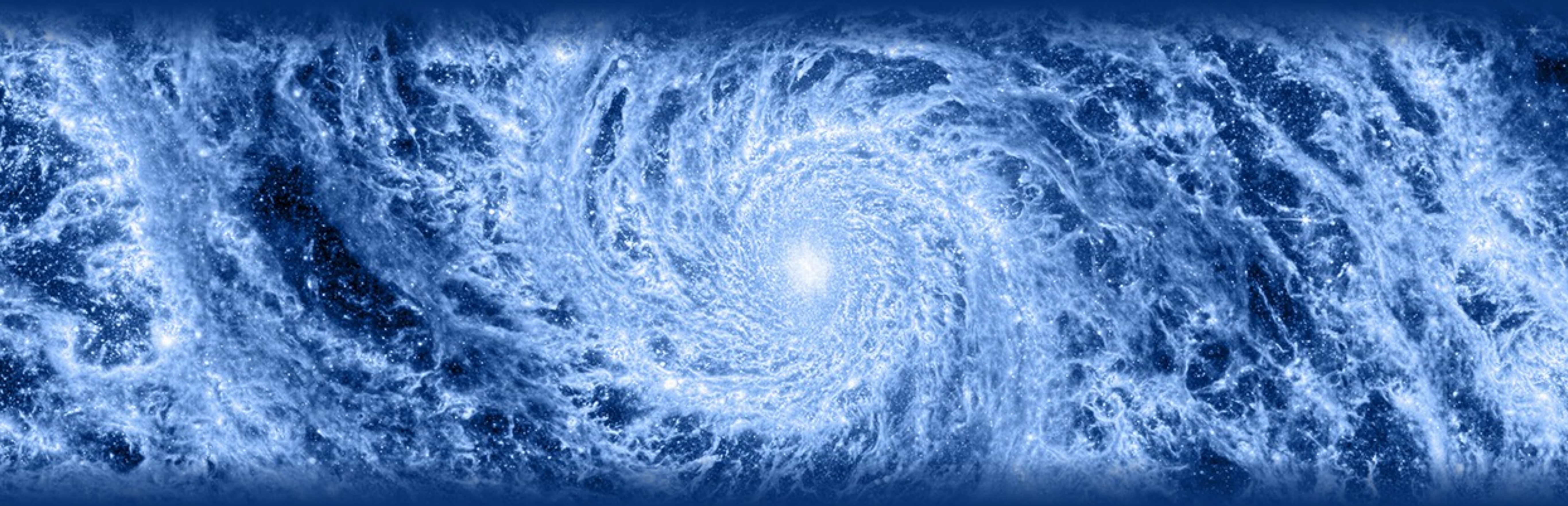
Galaxies

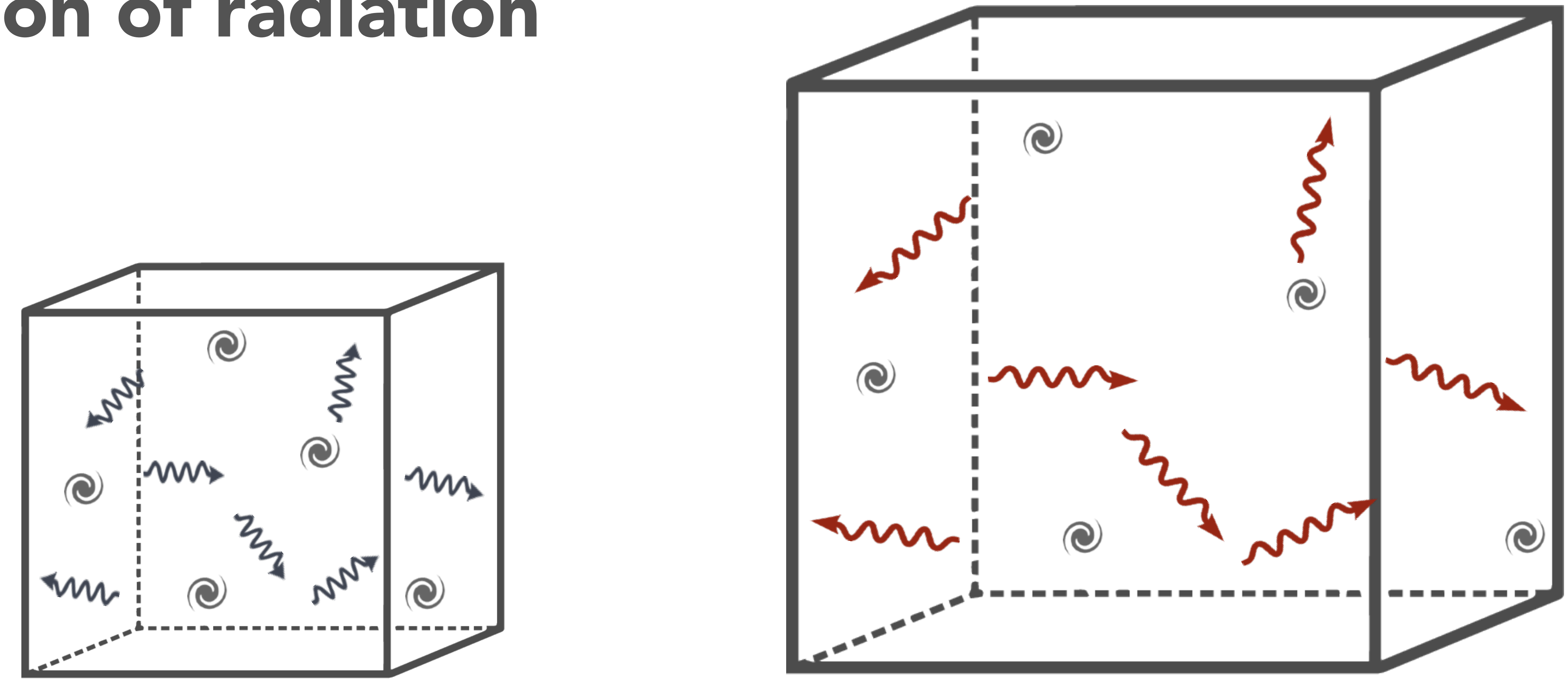
Prof. Benedikt Diemer



Chapter 3 • Density fluctuations in the early Universe

§3.1 • Fluctuations from inflation

Evolution of radiation



- Density of matter decreases as 1/volume = $1/a^3$
- Density of photons decreases the same way, but photons are also **redshifted**
- Since photon energy is proportional to frequency:

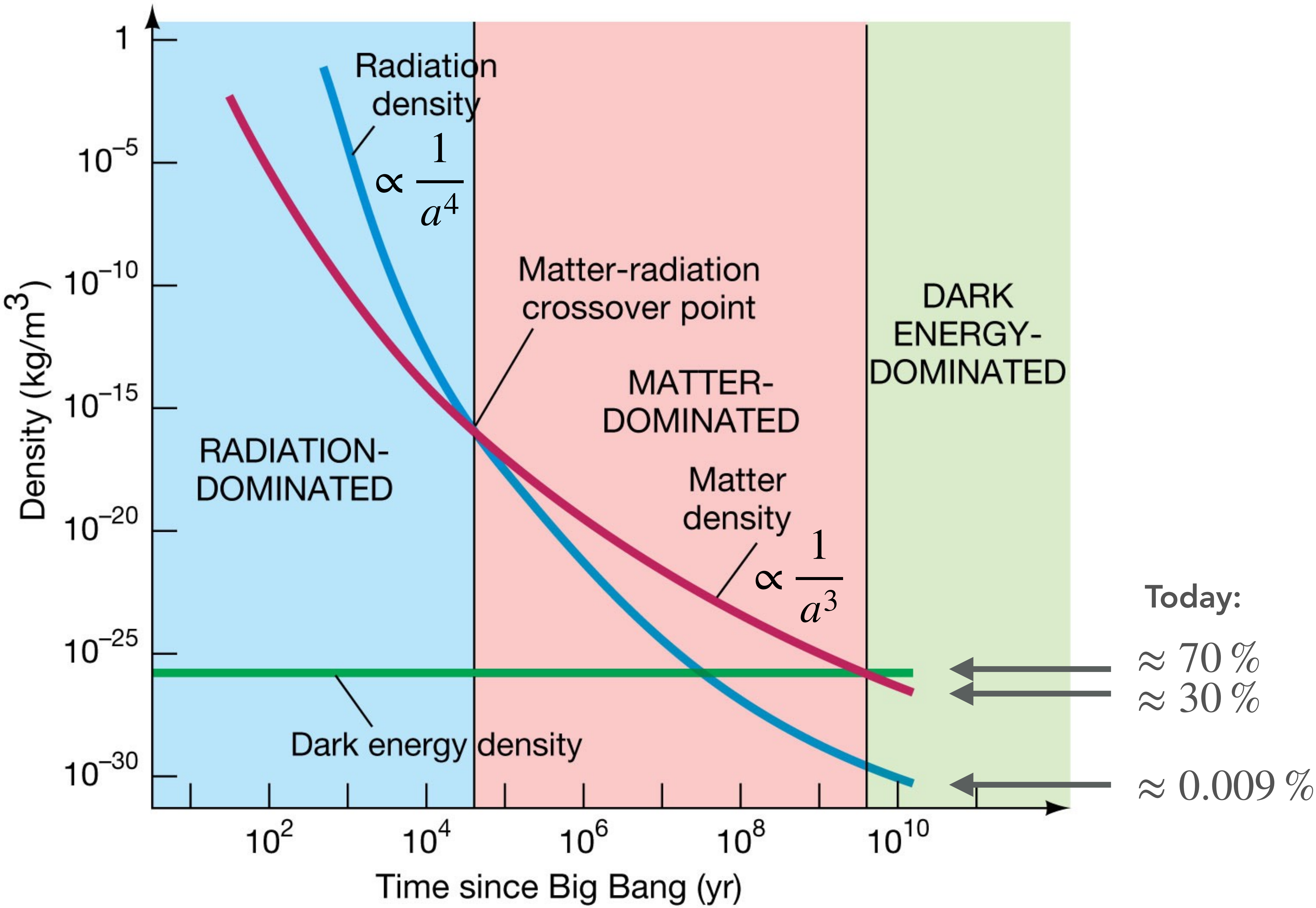
$$E = hf = \frac{hc}{\lambda} \implies E \propto \frac{1}{a}$$

- Thus, energy density of photons compared to today ($\rho_{\mathrm{r},0}$) goes down as $1/a^3 \times 1/a = 1/a^4$

$$\rho_{\mathrm{m}}(t) = \frac{\rho_{\mathrm{m},0}}{a^3}$$

$$\rho_{\mathrm{r}}(t) = \frac{\rho_{\mathrm{r},0}}{a^4}$$

What dominates the energy density?



University of Oregon / Pearson

Evolution of temperature

$$E_{\text{phot}} \propto \frac{1}{a}$$

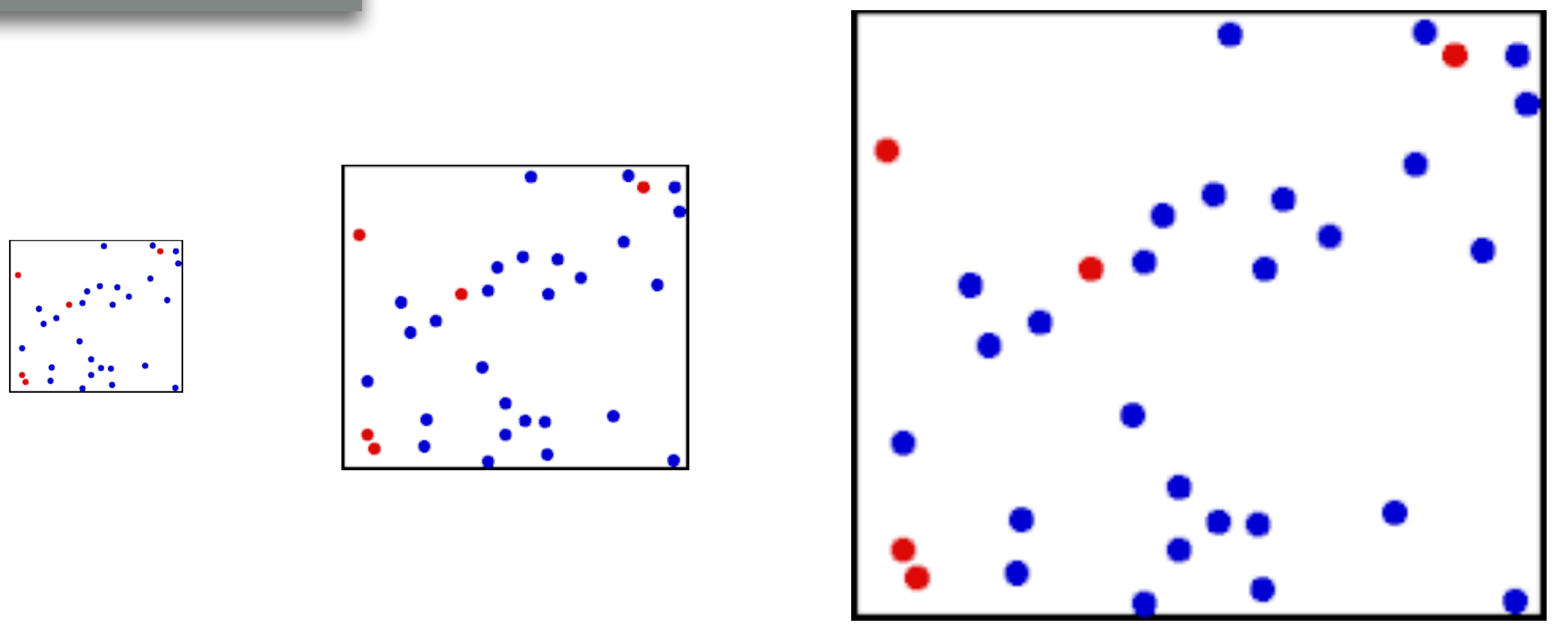
$$\langle E \rangle = \frac{3}{2} k_B T$$

$$\Longrightarrow$$

$$T(t) = \frac{T_0}{a}$$

CMB photon temperature today:

$$T_0 = 2.725\ \text{K}$$



Temperature and energy increase towards the Big Bang as 1/a

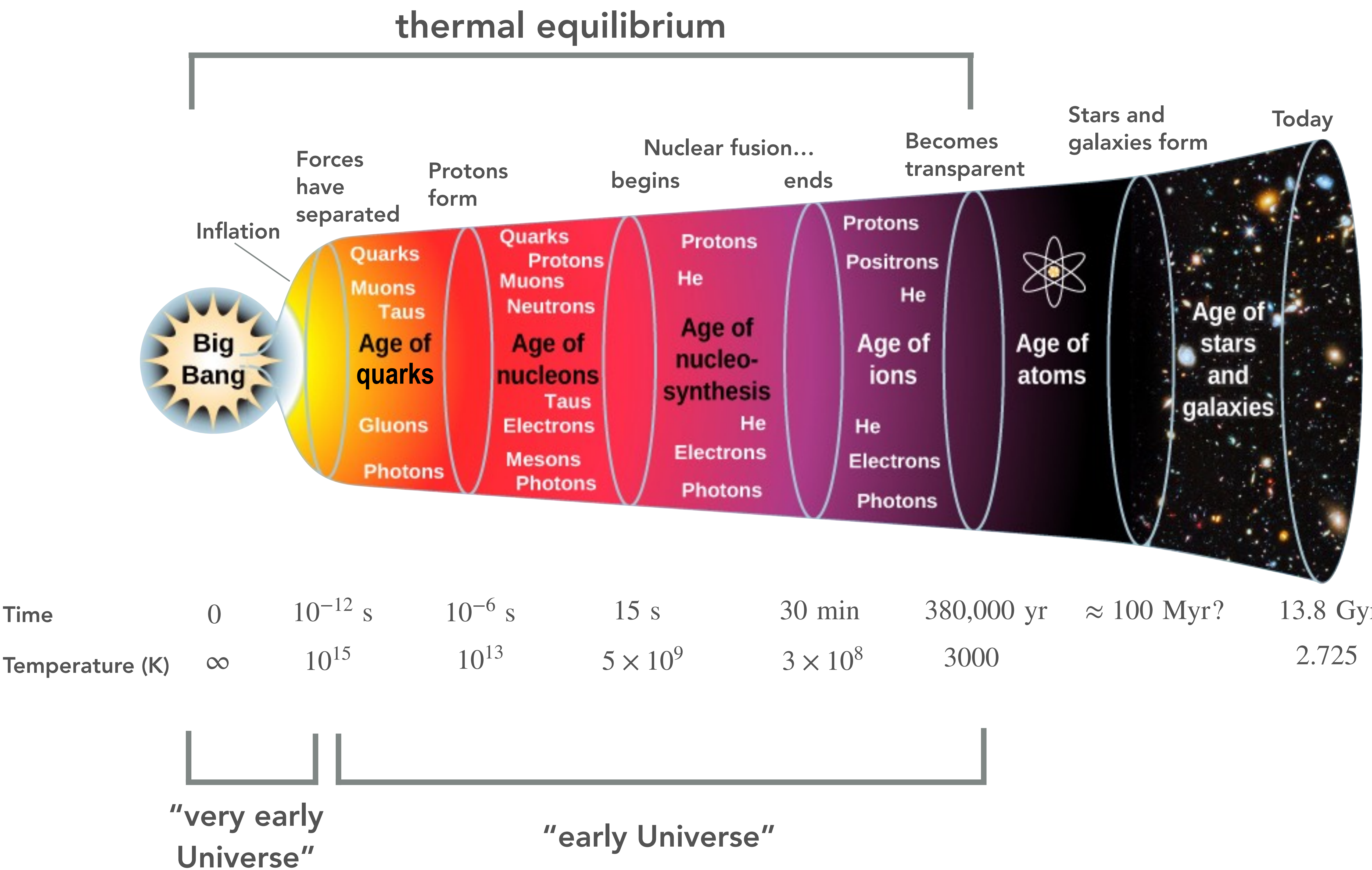
The hot Big Bang

- Lemaitre proposed Big Bang theory in 1927 (“primieval atom”)
- A hot early Universe was predicted in 1948 by George Gamow (with Alpher and Herman)
  - The idea: the universe started off very hot and cools as it expands
  - They predicted “relic radiation” with temperature of about 5K (close!)
  - Work not fully recognized until 1960s
- The **evolution of temperature** determines what happens
- In early Universe, temperature/energy was too high for electrons and nuclei to be bound as **atoms**
- In very early Universe, temperature/energy too high for protons and neutrons to remain bound as **nuclei**
- **No direct observations** to constrain theories

Lemaitre

Gamow

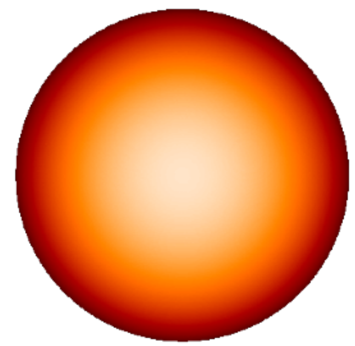
History of the Universe



Curvature and the fate of the Universe

Case 1: Closed Universe

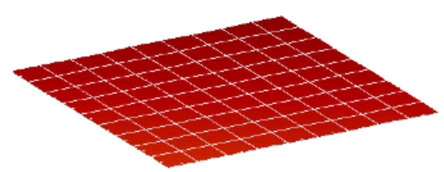
$$\Omega_{\mathrm{m}+\Lambda} > 1 \implies \Omega_{\mathrm{k}} < 0$$
$$\implies k > 0$$



Collapses eventually (big crunch)

Case 2: Flat Universe

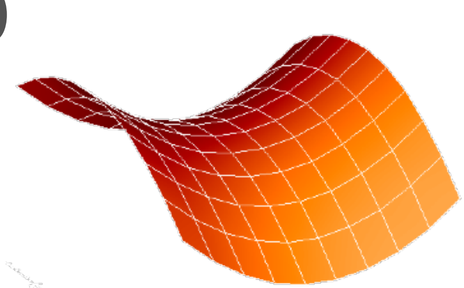
$$\Omega_{\mathrm{m}+\Lambda} = 1 \implies \Omega_{\mathrm{k}} = 0$$
$$\implies k = 0$$



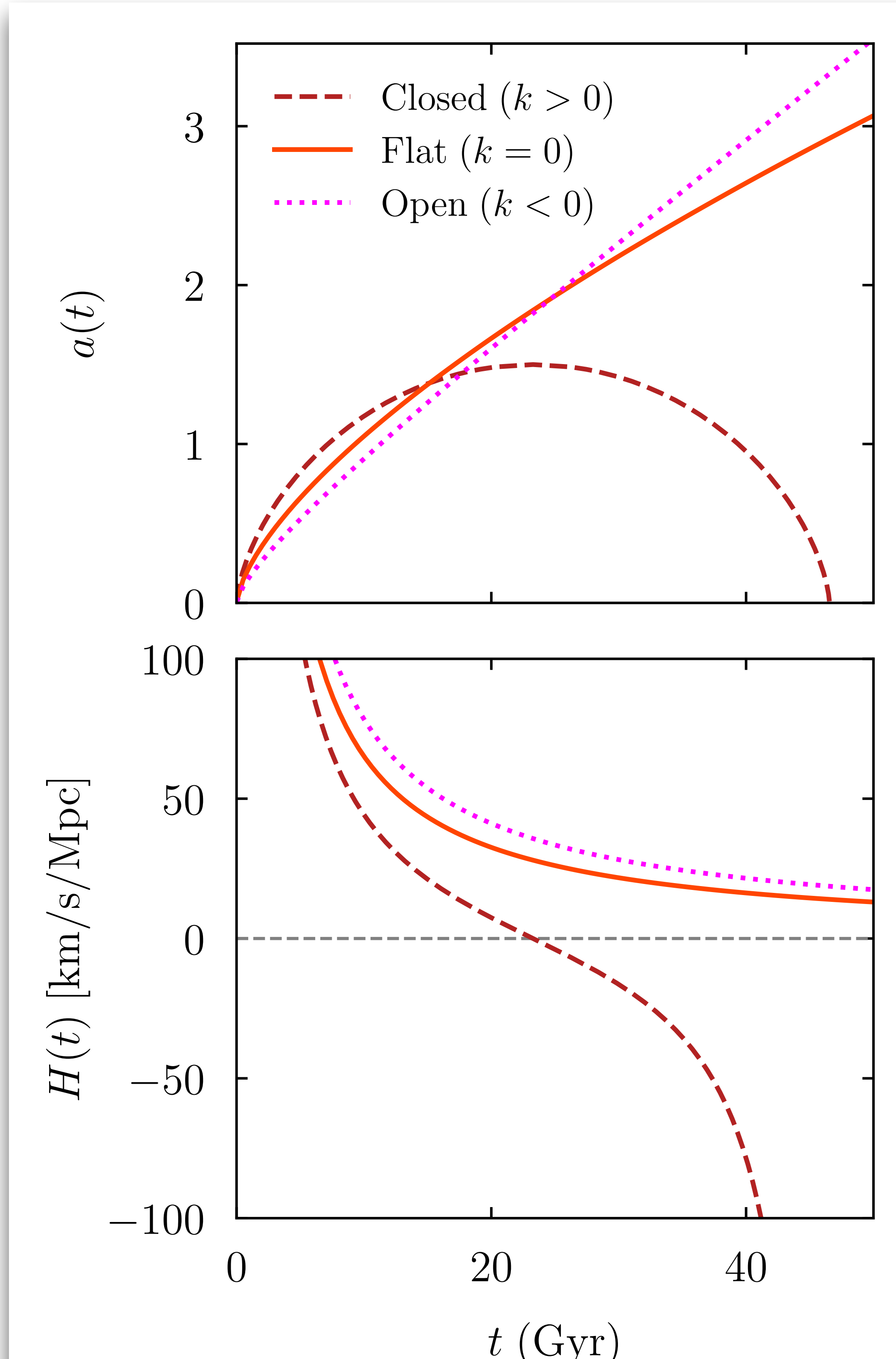
Slowly grinds to a halt

Case 3: Open Universe

$$\Omega_{\mathrm{m}+\Lambda} < 1 \implies \Omega_{\mathrm{k}} > 0$$
$$\implies k < 0$$



Expands forever



The flatness problem

Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\Omega_\mathrm{m} \equiv \frac{\rho}{\rho_\mathrm{c}}$$

Means that components must some to one:

$$\Omega_\mathrm{tot} \equiv \Omega_\mathrm{m} + \Omega_\Lambda + \Omega_\mathrm{k} = 1$$

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H^2}$$

$$\rho_\mathrm{c}(t) = \frac{3H^2(t)}{8\pi G}$$

$$\Omega_\mathrm{k} \equiv -\frac{Kc^2}{a^2H^2}$$

From cosmological data, e.g., Planck:

$$|\,\Omega_{\mathrm{k},0}\,| < 0.01$$

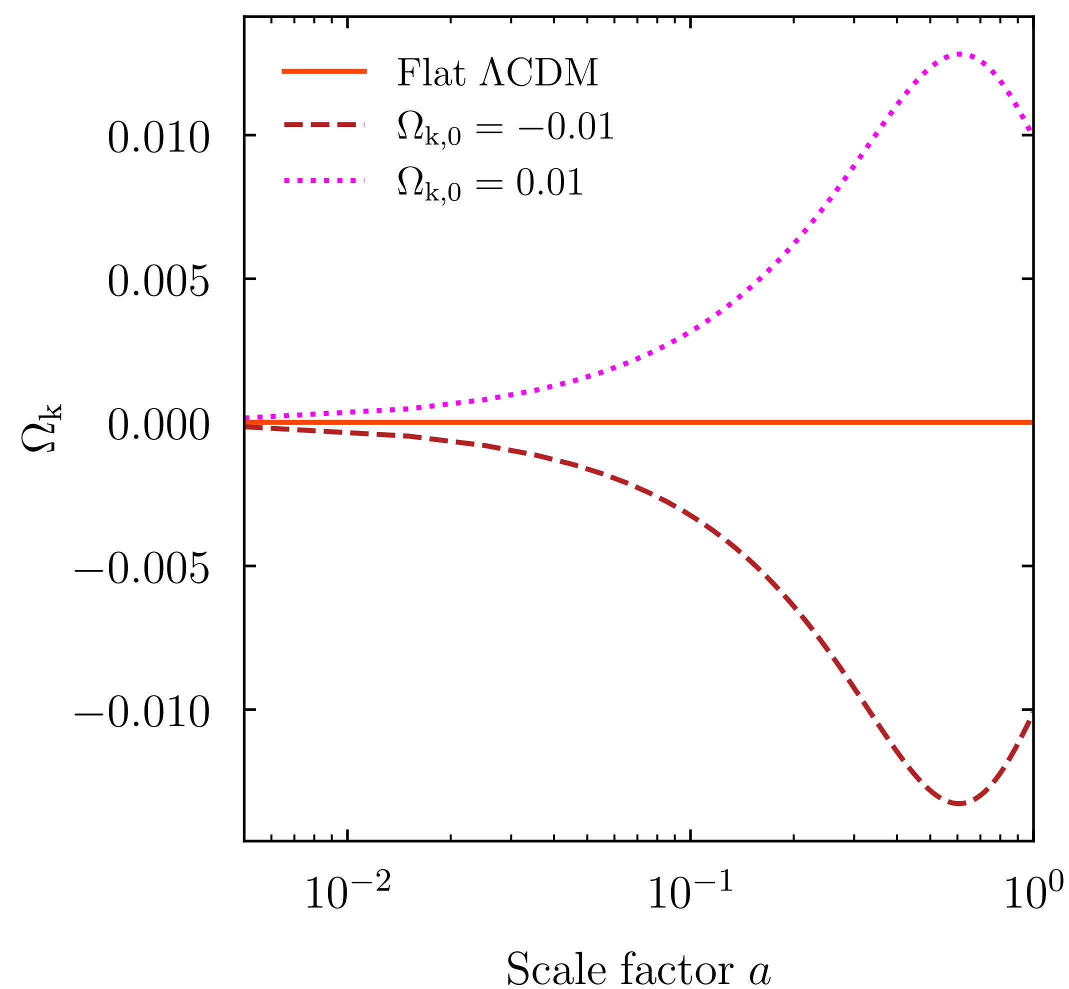
Question: doesn't it seem random that the Universe is flat, i.e., that $\Omega_{\mathrm{m},0} + \Omega_{\Lambda,0} \approx 1$ so that $\Omega_{\mathrm{k},0} \approx 0$?

The flatness problem

- The curvature term evolves as $\Omega_{\rm k} \propto 1/a(t)^2 H(t)^2$
- This scales with time between $\Omega_{\rm k} \propto t$ and $\Omega_{\rm k} \propto t^{2/3}$
  - If the Universe is slightly open in the beginning, that accelerates the expansion compared to flat, leading to lower density / even more open-ness
  - If the Universe is slightly closed in the beginning, that slows down the expansion compared to flat, leading to higher density / even more closed-ness
- If the Universe is close to $\Omega_{\rm k} = 0$ today, it must have been even much closer in the early Universe
- At end of the Planck epoch (10^{-43} s), $|\Omega_{\rm k}| < 10^{-60}$!
- This is a "**fine-tuning**" problem: why would $\Omega_{\rm m} + \Omega_\Lambda$ take on a value so close to one?
- Why do we care?
  - If $\Omega_{\rm m} + \Omega_\Lambda$ had been much above 1, would have recollapsed very early **before making galaxies**
  - If $\Omega_{\rm m} + \Omega_\Lambda$ had been much below 1, would have expanded so rapidly that **structures would not have formed**

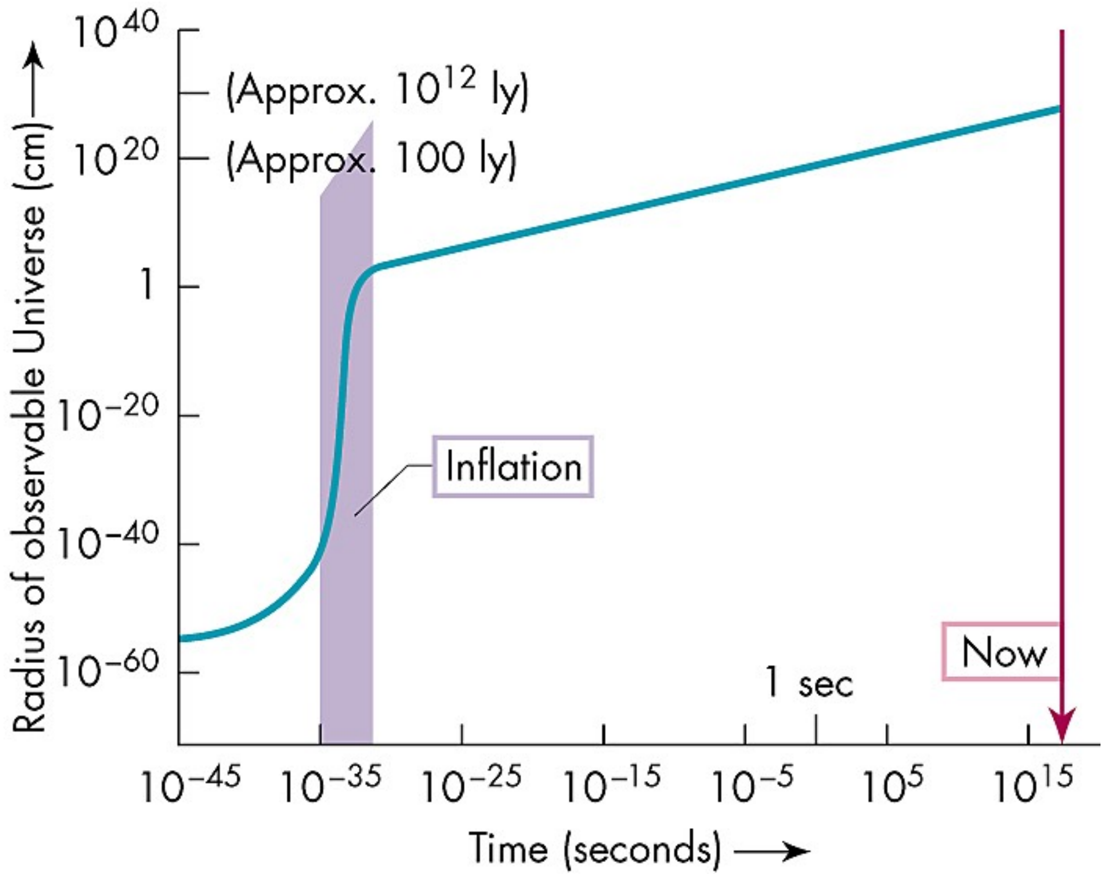
$$\Omega_{\rm tot} \equiv \Omega_{\rm m} + \Omega_\Lambda + \Omega_{\rm k} = 1$$

$$\Omega_{\rm k}(t) = -\frac{Kc^2}{a(t)^2 H(t)^2}$$



Inflation

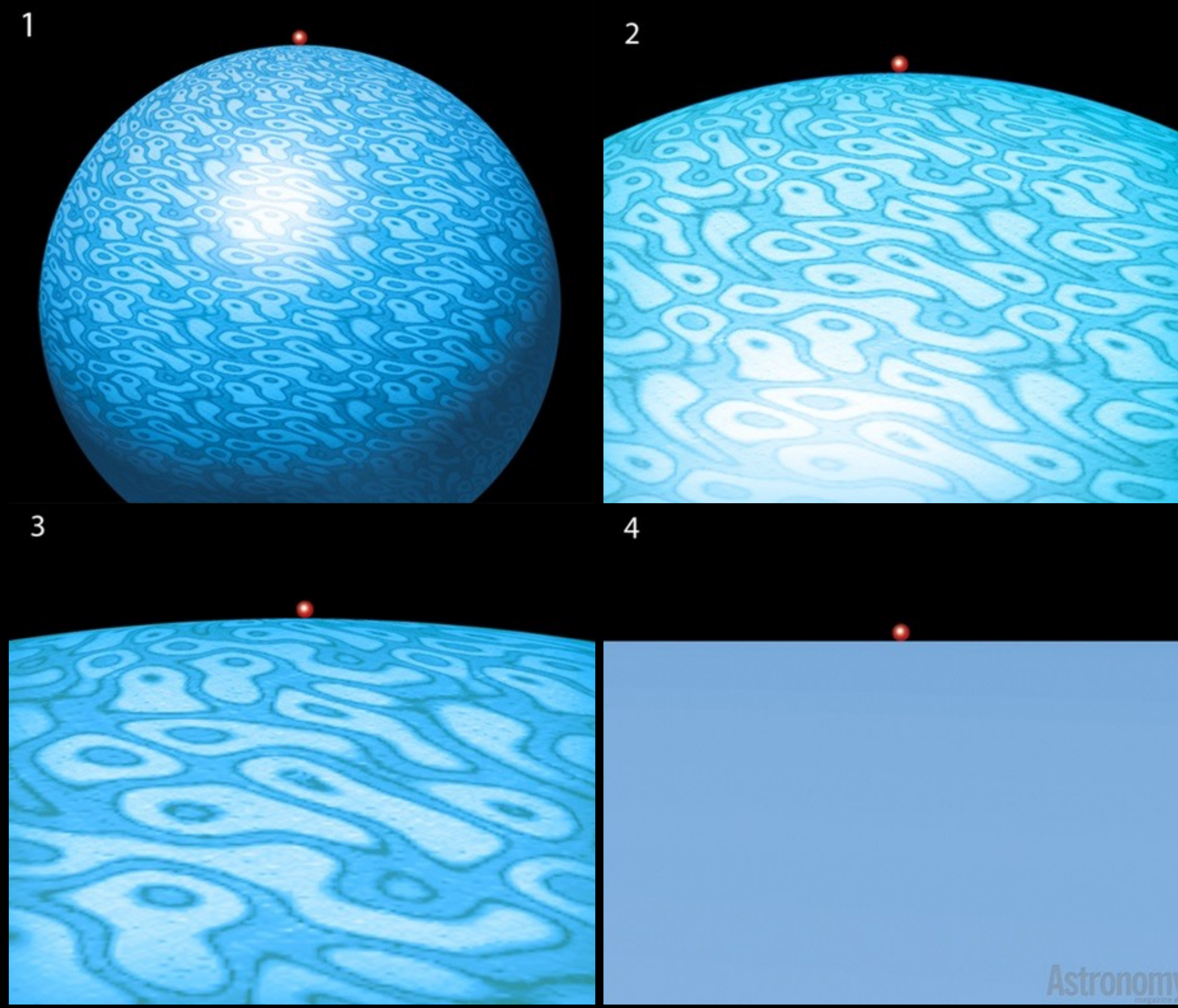
- Theory of cosmic inflation was first proposed by Alan Guth in 1982
- Inflation is a **very rapid expansion** of Universe at $t = 10^{-37}$-10^{-32} s after the Big Bang
- Universe expanded by a **factor of 10^{40} - 10^{100}** during this time!
- Looks a lot like **exponential expansion** due to dark energy, $a(t) \propto e^{Ht}$, just faster
- Super-light-speed expansion does not violate relativity because spacetime itself expands (same argument as for later Hubble expansion)



