragged around in the direction of rotation ("**frame dragging**")
- **Ergosphere** is region where space-time dragging is so intense that its impossible to resist rotation of black hole
- Event horizon of spinning (Kerr) black holes is not spherical, but close to it for moderate spins
- Light and even matter can escape the ergosphere
- Event horizon is **smaller for spinning black holes**, depending on the **dimensionless spin parameter** *a* (where J is the angular momentum)
- The spin parameter has to be $0 \leq a \leq 1$



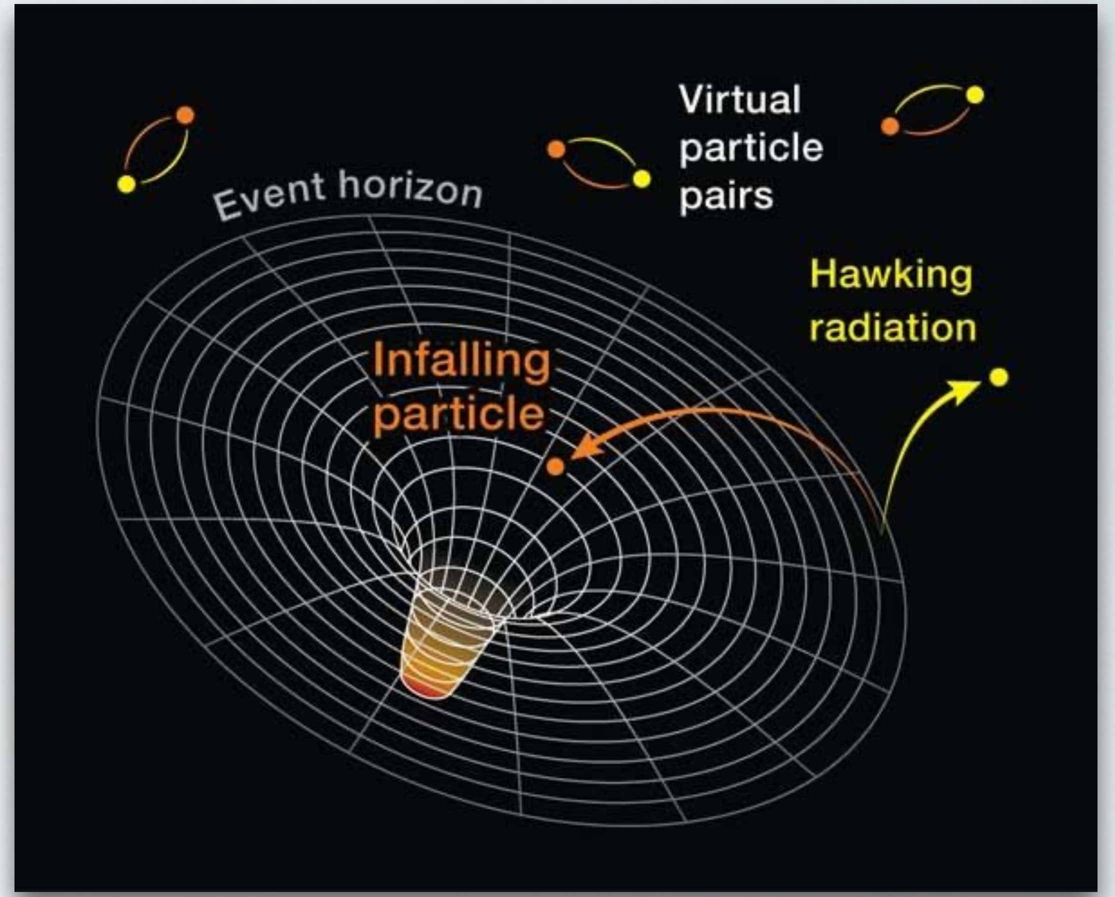
$$R_{s,\mathrm{Kerr}} = \frac{GM}{c^2}\left(1 + \sqrt{1 - a^2}\right)$$

$$a = \frac{cJ}{GM^2}$$

Image: Wikipedia

Hawking Radiation

- **Black hole slowly evaporates** due to quantum mechanics effects
- **Particle/antiparticle** pair is created near BH
- One particle falls into horizon; the other escapes
- Energy to create particles comes from gravity outside horizon
- Solar-mass black hole would take **10^{65} years** to evaporate
- Mini-black holes that could evaporate are not known to exist now, but possibly existed in early Universe
- The black hole does not have an internal temperature — only mass and spin (and charge)
- However, one can assign a temperature to a black hole via its Hawking radiation, which has a blackbody spectrum
- This temperature is extremely low: $T_{\mathrm{Hawking}} = 6 \times 10^{-8}\ \mathrm{K}\ (M_{\odot}/M)$



Part 2: Black holes in the Universe

Participation: Masses

TurningPoint:

How massive are the smallest black holes we know of?

