ASTR 670: Interstellar medium and gas dynamics

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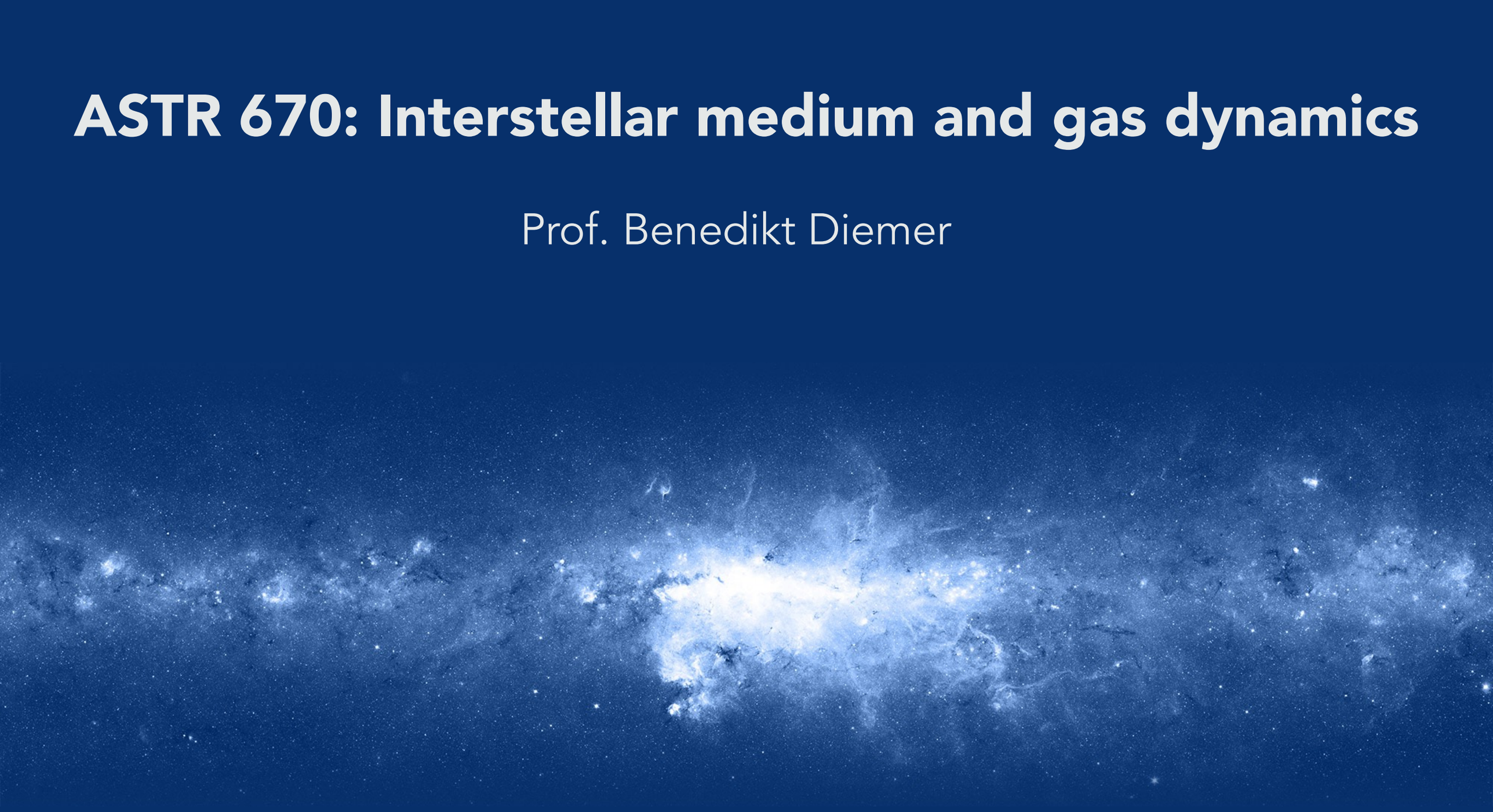
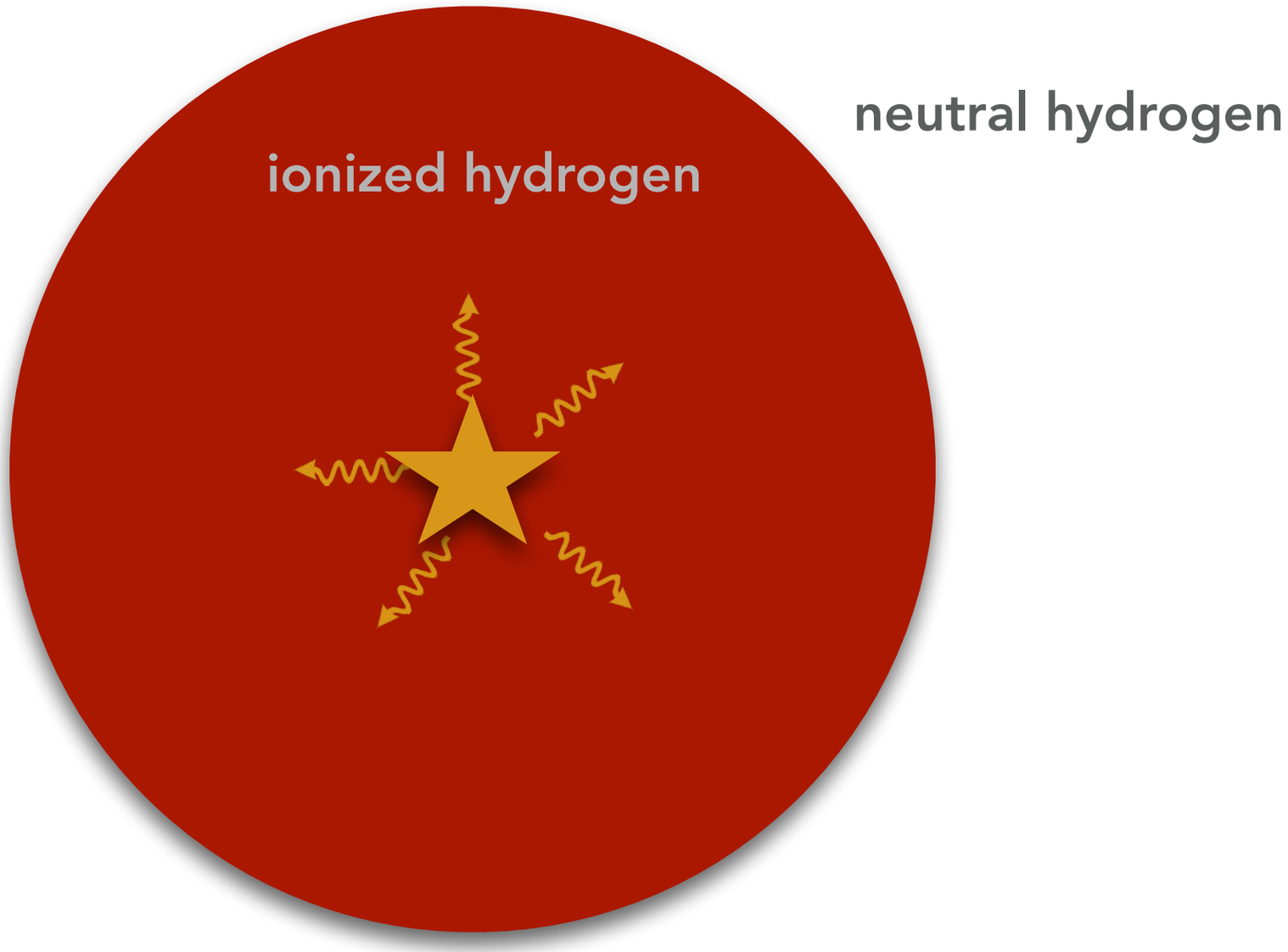