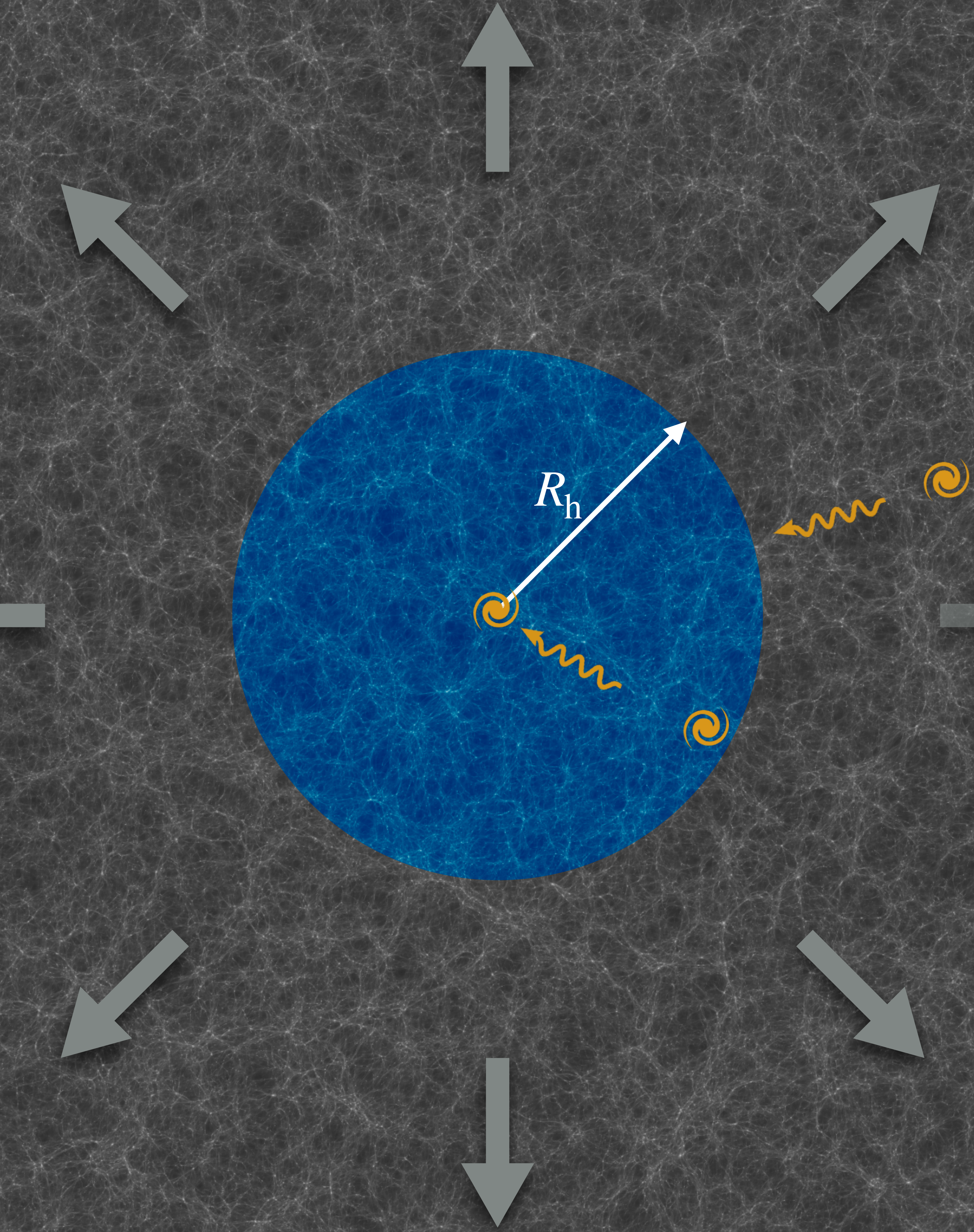
McGraw-Hill

The flatness problem

- Take any reasonably curved surface and expand it by a very large factor: it will look flat
- Inflation naturally **predicts a flat Universe**…
- …if the initial value of Ω_k was within some relatively generous range



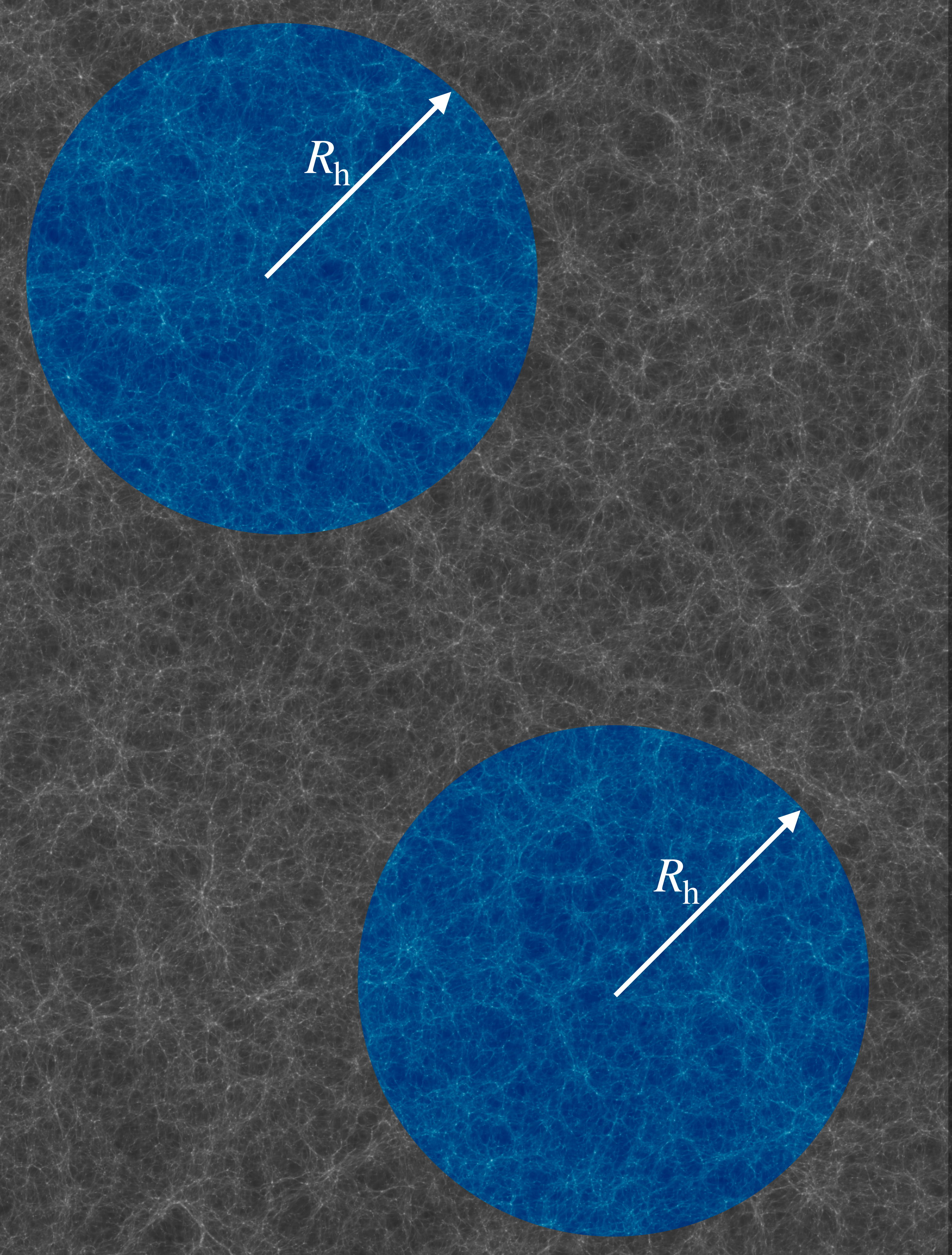
Astronomy Magazine



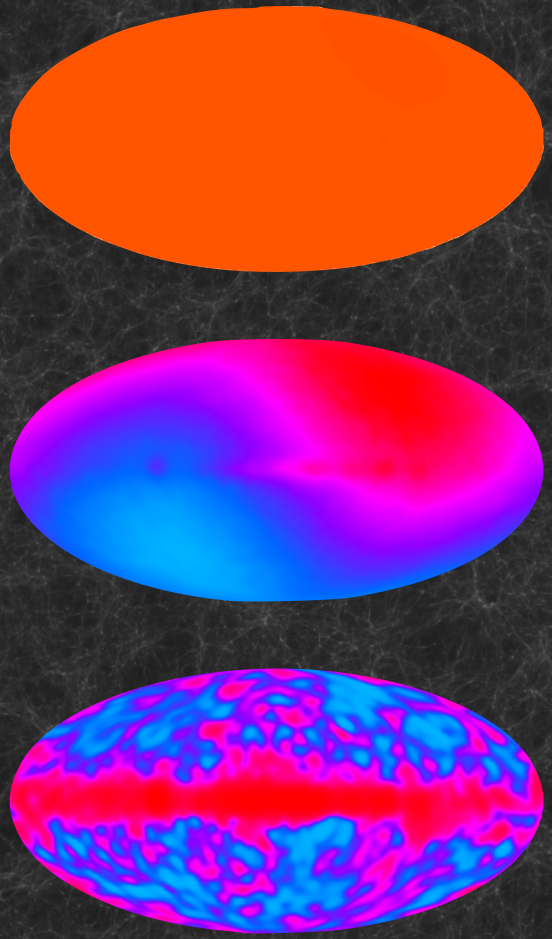
- Horizon is the distance that is **causally connected** (from which we can have received information)
- In a Universe without inflation, $R_{\mathrm{h}} \approx ct$ (the actual horizon is a little different due to the expansion history)
- At recombination, $t \approx 10^{13}$ s, the horizon would be

$$R_{\mathrm{h}} \approx ct \approx 10^{5}\ \mathrm{pc} = 100\ \mathrm{kpc}$$

- How big would this patch be today? It has grown with the scale factor since z = 1100, so 1100 times larger
- But that's only about **100 Mpc**! The distance to the CMB surface of last scattering is about 14 Gpc



- The CMB we see should be composed of many “patches” that had not been in causal contact before recombination
- How can they be so exactly the same? How can the temperature of the **Cosmic Microwave Background** be so homogeneous (1 part in 100,000)?
- This is called the **horizon problem**

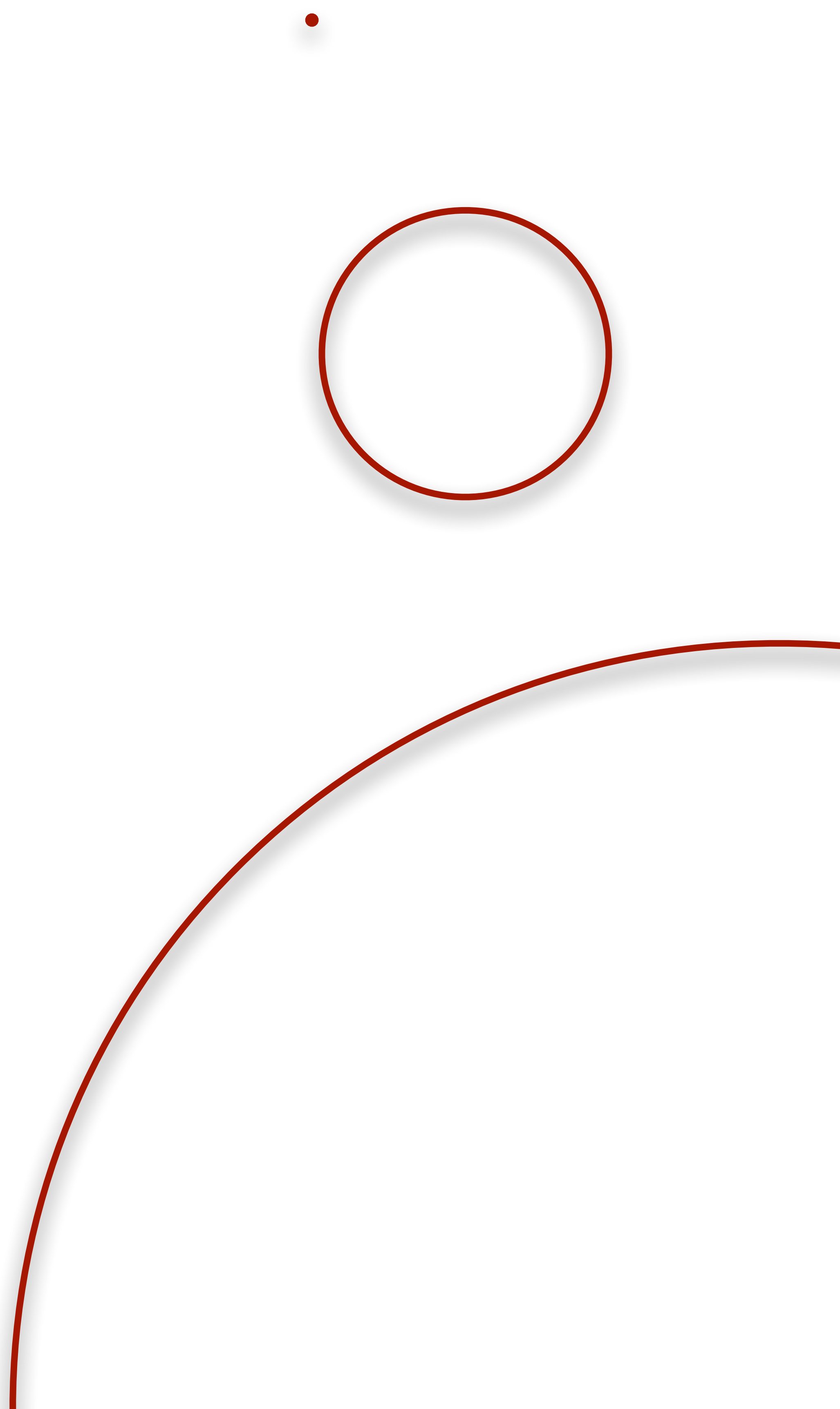


Inflation solves the horizon problem

- Before inflation:

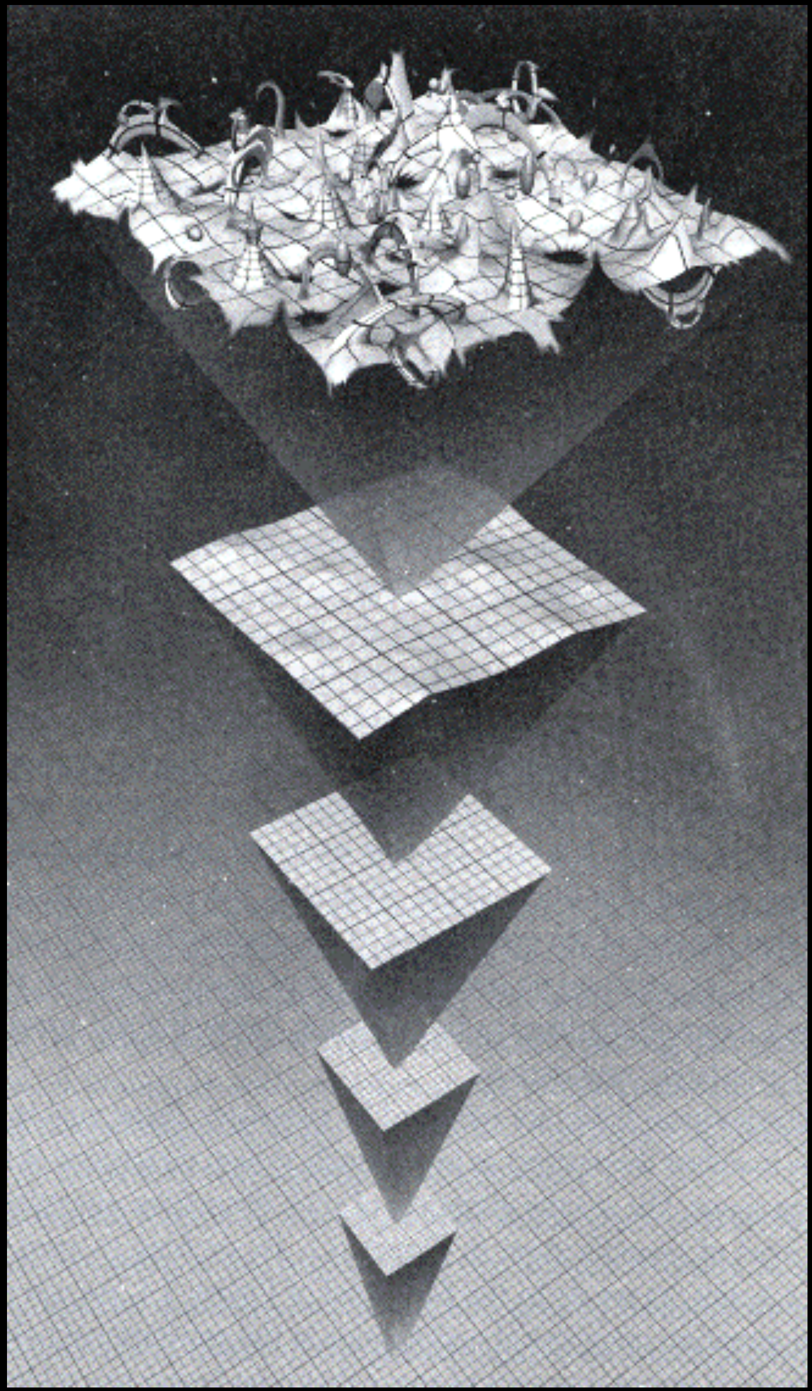
$$R_\mathrm{h} \approx ct = c \times 10^{-37}\ \mathrm{s} \approx 10^{-27}\ \mathrm{cm}$$

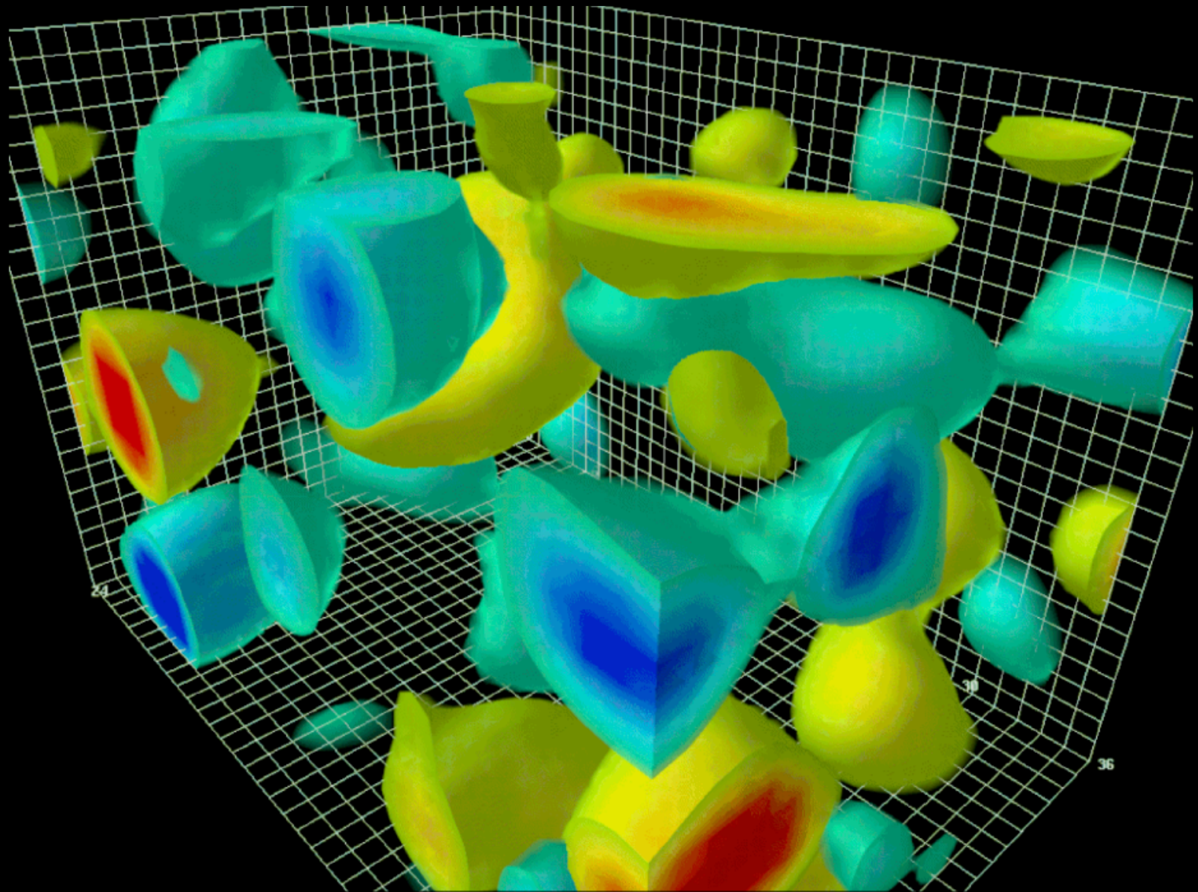
- Inflation **blows up causally connected regions** by a huge factor to at least 10^{13} cm (but possibly much more)
- Universe expands by 10^{22} by the time of recombination (380,000 years), so causally connected region is then about 10^{35} cm = 10^{7} Gpc or more
- The causally connected region then keeps growing as light travels, and is **much greater than the observable Universe**
- Thus, **inflation solves the horizon problem**



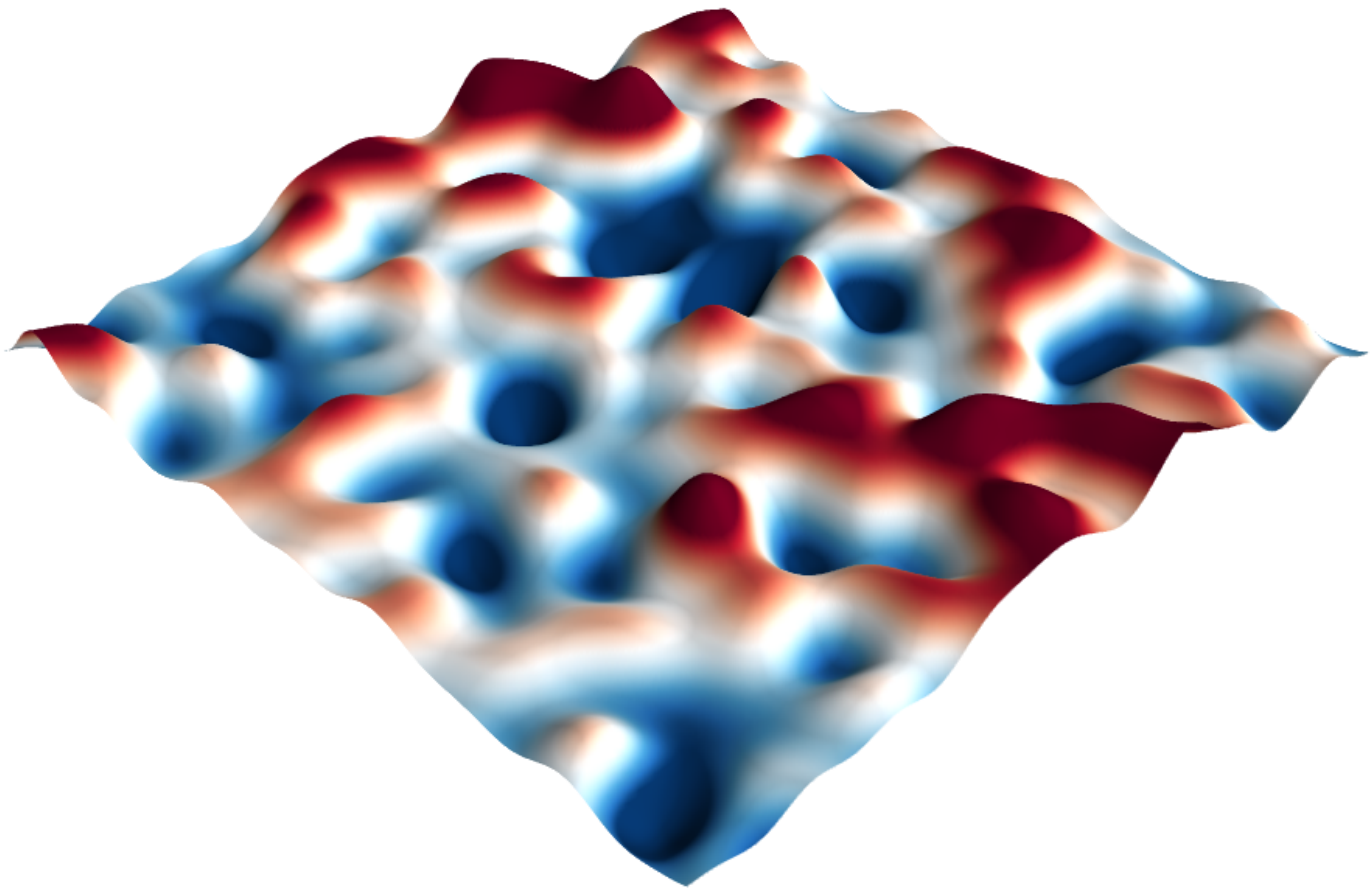
Quantum fluctuations

- Nature is fundamentally uncertain, which is called the "Heisenberg uncertainty principle"
- This principle implies that density can never be totally smooth
- The tiny fluctuations get amplified when the Universe rapidly expands during inflation
- The fluctuations are the origin of all structure in the Universe, such as galaxies!



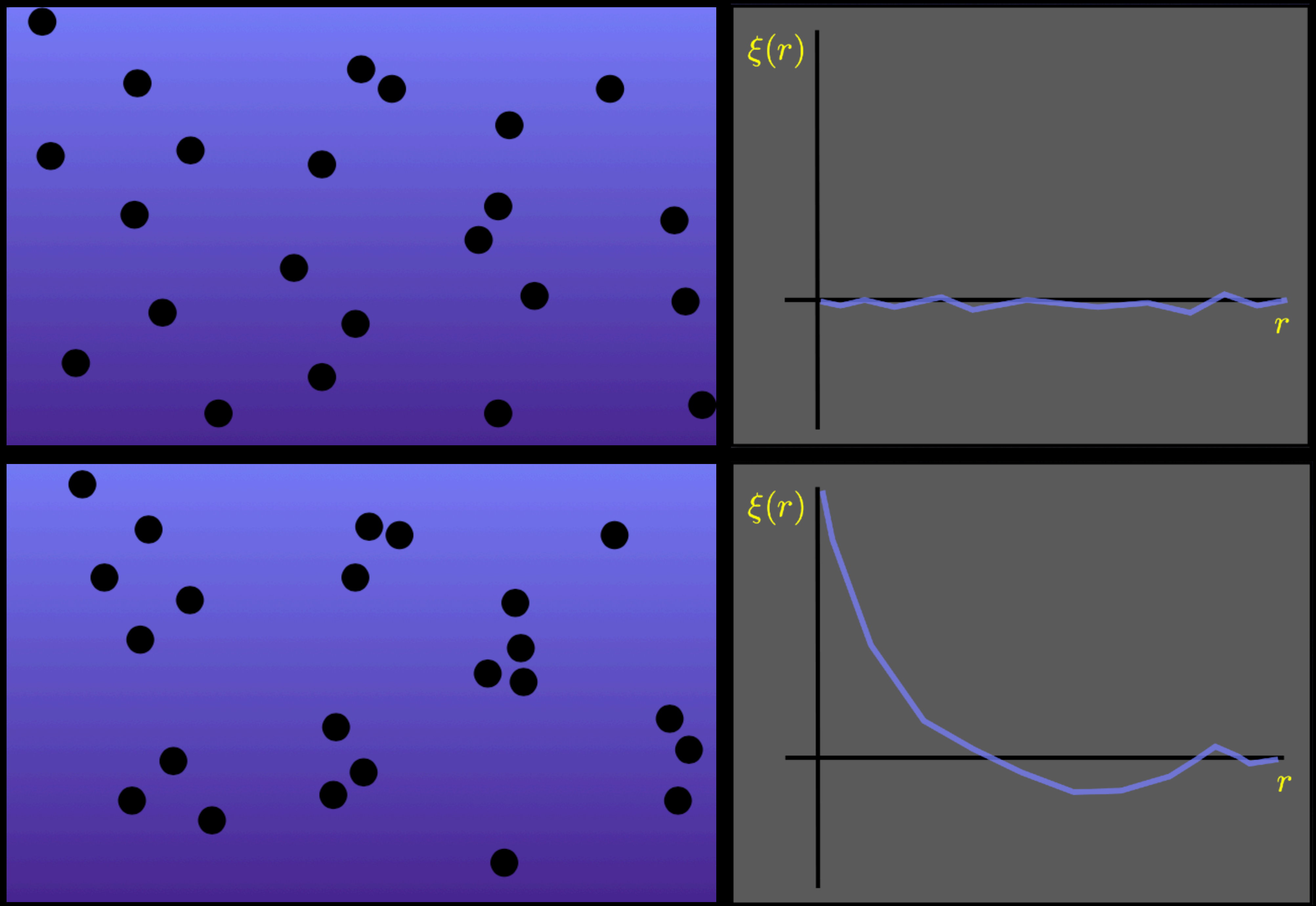


Overdensity field



§3.2 • Describing fluctuations with power spectra and correlation functions

Correlation function



$$1 + \xi(r) = \frac{n_{\text{pair}}(r \pm \mathrm{d}r)}{n_{\text{random}}(r \pm \mathrm{d}r)}$$