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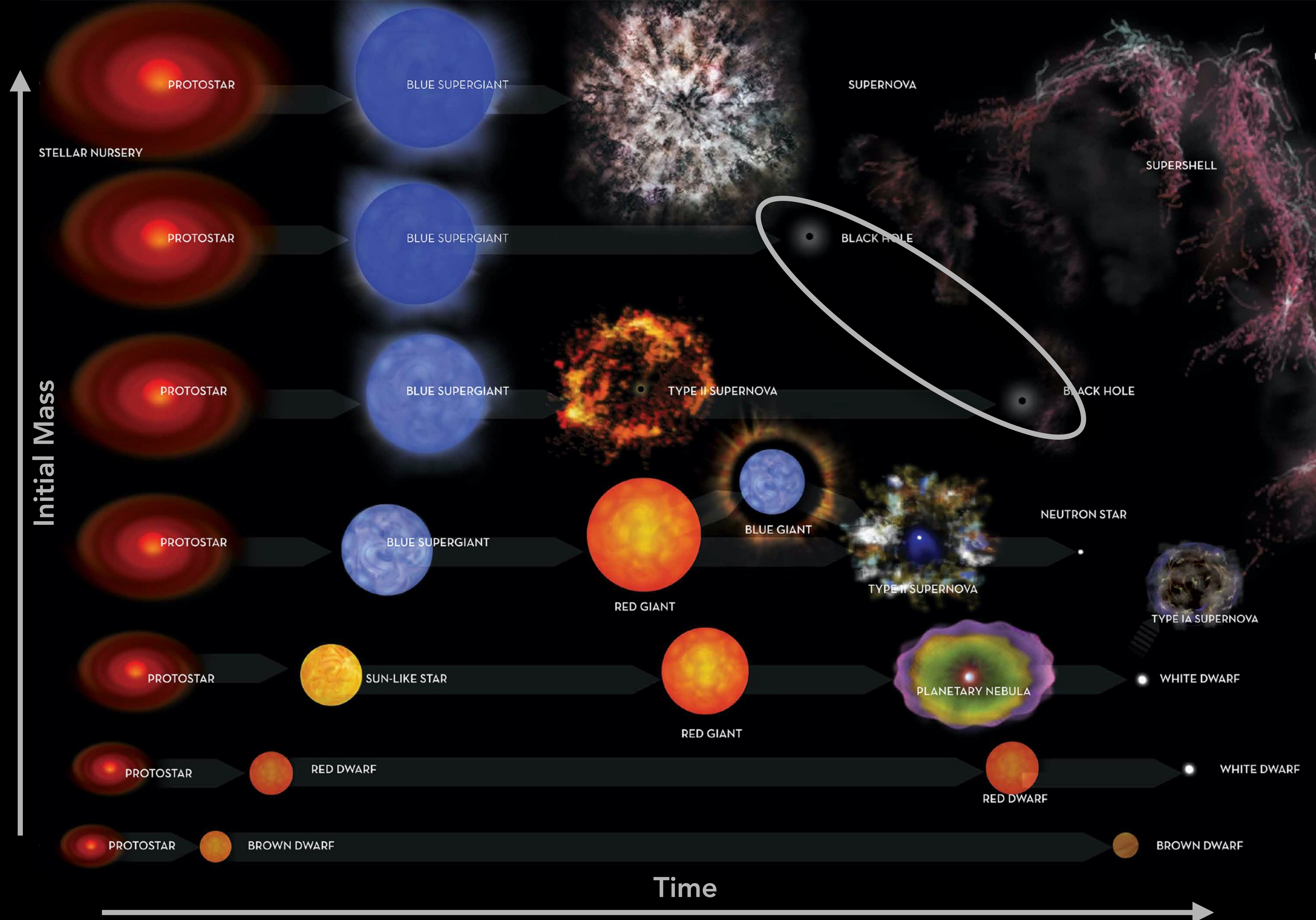
30 seconds



Summary of stellar evolution

NASA

Participation: Masses

TurningPoint:
How massive are the largest black holes we know of?

Session ID: diemer

30 seconds

Black holes in reality

- Theoretically, black holes can have any mass
- In reality, they span a large range:
  - **Stellar mass black holes**: left over from the collapse/implosion of a massive star (5 solar masses or greater)
  - **Intermediate-mass black holes**: suggested by recent observations (hundreds to thousand of solar masses)
  - **Supermassive black holes**: giants at the centers of galaxies (millions to billions of solar masses)
- Black holes grow either by **accreting gas** or by **merging** with other black holes (or compact objects)