Chapter 5 • Atomic physics II: Photoionization and recombination

Ionization balance



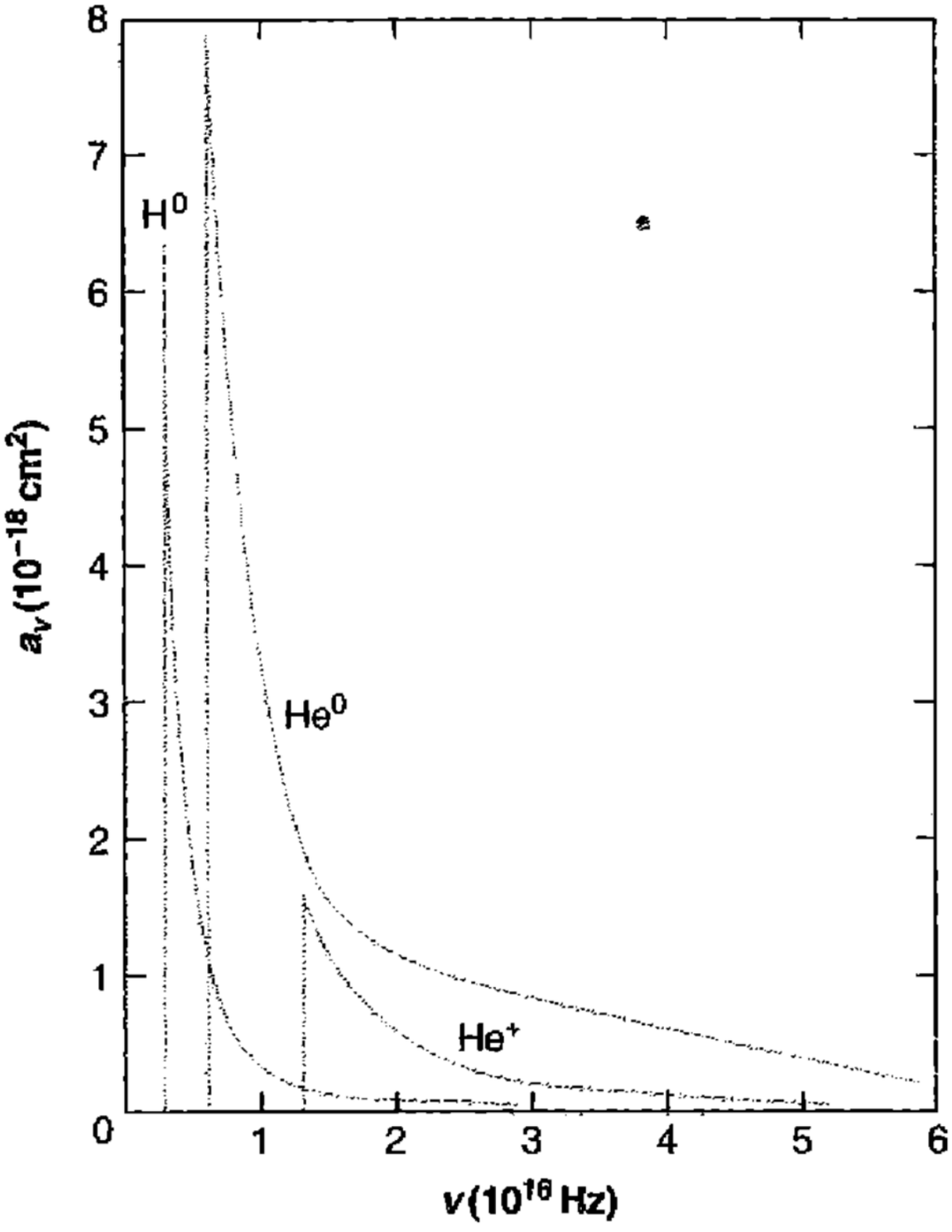
H II region

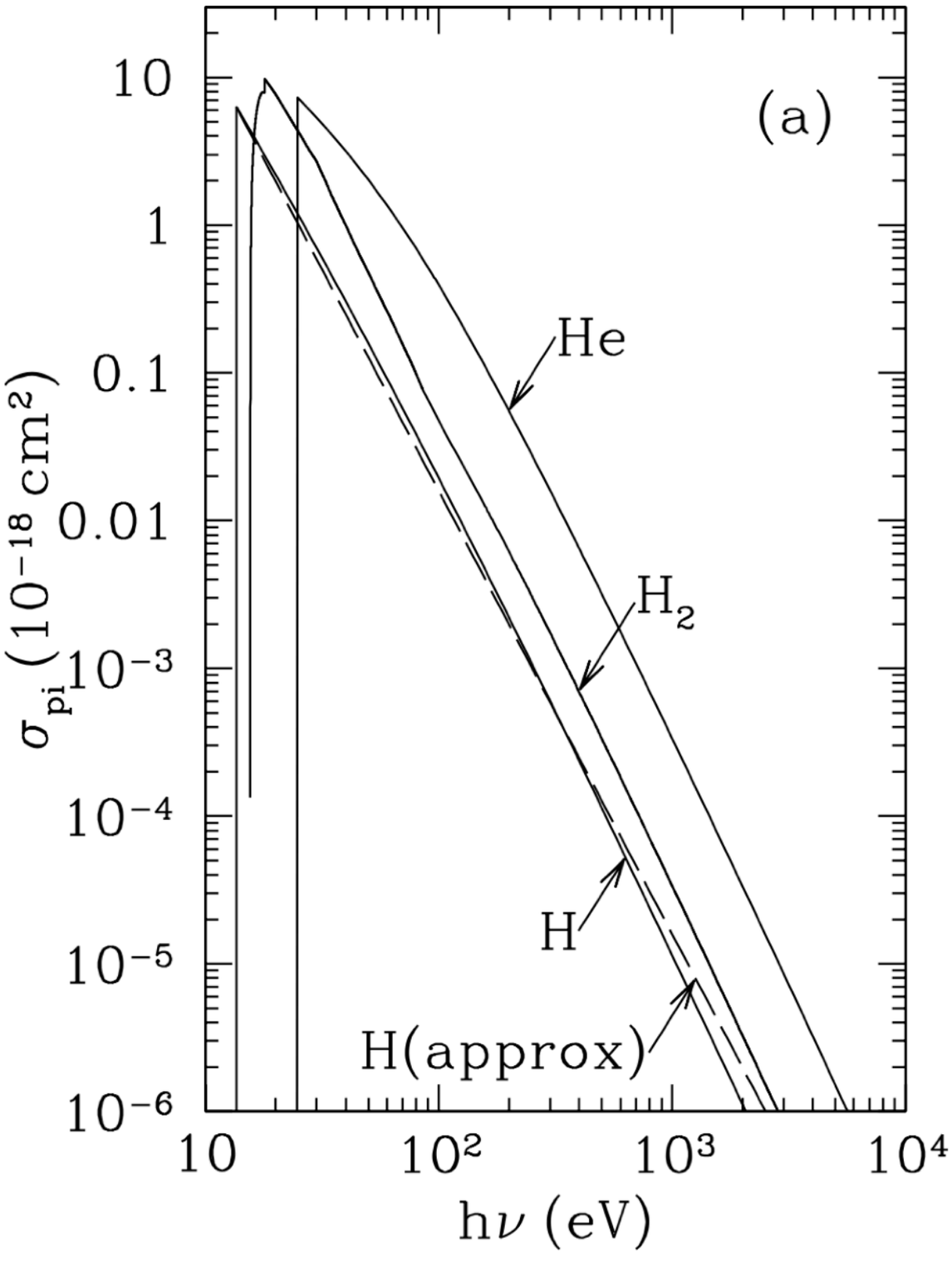
Rosette Nebula

Image: Greg Polanski

§5.1 • Photoionization

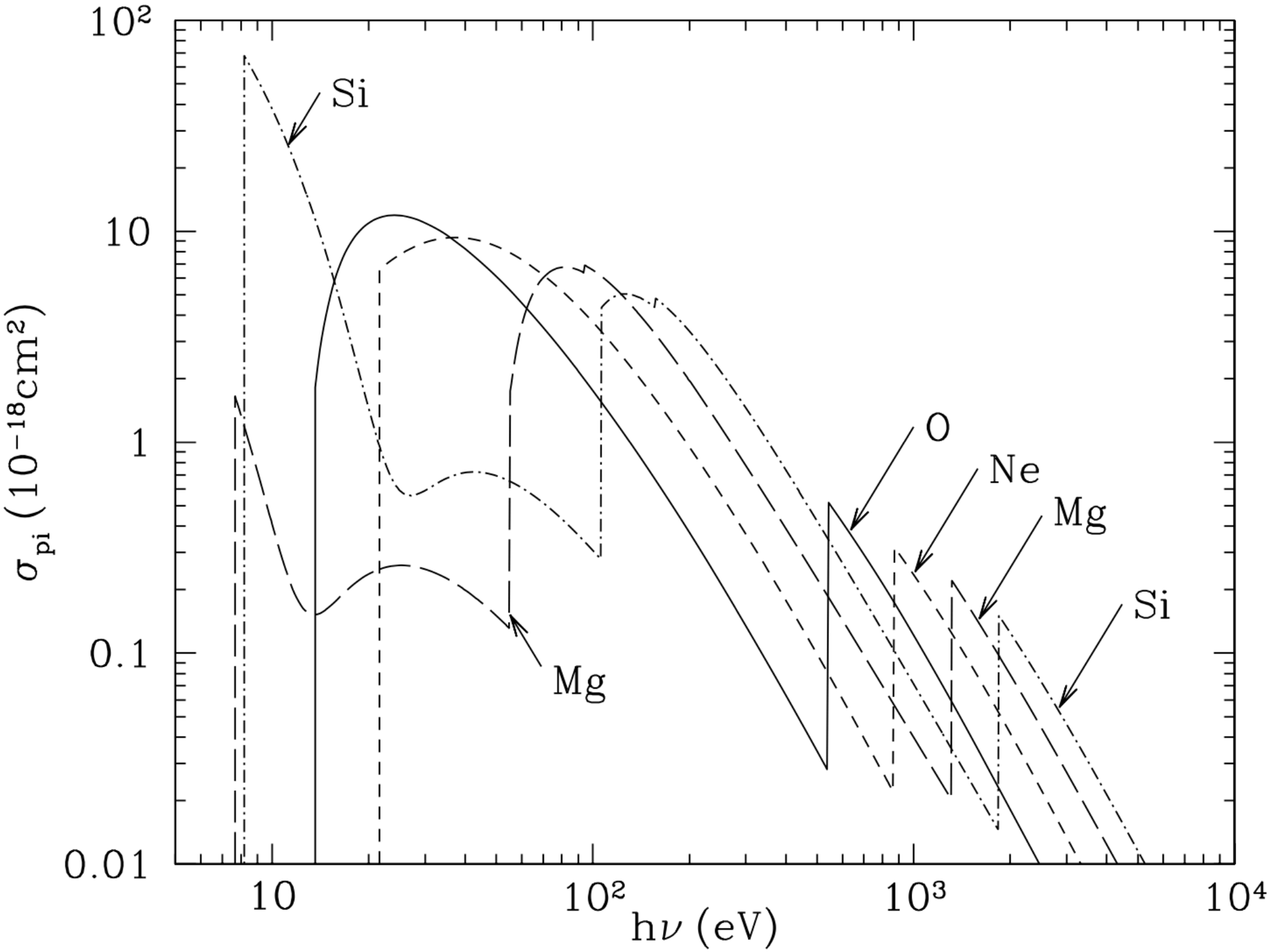
Ionization cross-section





Osterbrock & Ferland §2.2 • Draine §13.1

Ionization cross-section



Draine §13.1

§5.2 • Recombination

Radiative recombination coefficients

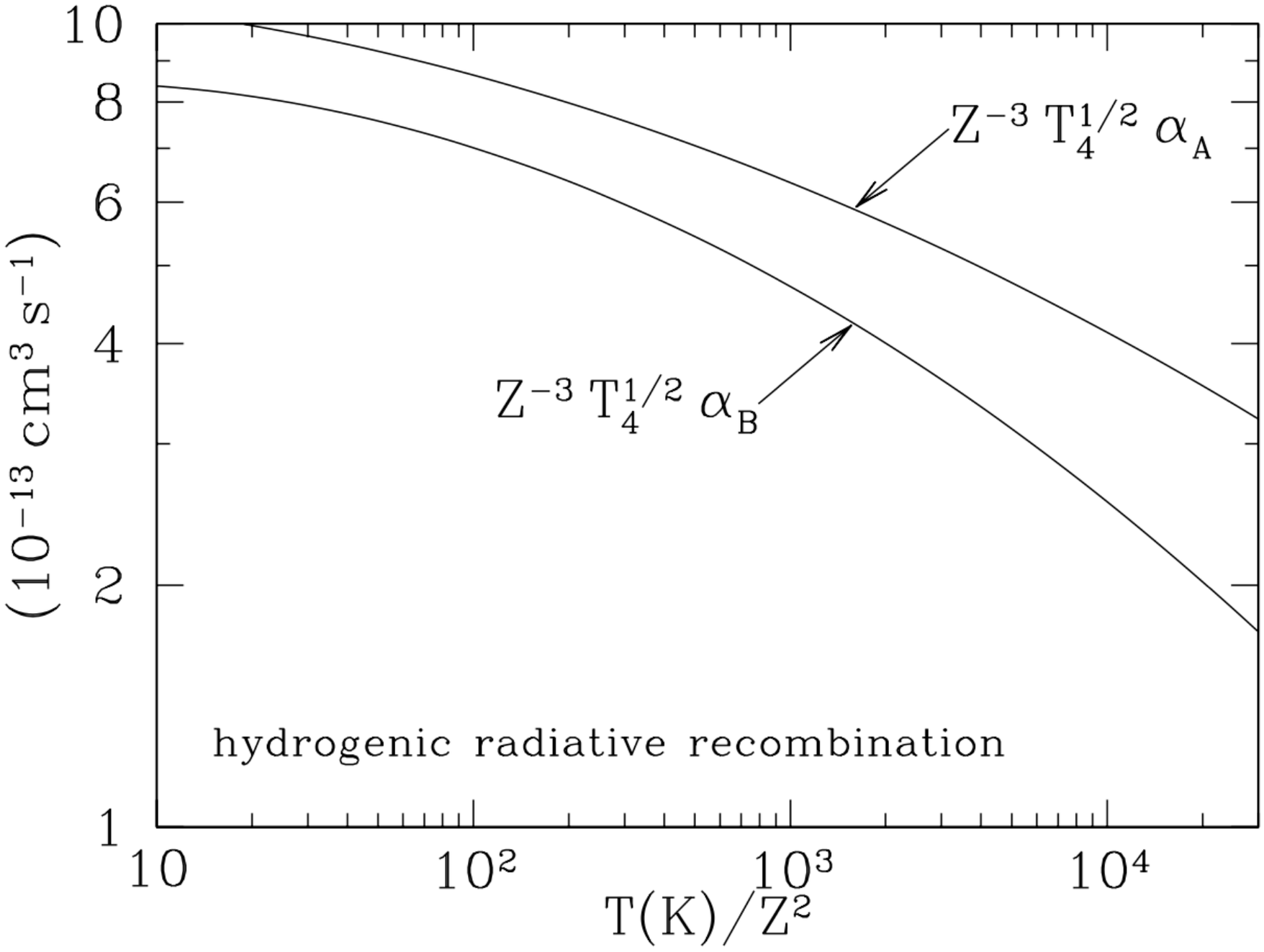