Images by Frank van den Bosch

Power spectrum

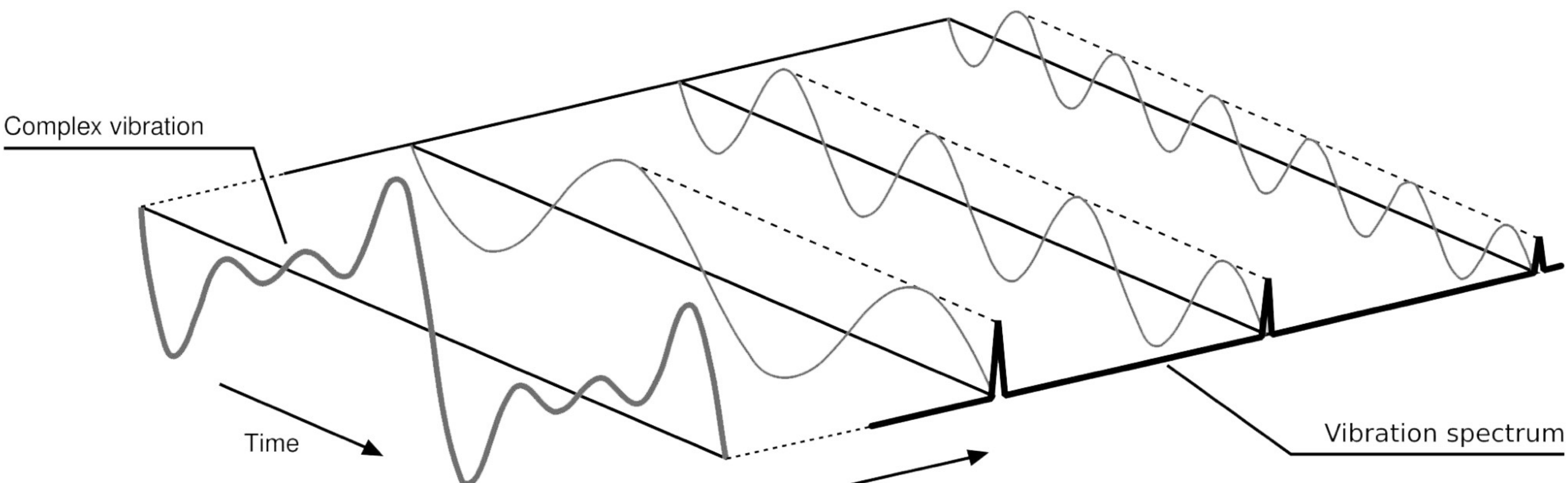
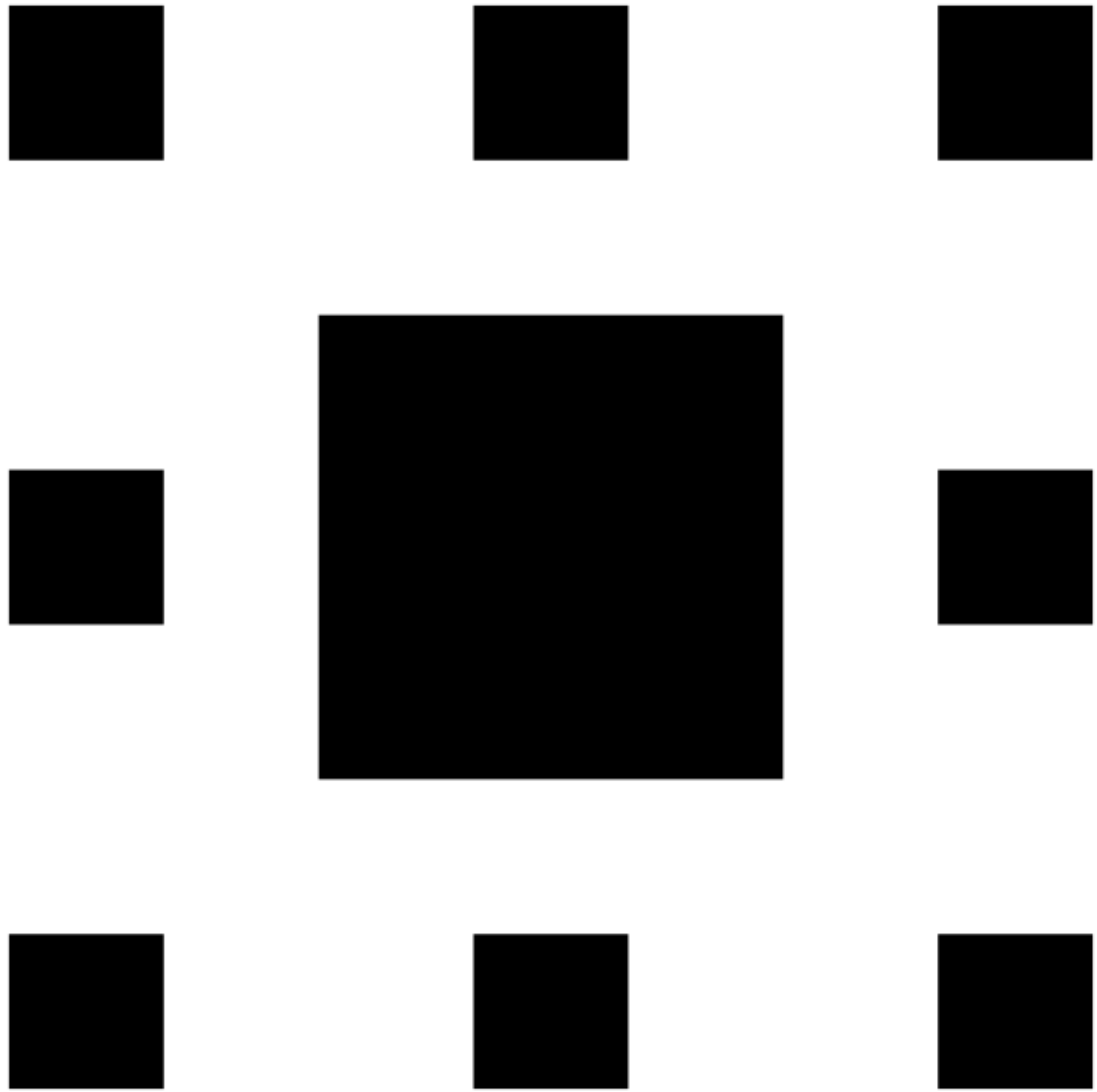
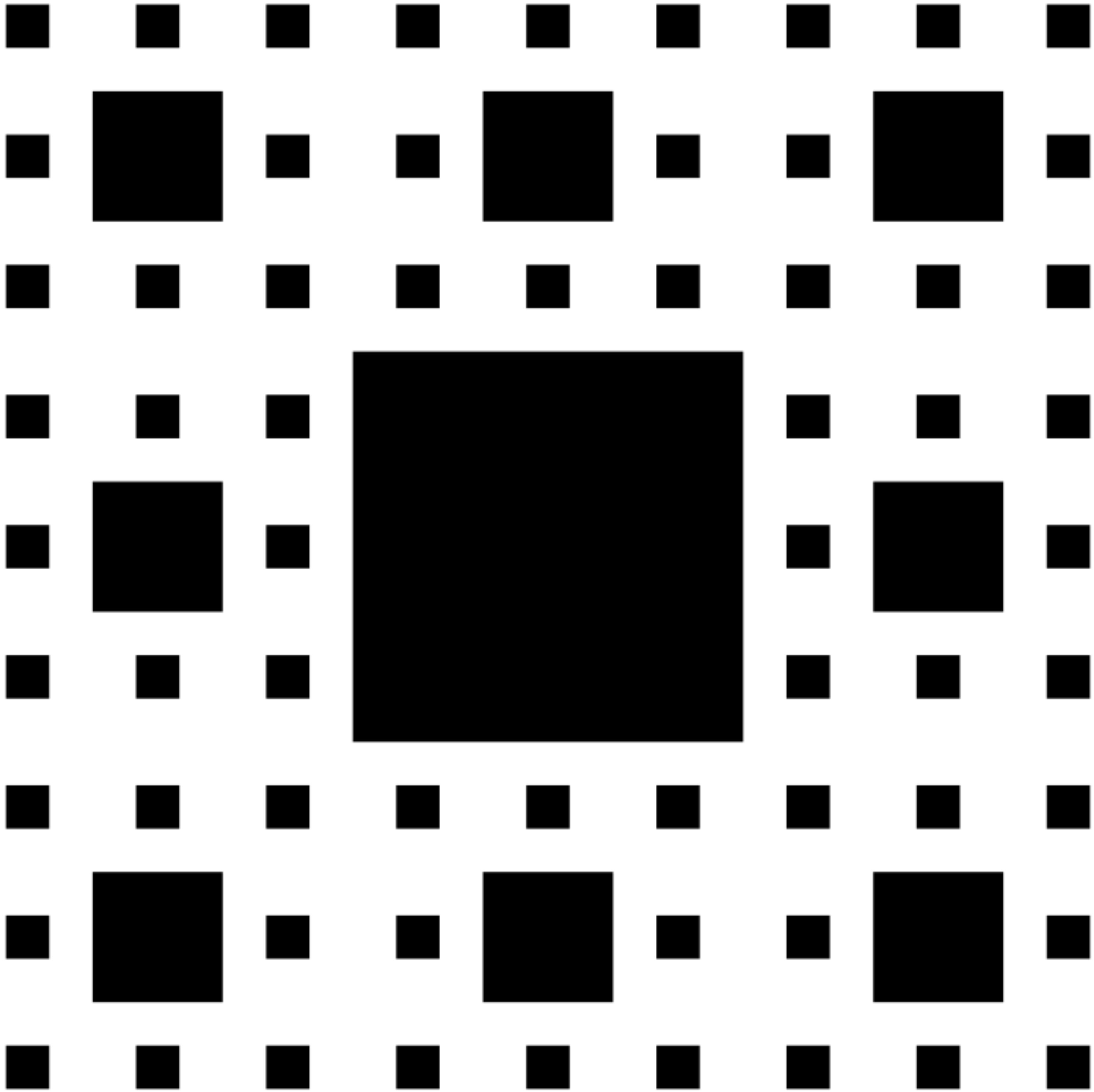


Image by Power-MI

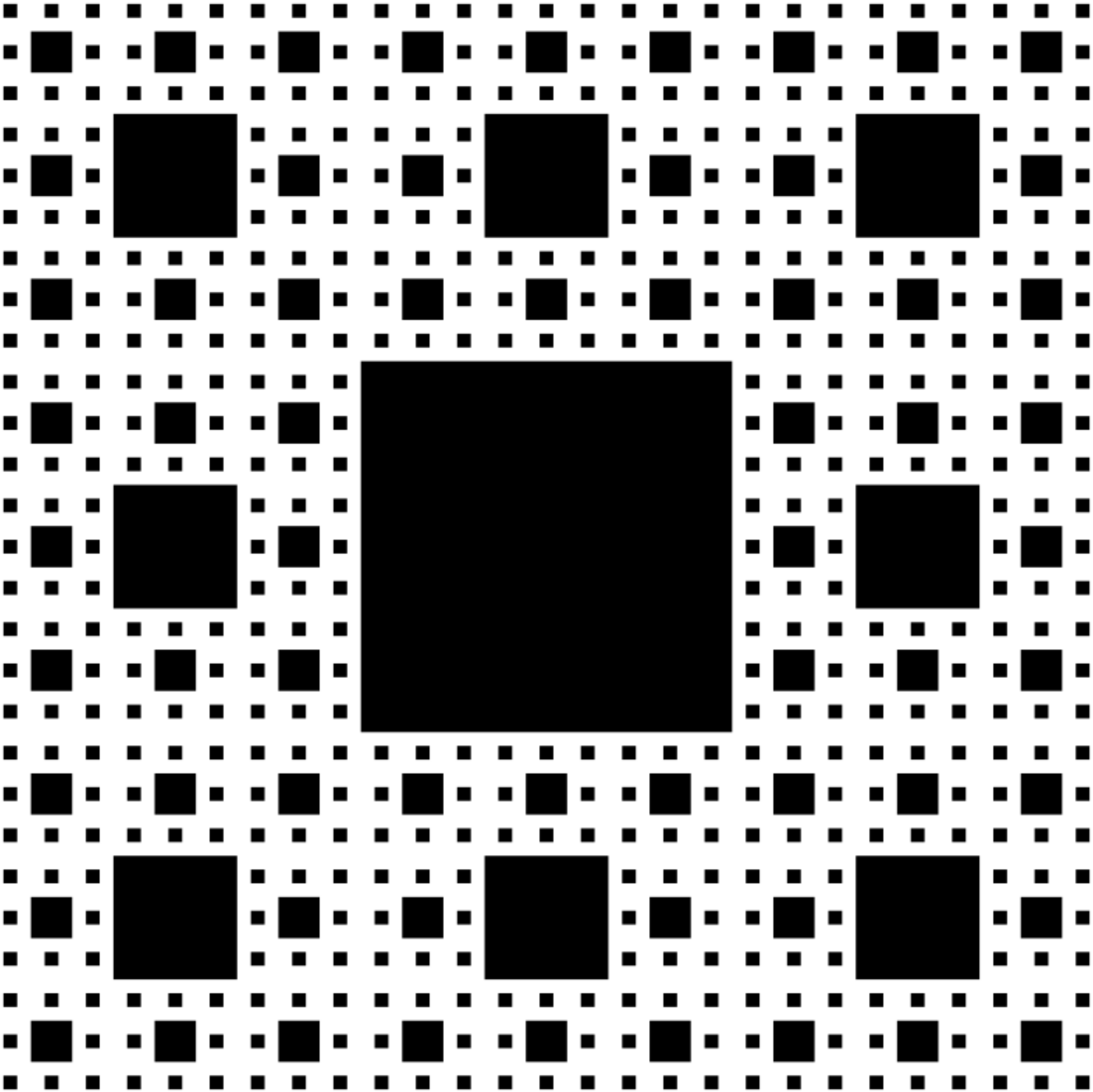
Sierpinski Carpet



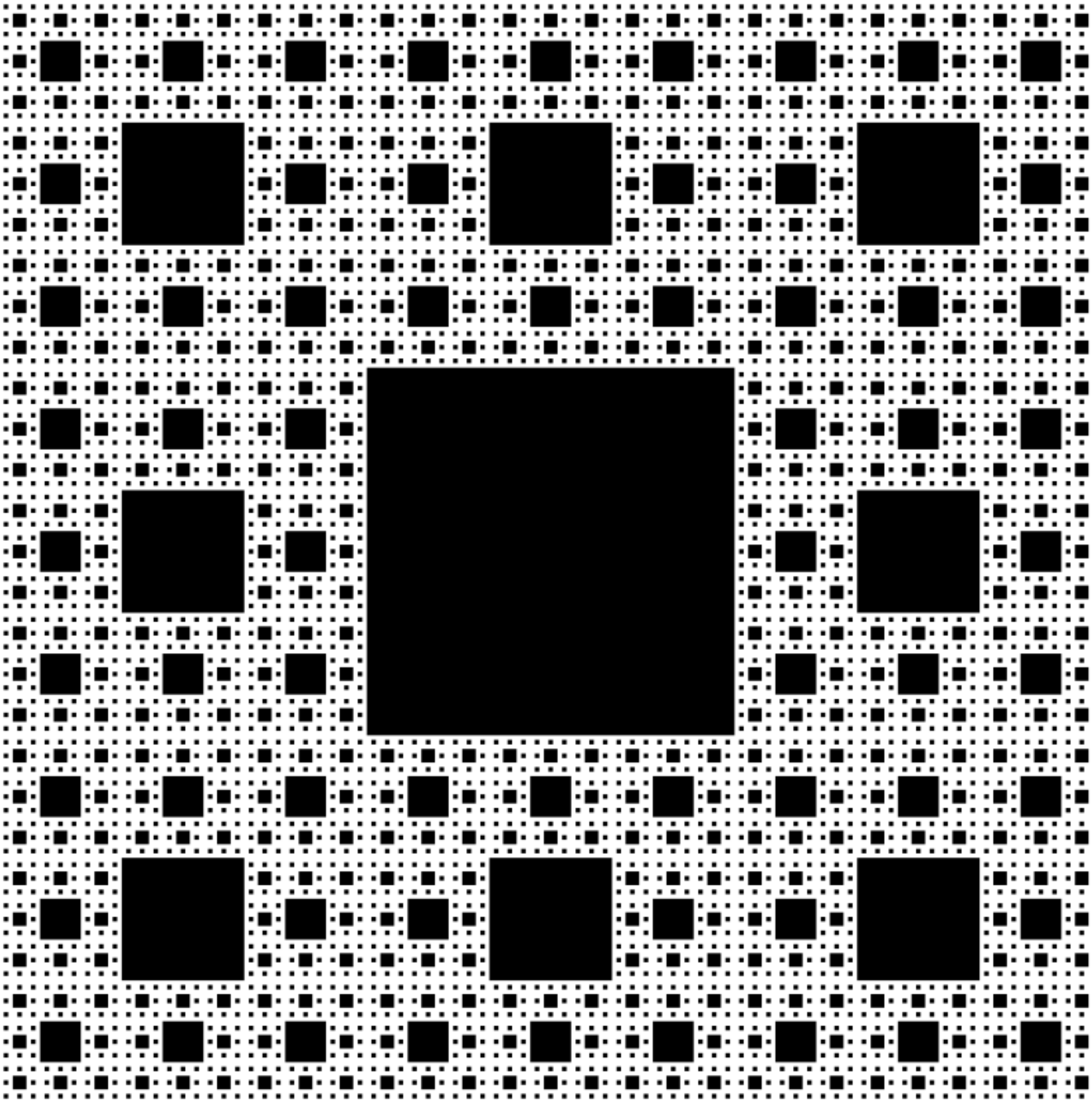
Sierpinski Carpet



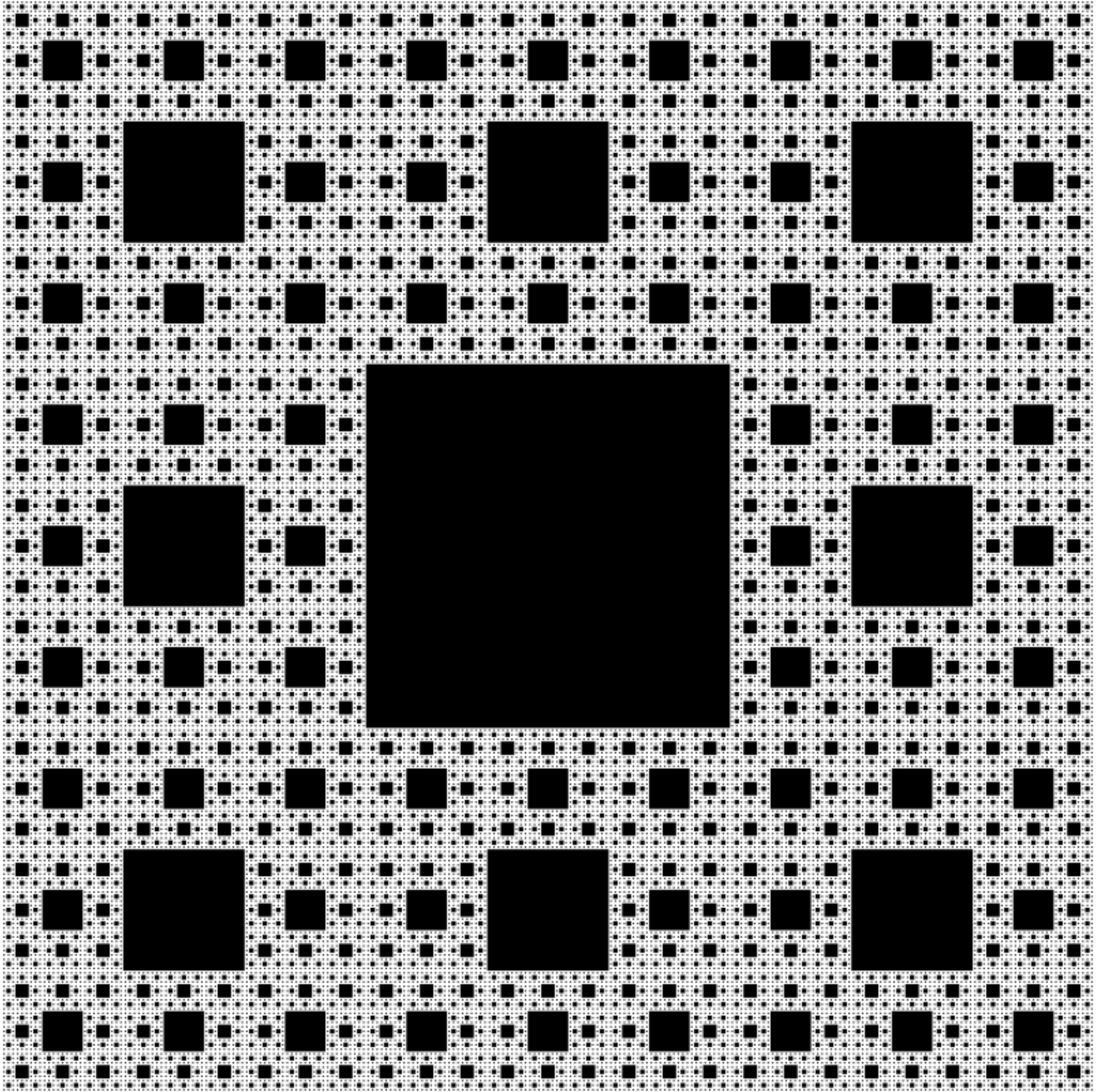
Sierpinski Carpet



Sierpinski Carpet



Sierpinski Carpet



Sierpinski Carpet

§3.3 • The linear evolution of fluctuations

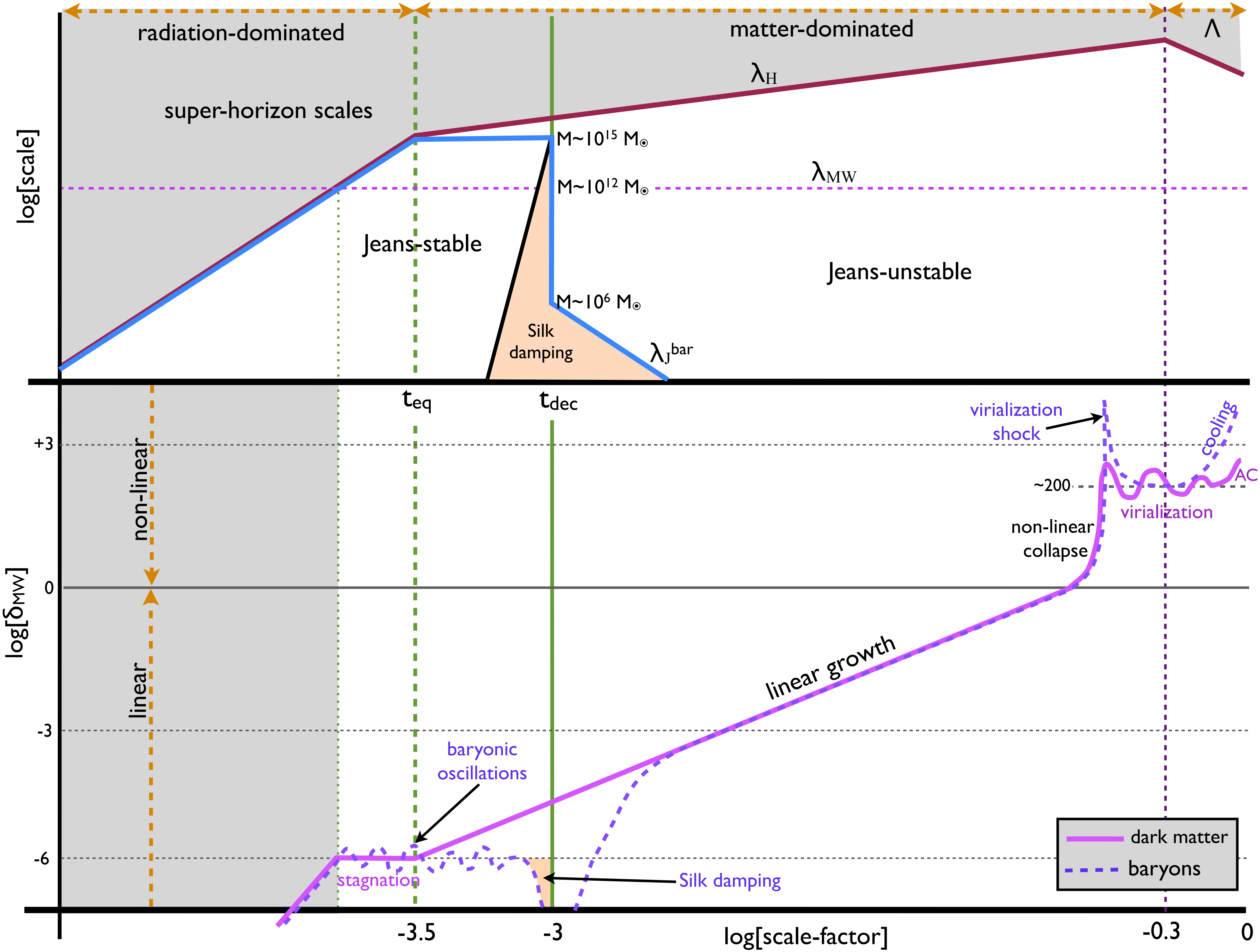
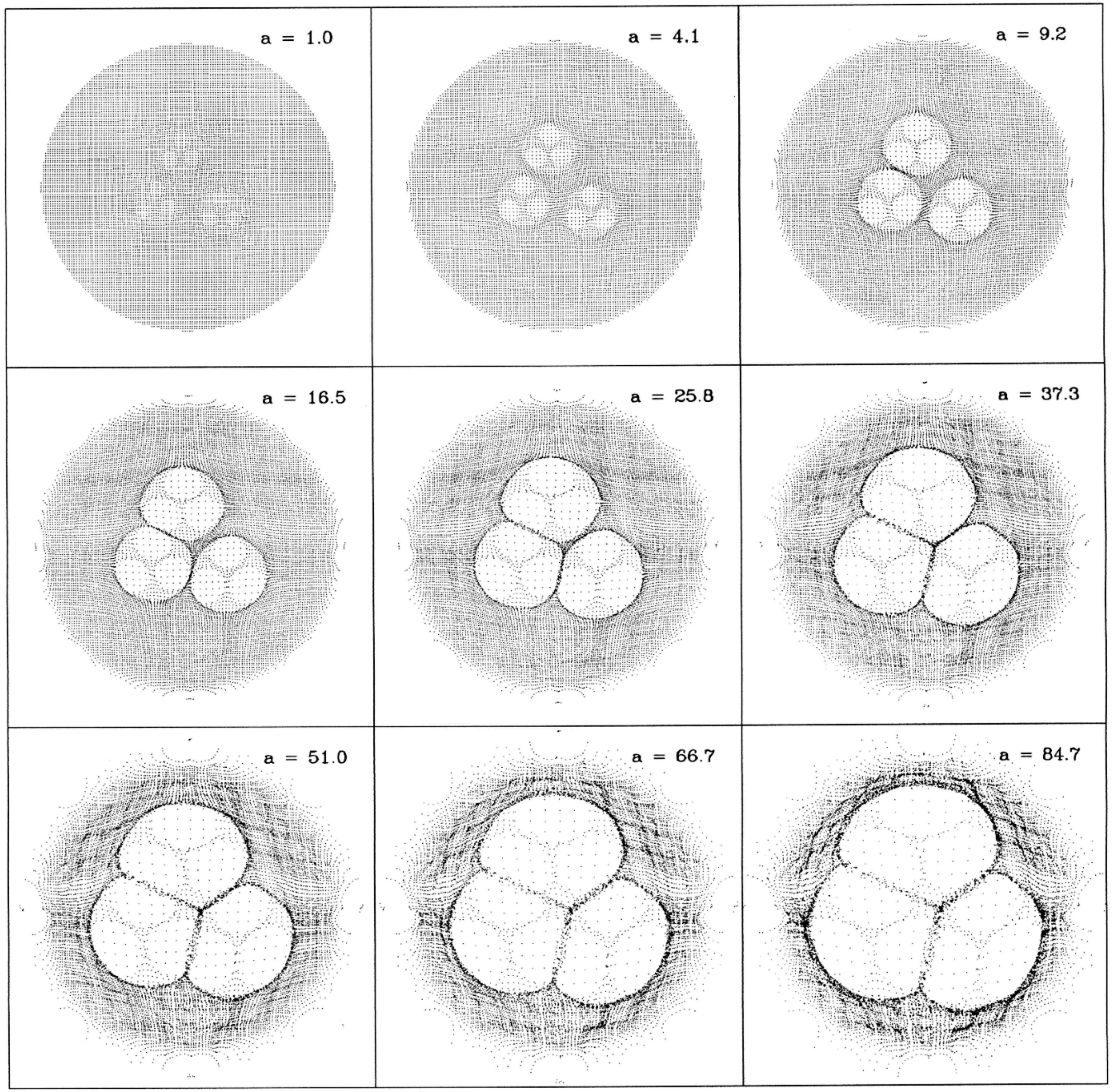


Figure by Frank van den Bosch

Evolution of collisionless underdensities



Dubinski et al. 1993

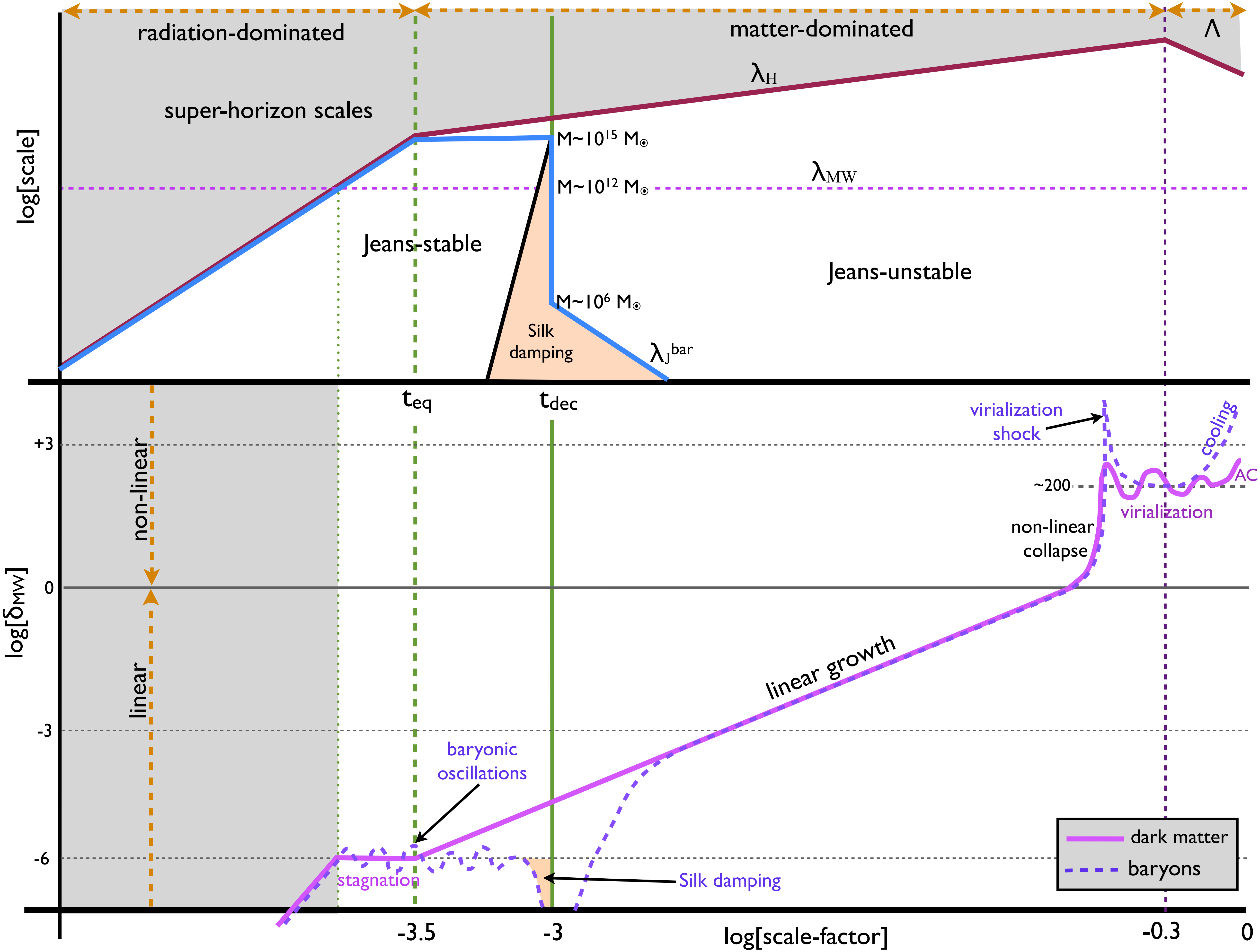


Figure by Frank van den Bosch

The Jeans Mass

- Gravity tries to collapse gas
- Pressure resists collapse
- Gravity wins if cloud is larger than Jeans length, or has mass larger than Jeans mass

Sir James Hopwood Jeans

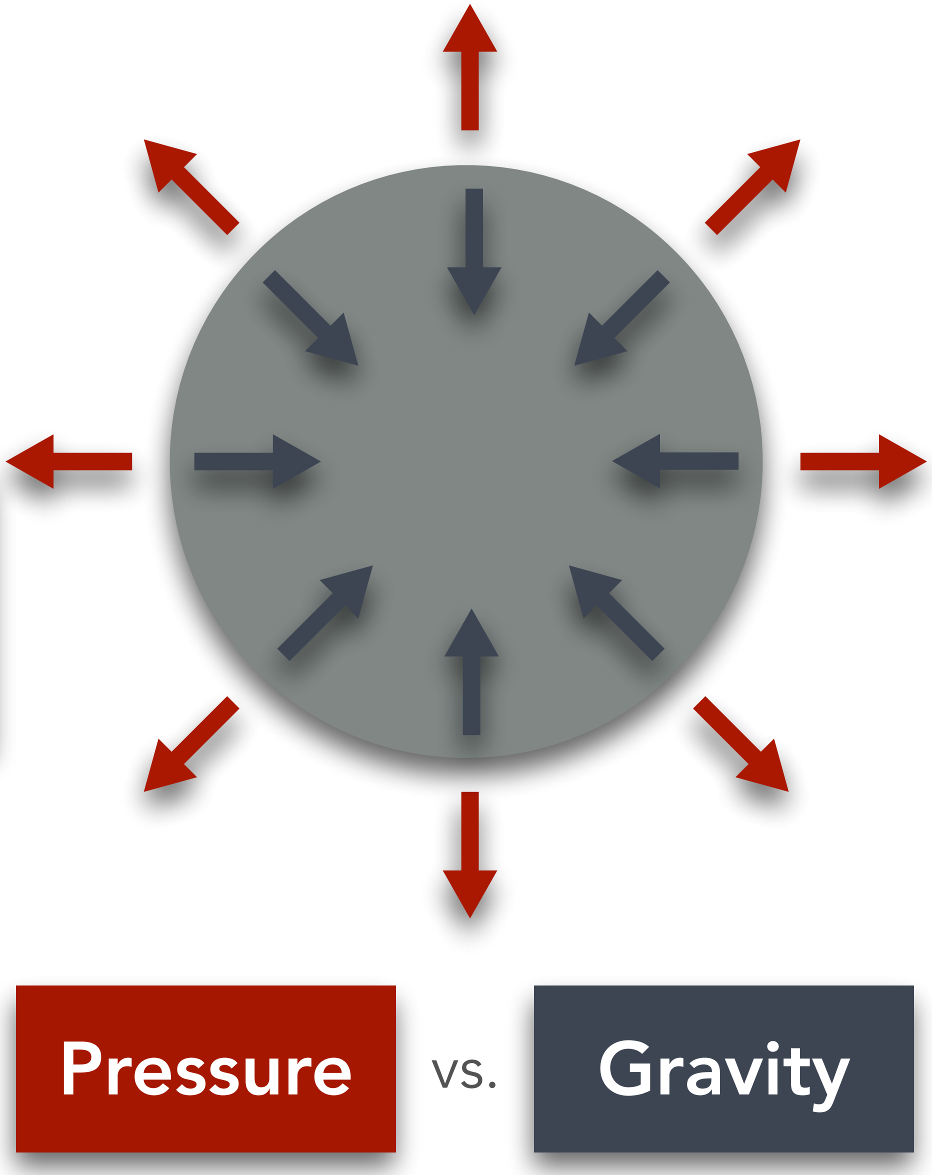
$$\lambda_J = \sqrt{\frac{5\pi}{3}\frac{k_B T}{G\, m_p\, \rho}} = 8.0 \times 10^7\ \mathrm{cm} \left(\frac{T}{K}\right)^{1/2} \left(\frac{\rho}{\mathrm{g/cm^3}}\right)^{-1/2}$$

$$M_J = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \rho \implies$$

$$M_J = \frac{\pi^{5/2}}{6}\left(\frac{5k_B T}{3Gm_p}\right)^{3/2} \rho^{-1/2} = 2.7 \times 10^{23}\ \mathrm{g} \left(\frac{T}{K}\right)^{3/2} \left(\frac{\rho}{\mathrm{g/cm^3}}\right)^{-1/2}$$