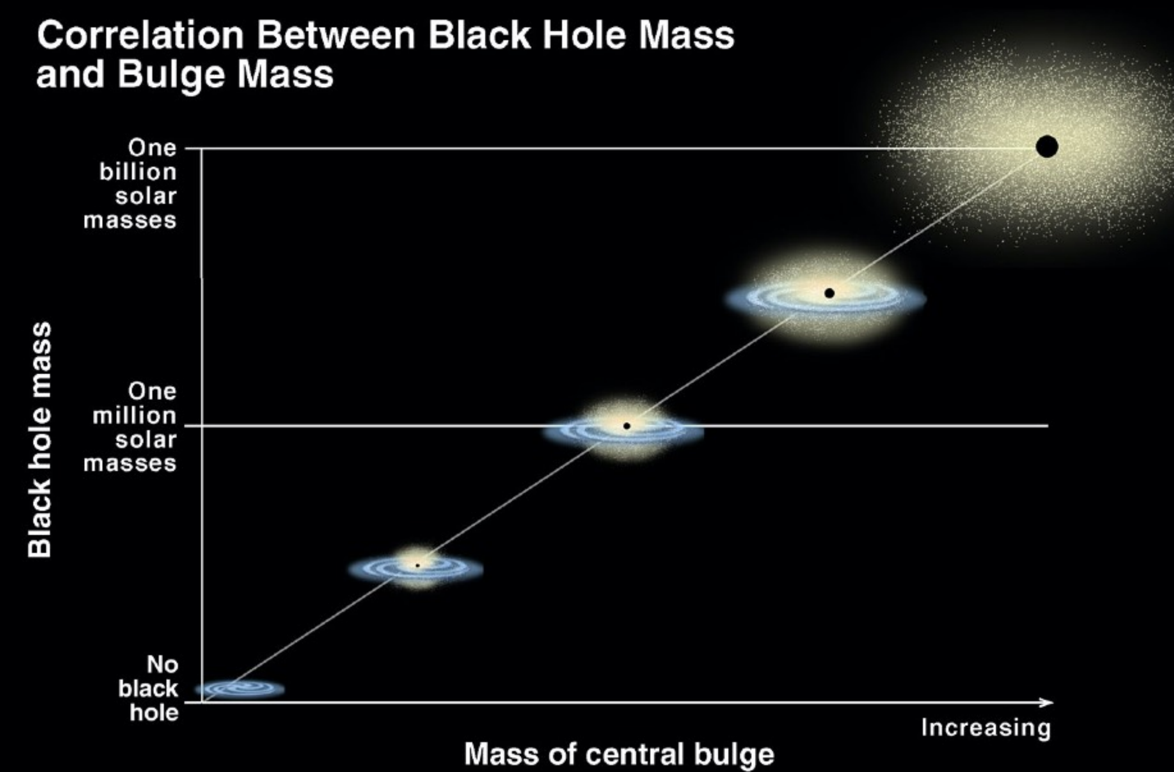
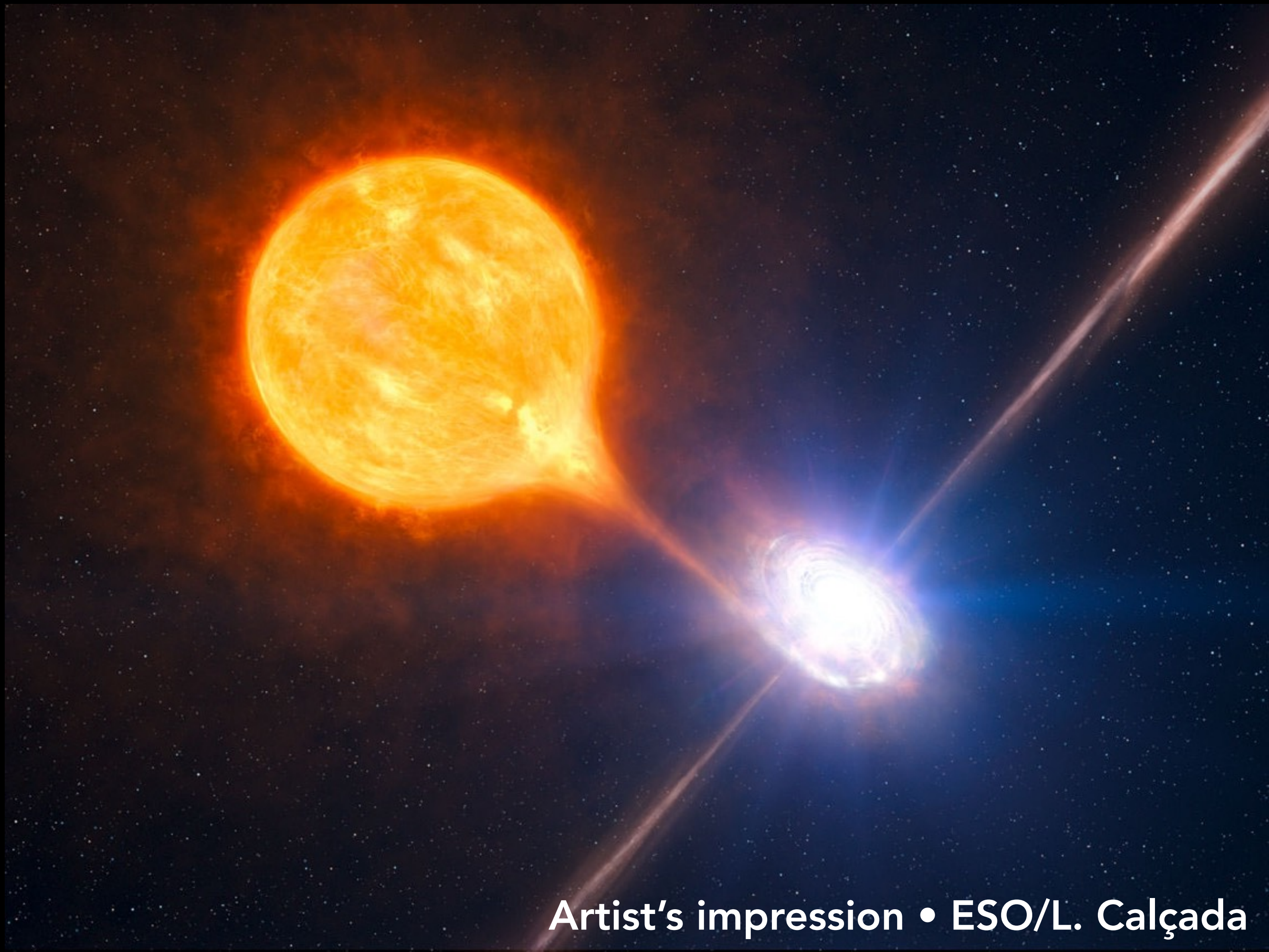