Table 14.1 Recombination Coefficients $\alpha_{n\ell}$ ($\mathrm{cm^3\,s^{-1}}$) for H.[a] The approximation formulae are valid for $0.3 \lesssim T_4 \lesssim 3$. For a broader range of T, see Eq. (14.5,14.6).

	Temperature T			
$\alpha_n(^2L)$	5×10^3 K	1×10^4 K	2×10^4 K	approximation
α_{1s}	2.28×10^{-13}	1.58×10^{-13}	1.08×10^{-13}	$1.58\times10^{-13}T_4^{-0.539-0.014\ln T_4}$
α_{2s}	3.37×10^{-14}	2.34×10^{-14}	1.60×10^{-14}	$2.34\times10^{-14}T_4^{-0.537-0.016\ln T_4}$
α_{2p}	8.33×10^{-14}	5.35×10^{-14}	3.24×10^{-14}	$5.35\times10^{-14}T_4^{-0.681-0.061\ln T_4}$
α_2	1.17×10^{-13}	7.69×10^{-14}	4.84×10^{-14}	$7.69\times10^{-14}T_4^{-0.637-0.045\ln T_4}$
α_{3s}	1.13×10^{-14}	7.81×10^{-15}	5.29×10^{-15}	$7.81\times10^{-15}T_4^{-0.548-0.021\ln T_4}$
α_{3p}	3.17×10^{-14}	2.04×10^{-14}	1.23×10^{-14}	$2.04\times10^{-15}T_4^{-0.683-0.068\ln T_4}$
α_{3d}	3.03×10^{-14}	1.73×10^{-14}	9.09×10^{-15}	$1.73\times10^{-14}T_4^{-0.869-0.087\ln T_4}$
α_3	7.33×10^{-14}	4.55×10^{-14}	2.67×10^{-14}	$4.55\times10^{-14}T_4^{-0.729-0.059\ln T_4}$
α_{4s}	5.23×10^{-15}	3.59×10^{-15}	2.40×10^{-15}	$3.59\times10^{-15}T_4^{-0.562-0.028\ln T_4}$
α_{4p}	1.51×10^{-14}	9.66×10^{-15}	5.81×10^{-15}	$9.66\times10^{-15}T_4^{-0.689-0.064\ln T_4}$
α_{4d}	1.90×10^{-14}	1.08×10^{-14}	5.68×10^{-15}	$1.08\times10^{-14}T_4^{-0.871-0.081\ln T_4}$
α_{4f}	1.09×10^{-14}	5.54×10^{-15}	2.56×10^{-15}	$5.54\times10^{-15}T_4^{-1.045-0.099\ln T_4}$
α_4	5.02×10^{-14}	2.96×10^{-14}	1.65×10^{-14}	$2.96\times10^{-14}T_4^{-0.803-0.059\ln T_4}$
α_A	6.82×10^{-13}	4.18×10^{-13}	2.51×10^{-13}	$4.18\times10^{-13}T_4^{-0.721-0.021\ln T_4}$
α_B	4.54×10^{-13}	2.59×10^{-13}	1.43×10^{-13}	$2.59\times10^{-13}T_4^{-0.833-0.034\ln T_4}$