$k_B = 1.38 \times 10^{-16} \frac{\mathrm{erg}}{K}$ Boltzmann constant

$m_p = 1.67 \times 10^{-24}\ \mathrm{g}$ Proton mass



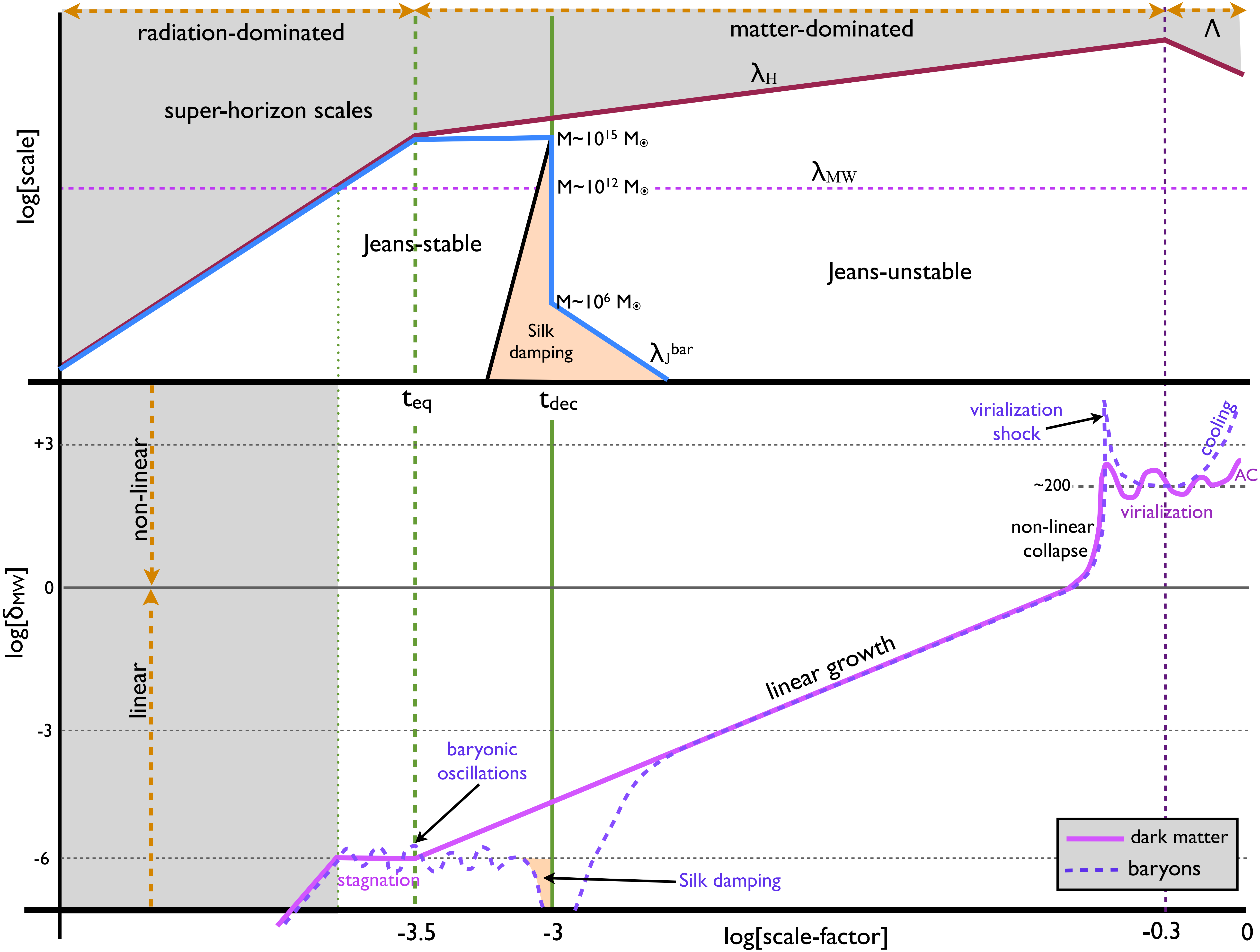
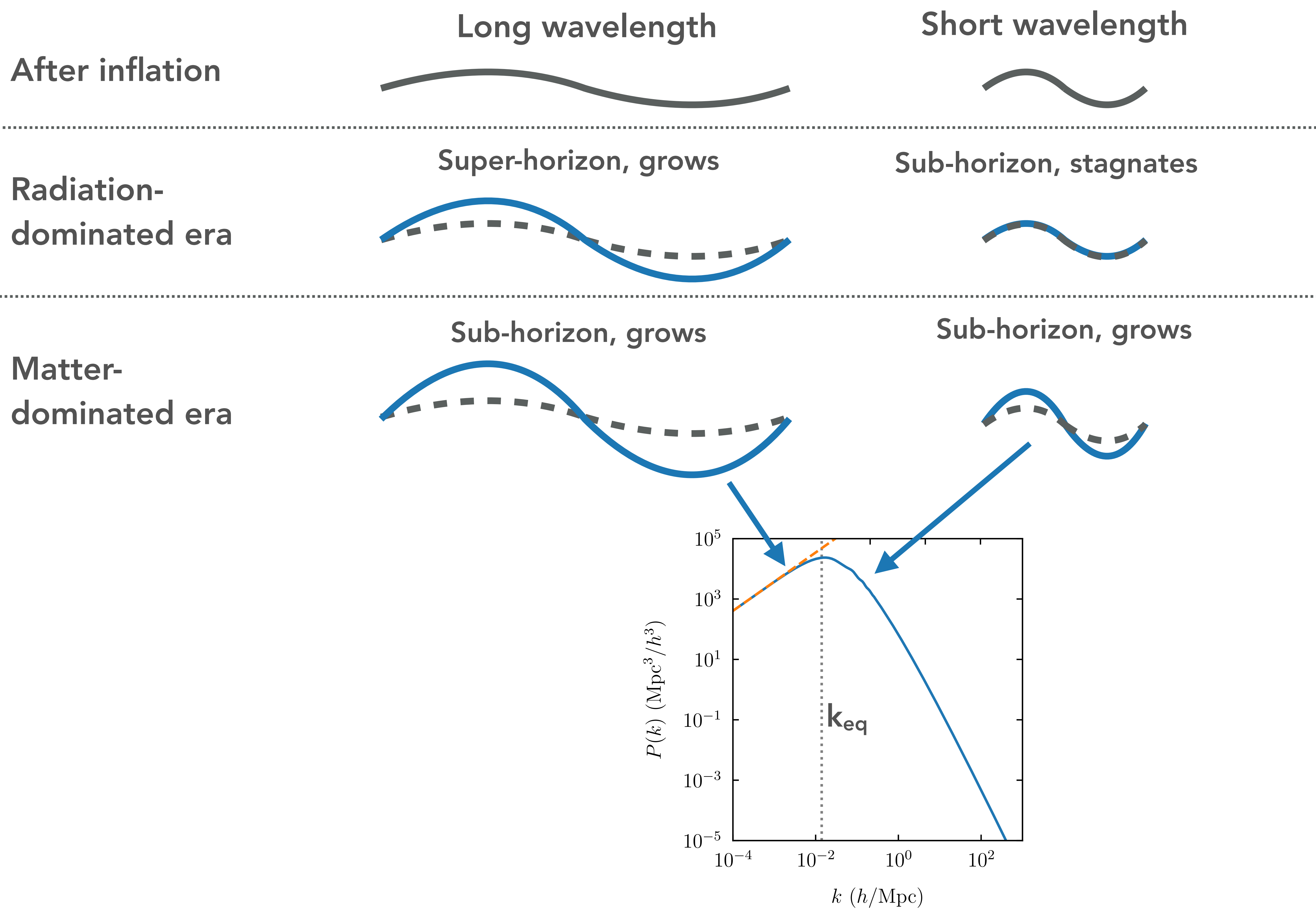


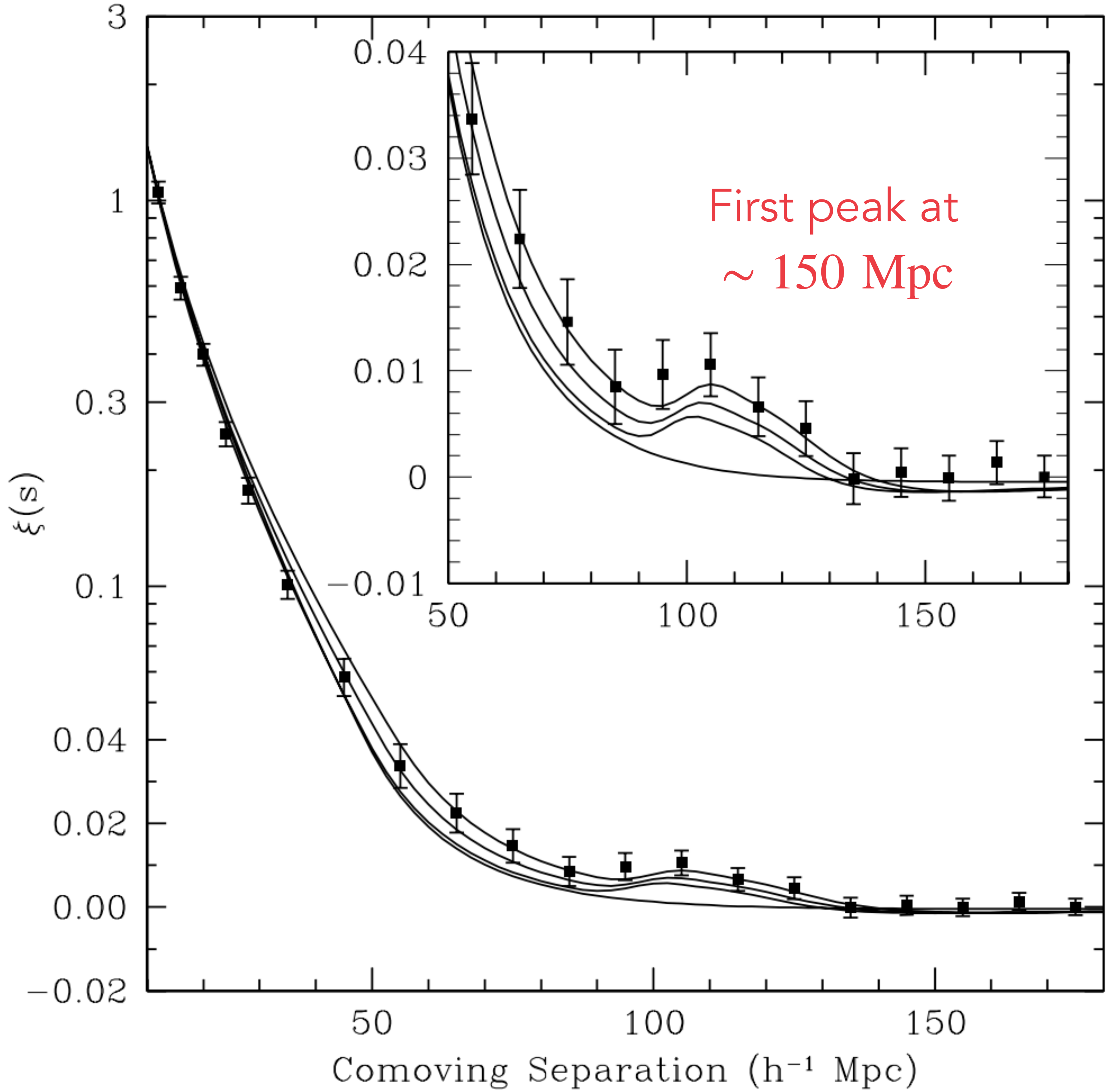
Figure by Frank van den Bosch

§3.4 • The power spectrum at recombination

The peak of the power spectrum



First detection of BAO feature



Eisenstein et al. 2005

Current observational constraints on P(k)

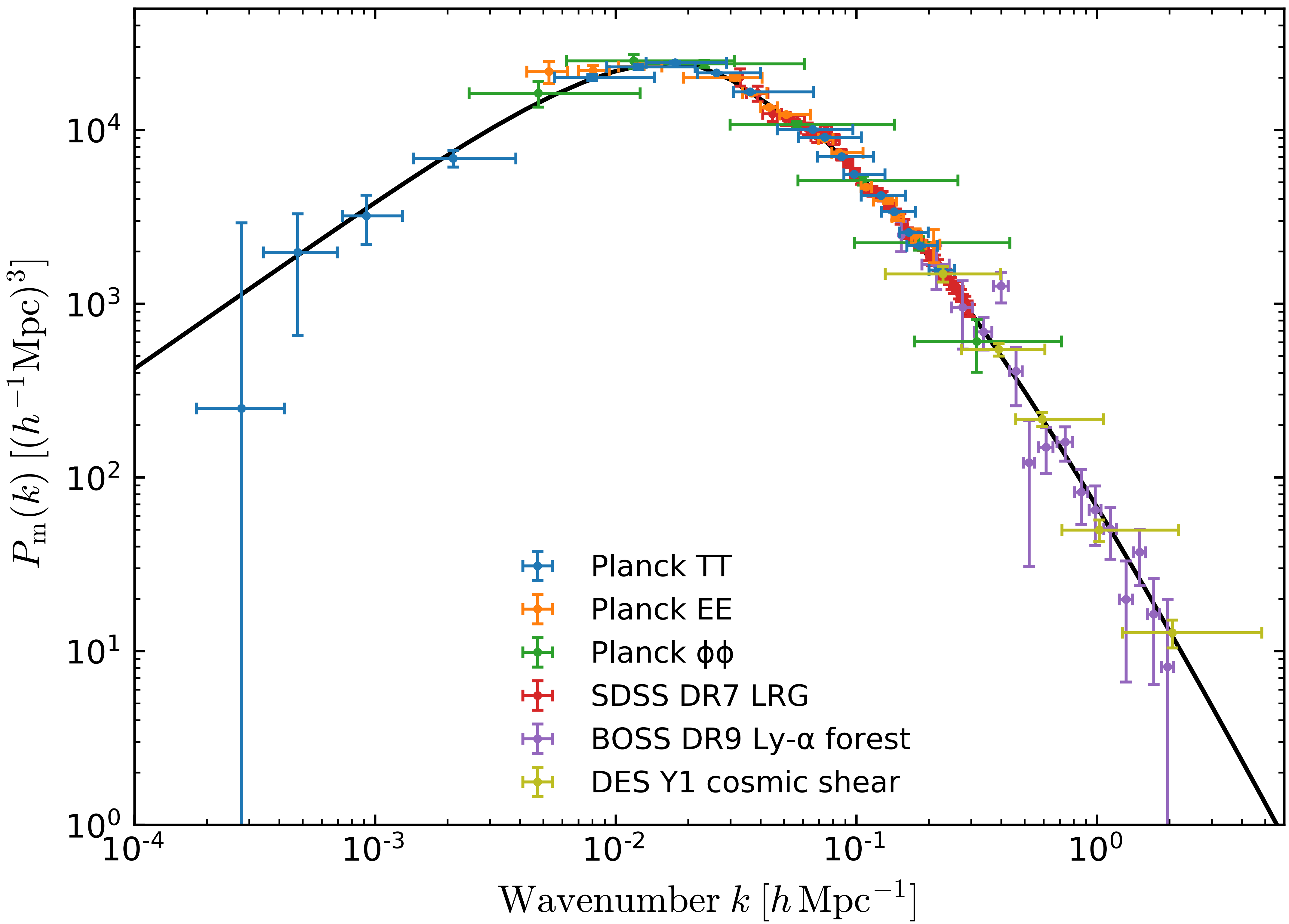
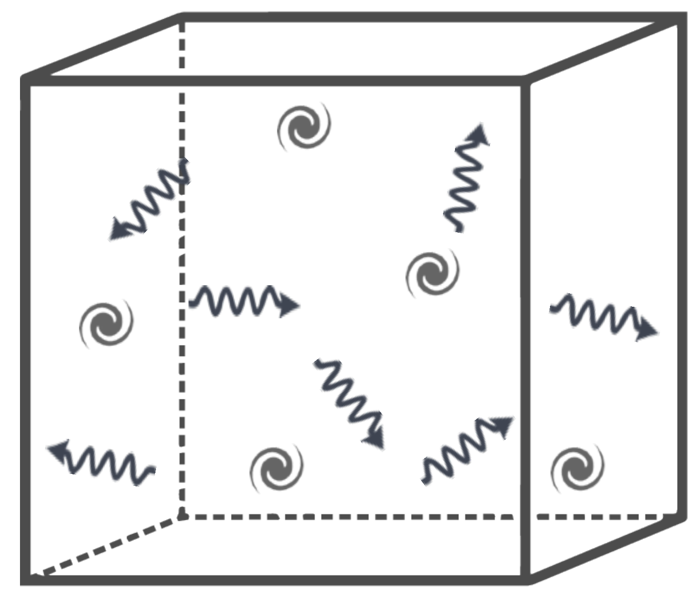
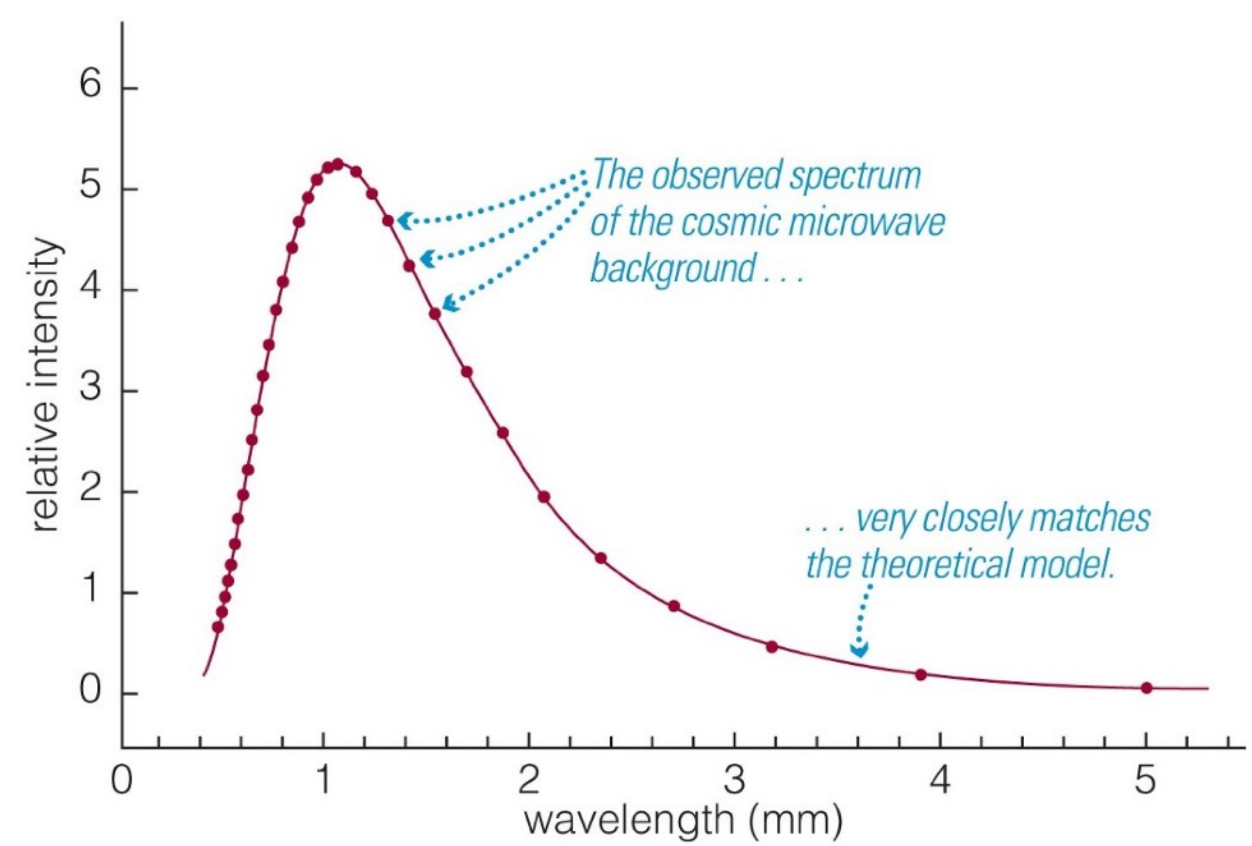


Figure by the Planck Collaboration

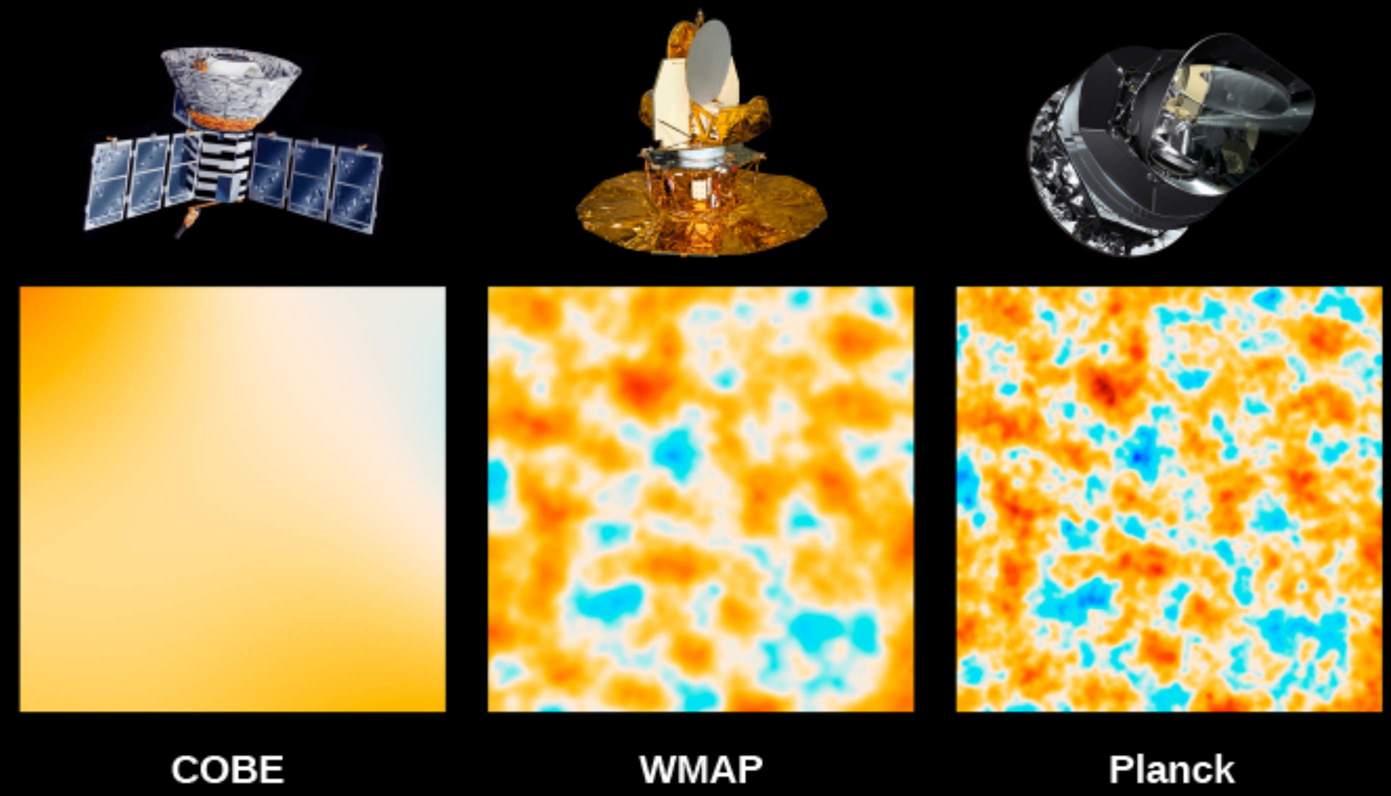
The Cosmic Microwave Background (CMB)



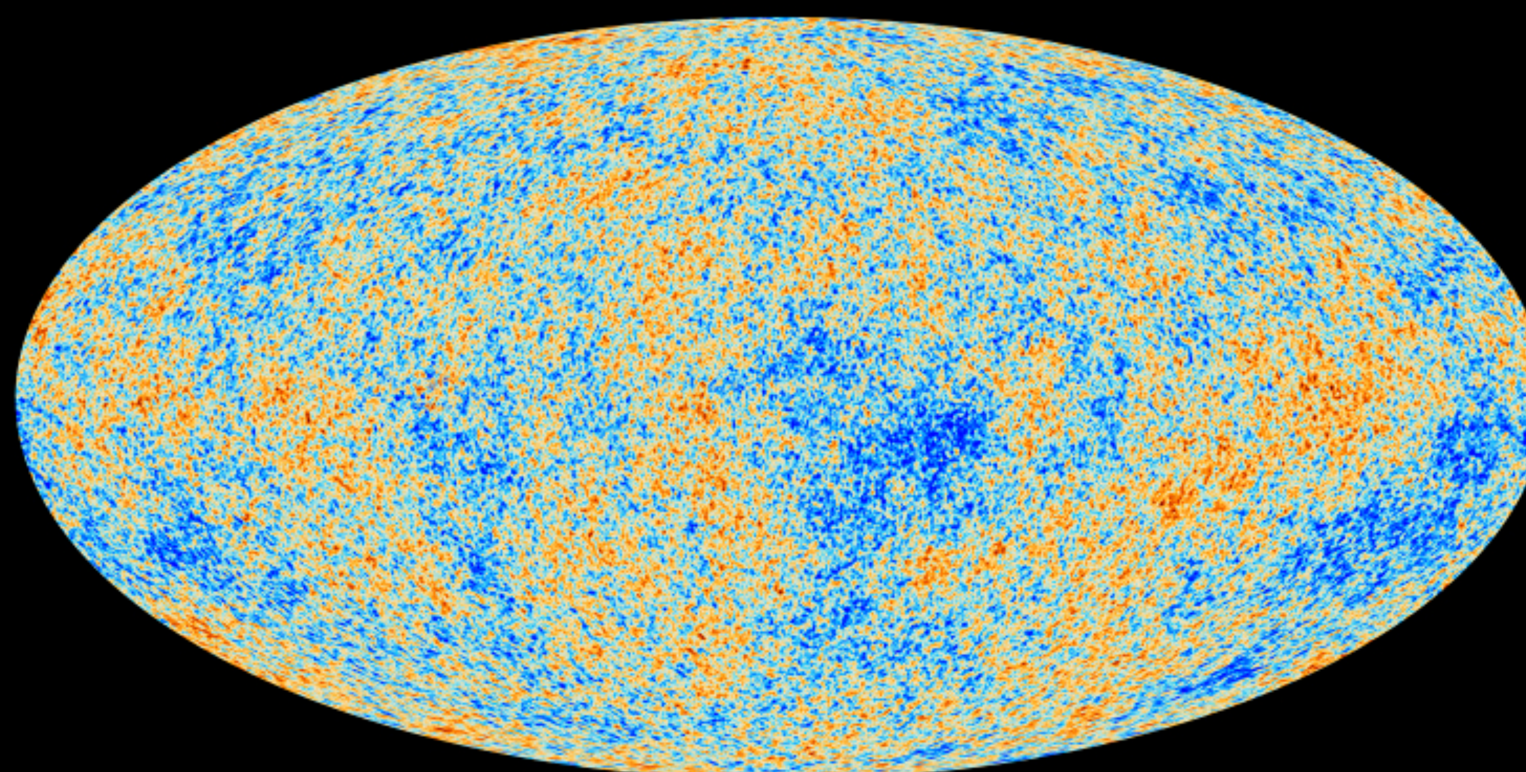
- Discovered in the 1960's
- Present density is 411 photons/cm^3
- Emitted at $z \approx 1100$
- Virtually perfect blackbody



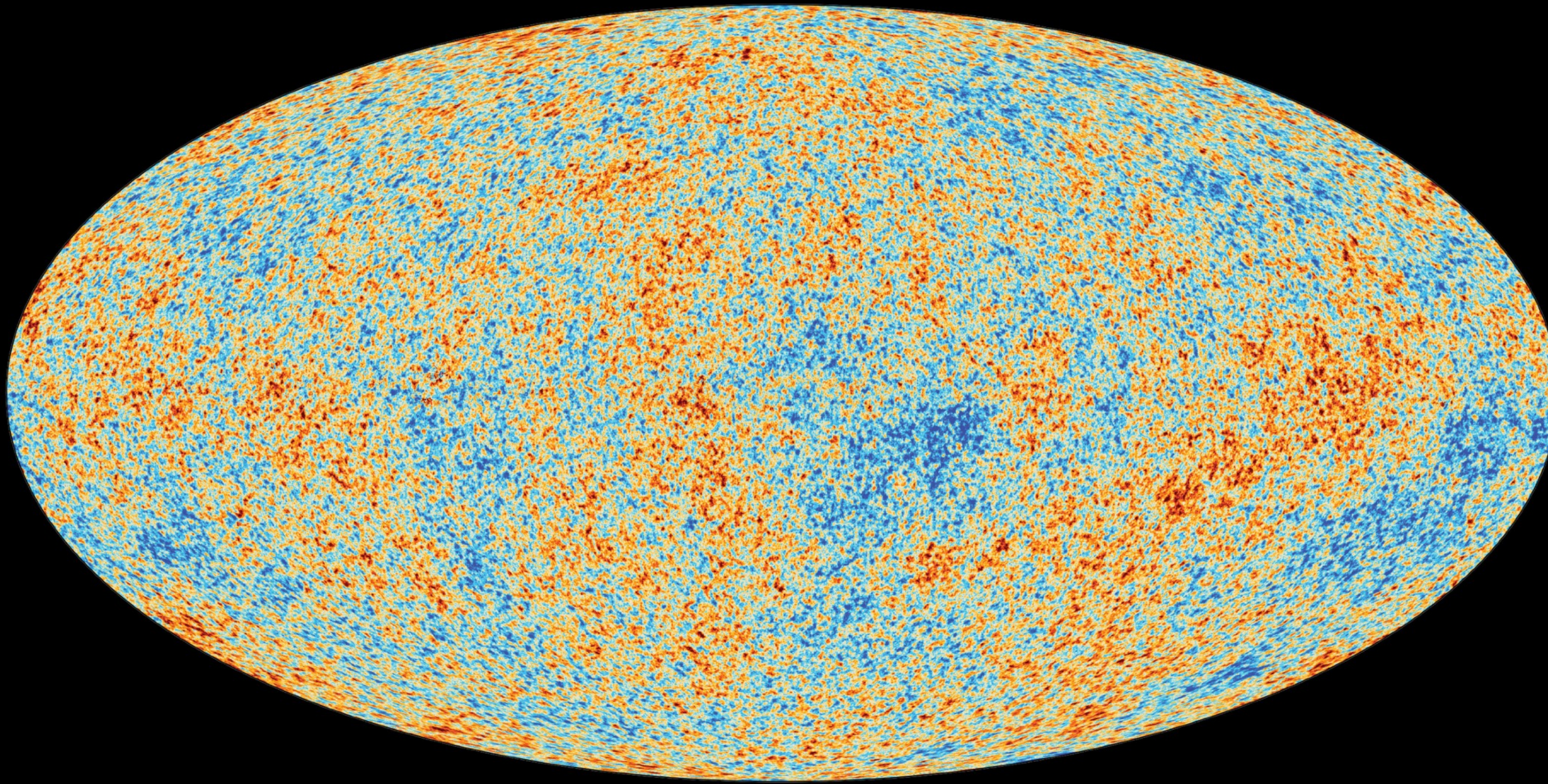
The Cosmic Microwave Background (CMB)



- The differences in temperature are roughly 1/100,000 K
- These fluctuations tell us a lot about the early Universe



Planck CMB Map

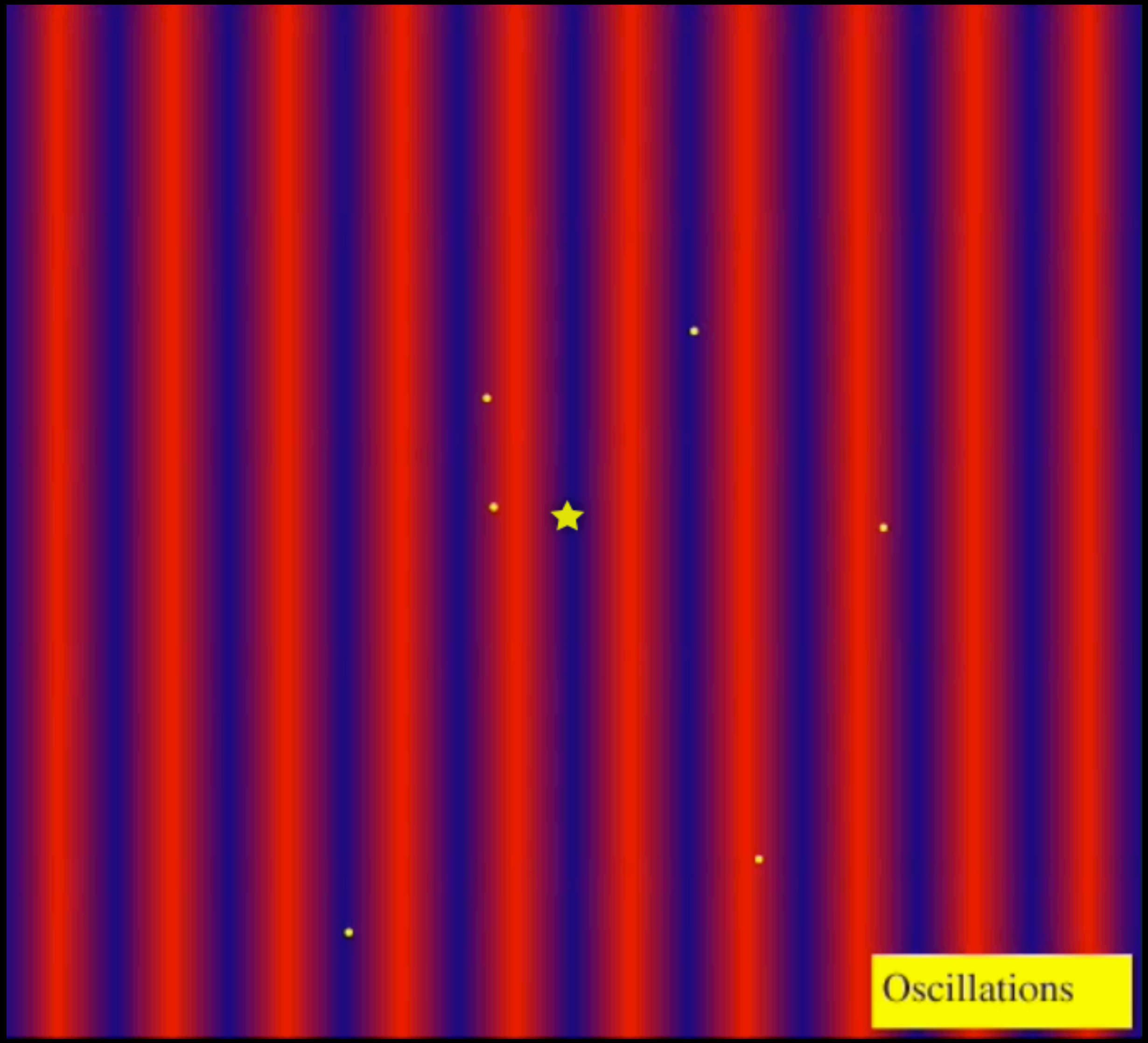


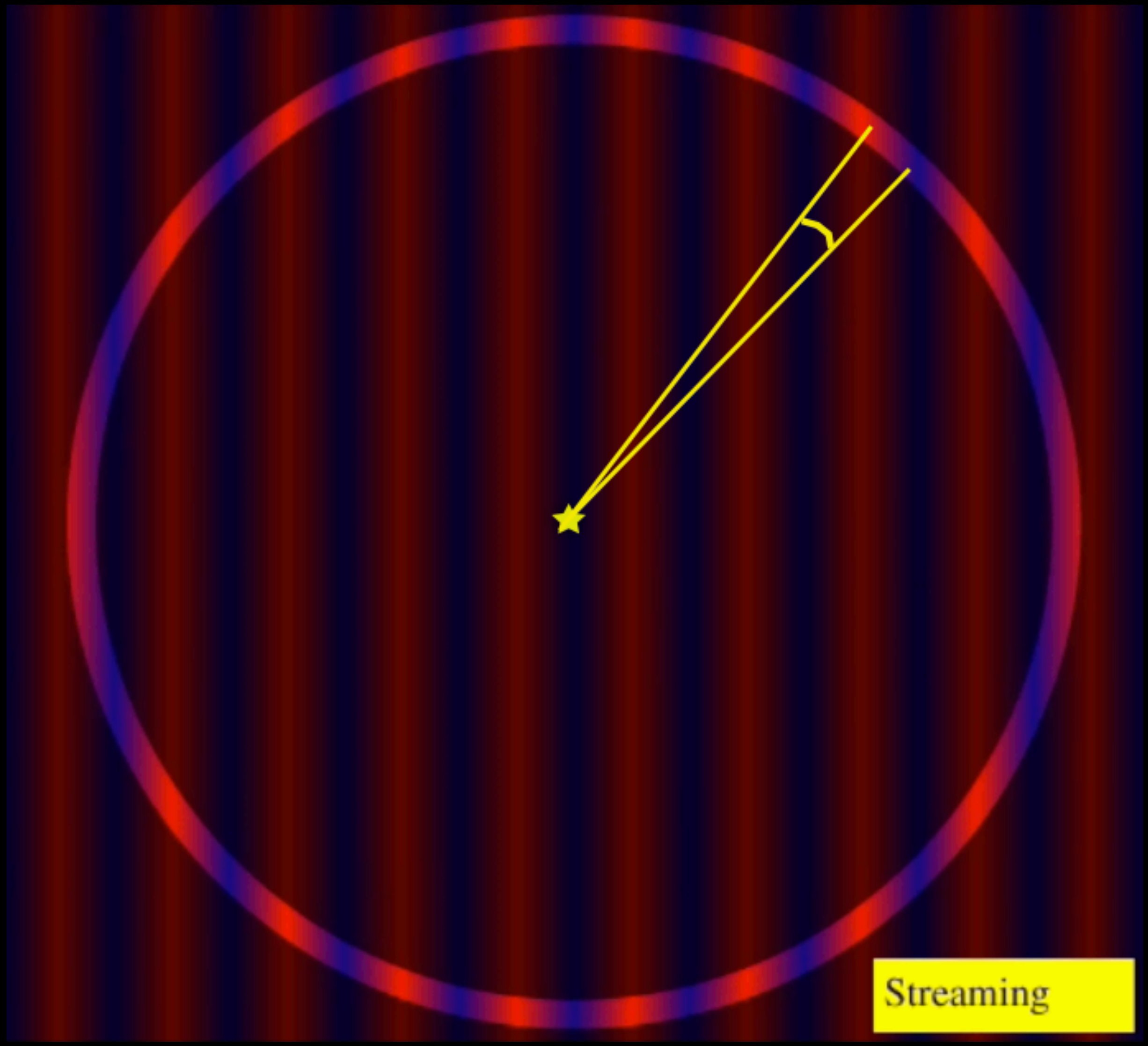
0.3 mK below mean (2.7252 K)

0.3 mK above mean (2.7258 K)

ESA / Planck

What do we see when we look at the CMB?