Image: K. Cordes, S. Brown

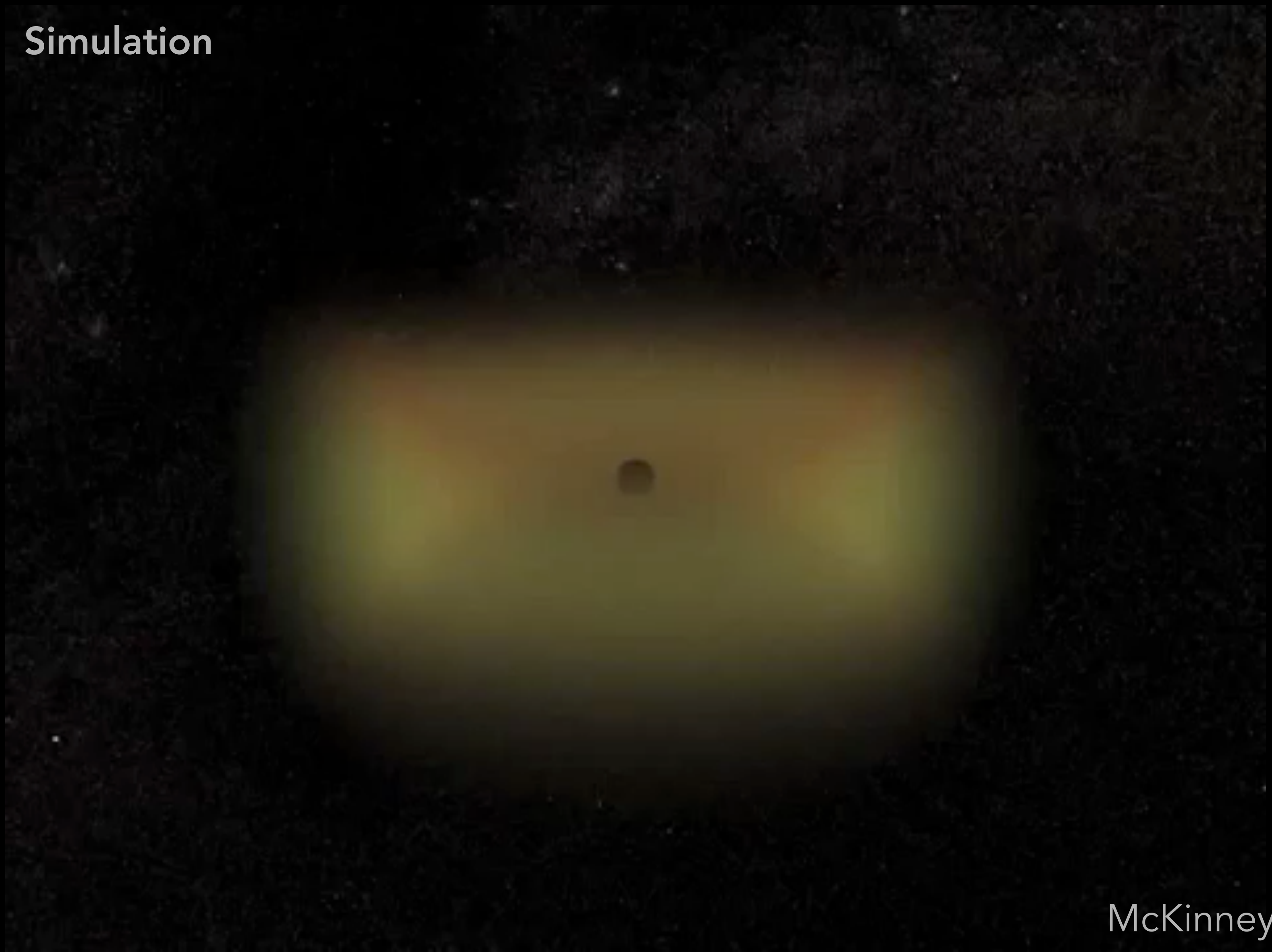
X-ray binaries

- Many stars exist in **binary** systems with another star
- If a black hole is formed in binary star system, tidal forces can rip matter from its companion
- The matter forms an **accretion disk** around the black hole; its gravitational energy heats the disk to millions of degrees
- The hot accretion disk radiates away energy as X-rays
- These systems are called **X-ray binaries**