[a] From Hummer & Storey (1987)

Draine §14.1

Radiative recombination coefficients



Case A: optically thin, recombination photons escape

Case B: optically thick above ionization energy, recombination photons to ground state ionize again

Draine §14.1

§5.3 • Ionization balance and the Milne equation

Deriving the Milne relation

Detailed balance:

$$n_0 \frac{4\pi J_\nu}{h\nu} \sigma_{\rm pi}(h\nu) {\rm d}\nu = n_{\rm e} n_{\rm p} u f_{\rm MB}(u) \sigma_{\rm nl}(u) [1 + \bar{n}_\gamma] {\rm d}u$$

Electron energy:

$$\frac{m_{\rm e} u^2}{2} = h(\nu - \nu_0) \quad \Longrightarrow \quad u = \sqrt{\frac{2h(\nu - \nu_0)}{m_{\rm e}}} \quad \Longrightarrow \quad {\rm d}u = \frac{h}{m_{\rm e} u} {\rm d}\nu \, .$$

Ratio between ionization and recombination cross-sections:

$$\begin{aligned}
\frac{\sigma_{\rm nl}(u)}{\sigma_{\rm pi}(h\nu)} &= \frac{n_0}{n_{\rm e} n_{\rm p}} \frac{4\pi J_\nu}{h\nu} \frac{1}{(1 + \bar{n}_\gamma)} \frac{1}{u f_{\rm MB}(u)} \frac{m_{\rm e} u}{h} \\
&= \frac{g_{\rm n}}{g_{\rm e} g_{\rm p}} \left(\frac{h^2}{2\pi m_{\rm e} k_{\rm B} T} \right)^{3/2} e^{h\nu_0 / k_{\rm B} T} \frac{4\pi}{h\nu} \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu / k_{\rm B} T} - 1)} \frac{1}{1 + \frac{1}{e^{h\nu / k_{\rm B} T} - 1}} \\
&\quad \times \frac{\sqrt{\pi}}{4} \left(\frac{2 k_{\rm B} T}{m_{\rm e}} \right)^{3/2} \frac{1}{u^2} e^{m_{\rm e} u^2 / 2 k_{\rm B} T} \frac{m_{\rm e}}{h} \\
&= \frac{g_{\rm n}}{g_{\rm e} g_{\rm p}} \frac{2\pi^{3/2} \nu^2 m_{\rm e}}{h c^2} \left(\frac{h^2}{\pi {m_{\rm e}}^2} \right)^{3/2} \frac{1}{u^2} \exp \left[\frac{h\nu_0}{k_{\rm B} T} - \frac{h\nu}{k_{\rm B} T} + \frac{h(\nu - \nu_0)}{k_{\rm B} T} \right] .
\end{aligned}$$

Derivation of ionized fraction

Assumptions | Equations | Inputs | Results

Single-electron atom

All ionizations happen from ground state

QM expression for cross-section for ionization $\sigma_{\rm pi}(\nu)$

Integral over photon energies

Total ionization rate $G_{\rm pi}$

Radiation field J_ν

Temperature

Density

Local thermodynamic equilibrium (LTE)

Balance between ionization and recomb.

Saha equation (balance between densities of states)

Maxwell-Boltzmann velocity distribution

Neutral fraction ξ

Milne rel. between σ's for ionization and recombination

Integral over electron velocities

Total Case A/B recomb. coefficient $\alpha_{\rm A}/\alpha_{\rm B}$

Detailed balance (equilibrium at each energy)

Recombination cross-section to level n,l $\sigma_{\rm nl}(u)$

Recombination coefficient to level n,l $\alpha_{\rm nl}$

Reading

Draine

- §13.1
- §14.1-2

O&F

- §2.1-2