Images by Frank van den Bosch

Multipoles

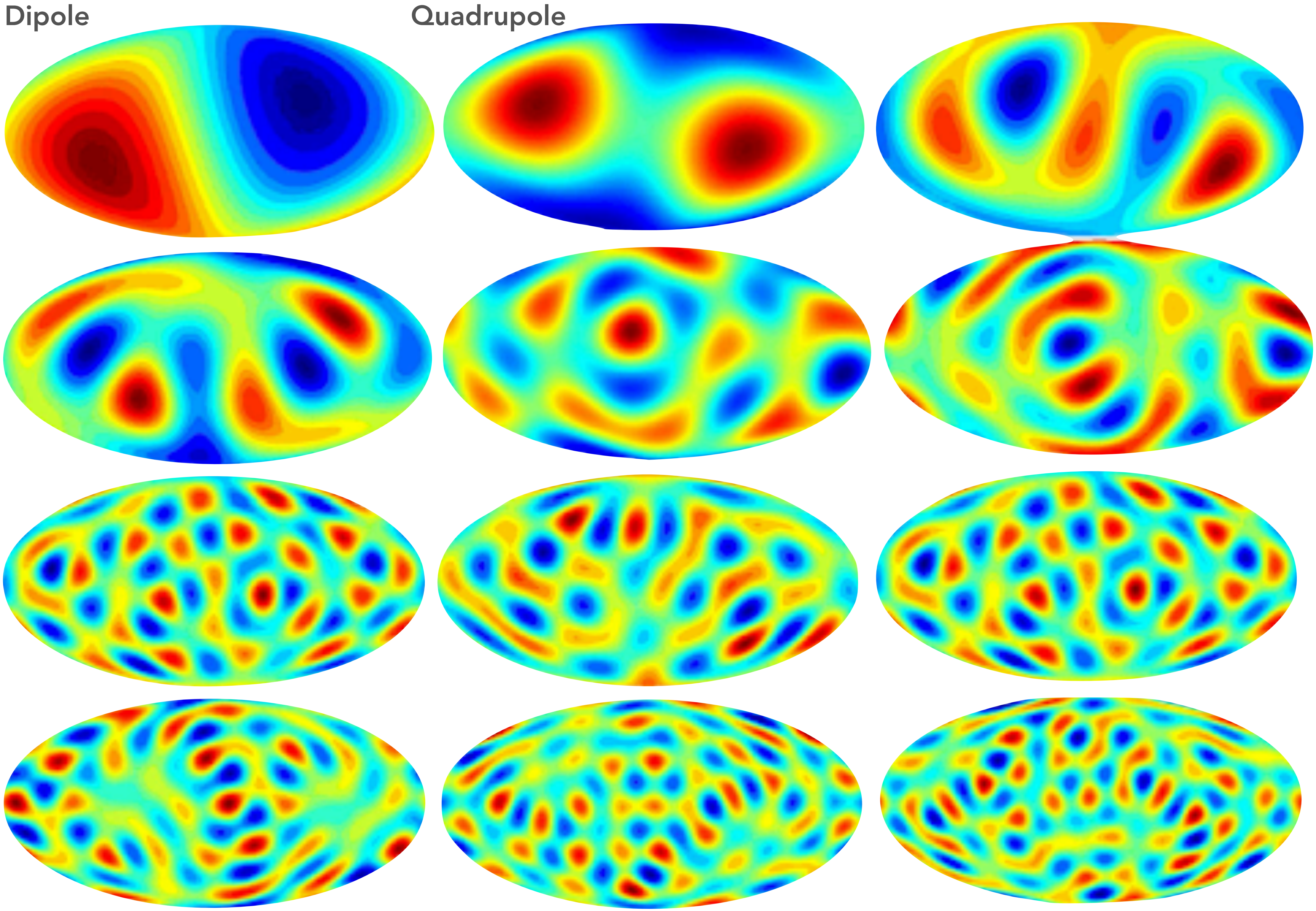
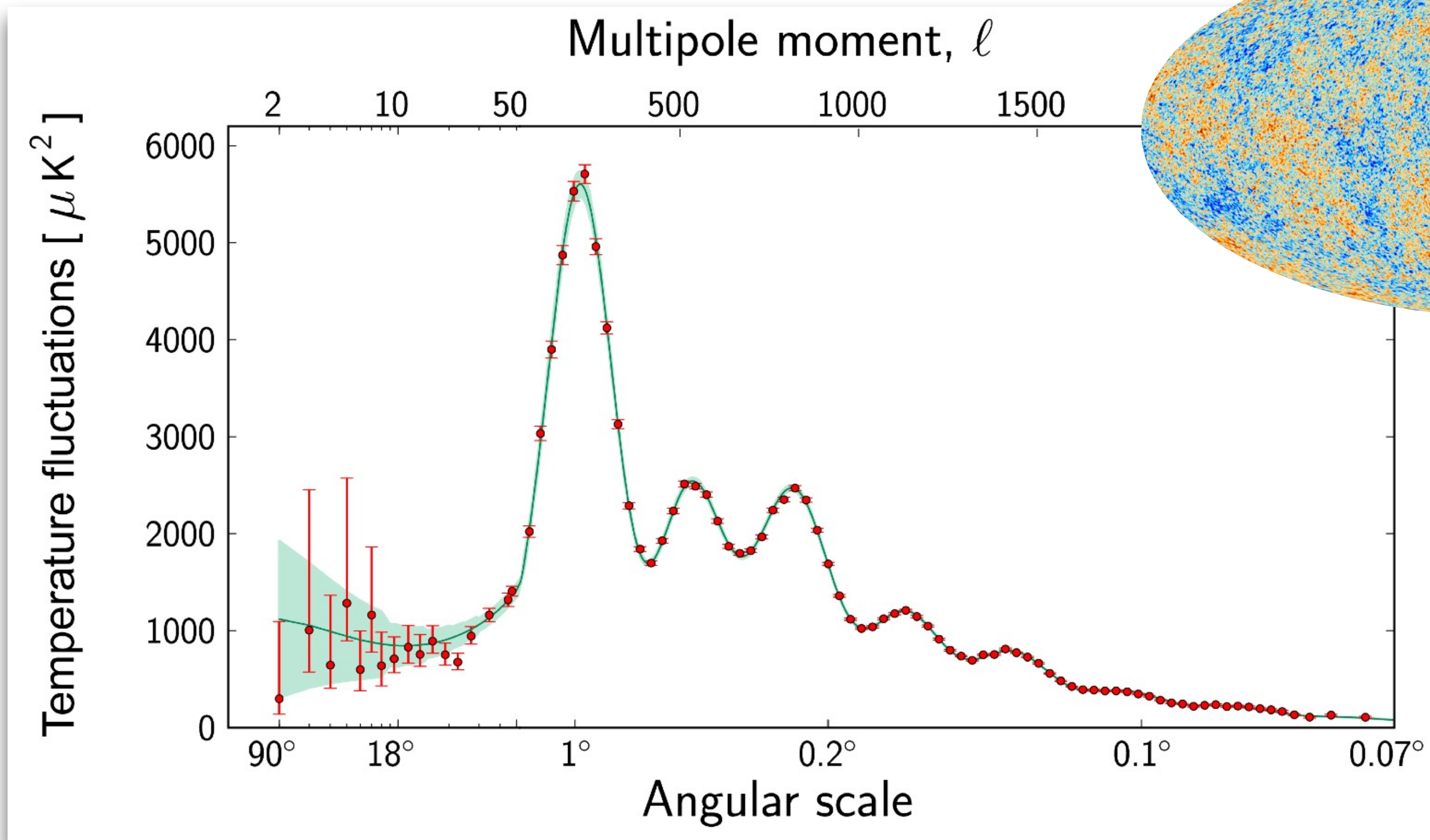
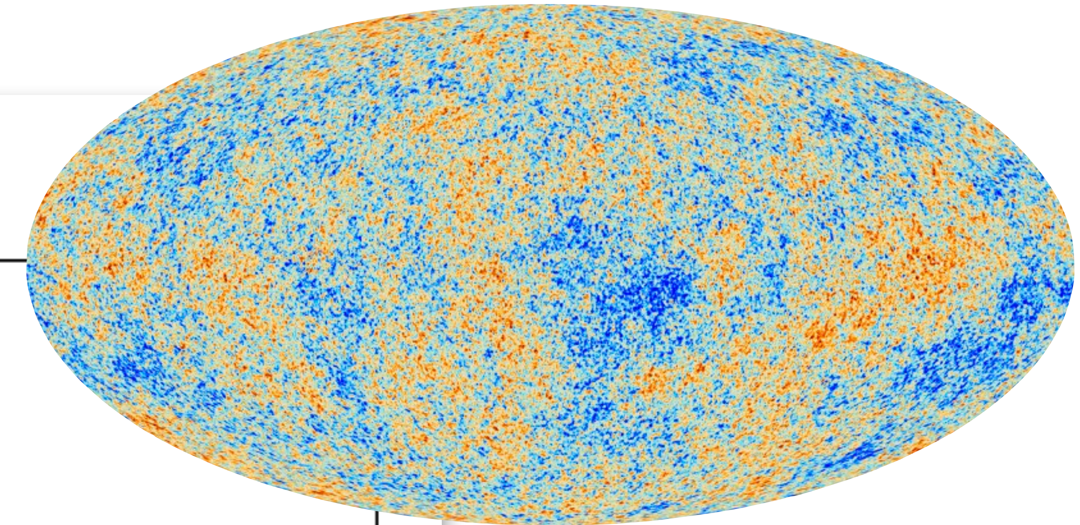


Image by Ville Heikkilä

Power spectrum of fluctuations





- First peak corresponds to about 1 degree
- LCDM cosmology matches the data extremely well (green line)
- Predictions are computed numerically

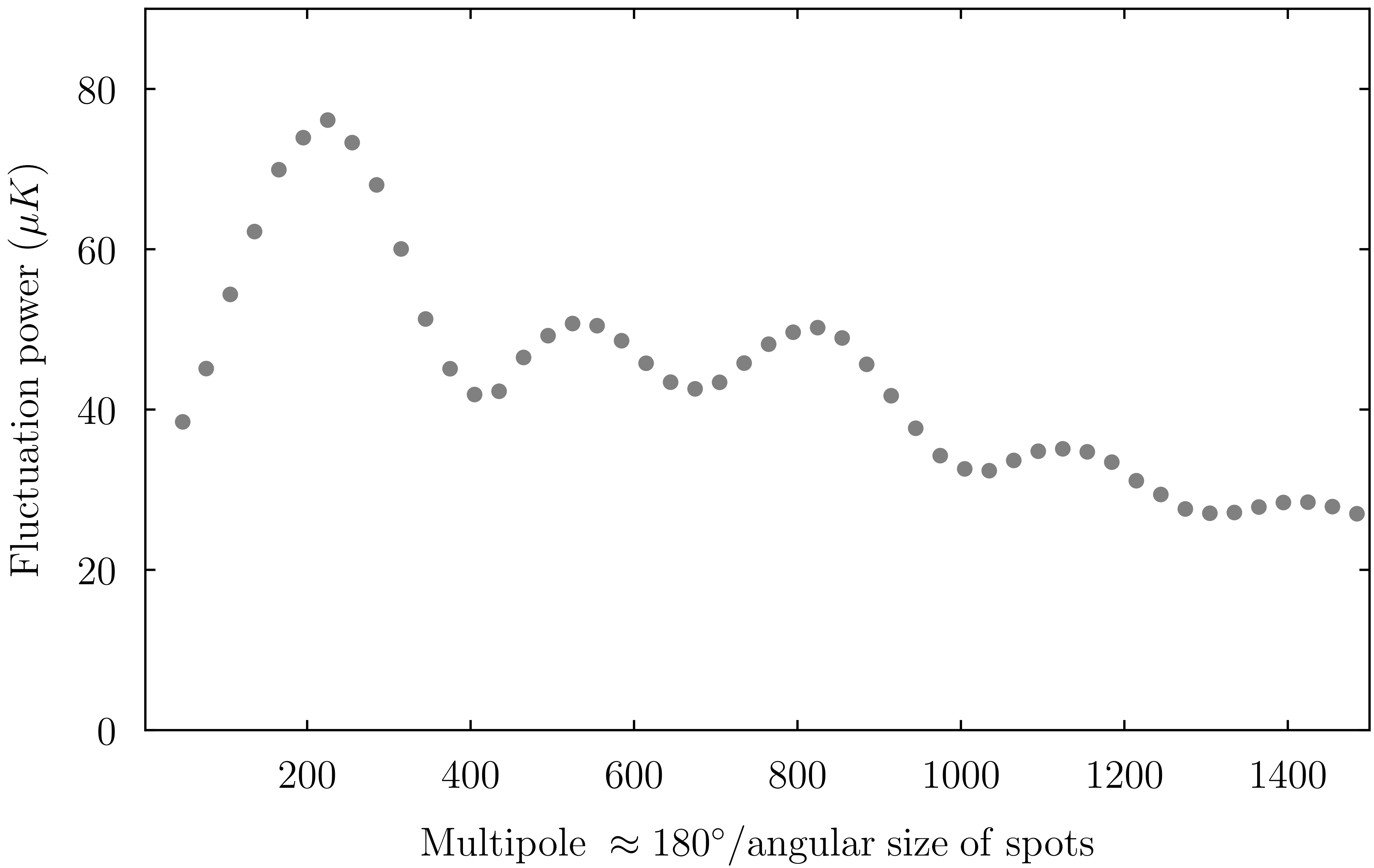
Parameters (flat ΛCDM)

measured by CMB

fixed or derived

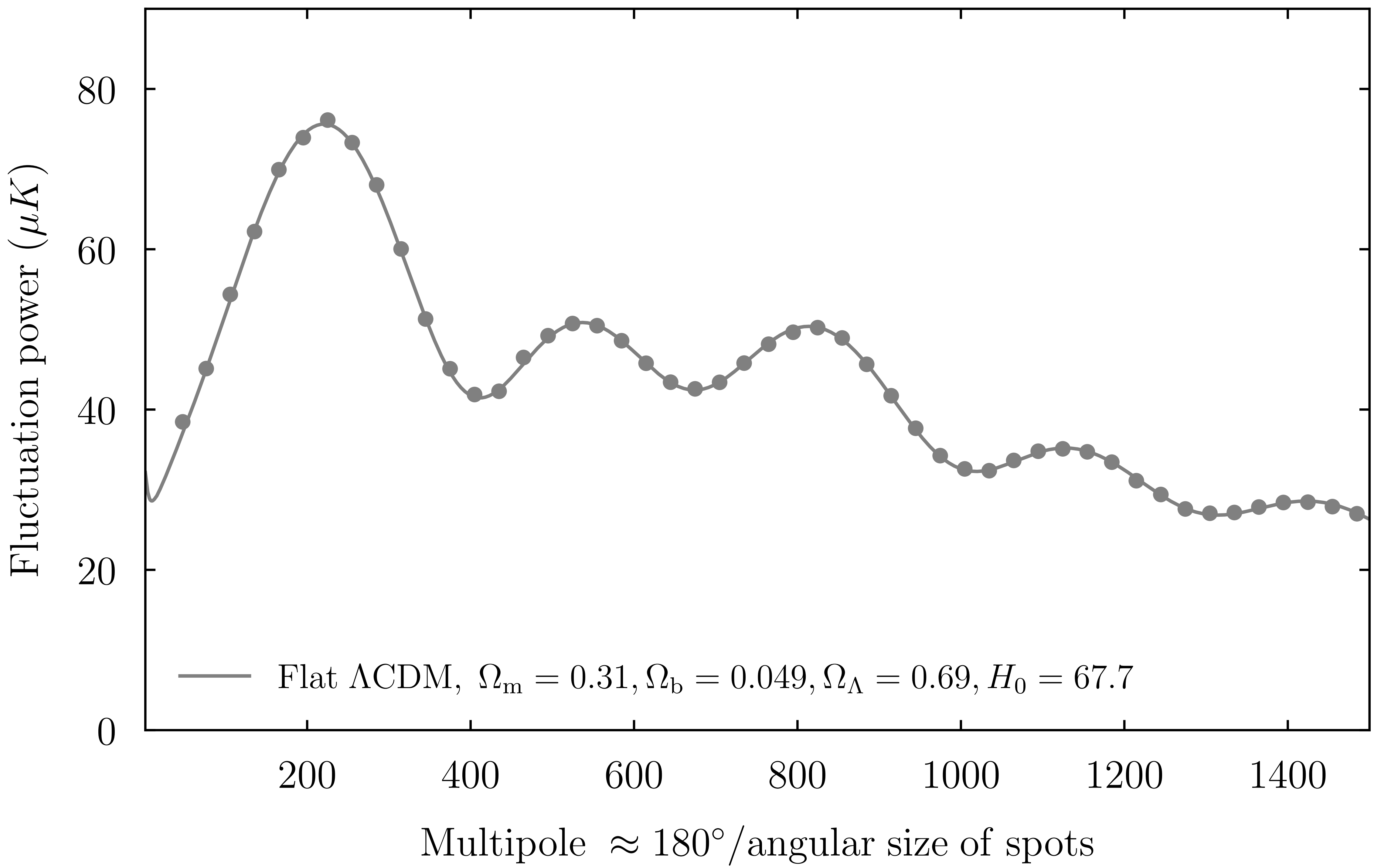
$\Omega_{\mathrm{m},0}$	Matter density in units of crit. dens. today	CMB, rotation curves, Supernovae, lensing…
$\Omega_{\mathrm{b},0}$	Baryon density in units of crit. dens. today	CMB, Big Bang nucleosynthesis, baryons in clusters
H_0	Hubble expansion rate today	CMB, low-z redshift-distance diagram…
τ	How many CMB photons are absorbed on the way to us	CMB
A_s	Strength of quantum fluctuations	CMB
n_s	Dependence of quantum fluctuations on scale	CMB
$\Omega_{\Lambda,0}$	Cosmological constant dens. in units of crit. dens. today	$\Omega_{\Lambda,0} = 1 - \Omega_{\mathrm{m},0}$
$\Omega_{\mathrm{k},0}$	Curvature density in units of crit. dens. today	Flat $\implies \Omega_k = 0$
t_0	Age of Universe	Derived
z_{rec}	Redshift of recombination	Derived