Active Galactic Nuclei revisited

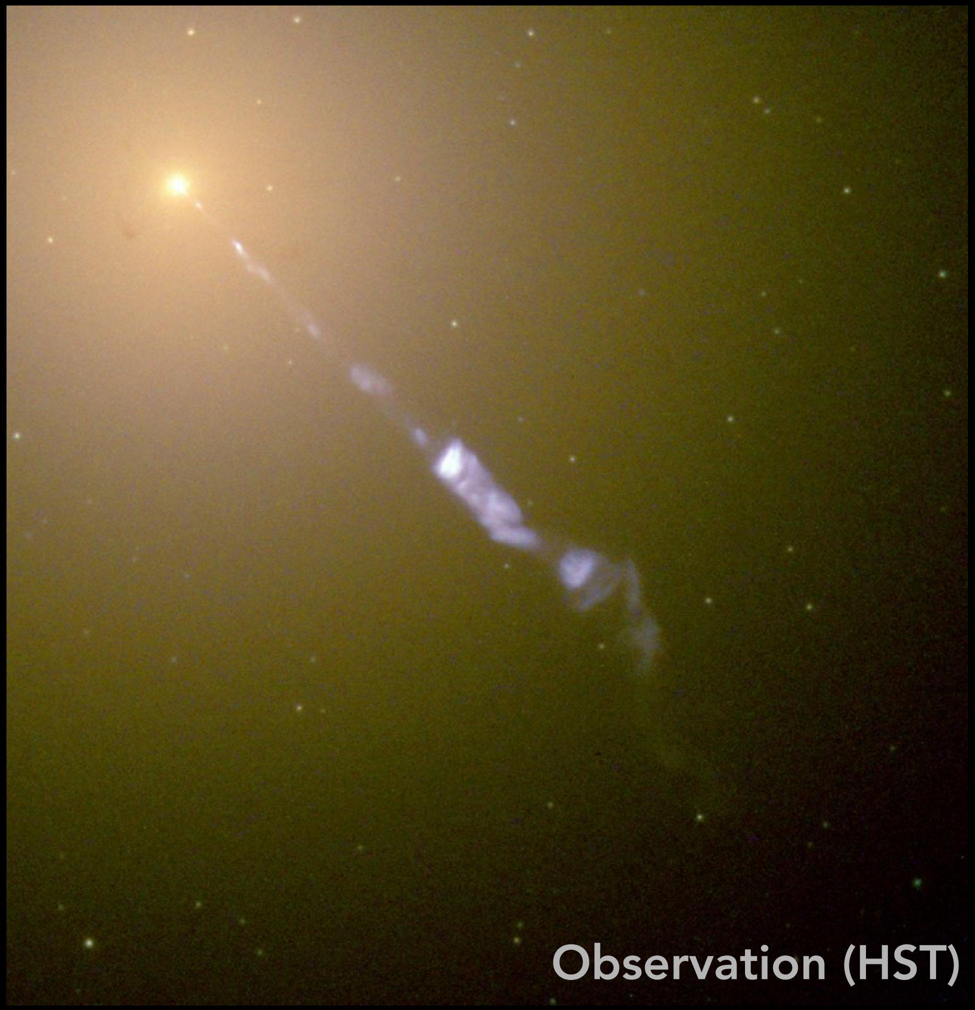
- AGN are supermassive black holes at the centers of galaxies and their accretion disks
- Similar mechanism as X-ray binaries: **energy released by accretion of matter** powers energetic phenomena
- Emission from **radio to gamma rays**, as well as relativistic **jets**
- Particularly powerful AGN are sometimes called **quasars**

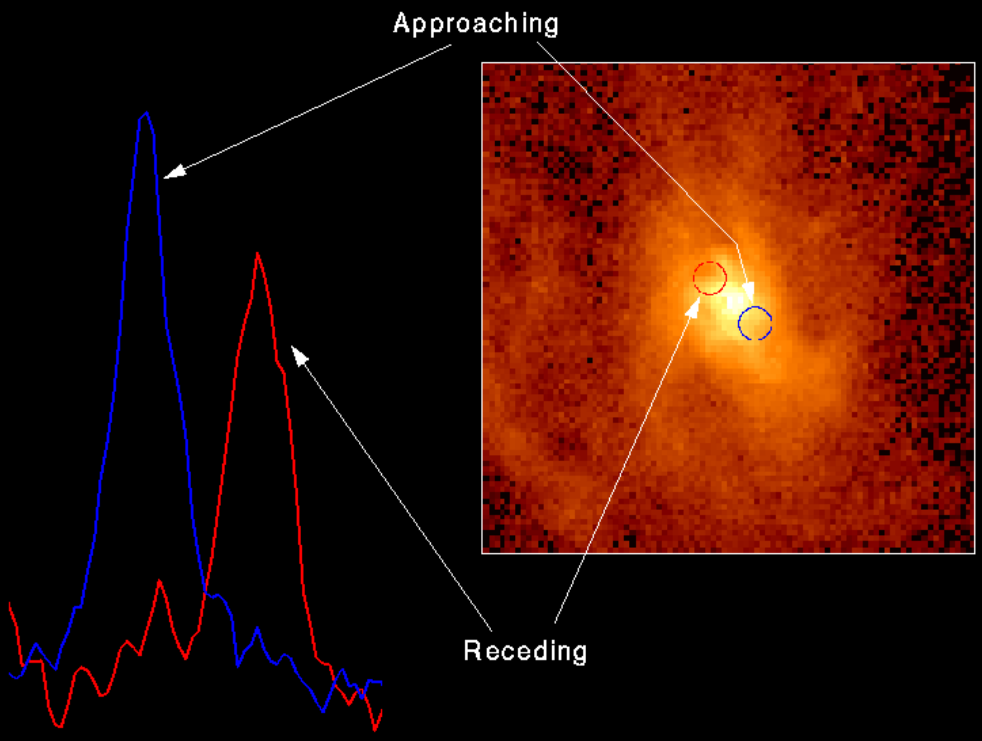


McKinney et al. 2012 • Ralf Kaehler

M87

- M87 is an example of an AGN where we see a jet
- Jet has special-relativistic $\gamma > 6$ (v > 0.9 c)!
- What powers the jet?
  - Accretion
  - Extraction of spin-energy from the black hole
- We also see a **spinning gas disk**
  - Measure velocities using the Doppler effect (red and blue shift of light from gas)
  - Need a **6 billion solar mass** black hole to explain gas disk velocities (and other observations)