CMB with different amounts of dark matter



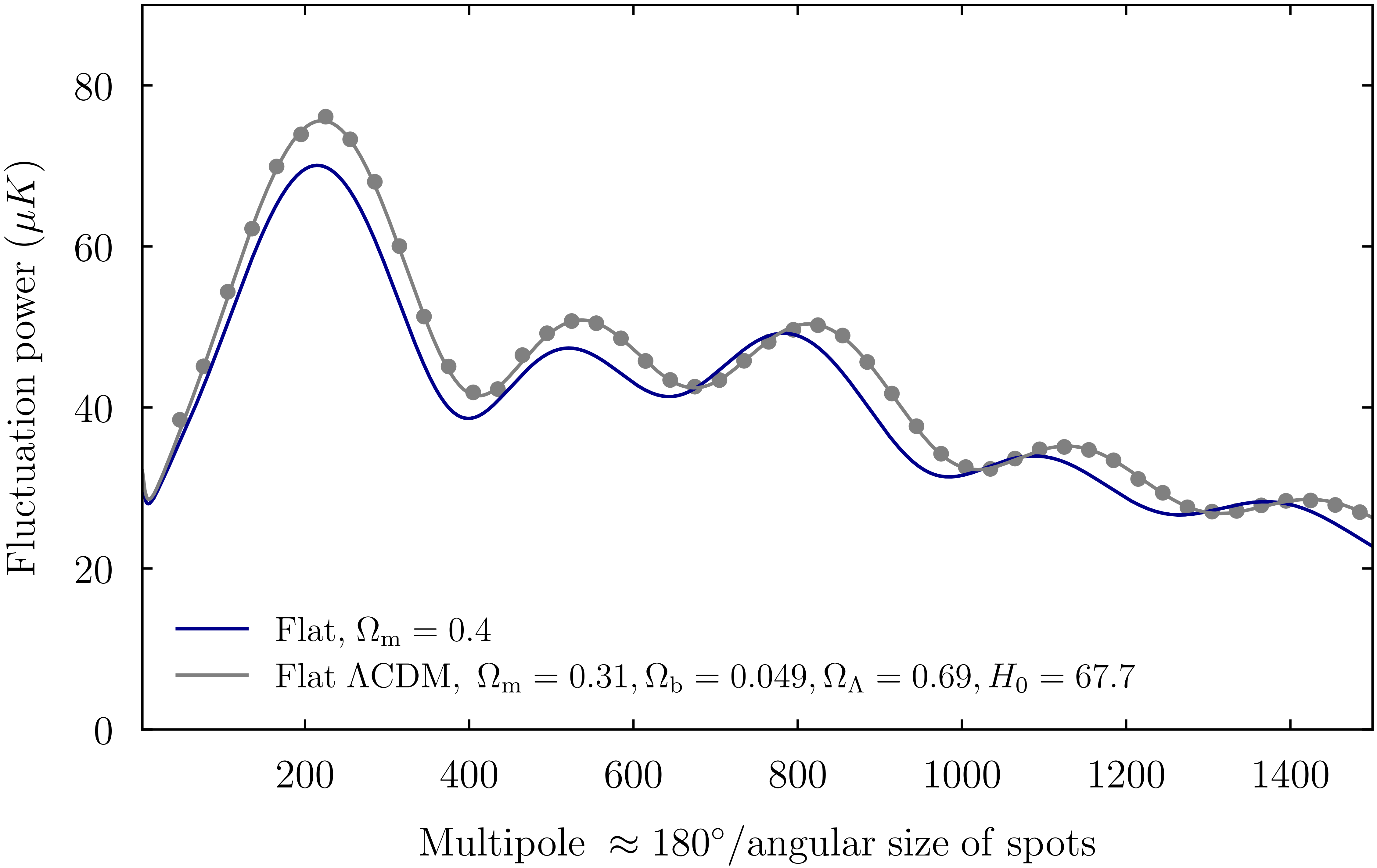
Made with CAMB • Planck data

CMB with different amounts of dark matter



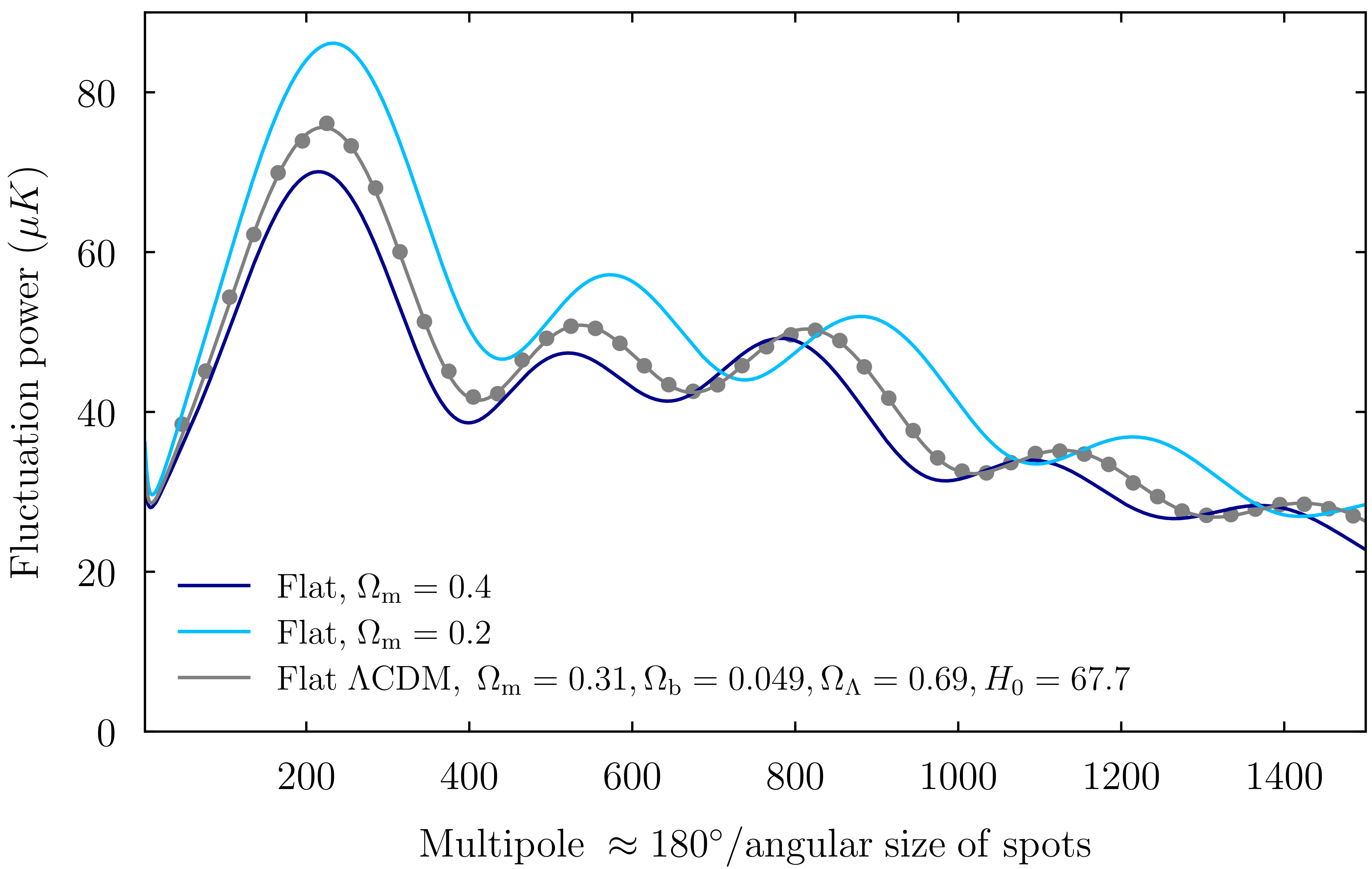
Made with CAMB • Planck data

CMB with different amounts of dark matter



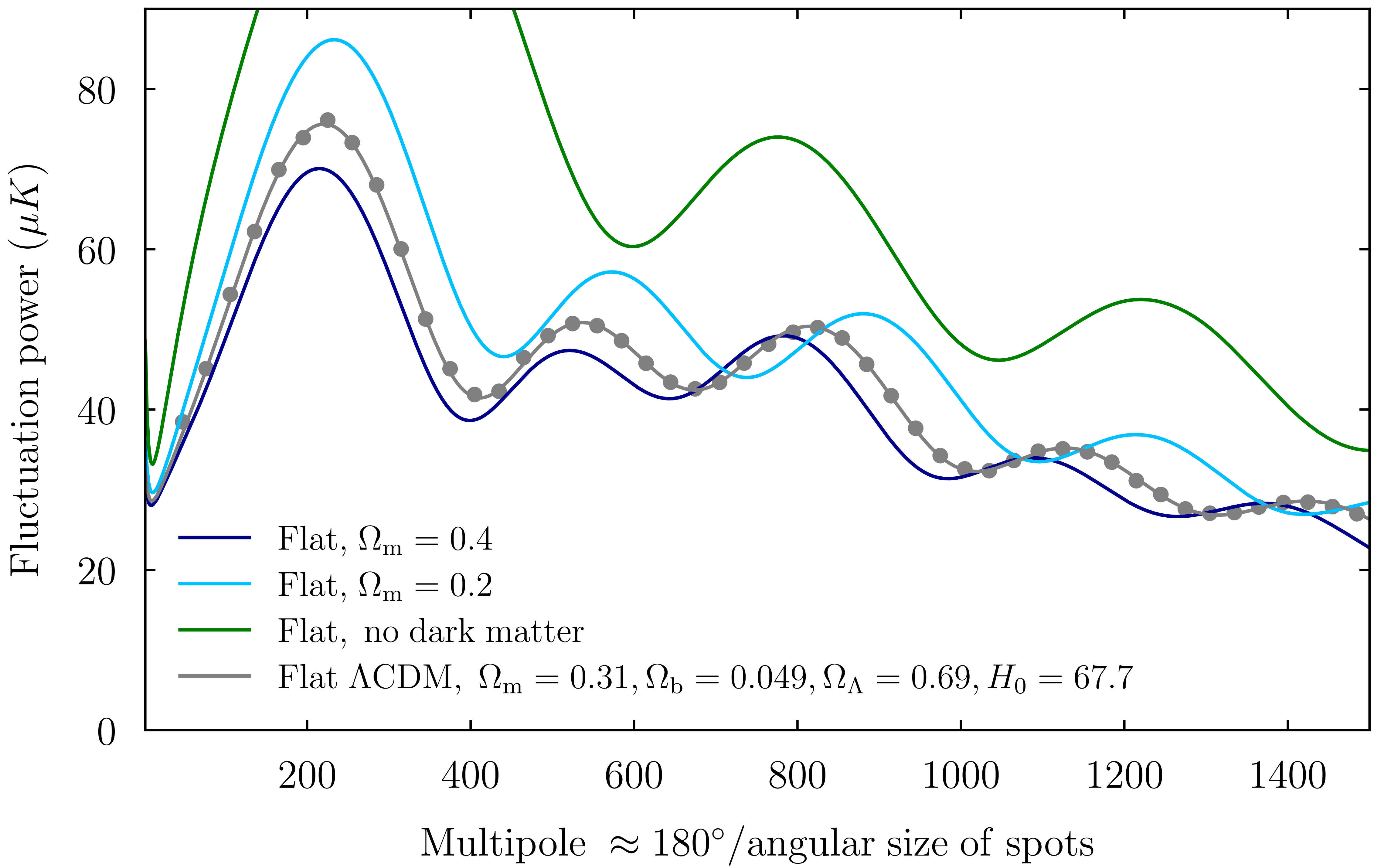
Made with CAMB • Planck data

CMB with different amounts of dark matter



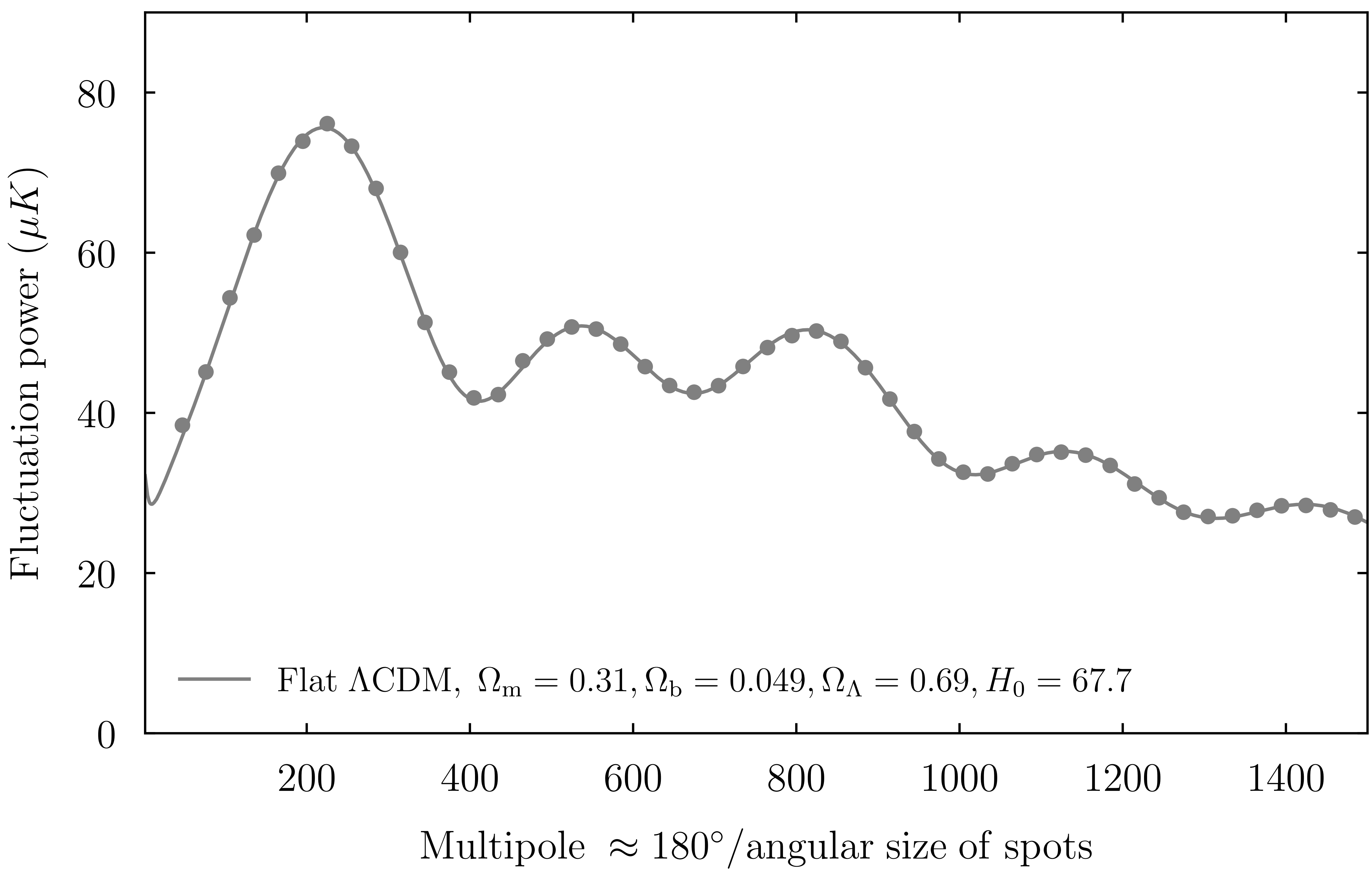
Made with CAMB • Planck data

CMB with different amounts of dark matter



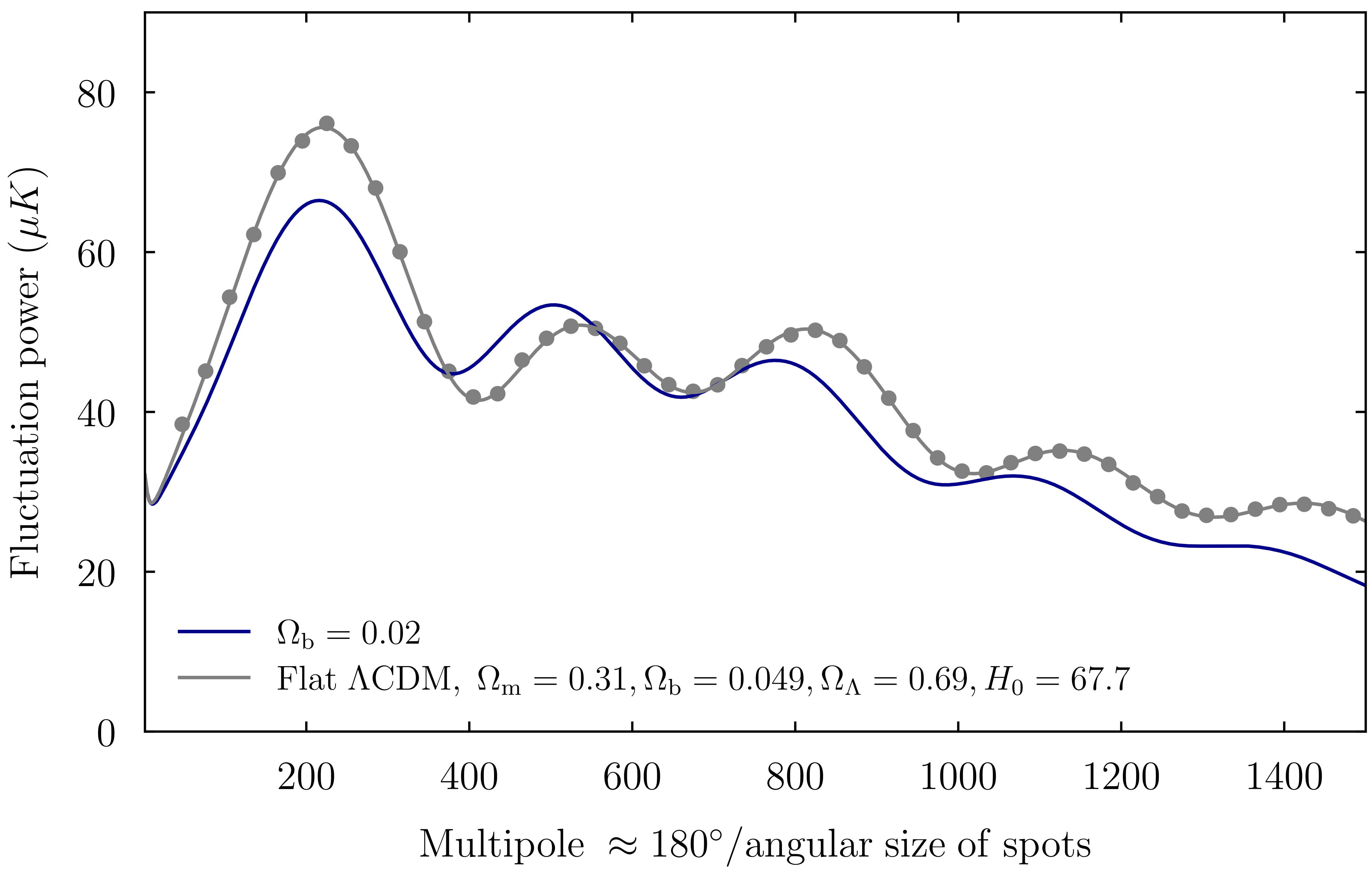
Made with CAMB • Planck data

CMB with different amounts of baryons



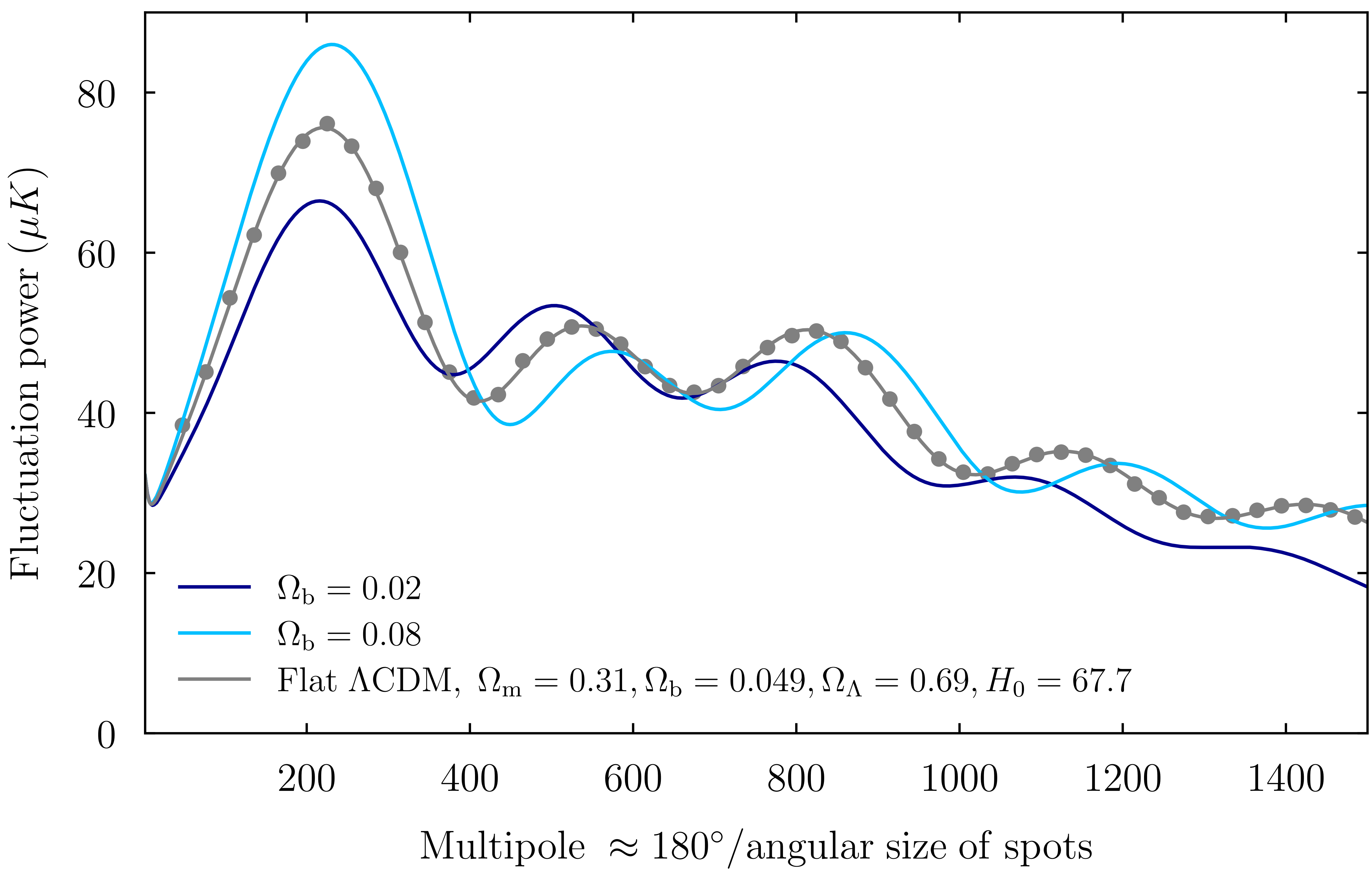
Made with CAMB • Planck data

CMB with different amounts of baryons



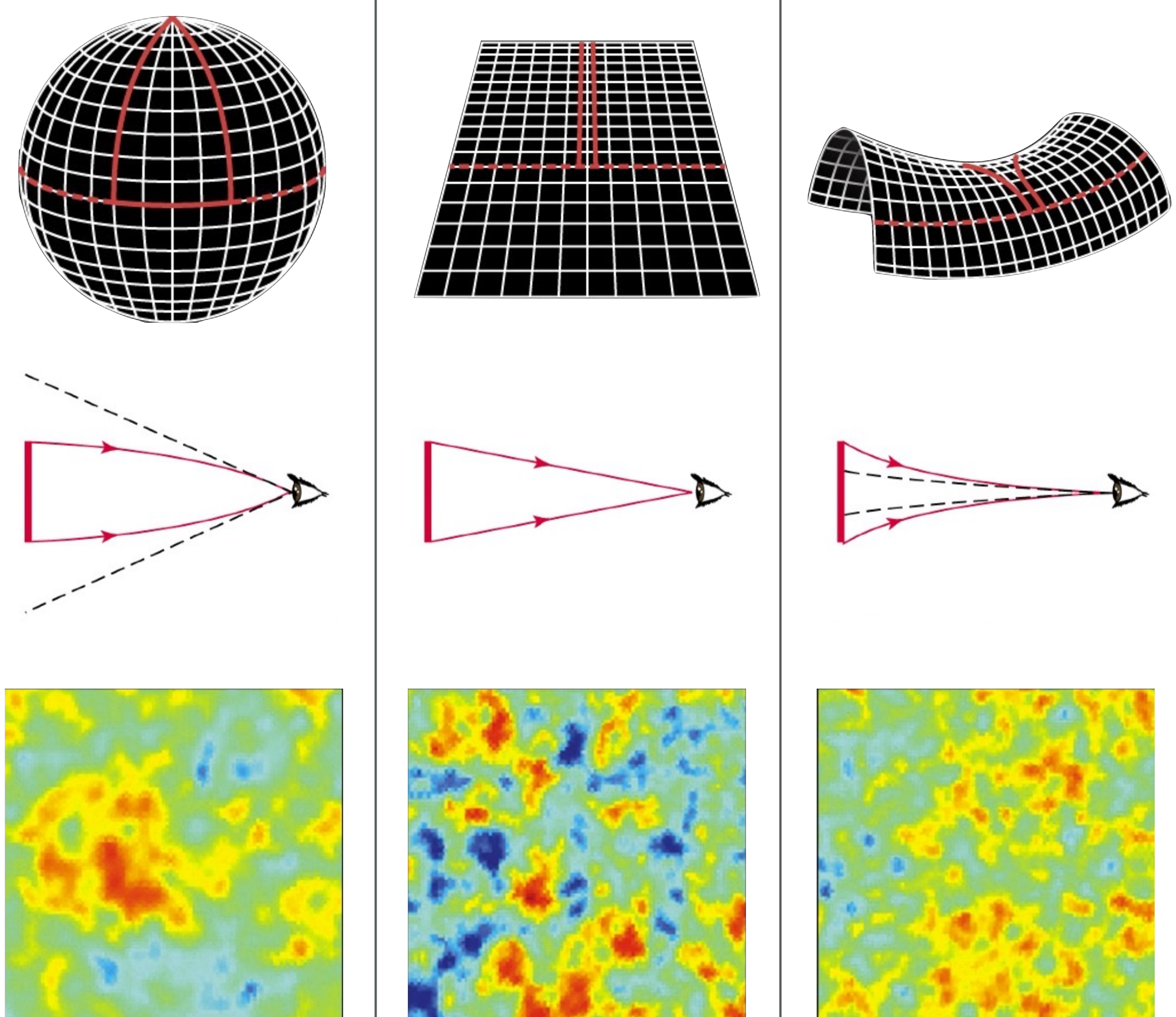
Made with CAMB • Planck data

CMB with different amounts of baryons



Made with CAMB • Planck data

Curvature of the Universe



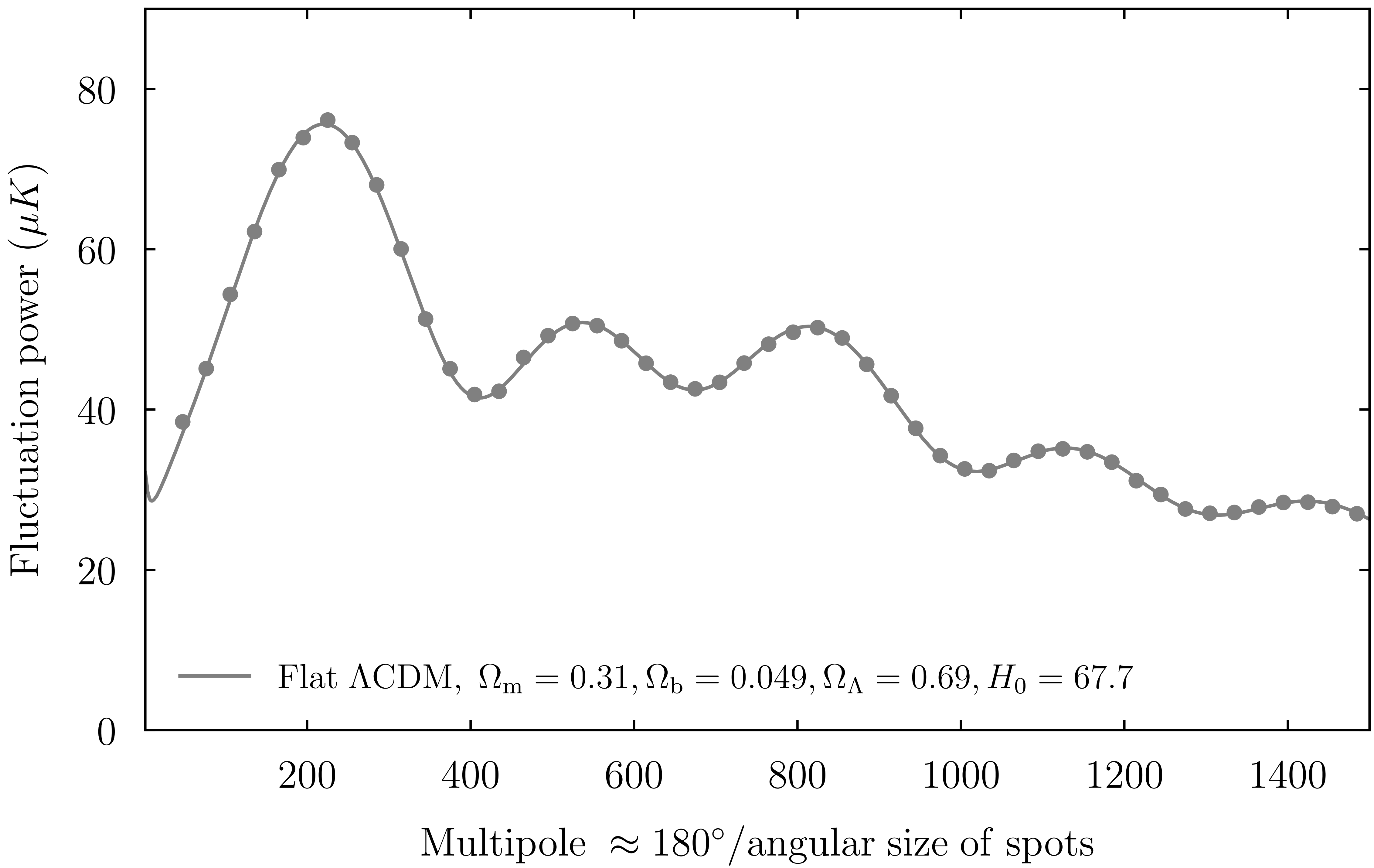
Closed (k > 0), spots appear larger

Flat (spots appear in actual size)

Open (k < 0), spots appear smaller

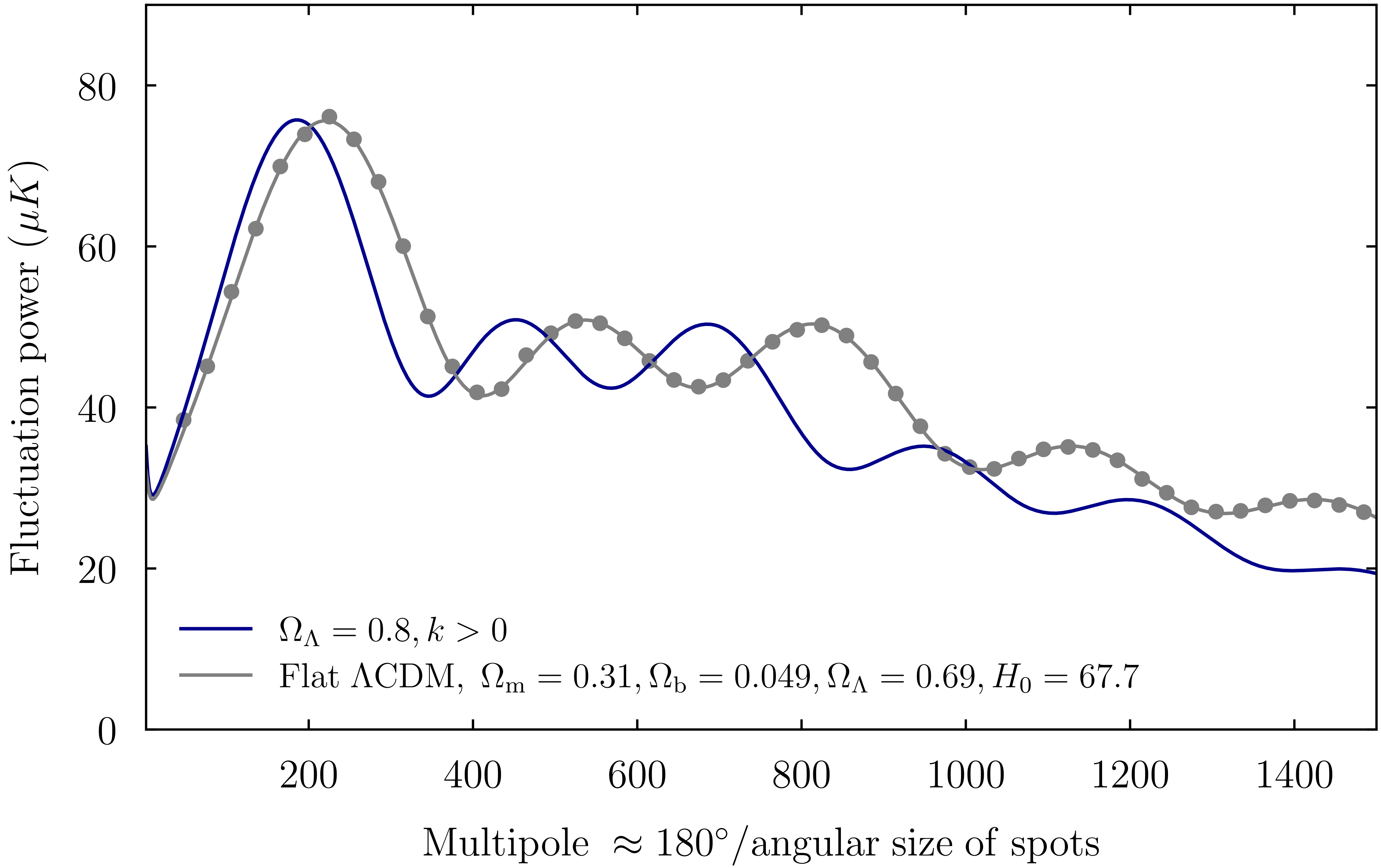
- We understand the physics of the CMB patches very well, so we know what size they should be
- If the Universe was strongly curved, the apparent size of the patches would change
  - Larger if positively curved, because lines converge
  - Smaller if negatively curved, because lines diverge
- All CMB measurements are compatible with k = 0 (flat)

CMB with different amounts of dark energy



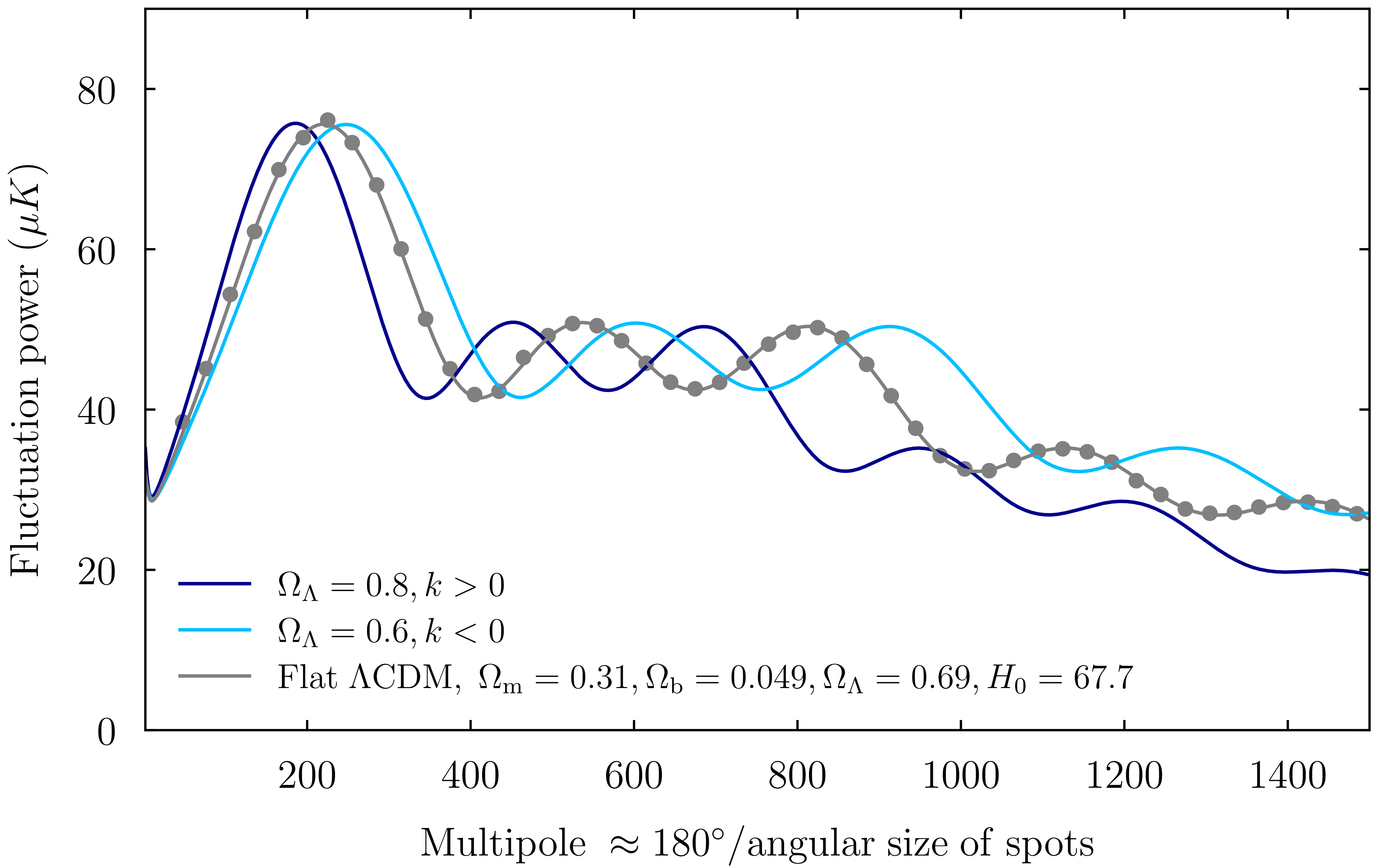
Made with CAMB • Planck data

CMB with different amounts of dark energy



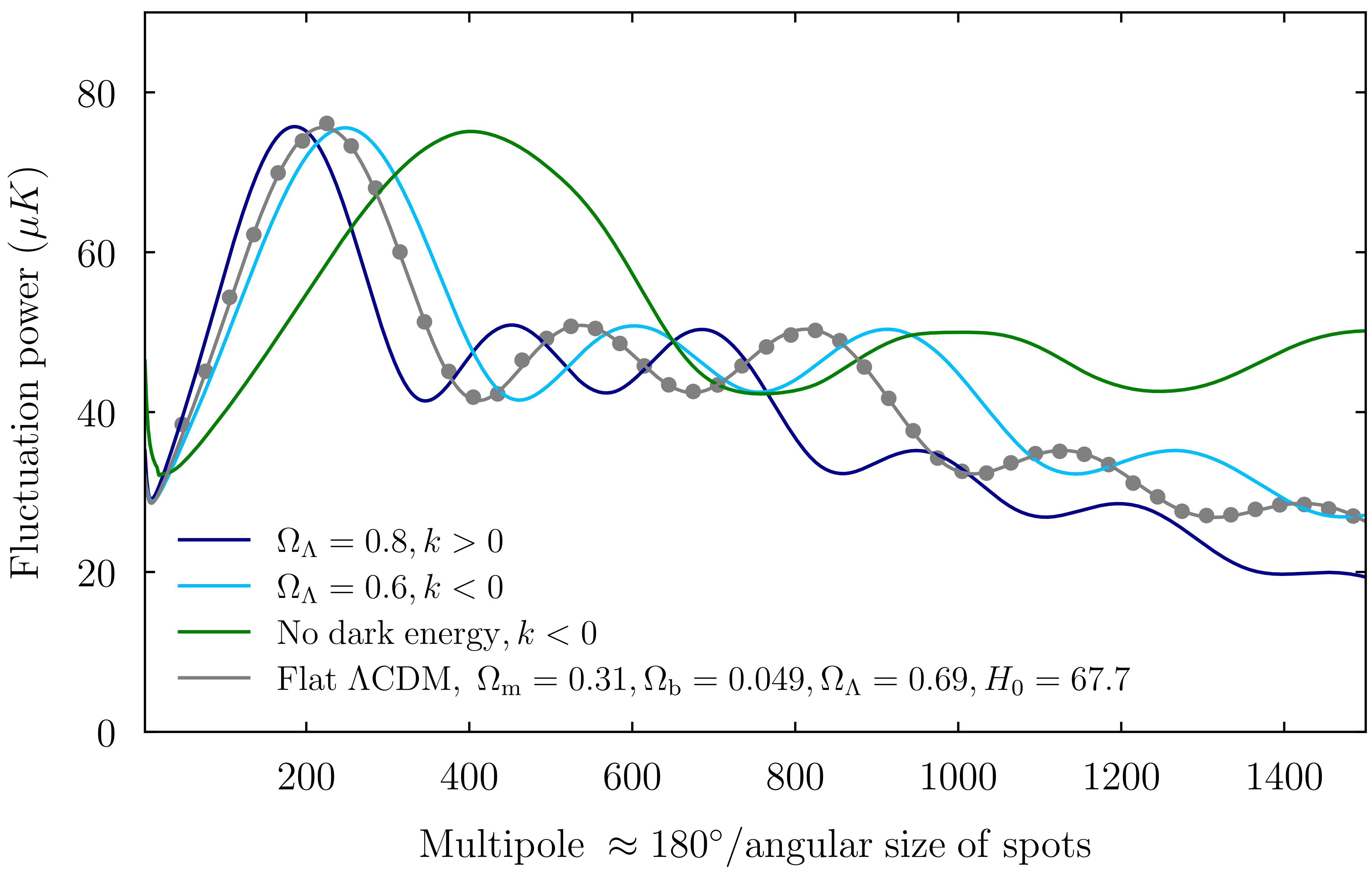
Made with CAMB • Planck data

CMB with different amounts of dark energy



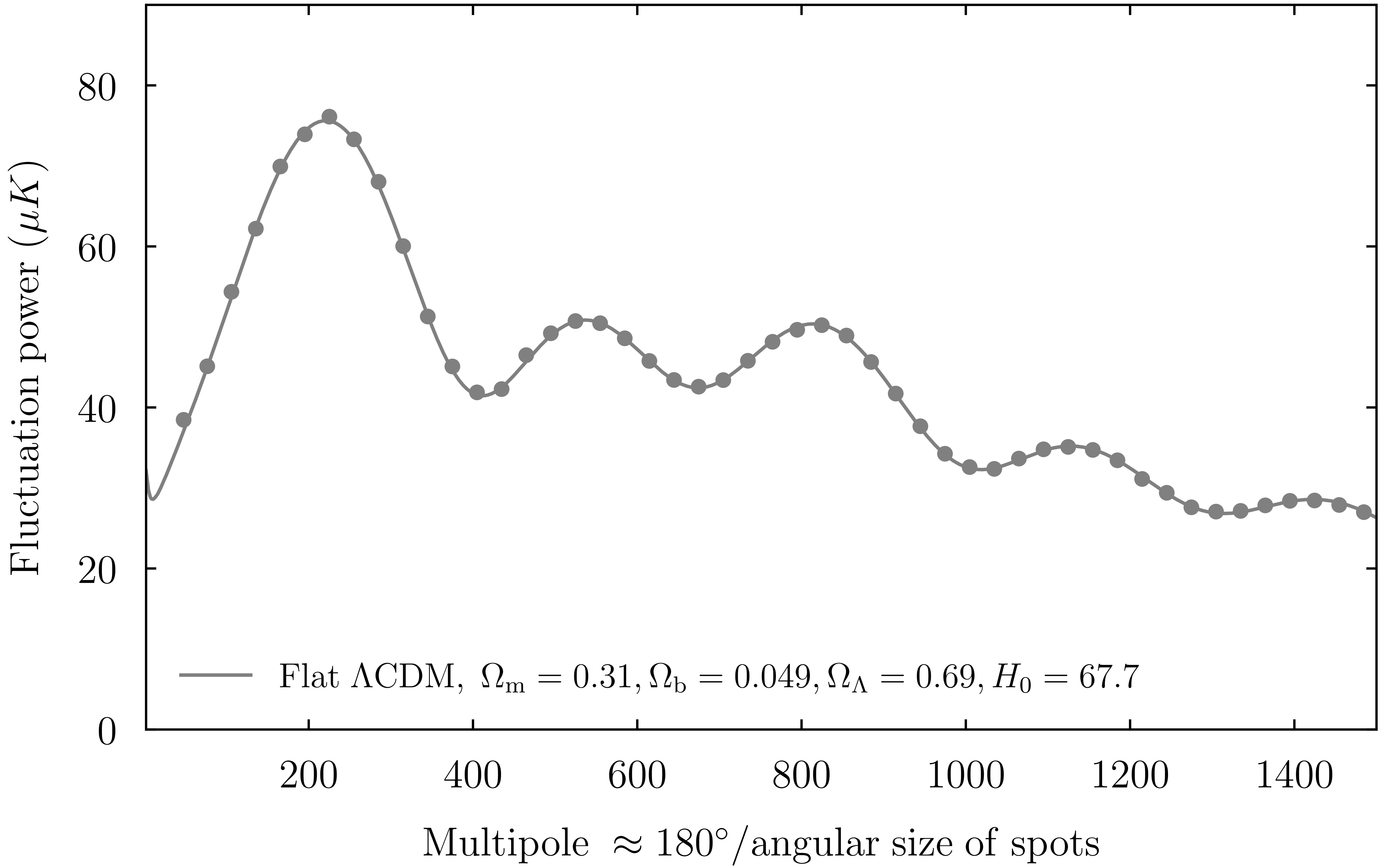
Made with CAMB • Planck data

CMB with different amounts of dark energy



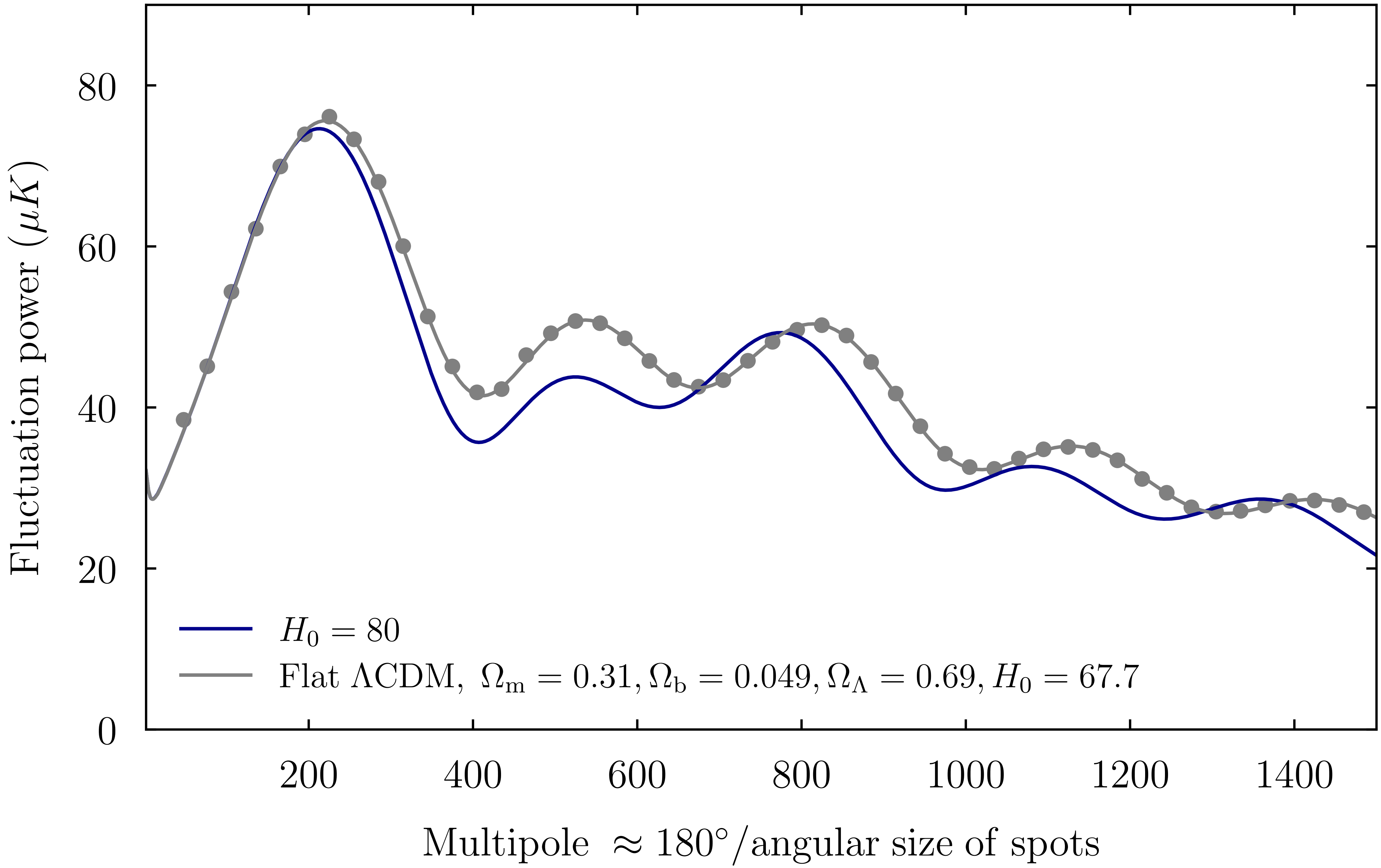
Made with CAMB • Planck data

CMB with different H0



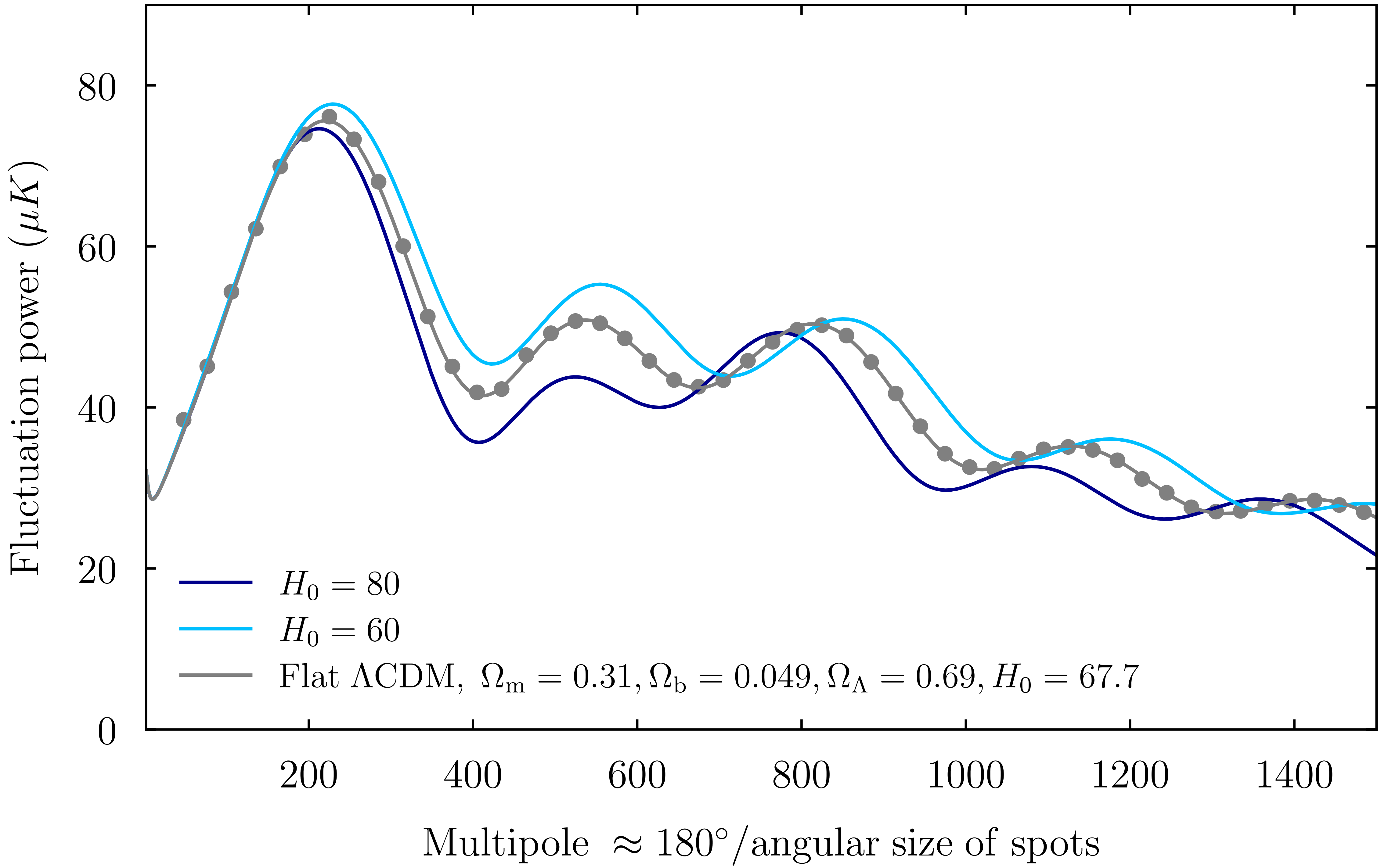
Made with CAMB • Planck data

CMB with different H0



Made with CAMB • Planck data

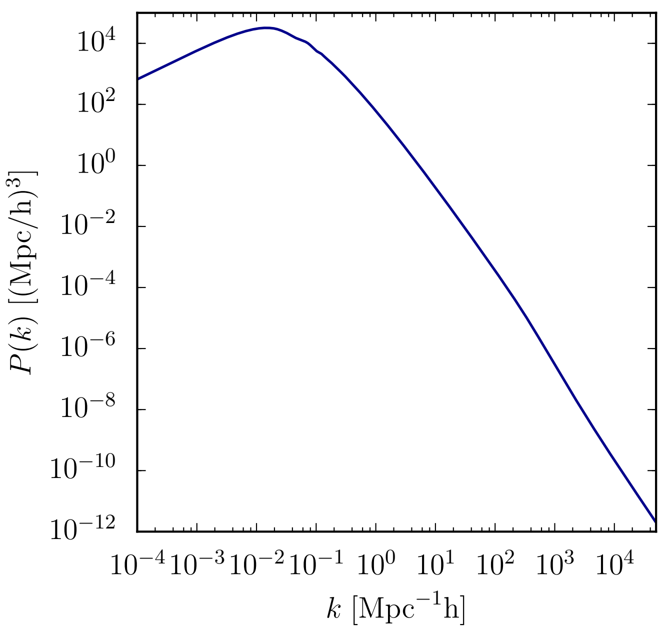
CMB with different H0

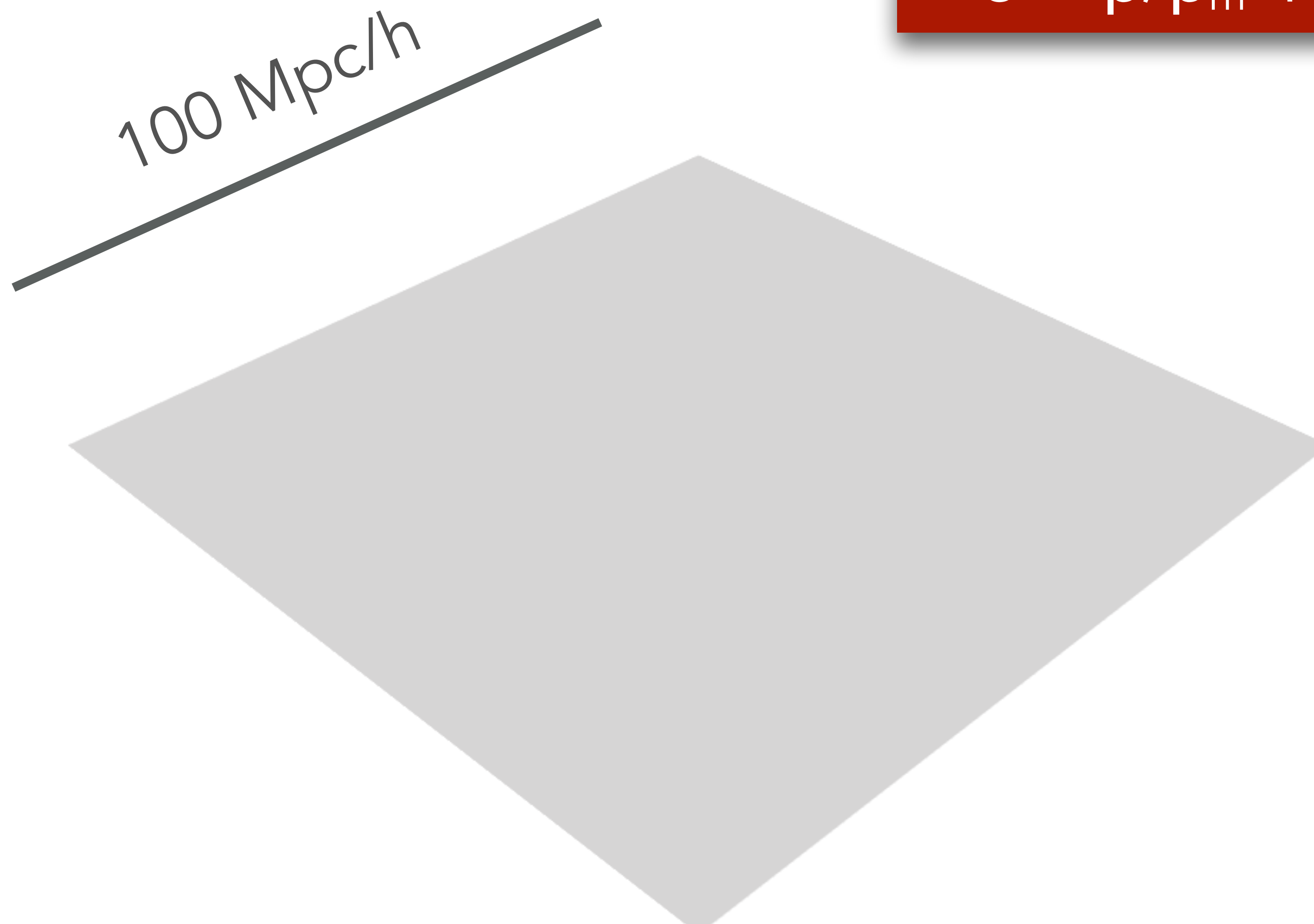


Made with CAMB • Planck data

§3.5 • The initial density field

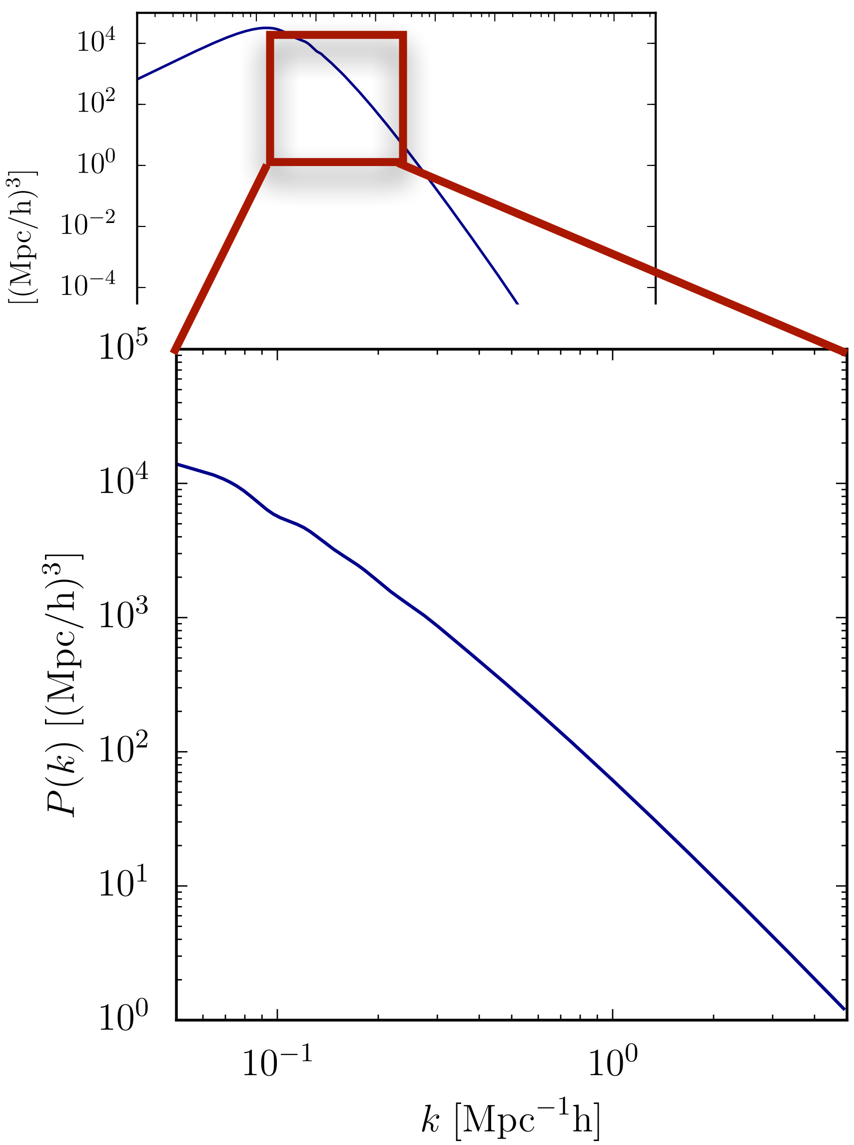
Gaussian random field

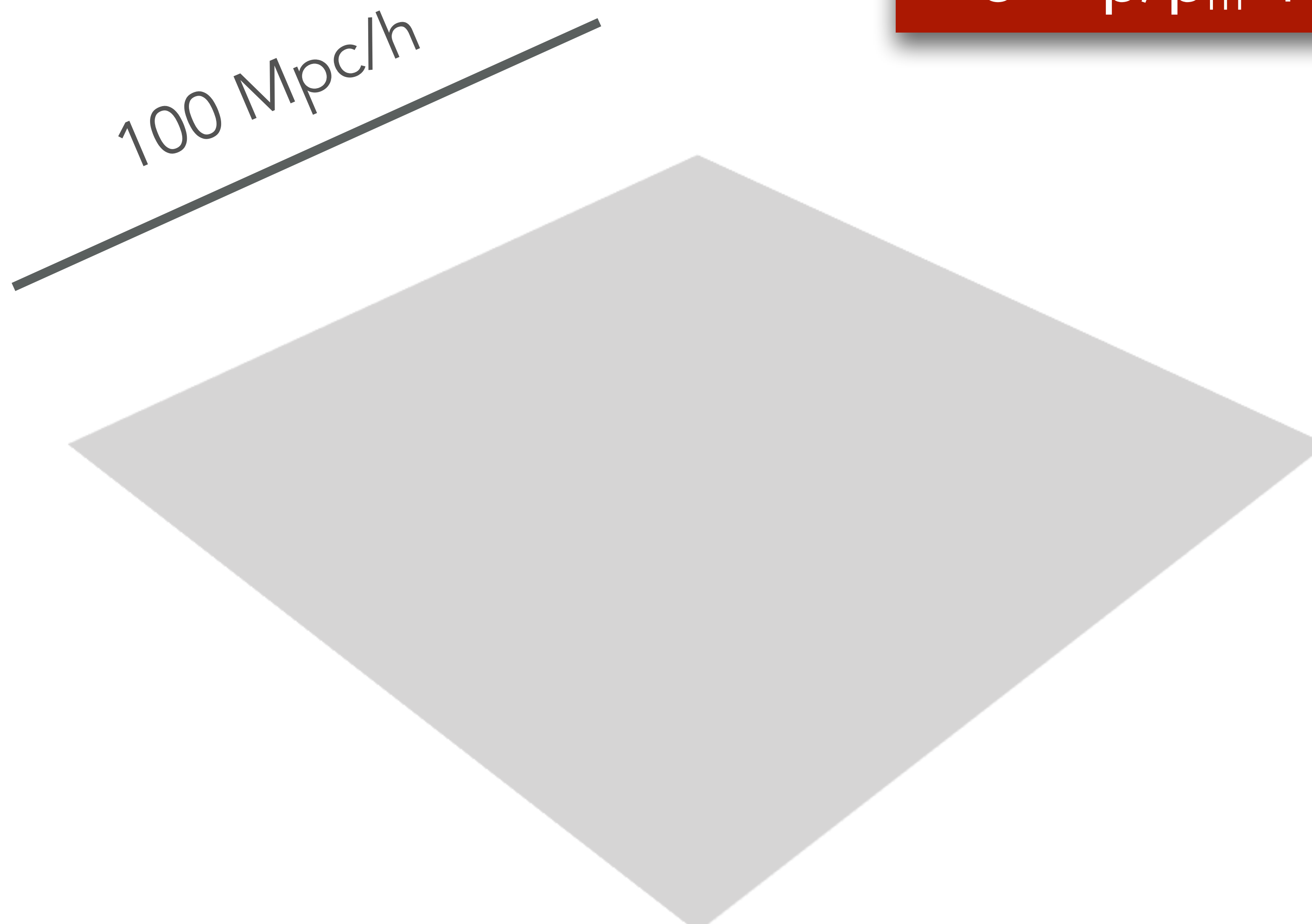




$\delta = \rho/\rho_m - 1$

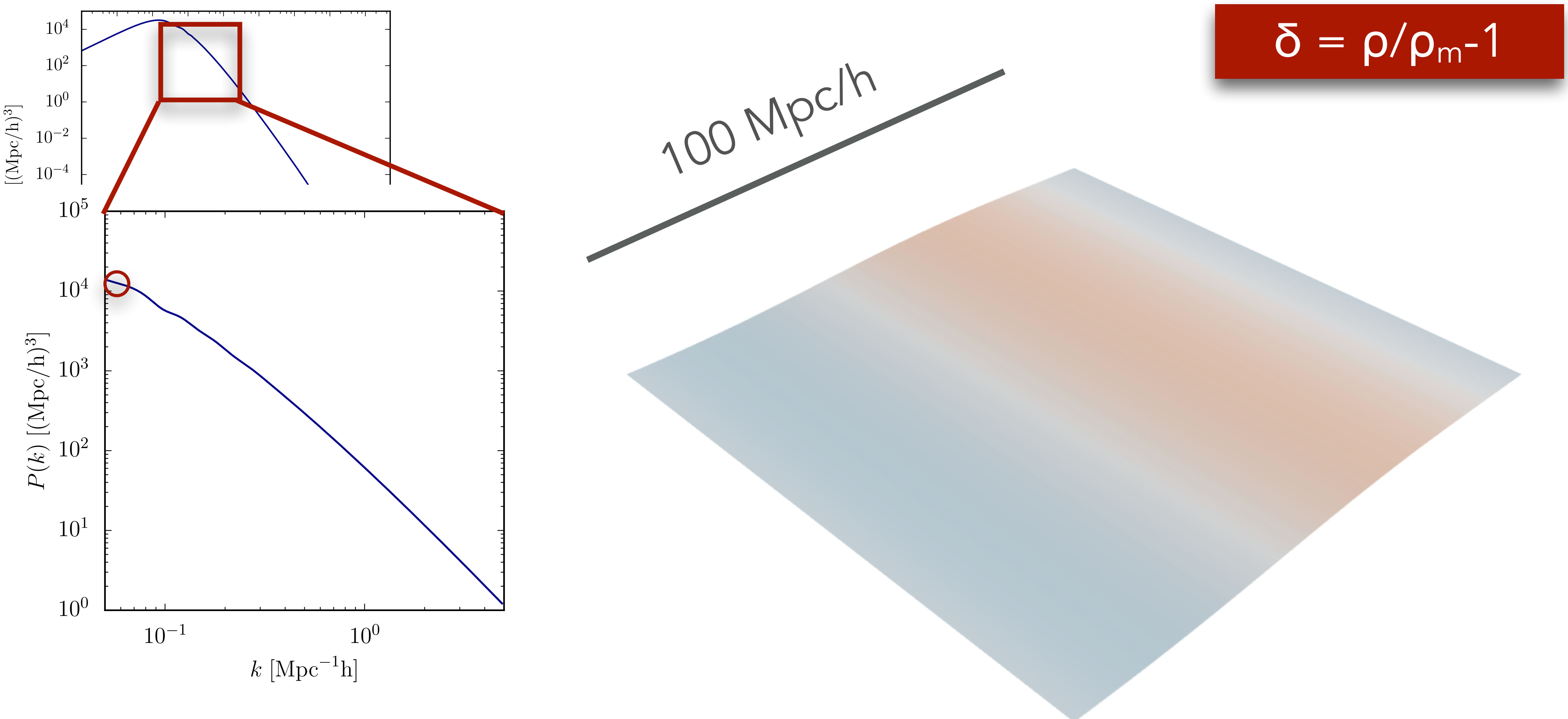
Gaussian random field



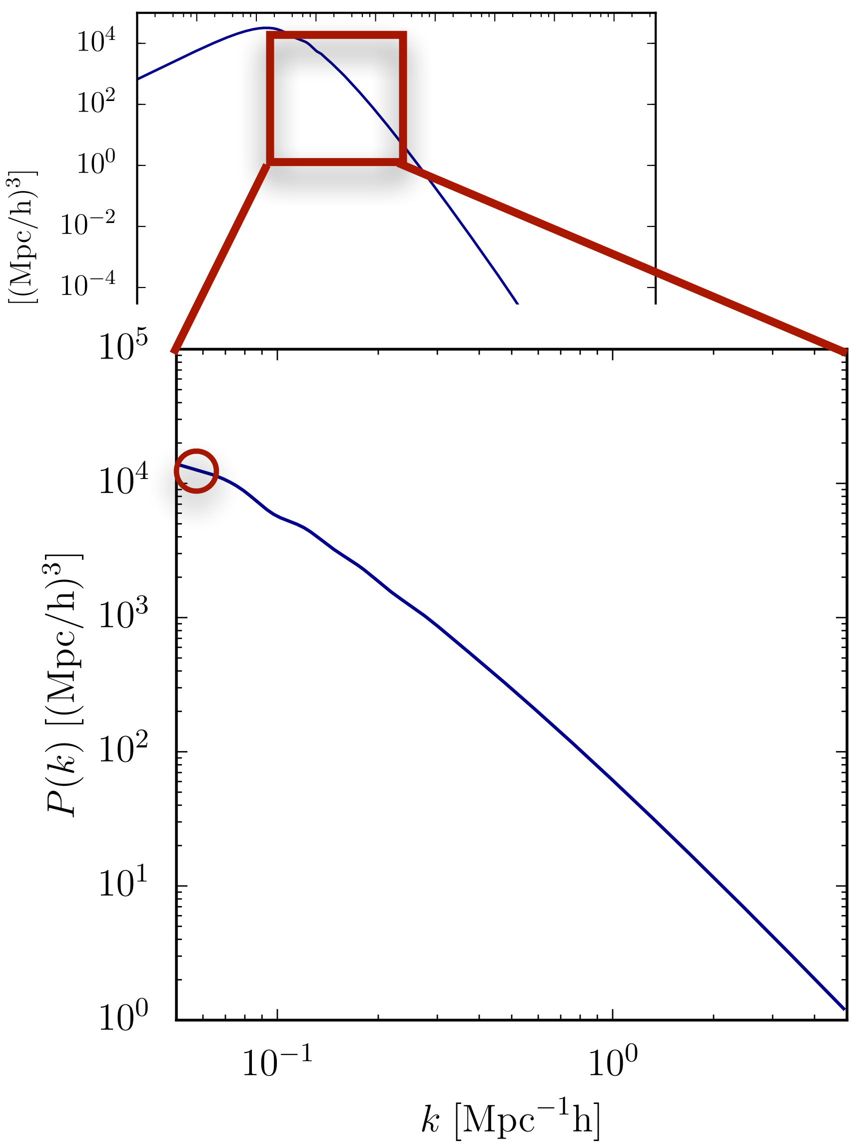