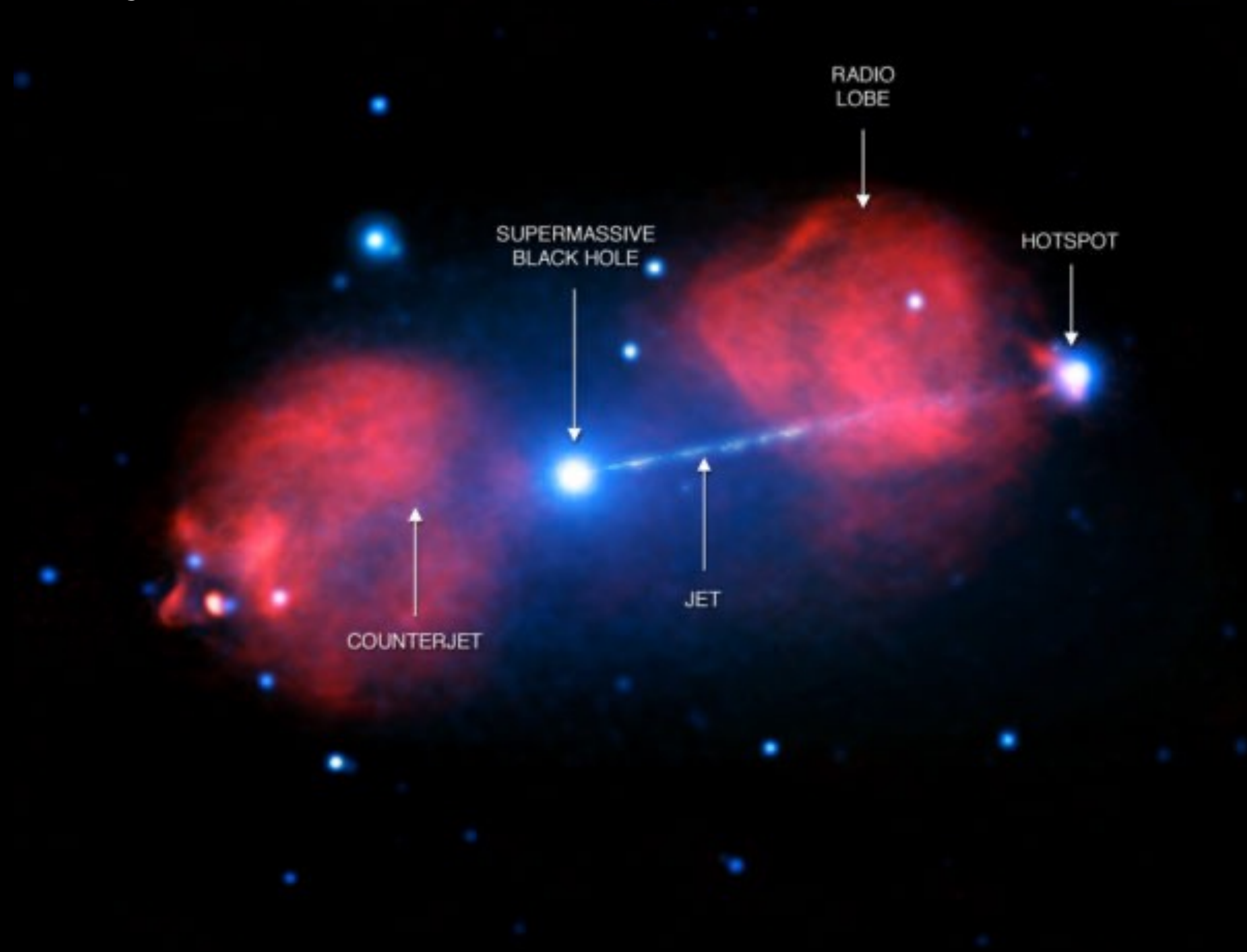




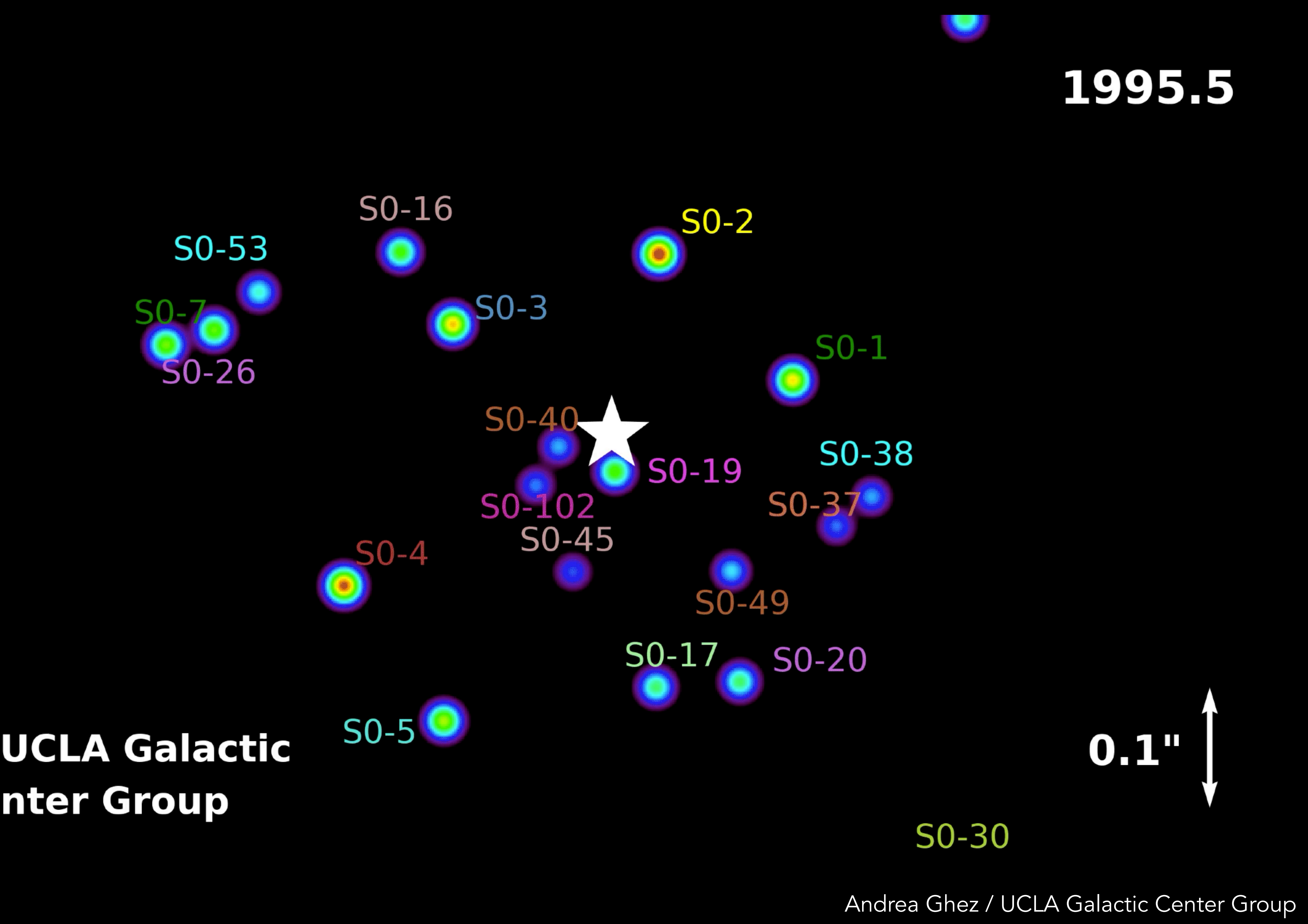
NASA / HST

AGN (in X-rays and radio)

Galaxy Pictor A (observations)