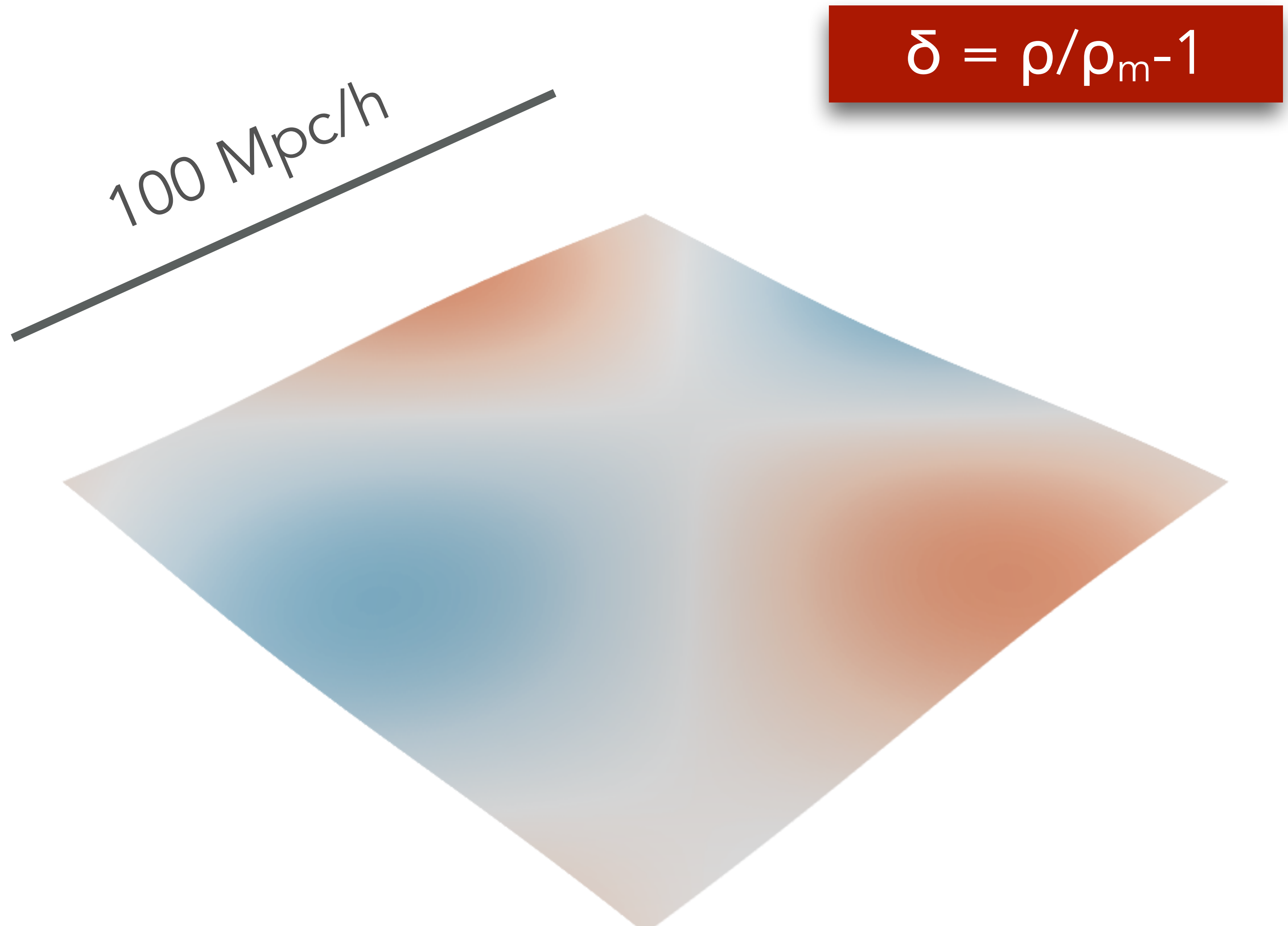
$\delta = \rho/\rho_m - 1$

Gaussian random field

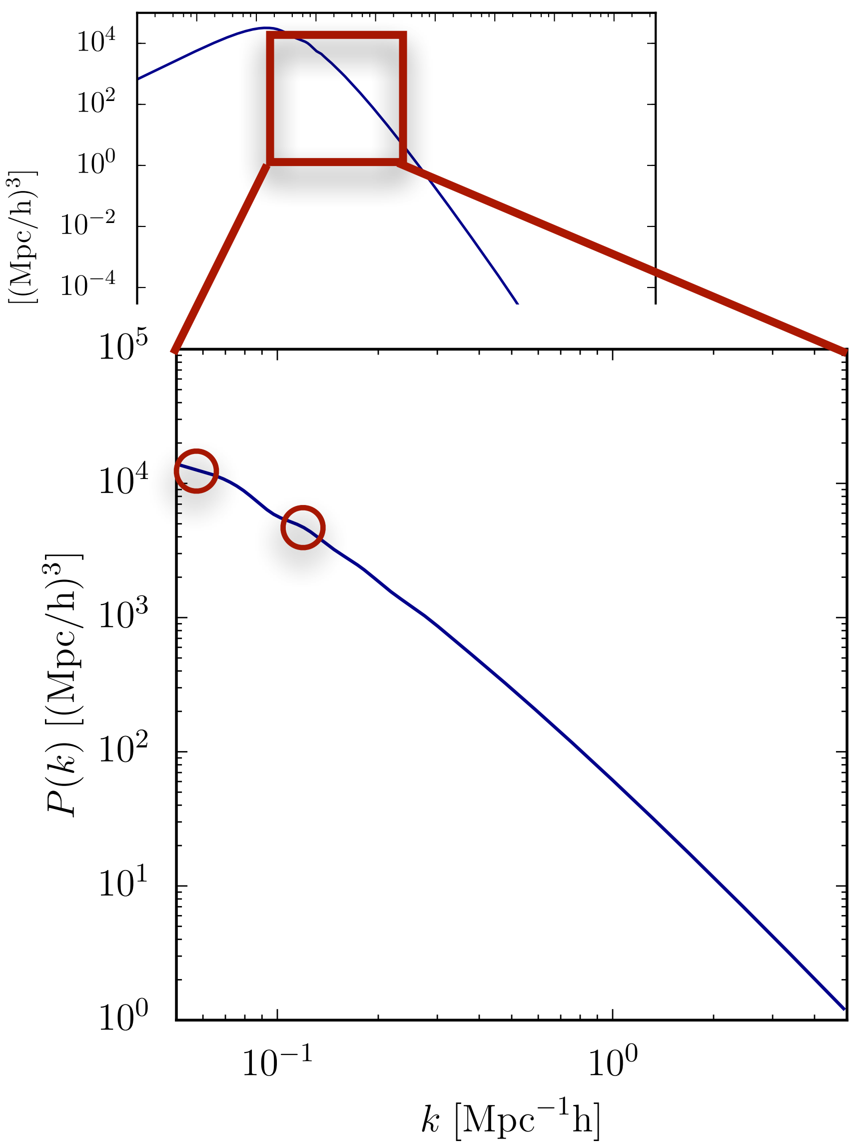


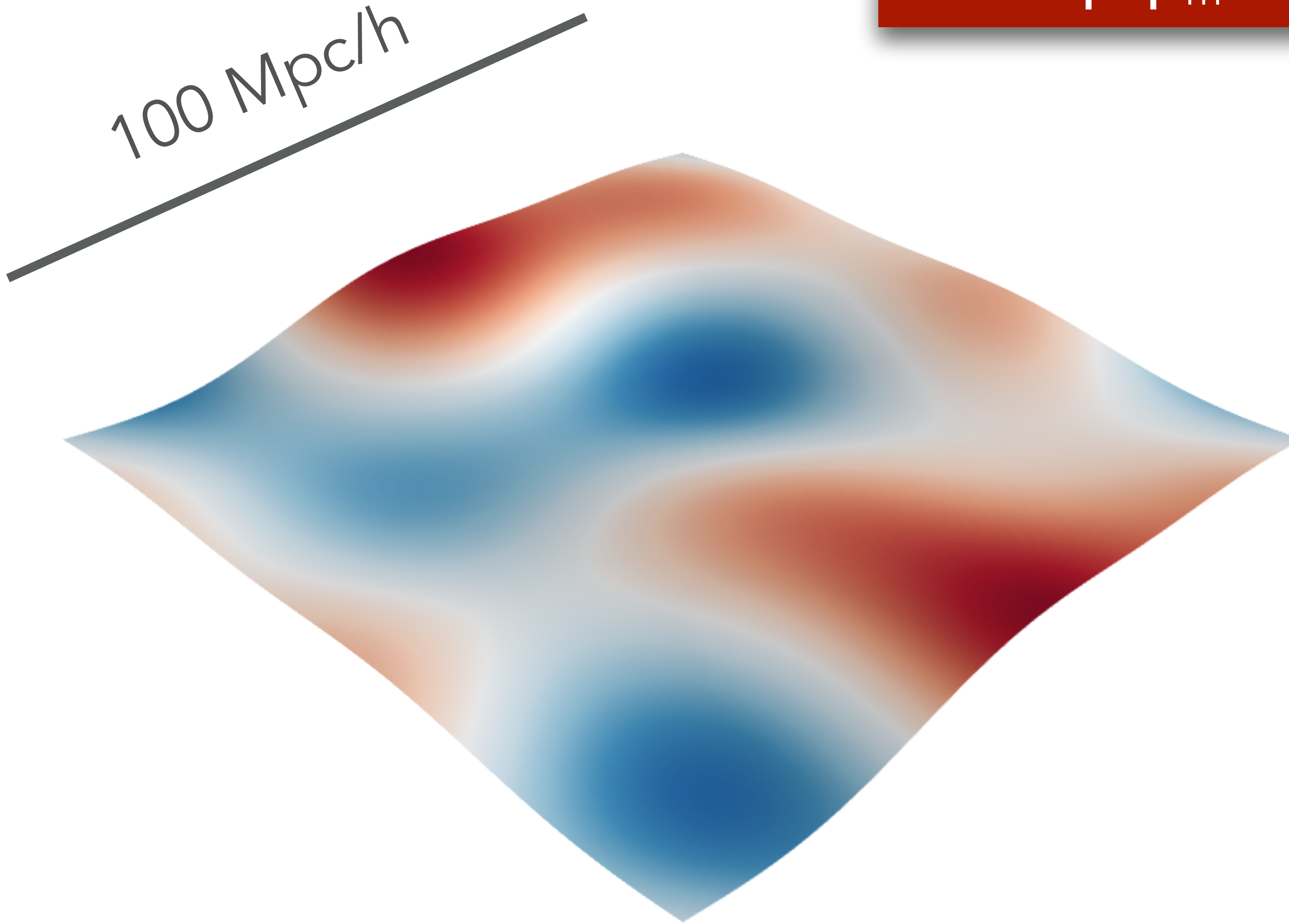
Gaussian random field





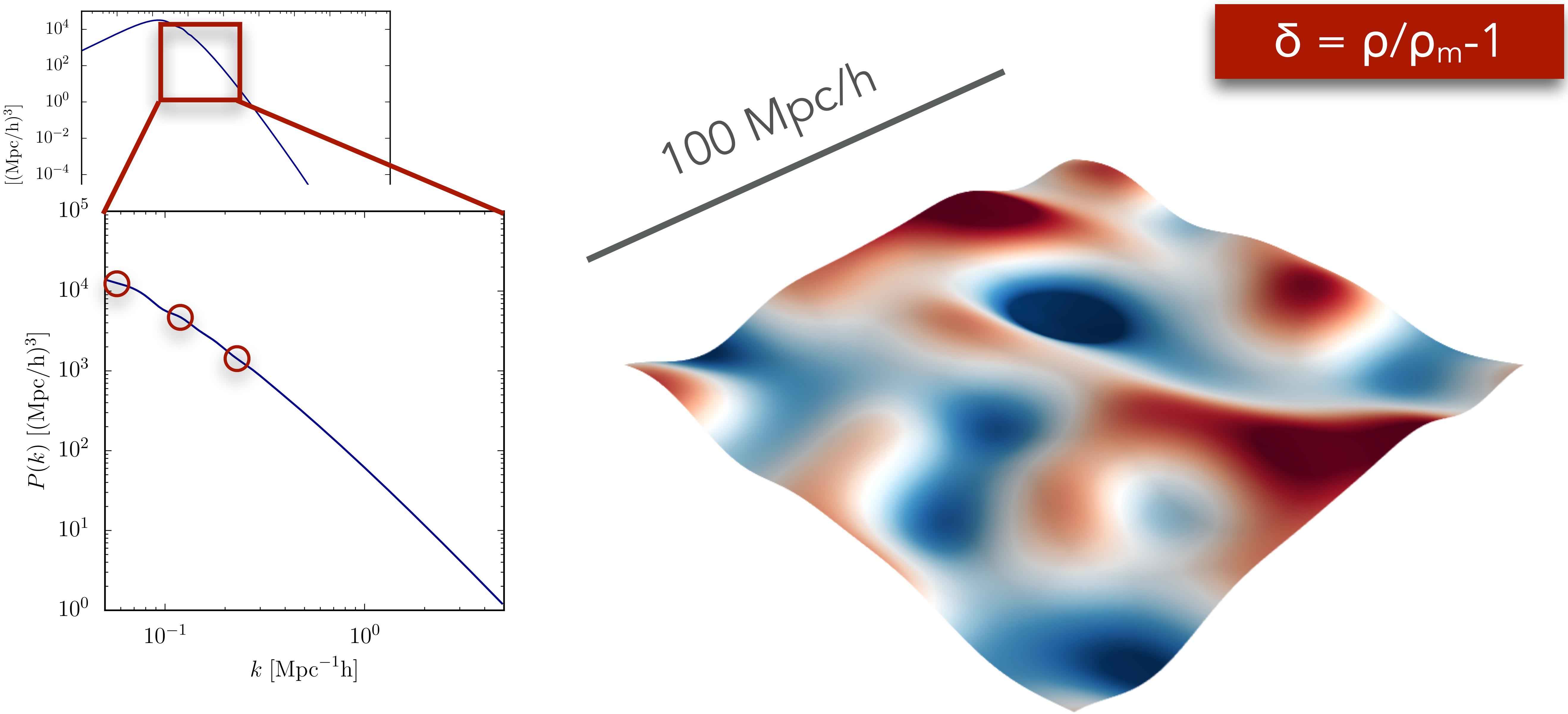
Gaussian random field



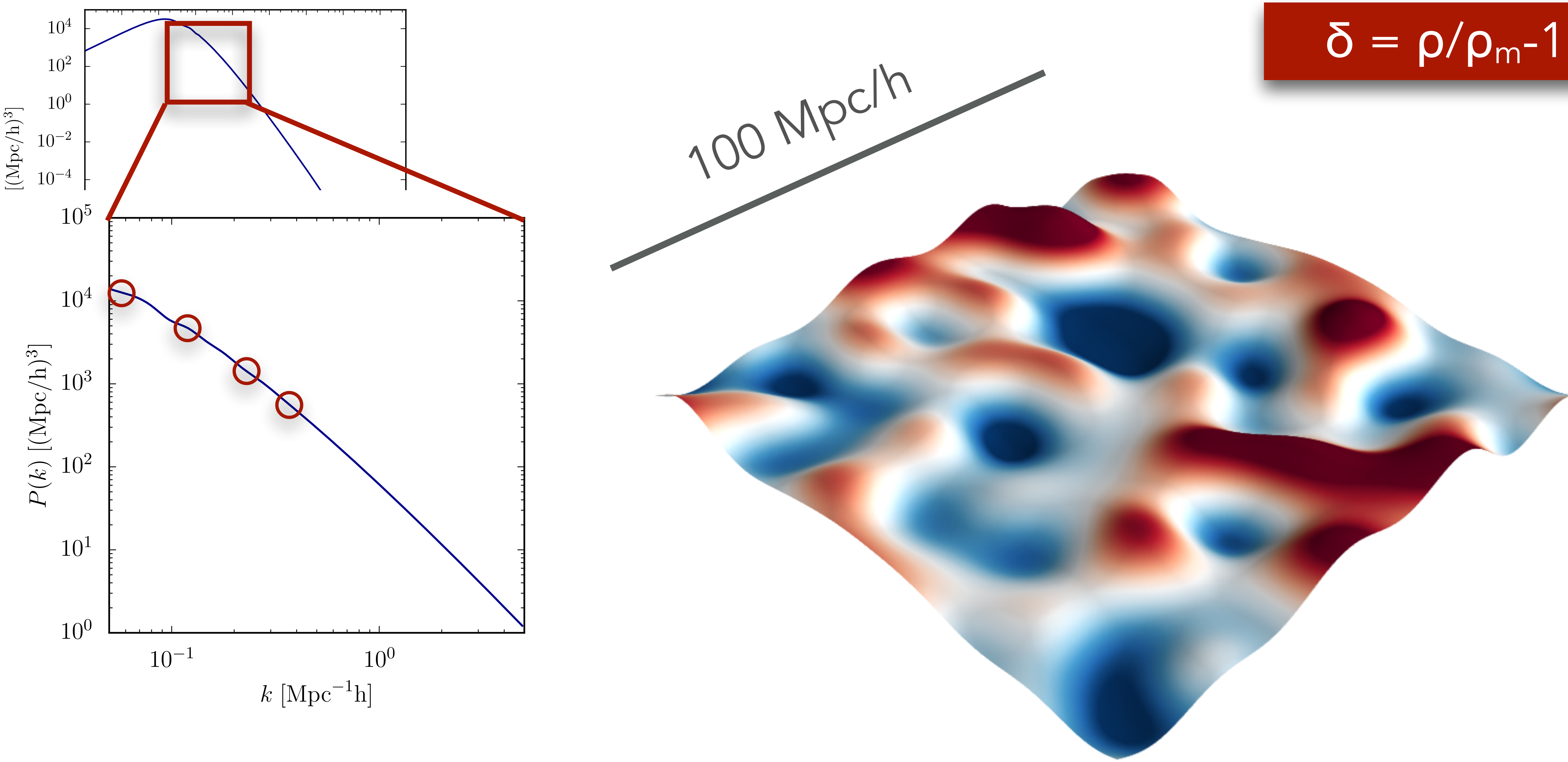


$\delta = \rho/\rho_m - 1$

Gaussian random field

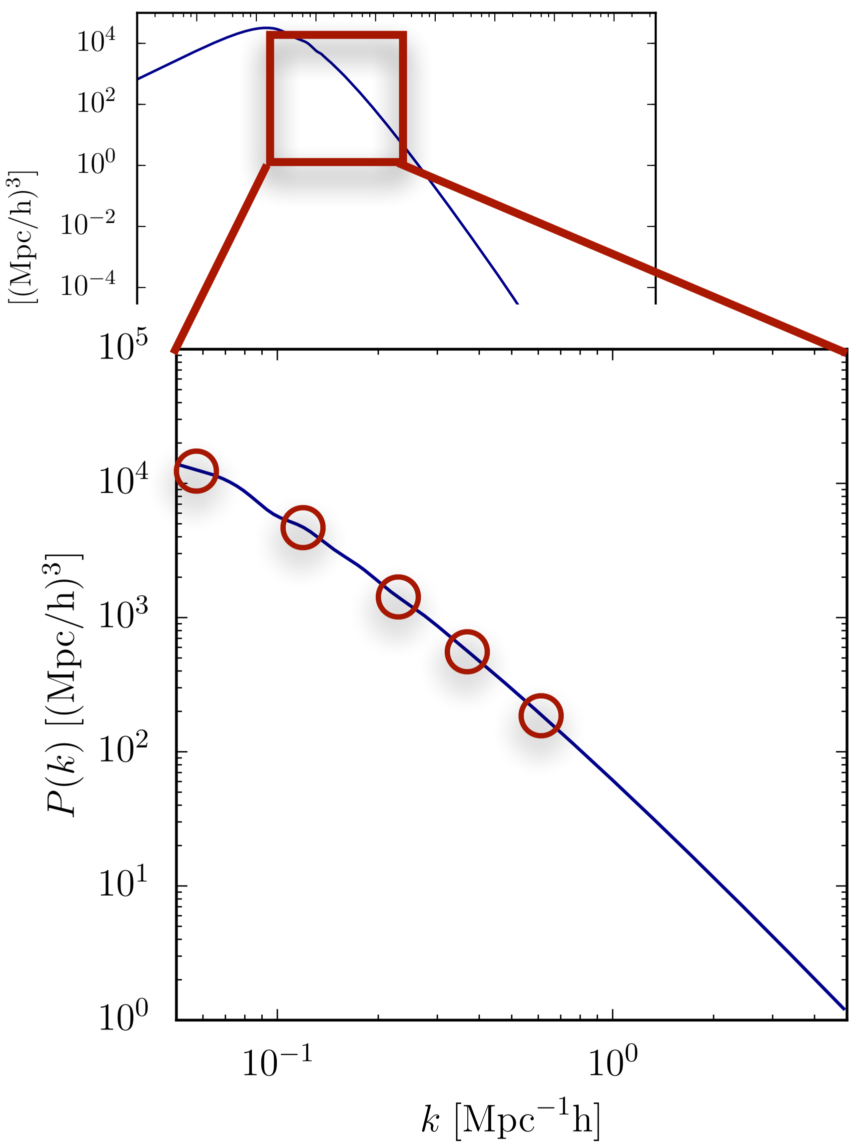


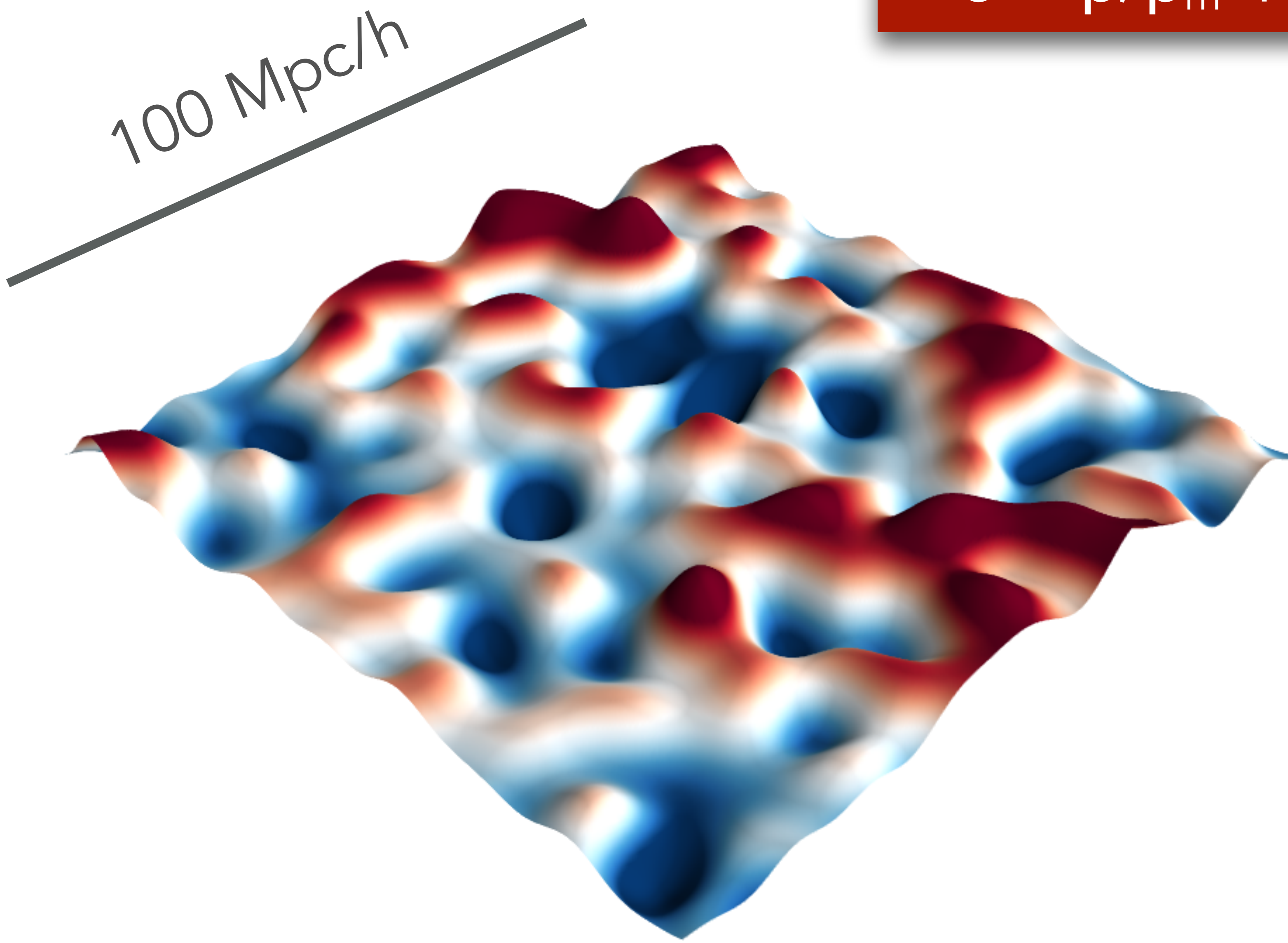
Gaussian random field



$\delta = \rho/\rho_m - 1$

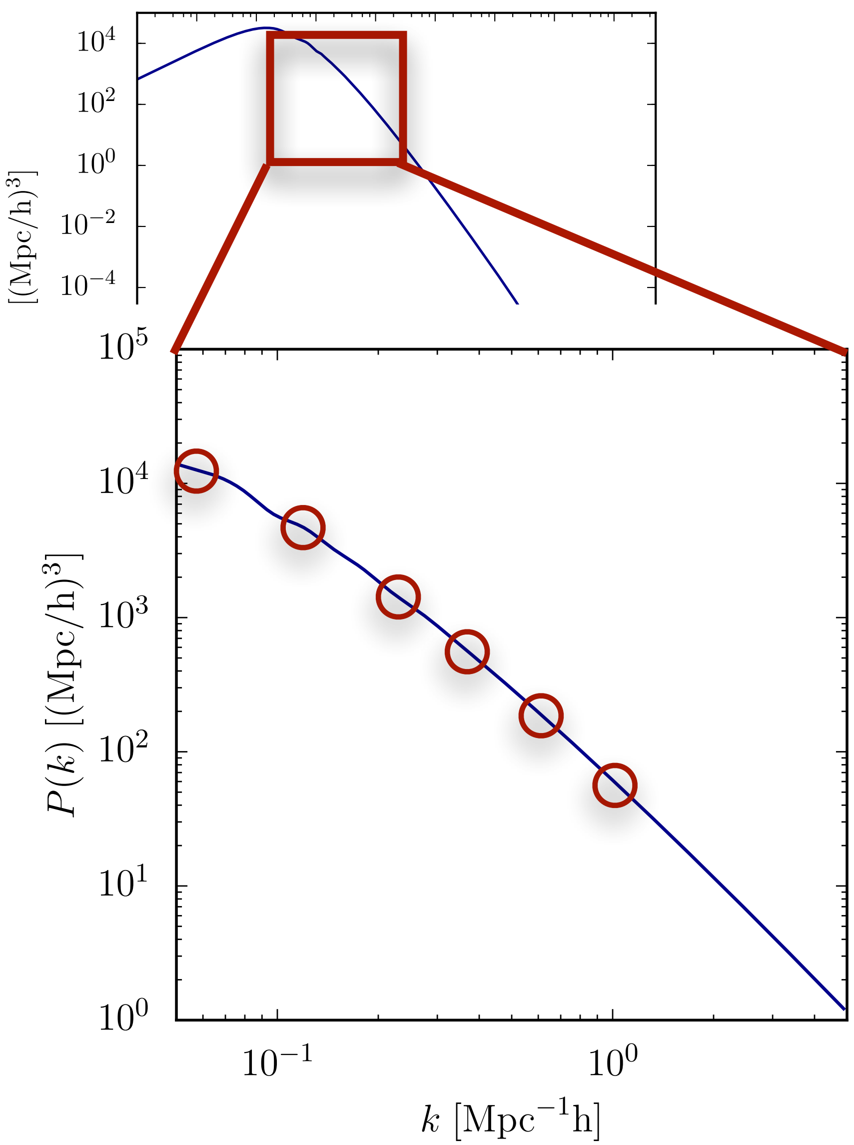
Gaussian random field

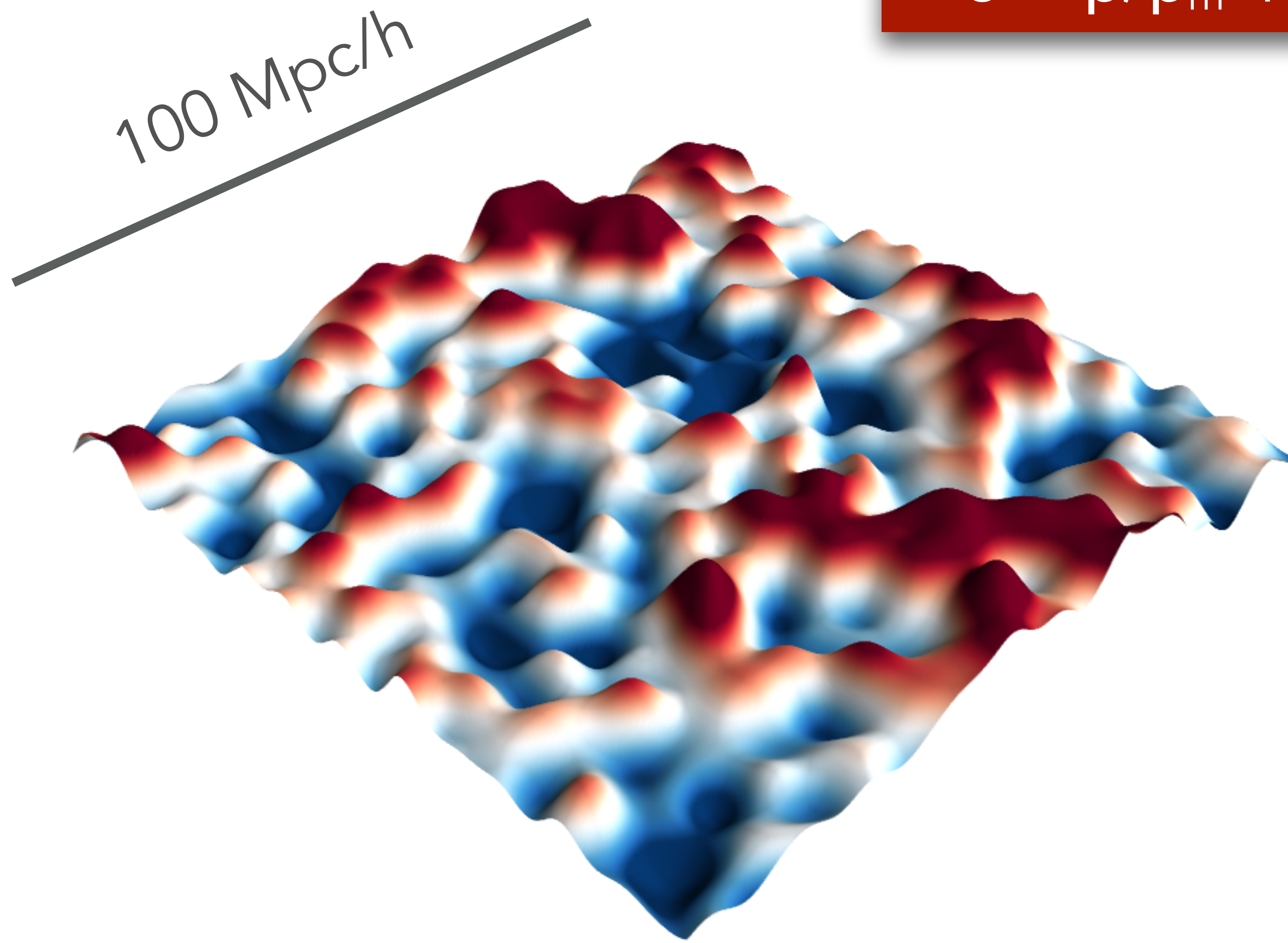




$\delta = \rho/\rho_m - 1$

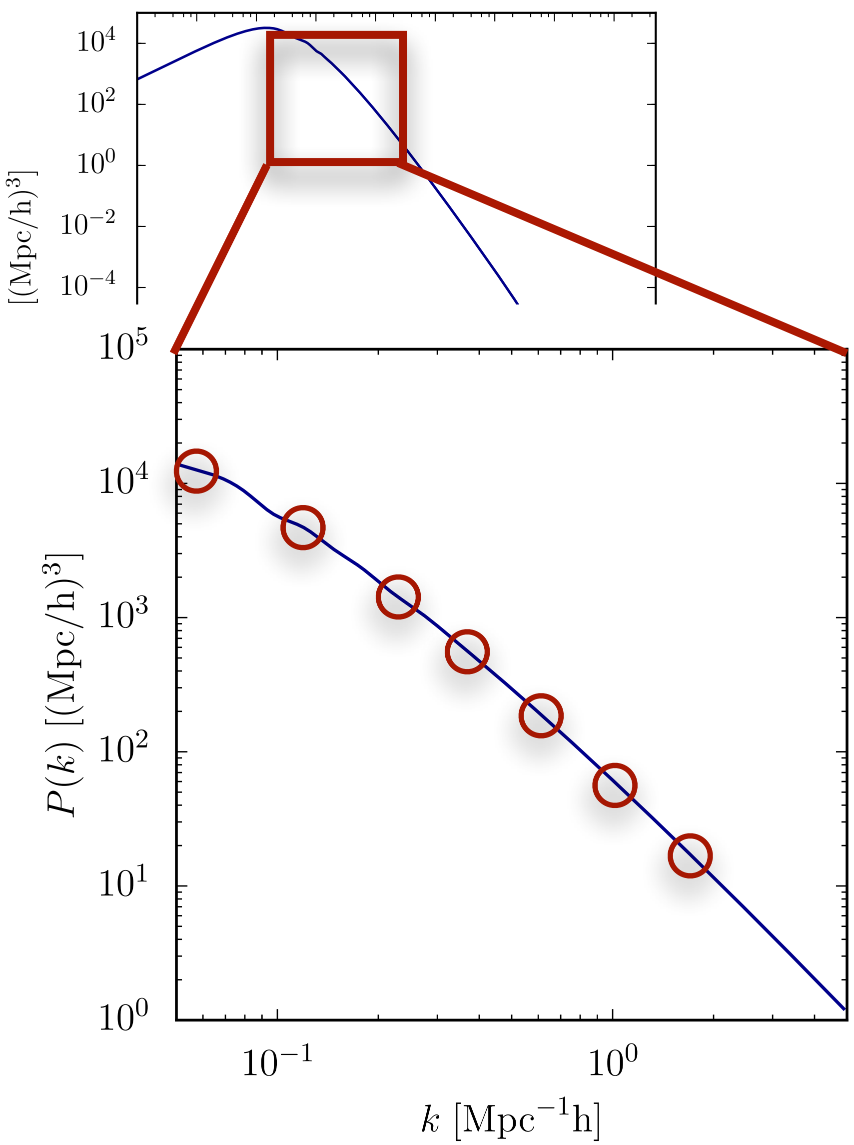
Gaussian random field

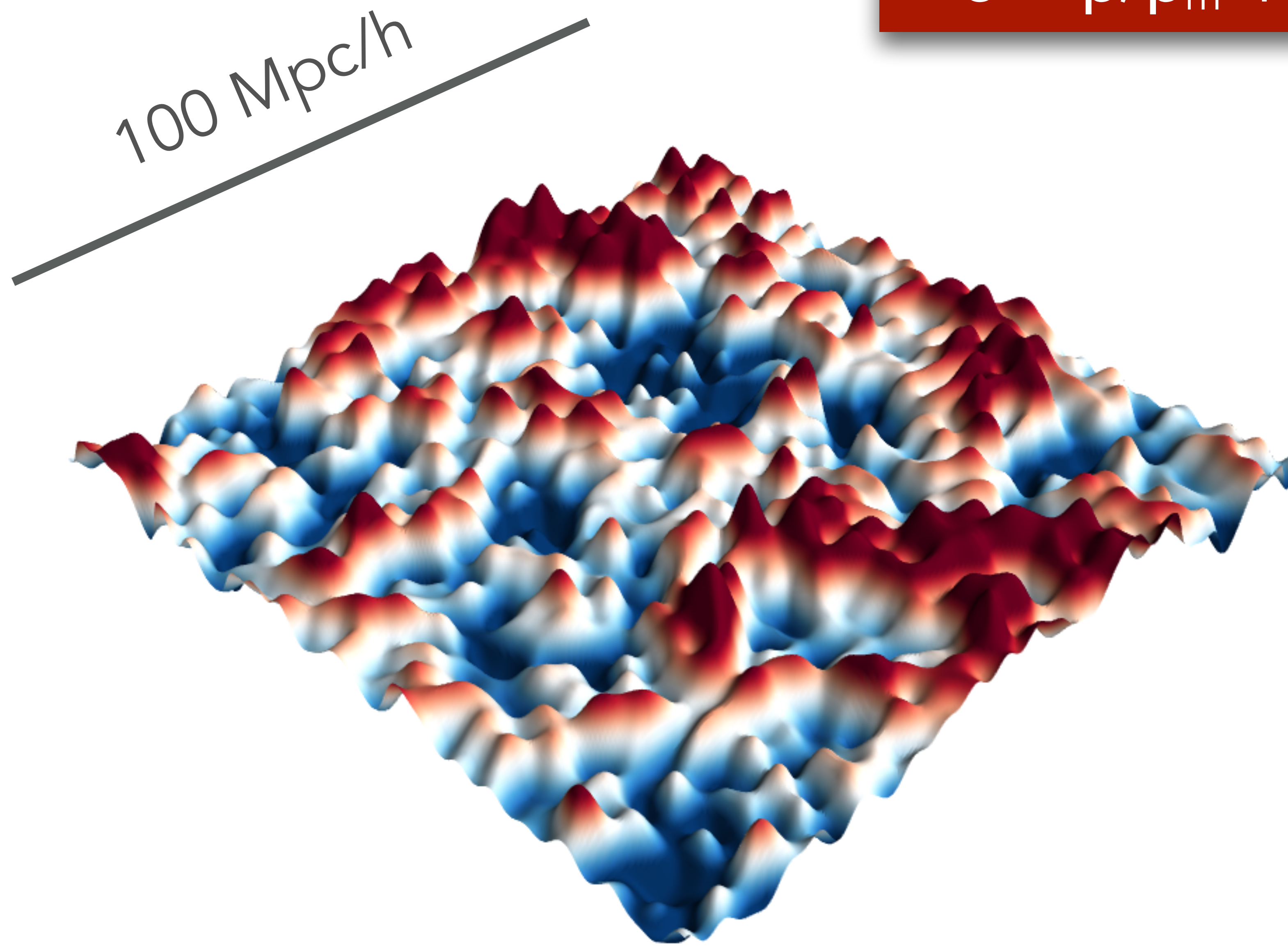




$\delta = \rho/\rho_m - 1$

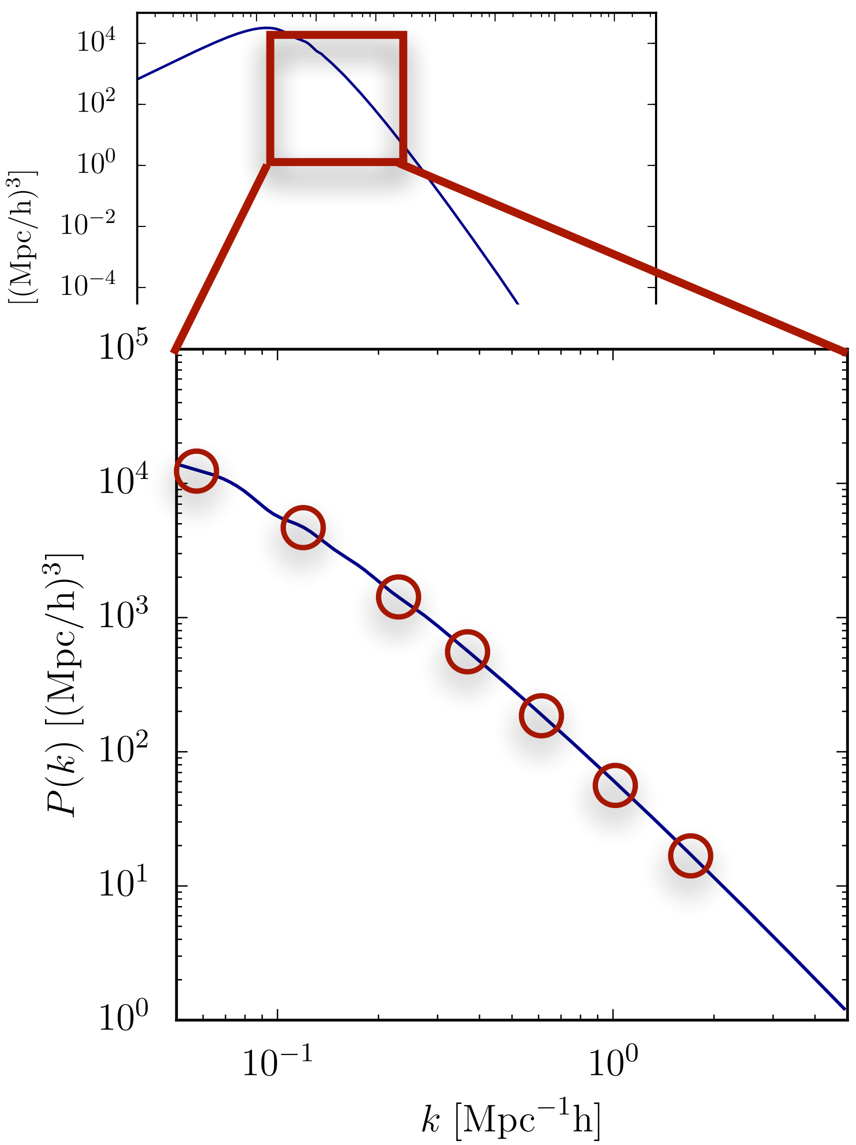
Gaussian random field

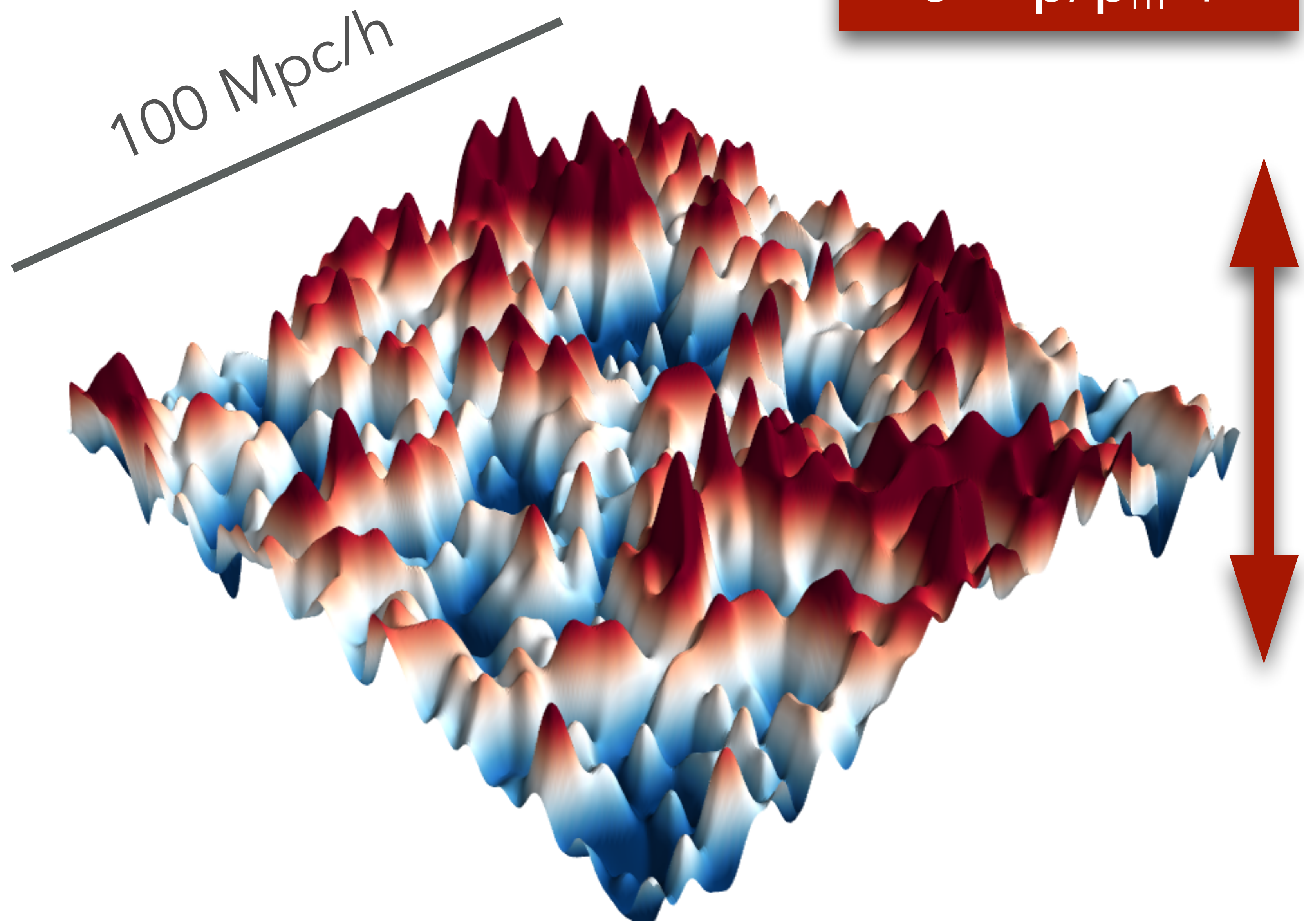




$\delta = \rho/\rho_m - 1$

Gaussian random field





$\delta = \rho/\rho_m - 1$

Reading

- CFN §2.4, §7.1-7.2, §7.3.1, §7.4.1-7.4.2
- MvdBW §4.1-4.1.6, §4.4.2, §4.4.4, §4.5