NASA / Chandra X-ray Center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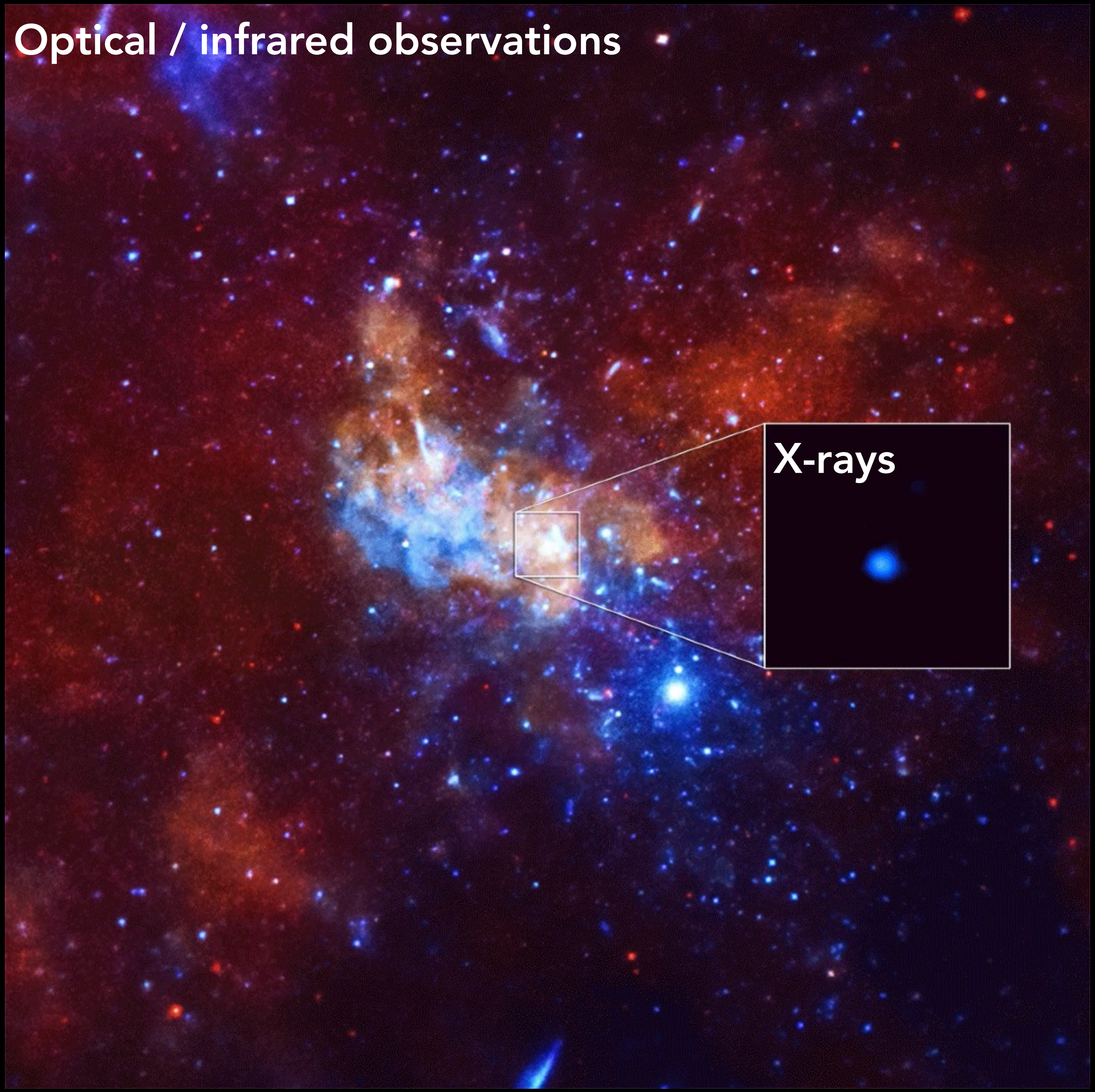
Andrea Ghez / UCLA Galactic Center Group

X-ray flare from Milky Way black hole

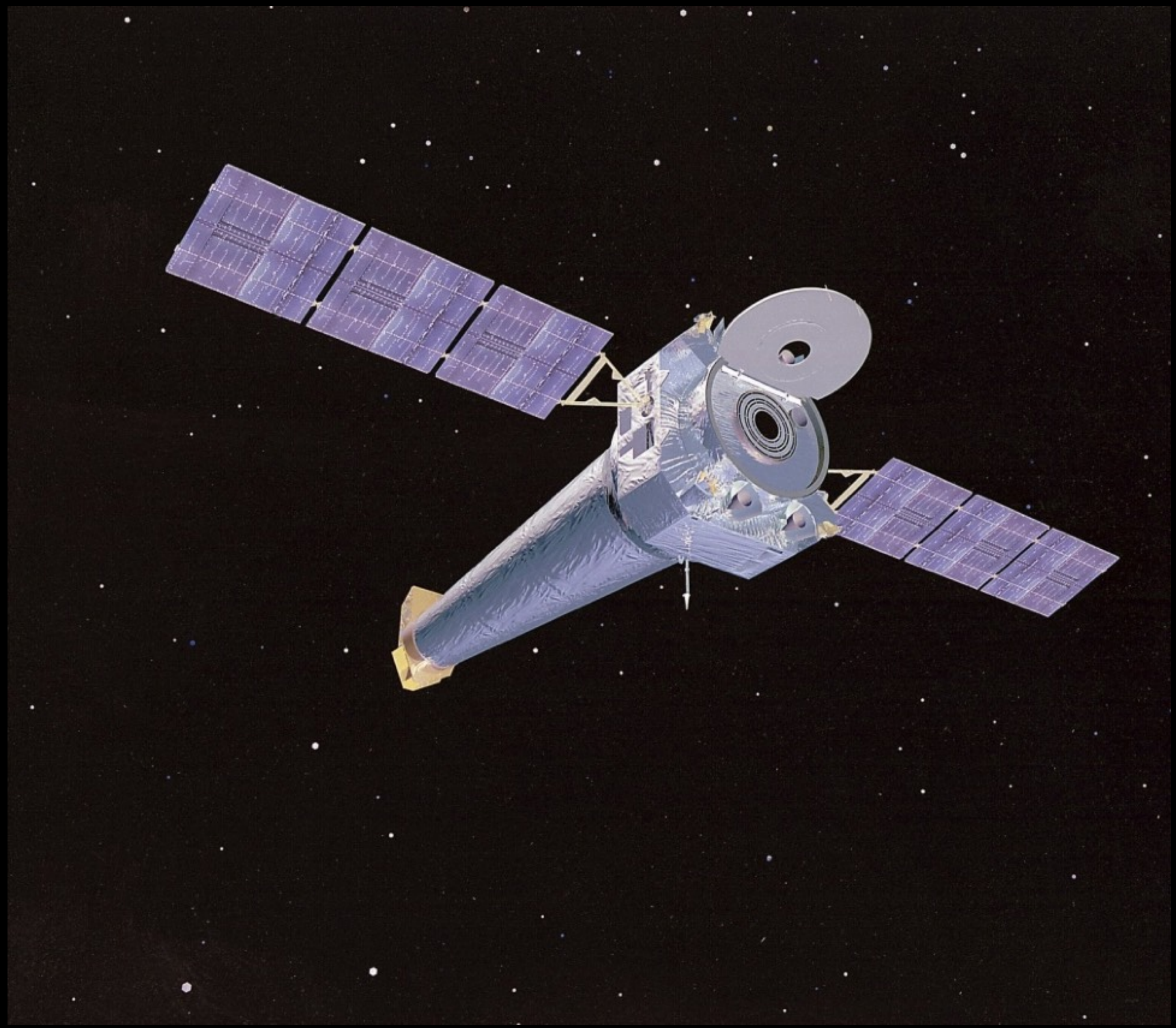
- Supermassive black holes also produce X-rays when they accrete matter
- Our black hole (Sagitarrius A*) sometimes flares in X-rays (image from 2014)



Chandra X-ray Center / I. Zhuravleva

Chandra X-ray Observatory

- In space since 1999
- Highest-resolution X-ray observatory



Participation: World Wide Telescope

Instructions

Go to Discussion #23 on Canvas and follow the instructions.

10 minutes

Tidal disruption event (simulation)

UNIVERSITY OF CALIFORNIA
SANTA CRUZ

- TDE occurs when a **star** gets too close to a black hole and is **ripped apart** by tidal forces
- Some stellar material falls into black hole, some is ejected
- Time shown in video is about 130 days

James Guillochon

Take-aways

- Black holes are a fundamental prediction of General Relativity, and fully described by their **mass and spin**
- Stellar-mass black holes are created when the **cores of massive stars** collapses at the end of their life; they grow via **accretion** of gas and **mergers**
- Material falling into black holes forms **accretion disks** that emit energetic radiation

Next time...

We'll talk about:

- Gravitational waves

Assignments

- Post-lecture quiz (by tomorrow night)
- Homework #5 (due 12/2)